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Bound State Description of Particles from a Quantum Field Theory of Fermions and Bosons, Compatible with Relativity

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Abstract

Both, the dilemma to find a quantum field theory consistent with Einstein's law of relativity and the problem to describe existing particles as bound states of matter has been solved by calculating bound state matrix elements from a dual fermion-boson Lagrangian. In this formalism, the fermion binding energies are compensated by boson energies, indicating that particles can be generated out of the vacuum. This yields quantitative solutions for various mesons ω (0.78 GeV) - Υ (9.46 GeV) and all leptons e, μ and τ, with uncertainties in the extracted properties of less than 1‰. For transparency, a Web-page with the address htpps://h2909473.stratoserver.net has been constructed, where all calculations can be run on line and also the underlying fortran source code can be inspected.

Keywords

Quantum Field Theory of Fermion and Boson Fields, Hadrons and Leptons Described as Bound States of Relativistic Fermions and Bosons, Leading to a Total Energy Equal to Zero

1. Introduction

Current theories of fundamental forces (apart from gravity) are based on first-order Lagrangians [1] [2]. Since they don't include the space-time degree of freedom, these theories are not compatible with Einstein's law of relativity. Further, they cannot describe particles as bound states of matter. As an example, in the Standard Model of particle physics [3] leptons are just described by massive fermions with a mass adjusted to the experimental values, and the structure of neutrinos is completely unknown. Nevertheless, it is surprising that the magnetic moments of leptons have been well described by Schwinger's higher order fermion-boson couplings in quantum electrodynamics [4], which established the correctness of relativistic field theory, at least in the description of electromagnetic phenomena.

Before the turn of the century a lot of work has been devoted to find the right Lagrangian for the description of fundamental forces. In particular, Lagrangians of (essentially) one variable but of different order have been studied, see e.g. articles of Simon [5], Foussats [6], Nesterenko [7] and others. Lagrangians of third or higher orders have many solutions not observed experimentally (ghosts); further, they need additional boundary and initial conditions. A more detailed discussion of higher order Lagrangians is also given in ref. [8]. To summarize these efforts, only first-order Lagrangians—as those in the Standard Model—with fermions (bosons are mediating only the interaction between fermions) have been found to be suited for the description of fundamental forces. But this is in strong disaccord to the requirement that particles are bound states of matter and compatible with Einstein's law of relativity.

2. Dual Fermion-Boson Lagrangian

The only way out of this dilemma is the use of a Lagrangian, in which bosons are more important. By realizing that the different fermion-boson couplings in Schwinger's higher order terms in the calculation of the magnetic moment may represent two degrees of freedom, those of fermions and bosons, it appears natural to replace this formalism by a field theory of relativistic fermions and bosons, which leads to the Lagrangian

$$\mathcal{L} = \frac{1}{\tilde{m}^2} \bar{\Psi} D_\nu i\gamma_\mu D^\mu D^\nu \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{1}$$

where $\tilde{m}$ is a mass parameter, Ψ a fermion 4-vector charge spinor $\Psi = \Psi^+$ and $\bar{\Psi} = \Psi^-$; further a 3-dimensional vector boson field A_μ with a charge coupling g between the fermion fields Ψ and $\bar{\Psi}$, contained in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$. Finally, the second part of Equation (1) is the Maxwell Lagrangian, containing Abelian field strength tensors $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

By inserting $D^\mu = \partial^\mu - igA^\mu$ and $D_\nu D^\nu = \partial_\nu \partial^\nu - ig\left(A_\nu \partial^\nu + \partial_\nu A^\nu\right) - g^2 A_\nu A^\nu$ in Equation (1), one obtains for the first term of $\mathcal{L}$

$$\begin{aligned}
\mathcal{L}_1 = \frac{1}{\tilde{m}^2} \bar{\Psi} i\gamma_\mu D^\mu D_\nu D^\nu \Psi &= \frac{i}{\tilde{m}^2} \bar{\Psi} \gamma_\mu \partial^\mu \partial_\nu \partial^\nu \Psi + \frac{g}{\tilde{m}^2} \bar{\Psi} \gamma_\mu A^\mu \partial_\nu \partial^\nu \Psi \\
&+ \frac{g}{\tilde{m}^2} \bar{\Psi} \gamma_\mu \partial^\mu A_\nu \partial^\nu \Psi + \frac{g}{\tilde{m}^2} \bar{\Psi} \gamma_\mu \partial^\mu \partial_\nu A^\nu \Psi - \frac{ig^2}{\tilde{m}^2} \bar{\Psi} \gamma_\mu A^\mu A_\nu \partial^\nu \Psi \\
&- \frac{ig^2}{\tilde{m}^2} \bar{\Psi} \gamma_\mu A^\mu \partial_\nu A^\nu \Psi - \frac{ig^2}{\tilde{m}^2} \bar{\Psi} \gamma_\mu \partial^\mu A_\nu A^\nu \Psi - \frac{g^3}{\tilde{m}^2} \bar{\Psi} \gamma_\mu A^\mu A_\nu A^\nu \Psi
\end{aligned} \tag{2}$$

with a gauge condition $\partial\left(\partial_\nu A^\nu\right) = 0$.

This Lagrangian is of third order, but there is only one third order term $\frac{i}{\tilde{m}^2} \bar{\Psi} \gamma_\mu \partial^\mu \partial_\nu \partial^\nu \Psi$, which cancels out. All other terms are of first or second order and are really needed in a bound state theory of fermions and bosons. A first complete evaluation of the Lagrangian (1) has been published in ref. [9].

3. Bound State Description of Particles from the Lagrangian

Observed particles, as hadrons, leptons or more complex systems, have to be understood as bound states of matter. For such a stationary system an equal time requirement allows to reduce the fermion four-vectors Ψ and the boson three-vectors A_μ by one dimension. Then, three-dimensional matrix elements $\mathcal{M}(q) \sim \bar{\mathcal{L}}_1(q)\mathcal{L}_1(q)$ can be calculated, in which the product of the overlapping fermion and boson fields Ψ and A_μ are replaced by dimensionless wave functions $\psi(q)$ and $w(q)$—which can be considered as the square root of their normalized probabilities in momentum space. For uncharged mesons the fermion wave functions are given by $\psi(q) \sim \frac{1}{\tilde{m}^3}\bar{\Psi}\Psi$, for leptons by $\psi(q) \sim \frac{1}{\tilde{m}^{9/2}}(\bar{\Psi}\Psi)\Psi$. The boson wave functions are given by $w(q) \sim \frac{1}{\tilde{m}^2}A_\mu A^\nu$, but one pair of boson fields has to be considered as interaction of vector structure $v_v(q) = \pm\alpha\frac{1}{\tilde{m}}A_\mu A^\nu$, acting between fermions and between bosons. Then, the matrix elements can be expressed in the form $\mathcal{M}(q) = \bar{\psi}(q)K(q)\psi(q)$, where the kernel $K(q) \sim \left[O_\mu^3(q)O^3(q)^\nu\right]$ contains boson fields A_μ and/or derivatives.

Contributions to static fermion potentials are obtained from the last terms of the Lagrangian (2) $\mathcal{L}_{1,7} = -\frac{ig^2}{\tilde{m}^2}\bar{\Psi}\gamma_\mu\partial^\mu A_\nu A^\nu\,\Psi$ and $\mathcal{L}_{1,8} = -\frac{g^3}{\tilde{m}^2}\bar{\Psi}\gamma_\mu A^\mu A_\nu A^\nu\,\Psi$. This leads to two matrix elements with wave functions of scalar or vector coupling. By removing further the γ-matrices by adding two matrix elements with interchanged μ and ρ (note that $\frac{1}{2}(\gamma_\mu\gamma_\rho + \gamma_\rho\gamma_\mu) = g_{\mu\rho}$), this yields

$$\mathcal{M}_{2g} = 3\alpha^2\tilde{m}\,\bar{\psi}(p')\partial^2 w_s(q)w_s(q)\psi(p) \tag{3}$$

and

$$\mathcal{M}_{3g} = -\alpha^3\,\bar{\psi}(p')w_{s,v}(q)v_v(q)w_{s,v}(q)\psi(p), \tag{4}$$

where $\alpha = g^2/4\pi$ and $v_v(q)$ is an attractive interaction. Importantly, the second order matrix element (3) yields a momentum displacement of the wave functions, which satisfies the condition of causality.

In addition, the parts of the Lagrangian (2) $\mathcal{L}_{1,3} - \mathcal{L}_{1,6}$ lead to matrix elements, which include the derivative of the fermion wave function $\partial\psi$

$$\mathcal{T}_{1g} = -3\alpha\tilde{m}\,\bar{\psi}(p')\partial^2 w_{s,v}(q)\partial\psi(p) \tag{5}$$

and

$$\mathcal{T}_{2g} = -\alpha^2\tilde{m}\,\bar{\psi}(p')w_{s,v}(q)w_{s,v}(q)\partial\psi(p). \tag{6}$$

These matrix elements are related to the kinetic energy of the system.

Finally, the parts $\mathcal{L}_{1,2} - \mathcal{L}_{1,4}$ lead to a matrix element with second derivative of the fermion wave function $\partial^2\psi$

$$\mathcal{B}_{1g} = 3\alpha\tilde{m}\,\bar{\psi}(p')w_{s,v}(q)\partial^2\psi(p), \tag{7}$$

which corresponds to an acceleration of the system.

In the boson sector there are additional matrix elements, a static component

$$\mathcal{M}^g = \alpha^3\, w_{s,v}(q)v_v(q)w_{s,v}(q), \tag{8}$$

which includes an interaction $v_v(q)$ between bosons, which is now repulsive. The other term is of dynamical structure

$$T^g = \frac{3\alpha^2\tilde{m}}{2} w_{s,v}(q)\partial^2 w_{s,v}(q). \tag{9}$$

From these matrix elements binding energies have been determined, see e.g. ref. [10], using vacuum expectation values $E_x = \langle\psi|\mathcal{M}_x|\psi\rangle$ with real fermion wave functions $|\psi\rangle$ (with $\bar{\psi}=\psi$), further boson energies $E_x^g = \langle w|\mathcal{M}_x^g|w\rangle$ with similar boson wave functions.

Fourier transformation (of the 3-dim. fermion and 2-dim. boson momenta) to r-space and using the eigenvalue relation (Virial equation) $E\psi(r) = \mathcal{M}\psi(r)$ leads to fermion binding energies E_{ng}

$$E_{ng} = 4\pi\int r^2 \mathrm{d}r\, M_{ng} = 4\pi\int r^2\mathrm{d}r\,\psi(r)V_{ng}(r)\psi(r) \tag{10}$$

with the potentials for $n = 2, 3$

$$V_{2g}(r) = \frac{\alpha^2(2s+1)(\hbar c)^2}{8\tilde{m}}\left(\frac{\mathrm{d}^2 w_s(r)}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}w_s(r)}{\mathrm{d}r}\right)\frac{1}{w_s(r)} + V_o \tag{11}$$

and

$$V_{3g}(r) = \frac{\alpha^2(\hbar c)}{\tilde{m}}\int \mathrm{d}r'\, w_{s,v}(r')v_v(r-r')w_{s,v}(r'), \tag{12}$$

with $s = 0$ for scalar and $s = 1$ for vector states and $v_v(r)$ an attractive interaction of the form $v_v(r) = -\alpha(\hbar c)w_v(r)$ for $w_v(r)\ge 0$ and $v_v(r)=0$ for $w_v(r)<0$. V_o is a very small constant, see e.g. the discussion in ref. [10] [11]. In the present formalism this constant may be interpreted as tiny energy, by which fluctuating bosons are bound in the vacuum (see also the discussion in sects. VI and V).

In a similar way the other terms of Equation (2) with derivatives of Ψ lead to kinetic energy contributions

$$E_{ng}^T = \frac{4\pi}{2}\int r^3\mathrm{d}r\, T_{ng} = \frac{4\pi}{2}\int r^3\mathrm{d}r\,\psi(r)V_{ng}^T(r)\frac{\mathrm{d}\psi(r)}{\mathrm{d}r} \tag{13}$$

with

$$V_{1g}^T(r) = \frac{\alpha(2s+1)(\hbar c)^3}{4\tilde{m}^2}\left(\frac{\mathrm{d}^2 w_s(r)}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}w_s(r)}{\mathrm{d}r}\right) \tag{14}$$

and

$$V_{2g}^T(r) = \frac{\alpha^2(\hbar c)^2}{\tilde{m}} w_{s,v}(r)w_{s,v}(r). \tag{15}$$

Further, terms in Equation (2) with second derivative of Ψ give rise to an acceleration term

$$\Delta E_{1g} = \frac{4\pi}{2}\int r^4 \mathrm{d}r\, B_{1g} = \frac{4\pi}{2}\alpha(\hbar c)\int r^4 \mathrm{d}r\, \psi(r) w_{s,v}(r)\frac{\mathrm{d}^2\psi(r)}{\mathrm{d}r^2}. \quad (16)$$

For simple particle bound states (hadrons) this term leads usually to spurious motion only (without real contributions), but in the case of leptons it gives rise to the existence of neutrinos, see ref. [12].

In the boson sector Fourier transformation of $\mathcal{M}^g$ leads to

$$E^g = 2\pi\int r\mathrm{d}r\, M^g(r) = 2\pi\alpha^2\int r\mathrm{d}r\, w_{s,v}(r) v_v^g(r) w_{s,v}(r) \quad (17)$$

with an interaction similar to fermions but repulsive $v_v^g(r) = \alpha(\hbar c) w_v(r)$ for $w_v(r) \geq 0$ and $v_v^g(r) = 0$ for $w_v(r) < 0$. In a similar way dynamical contributions can be obtained from Equation (2), a kinetic energy

$$E_T^g = \frac{2\pi}{2}\int r^2 \mathrm{d}r\, T^g(r) = 2\pi\frac{\alpha^2(\hbar c)}{4}\int r^2 \mathrm{d}r\, w_s(r)\frac{1}{r}\frac{\mathrm{d}w_s(r)}{\mathrm{d}r} \quad (18)$$

and an acceleration term

$$\Delta E^g = \frac{2\pi}{2}\int r^2 \mathrm{d}r\, B^g(r) = 2\pi\frac{\alpha^2(\hbar c)}{8}\int r^2 \mathrm{d}r\, w_s(r)\frac{\mathrm{d}^2 w_s(r)}{\mathrm{d}r^2}. \quad (19)$$

The structure of matrix elements shows two orthogonal fermion and boson wave functions $\psi_{s,v}(r)$ and $w_{s,v}(r)$ of scalar and vector structure, which should be of the same form $\psi_{s,v}(r) \sim w_{s,v}(r)$. These wave functions are normalized to $4\pi\int r^2 \mathrm{d}r\, \psi_{s,v}^2(r) = 1$ and $2\pi\int r\mathrm{d}r\, w_{s,v}^2(r) = 1$, their orthogonality is expressed by $4\pi\int r^2 \mathrm{d}r\, \psi_s(r)\psi_v(r) = 0$ and $2\pi\int r\mathrm{d}r\, w_s(r) w_v(r) = 0$.

An important point is that second derivative terms of the wave functions appear, which give rise to a displacement of the bound state and fulfill therefore the basic condition of causality.

The wave functions are determined in the following way: For fermions they should be solutions of Equations (10)-(12) for the static part, and also of Equations (13)-(16) for the dynamic part. In addition, for bosons the solutions should satisfy Equations (17)-(19).

Assuming the same radial form of $\psi(r)$ and $w(r)$ (denoted by $\phi(r)$), for fermions components are proportional to $\phi^4(r)\mathrm{d}\phi(r)/\mathrm{d}r$, to $\phi(r)\left[\mathrm{d}^2\phi(r)/\mathrm{d}r^2 + 2/r\,\mathrm{d}\phi(r)/\mathrm{d}r\right]$ and to $\phi(r)\mathrm{d}\phi(r)/\mathrm{d}r\left[\mathrm{d}^2\phi(r)/\mathrm{d}r^2 + 2/r\,\mathrm{d}\phi(r)/\mathrm{d}r\right]$. The boson wave functions should satisfy the relations $\phi(r)^2\,\mathrm{d}\phi(r)/\mathrm{d}r$, $\phi(r)1/r\,\mathrm{d}\phi(r)/\mathrm{d}r$ and $\phi(r)\mathrm{d}^2\phi(r)/\mathrm{d}r^2$. Solutions for components proportional to $\phi^n(r)$ would need only exponential wave functions $\sim \exp(-r/b)$. However, the terms $\sim\left[\mathrm{d}^2\phi(r)/\mathrm{d}r^2 + 2/r\,\mathrm{d}\phi(r)/\mathrm{d}r\right]$ require wave functions of the more restricted form $\sim \exp\left\{(-r/b)^\kappa\right\}$ (with $\kappa \sim \sqrt{2}$).

Interestingly, exactly this form of wave functions is needed, if we formulate a geometric criterion, requiring that the vector potential (product of the two vector distributions $v_v(r)$ and $w_v(r)$) must have a maximum overlap with the scalar density $w_s^2(r)$

$$\int r\mathrm{d}r\, V_{3g}^{v}(r) + \frac{1}{\alpha^3}\int r\mathrm{d}r\, w_s^2(r) = \delta \sim 0. \tag{20}$$

This leads to boson wave functions with orthogonal radial forms $w_s(r)$ and $w_v(r)$

$$w_s(r) = w_{s_o} \exp\left\{-(r/b)^{\kappa}\right\} \tag{21}$$

and

$$w_v(r) = w_{v_o}\left[w_s(r) + \beta R \frac{\mathrm{d}w_s(r)}{\mathrm{d}r}\right], \tag{22}$$

where $w_{(s,v)_o} = \left[2\pi\int r\mathrm{d}r\, w_{s,v}^2(r)\right]^{-1}$ and $\beta R = -\int r^2\mathrm{d}r\, w_s(r) \Big/ \int r^2\mathrm{d}r\, \mathrm{d}w_s(r)/\mathrm{d}r$. In this way unitarity is satisfied. The fermion wave functions are given similarly by $\psi_x(r) = w_x(r)\cdot\left[\int r\mathrm{d}r\, w_x(r) \Big/ 2\int r^2\mathrm{d}r\, \psi_x(r)\right]$. Because of the different constraints, the wave functions are for different systems, as hadrons, leptons, atoms or gravitational objects, see refs. [9] [12], of the same form (21) and (22).

Using these wave functions (21) and (22), in the analysis only three or four parameters are needed, shape and slope parameters κ and b and coupling constant α for electric binding; however, for magnetic binding an addition parameter (v/c) related to a rotation of the system is required. These parameters can be determined unambiguously by the geometric constraint (20) and the following boundary conditions:

1) Energy-momentum conservation (combining energy and momentum conservation) demands that the average fermion 3-momenta and the average boson 2-momenta cancel each other, equally the corresponding energies

$$\left\langle q_f^2\right\rangle^{1/2} - \left\langle q_g^2\right\rangle^{1/2} \sim E_f + E_g, \tag{23}$$

where $\left\langle q_f^2\right\rangle^{1/2}$ is taken as the root mean square momentum of the total fermion and boson distributions $\left\langle q_f^2\right\rangle^{1/2} = \left[\int q^4\mathrm{d}q\, \mathcal{M}_{3g}(q) \Big/ \int q^2\mathrm{d}q\, \mathcal{M}_{3g}(q)\right]^{1/2}$ and $\left\langle q_g^2\right\rangle^{1/2} = \left[\int q^3\mathrm{d}q\, \mathcal{M}^{g}(q) \Big/ \int q\mathrm{d}q\, \mathcal{M}^{g}(q)\right]^{1/2}$ for bosons. Further, E_f and E_g are the corresponding binding energies. Since $\left\langle q_f^2\right\rangle^{1/2}$ and $\left\langle q_g^2\right\rangle^{1/2}$ are different, these energies should be modified by recoil corrections $rec = 1/2\left(\left\langle q_f^2\right\rangle^{1/2} - \left\langle q_g^2\right\rangle^{1/2}\right) \Big/ \left(\left\langle q_f^2\right\rangle^{1/2} + \left\langle q_g^2\right\rangle^{1/2}\right)$, with sign negative for fermions and positive for bosons. Further, the interaction is attractive for fermions but repulsive for bosons; therefore the total energy $E_{tot} = E_f + E_g$ should vanish. If this condition is fulfilled, it indicates that all particles can be generated out of the vacuum.

The mean square radii are given similarly, $\left\langle r_f^2\right\rangle = \int r^4\mathrm{d}r\, \mathcal{M}_{3g}(r) \Big/ \int r^2\mathrm{d}r\, \mathcal{M}_{3g}(r)$

for fermions and $\langle r_g^2\rangle = \int r^3 \mathrm{d}r\, \mathcal{M}^g(r) \big/ \int r \mathrm{d}r\, \mathcal{M}^g(r)$ for bosons.

From relation (23) follows also that there is a coupling between space and time $\langle r_f^2\rangle^{1/2} - \langle r_g^2\rangle^{1/2} = v_{rot}\left(t_f - t_g\right)$, where v_{rot} is the mean rotation velocity. Such a space-time relation is demanded from Einstein's theory of relativity.

2) A mass-radius relation can be deduced from the potentials (11) and (12)

$$Rat_{2g} = \frac{(\hbar c)^2}{\tilde{m}^2 \langle r_g^2\rangle} = 1. \tag{24}$$

This relation shows that in the present formalism particles with various masses and radii can be described.

4. Bound State Solutions of Mesons

The Lagrangian (1) should give rise to fermion-antifermion states, which can be identified with simple mesons of spin and charge equal to zero. In such an analysis values of the shape parameter $\kappa = 1.375$ and the coupling constant $\alpha = 2.158$ gave optimal results for all systems, also for leptons discussed in sect. V. The slope parameter b was fitted to the different systems, by respecting the above constraints (23)-(24). As mentioned above, the acceleration term (7) does not contribute in this case to the fermion binding energy. In addition, the constant V_o in Equation (11) could be neglected for mesons.

It was found that the slope parameters in Table 1 gave rise to a rather good description of the masses ($M = -E_b$) of various mesons from ω (0.782 GeV) up to Υ (9.46 GeV). However, the boson energies were only 1/3 of those of fermions. Also it was found that the recoil corrections
$rec = \mp f_{rec}\left(\langle q_f^2\rangle^{1/2} - \langle q_g^2\rangle^{1/2}\right) \Big/ \left(\langle q_f^2\rangle^{1/2} + \langle q_g^2\rangle^{1/2}\right)$ should be much smaller than with the usual recoil factor $f_{rec} = 1/2$. If, however, the meson bound states contain three bosons, this leads to a decrease of f_{rec} to $1/2:3 = 1/6$. In addition, the confinement potential (11) is due to the motion of bosons and should not contribute to the fermion recoil. Without this part the fermion energy is only ~53% of the boson energy. Taking these corrections into account, one obtains equal absolute fermion and boson energies and $f_{rec} \sim 0.088$, which is in good agreement with the fitted value $fit_{rec} = 0.09$.

This leads to quantitative results, see Table 1. In all cases the masses $M = -E_f$ are in good agreement with experiment; however, even more important is that the fermion and boson binding energies balance each other with a total energy $E_{tot} \simeq 0$. This indicates that these mesons could be really generated out of the vacuum.

5. Lepton Analysis

Because of their charge a similar analysis of single leptons is not possible. Only lepton-antilepton systems can be related directly to the Lagrangian (1). Therefore, the wave functions of the total lepton-antilepton system have to be split up

Table 1. Meson results using $\kappa = 1.375$, $\alpha = 2.158$, a recoil factor $fit_{rec} = 0.09$ and assuming a three boson structure. For the given values of b extracted root mean square fermion radii and momenta as well as fermion, boson and total energies are given. All dimensions are in fm or GeV.

System	b	$\langle r_f^2 \rangle^{1/2}$	$\langle q_f^2 \rangle^{1/2}$	E_f	E_g	E_{tot}
ω (0.782 GeV)	$5.778 \cdot 10^{-1}$	$6.575 \cdot 10^{-1}$	1.053	−0.7821	0.7819	−0.0002
Φ (1.02 GeV)	$4.426 \cdot 10^{-1}$	$5.038 \cdot 10^{-1}$	1.374	−1.021	1.020	−0.001
J/ψ (3.097 GeV)	$1.458 \cdot 10^{-1}$	$1.660 \cdot 10^{-1}$	4.172	−3.098	3.098	−0.000
Υ (9.46 GeV)	$4.7744 \cdot 10^{-2}$	$5.435 \cdot 10^{-1}$	12.74	−9.462	9.460	−0.002

into two lepton wave functions of the form $\psi' \sim (\bar{\Psi}\Psi)\bar{\Psi}$ and $\psi \sim (\bar{\Psi}\Psi)\Psi$, whereas the form of boson wave functions can be left unchanged. To get the correct binding energy we have to assume also three boson pairs, as for mesons. The derived matrix elements (3)-(19), the geometric constraint (20) and the boundary conditions (23) and (24) can also be left unchanged. Also the ratio of fermion to boson energies is ~53% by excluding the confinement potential (11). This gives rise to a recoil factor $f_{rec} \sim 0.089$, in agreement with the fitted value $fit_{rec} = 0.105$ within the given uncertainties. However, an important difference from hadrons is that leptons can be bound electrically but also magnetically.

Electric binding

The obtained results are displayed in Table 2, which show masses ($M = -E_f$) for the μ- and τ-mesons in quantitative agreement with the data; further, a vanishing total energy E_{tot}. However, for the electron the absolute fermion binding energy is 4% smaller than the boson energy. This discrepancy can be explained by a contribution from V_o in Equation (11). To get equal absolute fermion and boson binding energies the value of V_o has to be −9.56 × 10^{-6} GeV, which gives rise to a negligible contribution for all other systems in question.

Finally, it is important to test, whether the matrix elements of the Lagrangian can really be spit up into two separate parts for single leptons. For the τ-lepton the normalized scalar density $(1/\alpha^3)\, w_s^2(r)$ (dot-dashed line) is compared in the upper part of Figure 1 to the vector potential $V_{3g}^v(r)$ (solid line). The difference between the dot-dashed and solid lines is hardly visible, indicating that the geometry condition (20) is also satisfied in the lepton analysis (for hadrons the same agreement has been established earlier).

Magnetic binding

Of particular interest is the magnetic binding, which yields additional information on magnetic moments and allows a verification of space-time coupling. Apart from the same values of $\kappa = 1.375$ and $\alpha = 2.158$, the binding is characterized by two parameters, slope b and velocity parameter (v/c), the latter related to the rotation orthogonal to the direction of interaction. This requires an additional factor $(v/c)^2$ in Equations (10), (13), (16), in energy-momentum conservation (23) and the mass-radius relation (24); further, a factor (v/c) in Equations (17)-(19). The results for magnetic binding are also given in Table 2. All

Table 2. Lepton results with κ and α as in **Table 1** and a recoil factor $fit_{rec} = 0.105$ for electric (*elec*) and magnetic (*mag*) binding. All dimensions are in fm or GeV.

System	$b / \frac{b}{v/c}$	$\langle r_f^2 \rangle^{1/2}$	$\langle q_f^2 \rangle^{1/2}$	E_f	E_g	E_{tot}
e elec	$8.835 \cdot 10^2$	1006	$0.50511 \cdot 10^{-3}$	$-0.5112 \cdot 10^{-3}$	$0.5110 \cdot 10^{-3}$	$-0.0002 \cdot 10^{-3}$
e mag	$\frac{2.125 \times 10^{-10}}{2.404 \times 10^{-13}}$	$2.418 \cdot 10^{-10}$	$0.5103 \cdot 10^{-3}$	$-0.5112 \cdot 10^{-3}$	$0.5108 \cdot 10^{-3}$	$-0.0004 \cdot 10^{-3}$
μ elec	4.275	4.866	0.1055	−0.1057	0.1056	−0.0001
μ mag	$\frac{3.917 \times 10^{-6}}{9.165 \times 10^{-7}}$	$4.459 \cdot 10^{-6}$	0.1055	−0.1058	0.1056	−0.0002
τ elec	0.254	0.2894	1.774	−1.777	1.776	−0.001
τ mag	$\frac{9.595 \times 10^{-3}}{3.775 \times 10^{-2}}$	$1.092\ 10^{-2}$	1.774	−1.778	1.776	−0.002

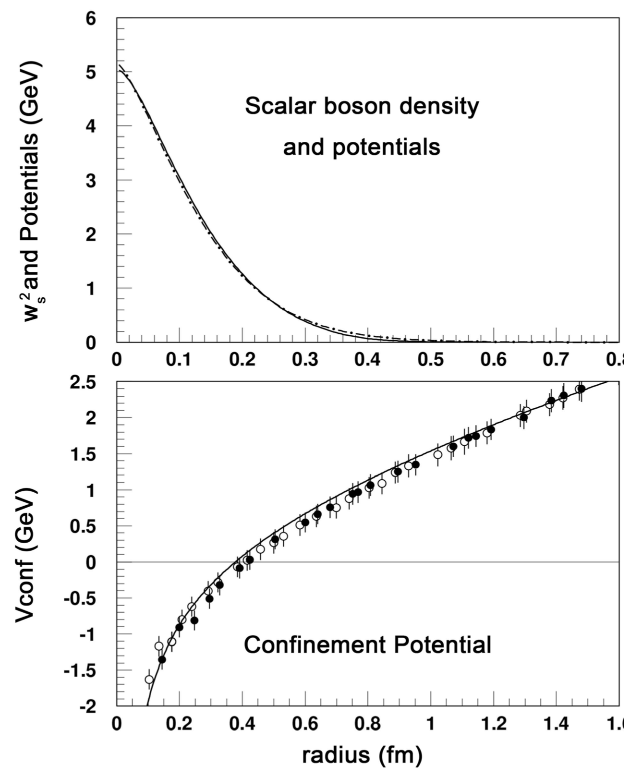

Figure 1. Upper part: Check of the geometry condition (20) for the τ-lepton by displaying the scalar density $w_s^2(r)$ (dot-dashed line) together with the vector potential $V_{3g}^v(r)$ (solid line). The difference between dot-dashed and solid line is hardly visible. Lower part: Potential $V_{2g}(r)$ in comparison with empirical confinement potentials from ref. [13] [14] with estimated uncertainties given by error bars.

masses $M = -E_f$ agree with the experimental data to < 1‰ and the total energy E_{tot} is zero within extremely small errors. This is the case also for the electron: here the radial extent of the boson wave function $w_s(r)$ is extremely small and thus the contribution from V_o is negligible.

The magnetic dipole moment of spin 1/2 particles is given by

$$\mathcal{M}1 = \frac{\langle r_f \rangle M}{2(\hbar c)(v/c)}, \tag{25}$$

where $\langle r_f \rangle$ is the linear mean fermion radius.

The deduced magnetic moments $\mathcal{M}1$ are 1.001160, 1.001038 and 1.001318 for electron, muon and tau, respectively, in excellent agreement with the experimental data.

It is also important to mention that the acceleration terms (16) and (19) give rise to neutrinos. This subject is discussed in detail in ref. [12].

Further, the rotation time is given by

$$t_{rot} = \frac{2\pi}{v_f}\langle r_f^2 \rangle^{1/2} = \frac{2\pi}{v_g}\langle r_g^2 \rangle^{1/2}, \tag{26}$$

where $v_{f,g}$ are the rotation velocities at the root mean square radii $\langle r_{f,g}^2 \rangle^{1/2}$. This time should be the same for electric and magnetic binding. This is indeed the case, $t_{rot} = 2.11 \times 10^{-20}$, 1.02×10^{-22} and 6.06×10^{-24} sec for electron, muon and tau, respectively.

Here one can show that the present bound state formalism is compatible with Einstein's law of relativity. If we write the (linear increasing) rotation velocity by $z = 2\pi r t_{rot}$ (redshift), the relativistic velocity $v/c \le 1$ is given by $v/c = \left[(z+1)^2 - 1\right] / \left[(z+1)^2 + 1\right]$. It should be mentioned that for simple systems only rotations exists. However, for complex systems also a compression or dilatation of the system is possible. In these cases the relativistic velocity is given by the same relativistic correction.

As a final point the confinement potential $V_{2g}(r)$ is given for the τ-lepton in the lower part of **Figure 1** together with empirical confinement potentials from ref. [13] [14] [15] [16], deduced from J/ψ and γ-meson spectroscopy, which shows a striking consistency. Further, a small contribution arises from the potential V_o, by which bosons are pulled out of the vacuum. It effects only the electric binding of electrons and amounts to a binding energy of -2.02×10^{-5} GeV.

6. Concluding Remarks

The long standing problem of the incompatibility of quantum field theories with Einstein's law of relativity has been resolved by introducing a dual fermion-boson Lagrangian to calculate bound state matrix elements of particles. In this description severe constraints have been introduced, as momentum and energy conservation, from which all model parameters were determined—leading to a description based on first principles. Then, hadron and lepton properties were obtained with uncertainties of less than 1‰.

These results indicate that the boson degree of freedom is of similar importance as that of fermions. Only if bosons are correctly taken into account, particle properties are quantitatively described, but also particles could emerge out of the vacuum. In this way a logic and natural description of the development and structure of the universe is warranted. Two popular scientific articles on these results are given in refs. [17] and [18].

All calculations can be run on line under the address https://h2909473.stratoserver.net, where also the underlying fortran source code (in gfortran) can be inspected.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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