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Gravitation, Density Upper Limit and Quantization of Space

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Abstract

The singularity at distance $r \to 0$ at the center of a spherically symmetric non-rotating, uncharged mass of radius *R*, is considered here. Under inverse square law force, the Schwarzschild metric, needs to be modified, to include Newton's Shell Theorem (NST). By including NST for $r < R$, both Schwarzschild singularity at $r = 2GM/c^2$ and at $r \to 0$ singularities are removed from the metric. Near $R \to 0$, the question of maximal density is considered based on Schwarzschild's modified metric, and compared to the quantum limit of maximal mass density put by Planck's quantum-based universal units. It is asserted, that General relativity, when combined with Planck's universal units, inevitably leads to quantization of gravity.

Keywords

Gravitation, Shell Theorem, Singularity, Schwarzschild Radius, CGH Physics: Planck's Scale

1. Introduction

Consider a spherically symmetric, non-rotating, uncharged mass (a stellar object).

Under spherical symmetry, and at a remote distance in empty space outside the object, the Schwarzschild metric [1]-[10] is given by:

$$g_{\mu\nu} = \begin{pmatrix} 1-\frac{2GM}{rc^2} & & & \\ & \frac{-1}{1-\frac{2GM}{rc^2}} & & \\ & & -r^2 & \\ & & & -r^2\sin^2\theta \end{pmatrix} \tag{1}$$

This metric describes the gravitational effects of that mass on gravitational field curvature, and dynamics of moving particles, in the empty regions outside the object.

It suffers of two problems:

1) At the Schwarzschild radius $r_s = \frac{2GM}{c^2}$, the metric has a singularity.

2) The metric diverges for $r \to 0$.

The Schwarzschild solution appears to have singularities at $r = 0$ and $r = r_s$, as some of the metric components diverge at these radii [1]-[12]. There is no problem as long as $R > r_s$. For ordinary stars and planets this is always true. For example, the radius of the Sun is approximately 700,000 km, while its Schwarzschild radius is only 3 km.

The singularity at $r = r_s$ divides the Schwarzschild coordinates in two disconnected patches. The exterior Schwarzschild solution with $r > r_s$ is the one that is related to the gravitational fields of stars and planets. The interior Schwarzschild solution with $0 \leq r < r_s$, which contains the singularity at $r = 0$, is completely separated from the outer patch by the singularity at $r = r_s$. The Schwarzschild coordinates therefore give no physical connection between the two patches, which may be viewed as separate solutions. The singularity at $r = r_s$ is an illusion however. It is an instance of what is called a coordinate singularity. As the name implies, the singularity arises from a bad choice of coordinates or coordinate conditions. When changing to a different coordinate system, the metric becomes regular at $r = r_s$ and can extend the external patch to values of r smaller than r_s. Using a different coordinate transformation one can then relate the extended external patch to the inner patch [9] [10].

The singularity in the case $r = 0$ is different, however. It is an inherent singularity at $r \to 0$ in any inverse square law central force problem. If one asks that the solution be valid for all r, one runs into a true physical singularity, or gravitational singularity, at the origin. To see that this is a true singularity one must look at quantities that are independent of the choice of coordinates. One such important quantity is the Kretschmann invariant, which is given by [11] [12] [13] [14] $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$. For a Schwarzschild black hole of mass M and radius R, the Kretschmann invariant is $K_r \frac{48G^2M^2}{c^4R^6} \approx \frac{842G^2}{\rho^2c^4}$ and obviously is independent of the radius R.

At $r = 0$ the curvature becomes infinite, if and only if, it represents a point particle of nonzero mass. One cannot compress a finite mass into an infinitesimal point ($r = 0$). Any finite mass should have a finite radius $R > 0$. Therefore, when looking into the point $r = 0$, one needs to consider the effects of the outer shells of matter around the $r = 0$ point.

A gravitational singularity, is a location in spacetime where the gravitational field of a celestial body is predicted to become infinite by general relativity in a way that does not depend on the coordinate system. The quantities used to measure gravitational field strength are the scalar invariant curvatures of spacetime, which includes a measure of the density of matter. Since such quantities become infinite at the singularity, the laws of normal spacetime break down.

Gravitational singularities are mainly considered in the context of general relativity, where density apparently becomes infinite at the center of a black hole, and within astrophysics and cosmology as the earliest state of the universe during the Big Bang. It is still undecided whether the prediction of singularities means that they actually exist (or existed at the start of the Big Bang), or that current knowledge is insufficient to describe what happens at such extreme densities.

General relativity predicts that any object collapsing beyond a certain point (for stars this is the Schwarzschild radius) would form a black hole, inside which a singularity (covered by an event horizon) would be formed.

Some theories, such as the theory of loop quantum gravity, suggest that singularities may not exist. This is also true for such classical unified field theories as the Einstein-Maxwell-Dirac equations. The idea can be stated in the form that due to quantum gravity effects, there is a minimum distance beyond which the force of gravity no longer continues to increase as the distance between the masses becomes shorter, or alternatively that interpenetrating particle waves mask gravitational effects that would be felt at a distance.

Recent results have shown that, when analyzed correctly, quantum gravitational collapse does not lead to the formation of the singularity in the heart of the black hole. In reality, what we now call a "black hole" turns out to be a very dense spherical shell formed by matter that condenses naturally on the Schwarzschild surface, which therefore turns out to be an apparent horizon rather than a real horizon. Something similar was originally found already by Einstein [10]. The first rigorous demonstration of this final state of collapse is due to Vaz [11]. Subsequently, Corda [12] [13] obtained the same result as Vaz by showing that these spherical shells have are markably similar quantum mathematical structure to the hydrogenatom in.

The first singularity is not a real one, as it can be removed by appropriate coordinate transformations [14] [15] [16] [17] [18].

The second singularity cannot be removed by coordinate transformation. It is an inherent singularity at $r \to 0$ in any inverse square law central force problem of gravitational field). One so far have not been able to relate to the divergency problem of force which is inversely proportional to the square of the distance r from center of the field source.

To solve this problem, we must investigate the behavior of the field inside the mass.

If any mass source is finite in size, the solution to the potential as a function of distance must be modified, so that it includes the region where $r < R$, the radius of the mass.

In addition, the Einstein field equation should be related to in non-empty space. But to a very good approximation, the Einstein equation in the inside of even the heaviest stellar objects can be considered same as empty space.

2. Shell Theorem

When considering gravitation at $r = 0$, one must consider the effect of the surrounding mass.

Due to the shell theorem, proven already by Isaac Newton [14]-[20], the gravitational potential of a spherically symmetric object of mass M and radius R, as a function of distance r, from object's center is given by:

$$\Phi(r) = -\frac{GM}{R^3}\begin{cases} \frac{1}{2}\left(3R^2 - r^2\right) & \text{for } r < R \\ \frac{R^3}{r} & \text{for } r \geq R \end{cases} \tag{2}$$

The result for $r < R$ is obtained by summation of the potential at some point p inside the mass from its spherical shell between R and r, and the remaining mass inside sphere of radius r.

The total potential at point p inside the sphere is a superposition of both $\Phi_{shell}(p)$, and $\Phi_{inner}(p)$. Thus, (for $r < R$):

$$\Phi_p(r) = -\frac{3GM}{2R^3}\left(R^2 - \frac{1}{2}r^2\right) \tag{3}$$

In Schwarzschild metric, the g_{00} term is a function of distance r and in a weak-field approximation one has:

$$g_{00}(r) \to \left(1 + \frac{2\Phi(r)}{c^2}\right) = \left(1 - \frac{2GM}{rc^2}\right) \tag{4}$$

Therefore, for anon-*weak-field*, we will assume for any r:

$$g_{00}(r) = 1 + \frac{2\Phi(r)}{c^2} \tag{5}$$

The Schwarzschild metric of such an object becomes:

$$g_{\mu\nu} = \begin{pmatrix} 1 + \frac{2\Phi(r)}{c^2} & & & \\ & \frac{-1}{1 + \frac{2\Phi(r)}{c^2}} & & \\ & & -r^2 & \\ & & & -r^2 \sin^2\theta \end{pmatrix} \tag{6}$$

With the correct presentation of $\Phi(r)$ as described above, the singularity at $r \to 0$ is removed.

$\Phi(r)$ is a function of distance r, and must have dimensions of c^2. It must obey the constraint of $\lim_{r\to\infty} \Phi(r) \to 0$ as should be the case at infinite distance in empty flat space.

Its units should be [G][Kg]/[m] in order to make $\frac{\Phi(r)}{c^2}$ dimensionless.

We therefore have a good reason to assume that $\Phi(r)$ is the gravitational potential due to a spherically symmetric object, independent of size and density of that object. Exactly as described by Equation (1).

Due to spherical symmetry, the metric must be of the form

$$g_{\mu\nu} = \begin{pmatrix} g(r) & & & \\ & h(r) & & \\ & & r^2 & \\ & & & r^2 \sin^2\theta \end{pmatrix} \tag{7}$$

where $g(r)$ and $h(r)$ are functions of distance r from the coordinate center (located at the center of mass.

So, the infinitesimal proper time interval $\mathrm{d}\tau$ between two events along a time-like path is given by

$$\mathrm{d}\tau^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = g(r)\mathrm{d}t - \frac{1}{c^2}\left(h(r)r^2\mathrm{d}r - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2\right) \tag{8}$$

With the flat space metric $g_{\mu\nu} = (+---)$.

3. Inside the Sphere ($r < R$)

In the case where $r < R$ (inside the mass), $\mathbb{K} \neq 0$ where $\mathbb{K} = \frac{16\pi G}{c^2}\rho$.

$G = 6.6743\times10^{-11}$ and so $\mathbb{K} \approx 37\times10^{-27}$, thus, for internal densities smaller than a magnitude order limit of 10^{27} Kg/m^3, the equation can be solved with the approximation of $\mathbb{K} = 0$.

Even for the heaviest neutron stars [21], overall densities are of magnitude order of 6×10^{17} [Kg/m^3], which are smaller yet by a 10^{-10} factor than the upper limit.

Elementary particles, the neutron for instance, has a density of approximately 3×10^{17} [Kg/m^3]. Nearly the same as the heaviest neutron star.

For all practical calculations one may assume $\mathbb{K} = 0$ inside the radius R of any known massive object (that is, for $r < R$) whose internal interactions are negligible.

Finding the solution of the homogeneous differential equation with $\mathbb{K} = 0$, will lead to the non-homogeneous solution with $\mathbb{K} = \text{constant}$. But we will concentrate on the homogeneous solution, since $\mathbb{K} \approx 0$ for all known cold objects except for hot stars and black holes.

Under the assumption of $\mathbb{K} \approx 0$, for the interior of the sphere, the time separation inside the sphere is given by $\mathrm{d}\tau^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$:

$$\mathrm{d}\tau^2 = \left(1 + \frac{2\Phi(r)}{c^2}\right)\mathrm{d}t^2 - \frac{1}{c^2}\left(h(r)r^2\mathrm{d}r - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2\right) \tag{9}$$

This is though limited to a spherically symmetric, non-rotating object, with no internal (electromagnetic) interactions. For hot objects, where internal interactions cannot be neglected, the solution will assume a certain constant value for $\mathbb{K}$ and the solution will be still valid based on the homogeneous solution for $\mathbb{K} = 0$.

4. Modified Schwarzschild Metric

Inserting the corrected (due to shell theorem) expression for $\Phi(r)$ inside the sphere ($r < R$), one obtains:

$$\mathrm{d}\tau^2 = \left(1 - \frac{4\pi\rho G R^2\left(1 - \frac{1}{3}\left(\frac{r}{R}\right)^2\right)}{c^2}\right)\mathrm{d}t^2 - \frac{1}{c^2}\left(h(r)r^2\mathrm{d}r - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2\right) \quad (10)$$

Picking the coordinate system in such a way that the radius r is along the x axis, the θ and φ terms are zero.

In other words, for $r < R$:

$$\mathrm{d}\tau^2 = \mathrm{d}t^2\left(1 - \frac{4\pi G\rho R^2}{c^2}\left(1 - \frac{1}{3}\left(\frac{r}{R}\right)^2\right)\right) \quad (11)$$

The term $h(r)$ is the inverse of $g(r)$ and vanishes for $r \to 0$. Therefore, it is not considered here.

There are now two separate issues ahead:

1) A fixed R, while $r \to 0$.

2) $R \to 0$ (both elementary particles and black holes).

In the first case of a fixed R, $\mathrm{d}\tau$ becomes:

$$\mathrm{d}\tau = \mathrm{d}t\sqrt{1 - \frac{4\pi\rho G}{c^2}R^2} \quad (12)$$

In case 2 with $r \to 0$, we see that the time component becomes independent of distance r from origin. In any case, the divergence at $r \to 0$ is removed.

We also notice that the expression in brackets in Equation (12) (valid for $r \ll R$) vanishes, when R is equal to the photon sphere radius, $\frac{3r_s}{2}$. In other words, in the unique case where the object's radius R is equal to its photon sphere radius, the proper time becomes zero, at $r \to 0$. The photon sphere is the physical radius, in which photons are trapped when reaching from outside.

Obviously then, at photon sphere radius:

$$\mathrm{d}\tau = 0$$

At the photon sphere time-stops and no photons can escape upon entrance.

This result is in accordance with our assumption on black holes, namely, that no light can escape if it is smaller than the photon radius.

The proper time $\mathrm{d}\tau$, must remain well defined. Therefore, apparently an upper limit on the density exists for a given radius R, otherwise the expression for $\mathrm{d}\tau$ becomes undefined.

Equation (15) leads to the following classical gravitation limit on the density:

$$\rho \le \frac{c^2}{4\pi G R^2} \quad (13)$$

In other words, for any object with radius R, there is an upper limit on its density ρ. When $R \to 0$, the density may increase indefinitely, but as will be shown next, there is an upper limit on the density even when $R \to 0$.

5. Density and Planck Universal Units

Under CGH physics [22] [23] [24] [25] there is a connection between the three universal constants: ***c***—the universal constant speed of propagation of electromagnetic field (light) in vacuum, ***G***—the universal gravitation constant and ***h***—Planck's universal quantum constant. Based on dimensional analysis, one can derive the Planck length ℓ_p, which is considered the smallest distance that has any physical plausibility.

In the following, it will be shown that the upper limit on density of any object can be approached from two different aspects of physics:

1) **Gravitation** (by using the Schwartz child metric.

2) **Planck** (by using the connection between *G* and *h*).

We will denote the gravitation derived upper limit by ρ_G and the CGH (Planck) derived upper limit by ρ_P.

Suppose there is a quantum minimum for distance. This is the Planck length and denote it by ℓ_p.

It is given by

$$\ell_p = \sqrt{\frac{\hbar G}{c}} = 1.616 \times 10^{-35} [\text{m}].$$

Under the assumption of Planck's distance. the minimal spherical volume possible is

$$V_p = \frac{4\pi \ell_p^3}{3}$$

Let $\mathcal{M}$ denote the mass of this volume. Its Planck density will be given by

$$\rho_P = \frac{3\mathcal{M}}{4\pi \ell_p^3}$$

Since by assumption ℓ_p is the minimal length possible in nature, then for any mass $\mathcal{M}$, ρ_P is the maximum density possible.

It can be showed, that for a classical spherically symmetric object of density ρ and radius *R*, the general relativistic limit gives

$$\mathrm{d}\tau = \mathrm{d}t \sqrt{\left(1 - \frac{4\pi \rho G}{c^2} R^2\right)} \tag{14}$$

Since the expression in brackets must be real, we arrive at the restriction:

$$\rho(R) \le \frac{c^2}{4\pi G R^2} \tag{15}$$

For an object of given mass $\mathcal{M}$ and radius *R* we have

$$\rho(R) = \frac{\mathcal{M}}{V} = \frac{3m}{4\pi R^3} \tag{16}$$

Since for any mass *m* of radius *R* one must, by the Schwarzschild metric, have

$$\rho(R) = \frac{3\mathcal{M}}{4\pi R^3} \le \frac{c^2}{4\pi G R^2} \tag{17}$$

The result is that for any mass $\mathcal{M}$

$$\mathcal{M}(R) \le \frac{Rc^2}{G}$$

Obviously, the smaller the radius *R*, the smaller the mass *m*, and irrespective of its density, $\lim_{R\to 0} \mathcal{M}(R) \to 0$.

The result is that $\rho_G = \frac{c^2}{4\pi G R^2}$ is the maximal possible density of an object of a given radius *R*.

6. Quantization by Planck's Units and Maximal Density Limit

The quantum gravitational effects become relevant at the Planck length $\ell_p = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35}$ [m]. Here $\hbar$ is the Planck universal constant which governs the scale of the quantum effects, G is the Newton universal constant which governs the strength of the gravitational force, and c is the universal speed of light, which governs the scale of the relativistic effects. The Planck length is many times smaller than what current technology is capable of observing. Because of this, we have no direct experimental guidance for building a quantum theory of gravity [21].

In the limit where $R \to 0$, one needs to consider quantum limits:

Planck's length $\ell_p = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35}$ [m] and Planck's mass $m_p = \sqrt{\frac{\hbar c}{G}} = 2.176 \times 10^{-8}$ [Kg].

Planck length is a theoretically derived number. It cannot be measured directly as it is a predicted quantum estimate.

(Recall though, that dimensional analysis can only determine the Planck's units up to a numerical factor. For instance, one may use h instead of $\hbar$).

Based on Planck's units one obtains Planck's maximal density ρ_P to be given by $\frac{\text{Planck's mass}}{\text{Planck's volume}}$ (assuming ℓ_p is the lowest possible physical distance, leads to the maximal possible physical density assumption).

$$\rho_P = \frac{\mathcal{M}_p}{4\pi \ell_p^3 / 3} = 1.23074 \times 10^{+96} \tag{18}$$

(If one uses the Planck constant instead of the reduced Planck's constant, the Planck density will be modified by a factor of 2π).

Since one must assume Planck's density to be the maximum possible theoretical density, we ask, what should the minimum classical radius R be, in order to always have $\rho_G \le \rho_P$.

Here ρ_G is the highest density possible, derived by gravitational arguments (Equation (17) above), so it is a measurable entity. (Unlike the Planck length which is a quantum theoretical number which can only be estimated).

whereas ρ_P is the maximum quantum mechanical density possible, based on

Planck's dimensional analysis (Equation (18) above).

Therefore:

$$\rho_G = \frac{c^2}{4\pi G R^2} \le \rho_P = \frac{3\mathcal{M}_p}{4\pi \ell_p^3} \tag{19}$$

The above result shows how the upper limit on measurable density by classical gravitation theory, is related to the upper limit on theoretical density predicted by quantum theory of Planck universal constants. This connects classical measurable gravity to quantum theory estimation.

Equation (19), with the help of the definitions of Planck length and Planck mass (see above) shows that $R \ge \frac{1}{\sqrt{3}}\ell_p$.

Apparently, the classical length may be smaller than the Planck length. To see where the $\frac{1}{\sqrt{3}}$ factor discrepancy comes from, let us assume the spherical quantum object has an unknown variable density $\rho(r)$, which varies with distance *r* between $r = 0$ to its outer estimated radius $r = \ell_p$.

Since this is a quantum object, we cannot specify its density at any specific radius *r*. Rather, we have to average.

We need to calculate the measurable radius *R* of this quantized object by its average normalized density.

By comparing thetota mass *M*, given by the integrated variable density $\rho(r)$ over the radius, to the mass of same object with average constant density ρ_0 one obtains:

$$M = 4\pi \int_0^R r^2 \rho(r)\,\mathrm{d}r = \frac{4\pi \rho_0 R^3}{3} \tag{20}$$

By definition, the average classical distance $\langle r^2 \rangle$ is given by the integral over the normalized density:

$$\langle r^2 \rangle \overset{\text{def}}{=} \frac{1}{R}\int_0^R r^2 \rho(r)/\rho_0 \,\mathrm{d}r \tag{21}$$

Thus

$$\langle r^2 \rangle = \frac{R^2}{3} \tag{22}$$

But, by definition, the quantum definition $\sqrt{\langle r^2 \rangle} \to \ell_p$ as $R \to 0$ (one cannot measure a definite radius for a quantum object). Therefore, for $R \to 0$

$$\ell_p \overset{\text{def}}{=} \sqrt{\langle r^2 \rangle} = \frac{1}{\sqrt{3}} R \tag{23}$$

Hence, the actual measured minimal classical radius *R* (gravitationally) is given by

$$R = \sqrt{3}\ell_p \tag{24}$$

Compared to Planck's length $\ell_p = 1.61625 \times 10^{-35}$ [m], the classically derived smallest possible radius *R* is larger than Planck's length ℓ_p, by a factor of $\sqrt{3}$

(= 1.732).

Therefore, the classical measurable radius, R, over an unknown, non-measurable density $\rho(r)$, must always be larger than the predicted Planck minimum length.

These calculations were based on the assumptions of negligible $\mathbb{K} = \frac{16\pi G}{c^2}\rho$, and $R \to 0$ ($r < R$).

It seems now, that the general relativistic solution to the metric of a spherically symmetric object, puts a lower limit on the measured radius R of a classical (gravitational only-no quantum) object, which is similar (up to a $\sqrt{3}$ factor) to the quantum Planck limit.

This derivation was based on putting an upper limit on density based on gravitation Schwarzschild metric on one hand, to the quantum limit based on the quantum based Planck's universal constants.

According to Vaz [11] and Corda [12] [13], the mass and energy spectra of Vaz's quantum shell have been obtained via a Schrodinger-like approach, by further supporting Vaz's conclusions that instead of a spacetime singularity covered by an event horizon, the final result of the gravitational collapse is an essentially quantum object, an extremely compact "dark star". This "gravitational atom" is held up not by any degeneracy pressure but by quantum gravity in the same way that ordinary atoms are sustained by quantum mechanics. By evoking the generalized uncertainty principle, the maximum value of the density of Vaz's shell has been estimated.

The estimated maximum value of the of Vaz's shell density, corresponds to the ground state of Vaz's shell, ≃ 0.0175 in Planck units. By recalling that the Planck density is roughly 10^{93} grams per cubic centimeter in standard units, one gets a value of $\rho_{\max} \simeq 1.752 \times 10^{91}$ grams per cubic cm, which is about two orders of magnitude less than the Planck density. This result is significantly smaller by 5 orders of magnitude than the Planck density ρ_P described above, but is in accordance with our restriction that the gravitational density ρ_G must be less than the Planck density:

$$\rho_G = \frac{c^2}{4\pi G R^2} \leq \rho_P = \frac{3\mathcal{M}_p}{4\pi \ell_p^3} \tag{25}$$

The deviation is due to the estimate on R the actual measured minimal classical radius R (gravitationally).

7. Conclusions

Using Newton's classical shell theorem, we modified the Schwarzschild metric. This removed the singularity at $r = 2MG/c^2$. It was further proved that singularity at $r \to 0$ is avoided because for $r < R$, the gravitation potential becomes linear with r.

For all practical matters, $r < R$ can be treated as an empty space even for the densest known stellar objects (neutron stars) and also for elementary particles (neutrons for instance).

This is of course limited to the hypothetical case where no internal electromagnetic interactions exist. Or, if they exist they are independent of radial distance from center (in other words a fixed constant). Recall, that even for hot stars where internal interactions may be huge, they can be assumed constant and hence the homogeneous solution can be easily extended to the non-homogeneous case.

It was shown how general relativity evidently leads to an upper limit on density, and same approach using Planck's universal constants lead to similar result on upper limit on density. Both classical and quantum mechanical limits on density give the same result.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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