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Unification of Gravitational and Strong Interaction Fields Using Partial Gauge Symmetry

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Abstract

We propose the new field potential by maintaining both the symmetry of the scalar gauge and the conservation law keeping Nöether's theorem, while disregarding the symmetry of the vector gauge. The new potential forms like the well-type potential where a particle behaves almost freely but is very hard to escape without external energy, which can be interpreted as local confinement and asymptotic freedom. By assuming a 2-dimensional metric tensor in 4-dimensional space-time, we suggest the existence of 3 kinds of particles that resemble QCD with 3 color charges. We also show that the mass term exists but comes to zero and derive the charge and spin values. We can regard the particle with this new potential as a gluon, and the interaction in this well-type potential as a strong interaction for the properties of mass, charge, spin, and its behavior. We suggest the eight-fold way with this new particle, which is similar to the existing method based on SU (3) symmetry. Even though the strong interaction has been analyzed in the standard model and string theory, we build a new consistent model based on the theory of relativity including Riemann geometry, and show the unification of gravitational and strong interactional field.

Keywords

Strong Interaction, Gauge Symmetry, Relativity, QCD, Confinement, Asymptotic Freedom

1. Introduction

Among the four fundamental forces in nature, the theory of the strong force has been successfully explained through the Standard Model and QCD, with Quark and Gluon. We have studied the integrated mathematical theory consistent with the existing theories that govern the related phenomenon starting from the general field equation. Especially we have concentrated on the gauge symmetry and the field equation. In this paper, we aim to discuss the mathematical development of a new perspective on the strong force as an intermediate result of the study.

In the previous work, we have proposed two gauges named "vector gauge" and "scalar gauge" to obtain the full rank-2 linear tensor field equation [1]. In particular, the scalar gauge with respect to the metric tensor having its trace value of 2 (thus it can be regarded as a 2-dimensional gauge) has been newly suggested to obtain all the coefficients of the linear approximated field equation. The symmetries of the two gauges lead to the conservation of the energy-momentum tensor, which meets the well-known Nöether's theorem. These two symmetries and one conservation condition can make the full linear gravity equation which is massless as is well-known.

Like the way we have dealt with gravity, we will adopt some other conditions maintaining both the symmetry of only the scalar gauge and the conservation of the energy-momentum tensor, which is still keeping Nöether's theorem [2] [3]. This way, we will obtain the new field equation and investigate the features.

By researching the basic values of the physical quantities and the properties of the new field potential, we can conclude that the new particle specifies the gluon and that the interaction is the strong field interaction from the Standard Model. We will discuss the special claim that the gravitational field and the strong field posed on the extreme ends can be generated and explained from the same rank-2 field equation by applying certain gauge conditions.

2. New Equation Derived from Rank-2 Linear Field Equation

2.1. The First Equation as a Gravity Equation

The general rank-2 linear approximated field equation is (2-1) [4].

$$\begin{aligned}&\partial_\lambda\partial^\lambda h^{\mu\nu}+a\partial^\mu\partial^\nu h+b\left(\partial_\lambda\partial^\nu h^{\mu\lambda}+\partial_\lambda\partial^\mu h^{\nu\lambda}\right)+c\eta^{\mu\nu}\partial_\lambda\partial^\lambda h+d\eta^{\mu\nu}\partial_\lambda\partial_\sigma h^{\lambda\sigma}\\&+eh^{\mu\nu}+f\eta^{\mu\nu}h=-kT^{\mu\nu}\end{aligned}\tag{2-1}$$

where *a*, *b*, *c*, *d*, *e*, and *f* are unknown coefficients and $k=8\pi G/c^4$.

Here, through Nöether's theorem [2] [3], it was found that the linear approximated gravitational equation has two gauges as follows. [1]

$$h^{\mu\nu}\to h^{\mu\nu}+\partial^\nu\Lambda^\mu+\partial^\mu\Lambda^\nu+\eta'^{\mu\nu}\Lambda\tag{2-2}$$

where Λ is an arbitrary scalar function and $\eta'^{\mu\nu}$ is a metric tensor with a trace value of 2 [1]. We have named $\partial^\nu\Lambda^\mu+\partial^\mu\Lambda^\nu$ as a "vector gauge" and $\eta'^{\mu\nu}\Lambda$ as a "scalar gauge". With a vector gauge, it was shown that gravity and electromagnetic fields have been unified [5] [6]. By finding a new gauge, the scalar gauge [1], we have acquired a cornerstone to step to a new potential. Nöether's theorem plays a role in the process of finding all coefficients in (2-1) [1].

In the first equation, both gauges (vector and scalar) satisfy the symmetries, which also include conservation law, all coefficients are determined as follows. [1]

$$\begin{aligned} &a = 1 \\ &b = -1 \\ &c = -1 \\ &d = 1 \\ &e = f = 0 \end{aligned} \tag{2-3}$$

The field equation with these coefficients corresponds to the equation of gravity. Here, when describing gravitational waves (gravitational particles), the following equation can be used [4] (pp. 241-294).

$$\Phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h \tag{2-4}$$

Using the above equation, the linear approximated gravitational equation can be expressed as follows [4] (pp. 241-294).

$$\partial_\lambda \partial^\lambda \Phi^{\mu\nu} = -\kappa T^{\mu\nu} \tag{2-5}$$

$$\partial_\mu \Phi^{\mu\nu} = 0 \tag{2-6}$$

Equation (2-4) is called the Hilbert gauge condition, and it allows for the formation of a wave equation and description of gravitational waves. The reason for focusing only on the linear equation is that we want to deal with local gauges that are applicable in linear space-time. On the other hand, global gauges apply to curved space-time [1].

The second equation that we can do further is to adopt the symmetry of only a scalar gauge while keeping the conservation law. We will examine what happens to the newly created potential and its meaning in the following chapters.

2.2. The Second Equation in Case of Scalar Gauge Symmetry and Conservation Law

As mentioned in the previous chapter, gravity is a case where both gauges are satisfied. This chapter addresses the case where a scalar gauge maintains symmetry but a vector gauge does not require symmetry. We summarize this in **Table 1** [1].

When Case (1) and Case (2) were both satisfied (blue box), then Case (3) was automatically satisfied [4]. Now let us consider the cases where Case (2) and Case (3) are satisfied (red box). This is a case where the conservation law (Case (3)) is satisfied and only one of the two symmetries (Case (2)) is satisfied. In this case, the values of the coefficients (a, b, c, d, e, and f) can be fixed through the equations in **Table 1** [1] (green box).

$$\begin{aligned} &a = 1 \\ &b = -1 \\ &c = -1 \\ &d = 1 \\ &e + 2f = 0 \end{aligned} \tag{2-7}$$

Table 1. Coefficient relations for each case.

Case	Case (1): vector gauge symmetry	Case (2): scalar gauge symmetry	Case (3): conservation
Conditions	$h^{\mu\nu} \to h^{\mu\nu} + \partial^{\nu}\Lambda^{\mu} + \partial^{\mu}\Lambda^{\nu}$	$h^{\mu\nu} \to h^{\mu\nu} + \eta'^{\mu\nu}\Lambda$	$\partial_{\mu}T^{\mu\nu} = 0$
Relation between coefficients	$1+b=0$ $a+b=0$ $c+d=0$ $e+f=0$	$1+2c+d=0$ $a+b=0$ $e+2f=0$	$1+b=0$ $a+c=0$ $b+d=0$ $e\partial_{\mu}h^{\mu\nu} + f\eta^{\mu\nu}\partial_{\mu}h = 0$
Unknown coefficients	c,d,e,f	a,b,c,d,e,f	a,c,e,f

Except for e and f, all coefficients are equal to (2-3). When the coefficients determined in (2-7) are put into Equation (2-1), we have:

$$\partial_{\lambda}\partial^{\lambda}h^{\mu\nu} + \partial^{\mu}\partial^{\nu}h - \left(\partial_{\lambda}\partial^{\nu}h^{\mu\lambda} + \partial_{\lambda}\partial^{\mu}h^{\nu\lambda}\right) - \eta'^{\mu\nu}\partial_{\lambda}\partial^{\lambda}h + \eta'^{\mu\nu}\partial_{\lambda}\partial_{\sigma}h^{\lambda\sigma} + e\left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu}h\right) = -\kappa T^{\mu\nu} \tag{2-8}$$

According to the symmetry of $h^{\mu\nu} \to h^{\mu\nu} + \eta'^{\mu\nu}\Lambda$ [1], so all $\eta^{\mu\nu}$ are modified to $\eta'^{\mu\nu}$ in (2-8) [1]. When we use the conservation law, we have:

$$\partial_{\mu}\left(\partial_{\lambda}\partial^{\lambda}h^{\mu\nu} + \partial^{\mu}\partial^{\nu}h - \left(\partial_{\lambda}\partial^{\nu}h^{\mu\lambda} + \partial_{\lambda}\partial^{\mu}h^{\nu\lambda}\right) - \eta'^{\mu\nu}\partial_{\lambda}\partial^{\lambda}h + \eta'^{\mu\nu}\partial_{\lambda}\partial_{\sigma}h^{\lambda\sigma}\right) + e\partial_{\mu}\left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu}h\right) = -\kappa\partial_{\mu}T^{\mu\nu} = 0 \tag{2-9}$$

The blue terms in Equation (2-9) are self-terminated and can also be seen in the conservation law for the original linear gravity equation. Then, the last black terms on the left side in Equation (2-9) satisfy the following condition:

$$\partial_{\mu}\left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu}h\right) = 0 \quad \text{or} \quad \partial_{\nu}\left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu}h\right) = 0 \tag{2-10}$$

Although Equation (2-10) is similar to the existing Hilbert gauge condition [4], it is not the same. Be careful that the metric tensor in Equation (2-10) is not the conventional $\eta^{\mu\nu}$, but a metric tensor $\eta'^{\mu\nu}$ with a trace value of 2 [1]. We may call it a "new gauge conditional expression" or "Hilbert-like gauge condition". When the trace of $\eta'^{\mu\nu}$ is 2, there can be three cases, as shown in **Table 2**, depicted in four dimensions of space-time.

Because the trace of all the tensors in **Table 2** is 2, whose components are one space element and the other time element, the field looks to operate only in one space dimension. This one-dimensional field can explain the force line that binds quarks together, which will be interpreted mathematically in a later chapter. The meaning of Equation (2-9) is that $h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu}h$ makes the new

Table 2. All kinds of $\eta'^{\mu\nu}$.

$\eta_1'^{\mu\nu}$	$\eta_2'^{\mu\nu}$	$\eta_3'^{\mu\nu}$
$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

conserved physical quantity. $h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu} h$ can be one of the above three metric tensors in Table 2. When (2-10) is applied to (2-8), we have (2-11).

$$\begin{aligned} &\partial_\nu h^{\mu\nu} = \frac{1}{2}\partial^\nu h \\ &\partial_\lambda \partial^\lambda h^{\mu\nu} + \partial^\mu \partial^\nu h - \left(\frac{1}{2}\partial^\mu \partial^\nu h + \frac{1}{2}\partial^\mu \partial^\nu h\right) - \eta'^{\mu\nu}\partial_\lambda \partial^\lambda h \\ &+ \eta'^{\mu\nu}\frac{1}{2}\partial_\sigma \partial^\sigma h + e\left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu} h\right) = -\kappa T^{\mu\nu} \end{aligned} \tag{2-11}$$

The red terms are erased, and the blue terms are left. Rearranging this, we have (2-12).

$$\partial_\lambda \partial^\lambda h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu}\partial_\sigma \partial^\sigma h + e\left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu} h\right) = -\kappa T^{\mu\nu} \tag{2-12}$$

or

$$\partial_\lambda \partial^\lambda \left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu} h\right) + e\left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu} h\right) = -\kappa T^{\mu\nu} \tag{2-13}$$

(2-13) is a new field equation different from the gravitational equation, and the 2nd term, including the coefficient e in (2-13), is the new term, which can be explained as a mass term. We note that the mass term is obtained by breaking the symmetry of case (1) in Table 1. We emphasize that the conservation law is satisfied even with this mass because case (3) in Table 1 is still valid. We attempt to find the solution of this new field equation in the next chapter.

3. Simple Solution of the New Equation

We define $\Phi^{\mu\nu}$ as in (3-1).

$$\Phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu} h \tag{3-1}$$

Then, (2-13) becomes (3-2), and (2-10) becomes (3-3).

$$\partial_\lambda \partial^\lambda \Phi^{\mu\nu} + e\Phi^{\mu\nu} = -\kappa T^{\mu\nu} \tag{3-2}$$

$$\partial_\mu \Phi^{\mu\nu} = 0 \tag{3-3}$$

If we look at Equations (3-1), (3-2), and (3-3) in Table 3, they are roughly similar in shape to Equations (2-4), (2-5) and (2-6), respectively, used to describe gravitational waves. However, there are differences in the metric tensor as

Table 3. Summary of equations of graviton and new particle.

	Gravitational wave		**New field**	
Notation	$\Phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h$	(2-4)	$\Phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu}h$	(3-1)
Field equation	$\partial_\lambda\partial^\lambda\Phi^{\mu\nu} = -\kappa T^{\mu\nu}$	(2-5)	$\partial_\lambda\partial^\lambda\Phi^{\mu\nu} + e\Phi^{\mu\nu} = -\kappa T^{\mu\nu}$	(3-2)
gauge condition	$\partial_\mu\Phi^{\mu\nu} = 0$ (Hilbert)	(2-6)	$\partial_\mu\Phi^{\mu\nu} = 0$ (Hilbert-like)	(3-3)

we have already mentioned in the previous chapter. Solving (3-2) from now on can make a wave equation for a new particle different from a gravitational wave. The new field will not act on a mass because it is not gravitational. When a particle subjected to a force is a point particle, the potential becomes source-less in a vacuum; thus, we can write (3-4).

$$\partial_\lambda\partial^\lambda\Phi^{\mu\nu} + e\Phi^{\mu\nu} = 0 \tag{3-4}$$

Although the form of (3-4) is similar to the Klein-Gordon Equation (3-5) [7], it is not the same because the Klein-Gordon equation is spatially three-dimensional, but (3-4) is one-dimensional.

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \mu^2\right)\Phi(x,t) = 0 \tag{3-5}$$

Here, we have to approach and solve it differently. In Equation (3-2), since the coefficient sign is selected as before $T^{\mu\nu}$, it is taken as $\partial_\lambda\partial^\lambda = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$. In Equation (3-2), if $e = 0$:

$$\partial_\lambda\partial^\lambda\Phi^{\mu\nu} = -\kappa T^{\mu\nu} < 0 \tag{3-6}$$

Accordingly, for $\partial_\lambda\partial^\lambda\Phi^{\mu\nu}$ to always remain negative regardless of *e*, *e* must be positive (>0). Considering the dimension of *e*, we put the square of the mass as $e = m^2$ in (3-7).

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\Phi^{\mu\nu} = 0 \tag{3-7}$$

Since it is spatially one-dimensional, we take a simple exponential function such as a plane wave as a solution in (3-8a) or (3-8b).

$$\Phi^{\mu\nu} = C^{\mu\nu}\exp\left(i\left(\mp Et \pm Pr\right)\right) \quad \left(C^{\mu\nu}: \text{constant tensor}\right) \tag{3-8a}$$

or

$$\Phi^{\mu\nu} = \begin{bmatrix} C^{00}\exp\left(i\left(\mp Et \pm Pr\right)\right) & 0 \\ 0 & C^{11}\exp\left(i\left(\mp Et \pm Pr\right)\right) \end{bmatrix} \tag{3-8b}$$

where E is energy, t is time, P is momentum, and r is one-dimensional distance. According to the logic of **Table 2**, (3-8b) has 3 kinds of $\Phi^{\mu\nu}$ spatially.

Let us apply the above formula to (3-3). The (0, 0) and (0, 1) components in the first row are differentiated by *t*, and the (1, 0) and (1, 1) components in the second row are differentiated by *r*. We obtain the following expressions (3-9) and (3-10), respectively.

$$C^{00} iE\left(\exp\left(i\left(\mp Et \pm Pr\right)\right)\right) + 0 = 0 \quad \left(\text{first row}\right) \tag{3-9}$$

$$0 + C^{11} iP\left(\exp\left(i\left(\mp Et \pm Pr\right)\right)\right) = 0 \quad \left(\text{second row}\right) \tag{3-10}$$

To have C^{00} and C^{11} as non-zero. The condition is $E = P = 0$; then, we have (3-11).

$$\Phi^{\mu\nu} = \begin{bmatrix} C^{00} & 0 \\ 0 & C^{11} \end{bmatrix} \tag{3-11}$$

(According to the logic of Table 2, (3-11) has 3 kinds of $\Phi^{\mu\nu}$ spatially.) Of course, there is a more general way in which E and P are not 0 while satisfying Equation (3-3), that is, (3-12).

$$Et - Pr = \text{constant} \tag{3-12}$$

In the condition of (3-12), (3-9), and (3-10) make (3-11). The speed of the new particle is that of light. There is one more thing to know about $\Phi^{\mu\nu}$. Applying $\eta'^{\mu\nu}$ on both sides of (3-1), there is a new conditional expression.

$$\Phi = h - \frac{1}{2} 2h = 0 \tag{3-13}$$

If (3-11) is applied to (3-13), we get:

$$C^{00} - C^{11} = 0 \tag{3-14}$$

If we take $C^{00} = C^{11} = E$, then (3-11) becomes (3-15).

$$\Phi^{\mu\nu} = \alpha \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix} \quad \text{at} \quad Et - Pr = \text{constant} \tag{3-15}$$

where α is a proportional constant. (According to the logic of Table 2, (3-15) has 3 kinds of $\Phi^{\mu\nu}$ spatially.)

According to (3-9) and (3-10), in the case that $Et - Pr$ is not constant, we have (3-16).

$$\Phi^{\mu\nu} = \alpha \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{at} \quad Et - Pr \neq \text{constant} \tag{3-16}$$

(According to the logic of Table 2, (3-16) has 3 kinds of $\Phi^{\mu\nu}$ spatially.) In the region where $Et - Pr = \text{constant}$, the propagation velocity of $\Phi^{\mu\nu}$ is constant as shown in (3-15), and the principle of uncertainty $\Delta E \Delta t = \Delta P \Delta r > \frac{\hbar}{2}$ is meaningful. On the other hand, in the region where $Et - Pr \neq \text{constant}$, $\Phi^{\mu\nu}$ becomes zero as shown in (3-16) due to (3-9) and (3-10). The range where (3-15) is valid is calculated by the uncertainty principle as shown in (3-17).

$$\Delta r = c\Delta t \approx c\left(\frac{\hbar}{\Delta E}\right) \approx c\left(\frac{\hbar}{m_p c^2}\right) \approx 10^{-15}\ \text{m} \tag{3-17}$$

where m_p is the proton mass.

In the case of a proton, $\Phi^{\mu\nu}$ is confined to less than 1 fm. The dynamics in the confinement region is described below.

Since the value of $\Phi^{\mu\nu}$ is constant, the value is not determined whether it is positive or negative. To make it physically stable, when we assume it to be negative, it would be a kind of potential well, as shown in **Figure 1**.

When the particle is trapped in this potential well, it moves like a free particle before it meets the well, which can be explained as a property of the known asymptotic freedom [8].

When a particle moves away from a certain distance from another particle, it receives a strong inward trapping force because the two particles make the well-type potential. There is an interaction mechanism of the particles inside the well. This will be discussed in the next chapter.

4. Physical Properties of New Particle

4.1. Derivation of Mass

From the above equations, we can find the exact mass of the particle. When we put $\Phi^{\mu\nu} = C^{\mu\nu} \exp\left(i\left(\mp Et \pm Pr\right)\right)$ into $\Phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu} h$ (3-1), we have

$$\left(0 - 0 + m^2\right)\Phi^{\mu\nu} = 0 \tag{4-1}$$

Therefore, we have

$$m = 0 \tag{4-2}$$

According to Equation (3-12), the speed of this particle is that of light, as shown in (4-3). Thus, the rest mass $m = 0$ is a correct result.

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{E}{P} = \frac{Pc}{P} = c \tag{4-3}$$

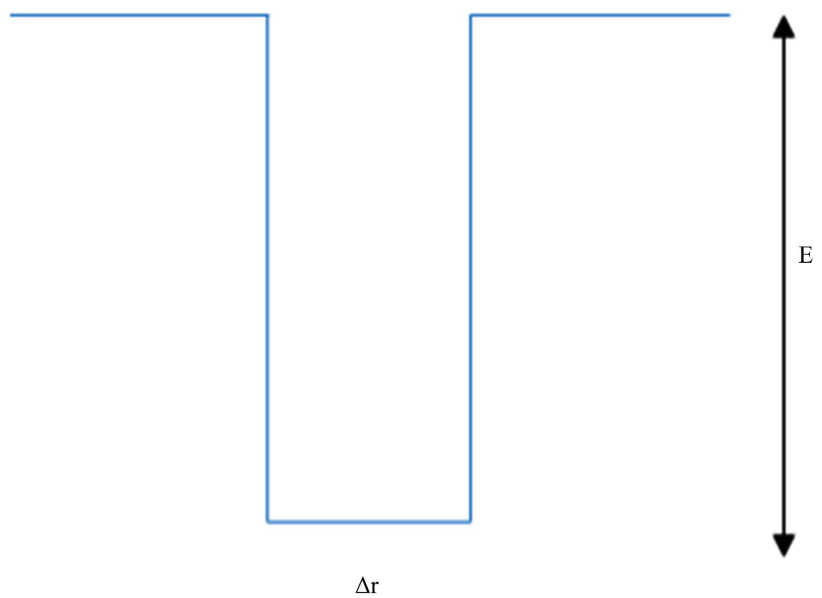

Figure 1. Potential well of new field.

4.2. Derivation of Spin

In this chapter, we will determine the spin value of the new particle. The spin of the gravitational particle (graviton) is 2. We can also infer the spin of the new particle by reasoning from the spin of the graviton. Let's consider a gravitational wave traveling along the Z-axis [4] (pp. 241-294) [9].

$$h_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & h_+ & h_x & 0 \\ 0 & -h_x & h_- & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4-4}$$

The time elements of the metric perturbation h_{00} and h_{0i} in (4-4) remain invariant when rotated about the Z-axis by any angle ϕ, while the spatial element h_{ij} transforms as follows:

$$h_{ij} = \boldsymbol{R}(\phi)_{ia} h_{ab} \boldsymbol{R}(\phi)_{jb} \tag{4-5}$$

where $\boldsymbol{R}(\phi)_{ij}$ is a rotation matrix given by:

$$\boldsymbol{R}(\phi)_{ij} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4-6}$$

By working through the algebra and using various trigonometric identities, we can show that h_{ij} is invariant under a rotation by π, which is a characteristic feature of a spin-2 particle. In general, after a rotation, we find that the components of h_{ij} are multiplied by functions of 2ϕ, another characteristic sign of a spin-2 particle. However, in the case of $h^3_{\mu\nu}$ (Z-axis) when the trace of the metric tensor is 2, performing the calculation as in (4-5) results in invariance in terms of ϕ, while in Equation (3-8), it demonstrates symmetry under 2π rotations. This implies the spin becoming 1, akin to the characteristics of an electromagnetic wave. Table 4 shows this explanation.

Table 4. Characteristics of spin 1.

From	Transformation	Result
(3-8)	$\Phi^{\mu\nu} = \begin{bmatrix} C^{00}\exp\left(i\left(\mp Et \pm Pr\right)\right) & 0 \\ 0 & C^{11}\exp\left(i\left(\mp Et \pm Pr\right)\right) \end{bmatrix} = \begin{bmatrix} C^{00}\exp\left(i\left(\mp Et \pm Pr + 2\pi\right)\right) & 0 \\ 0 & C^{11}\exp\left(i\left(\mp Et \pm Pr + 2\pi\right)\right) \end{bmatrix}$	2π **symmetry**
Table 5	$h^3_{\mu\nu} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ where $a = C^{00}\exp\left(i\left(\mp Et \pm Pr\right)\right)$ where $b = C^{11}\exp\left(i\left(\mp Et \pm Pr\right)\right)$	ϕ **independent**

(According to the logic of Table 2, (3-8) in Table 4 has 3 kinds of $\Phi^{\mu\nu}$ spatially. $h^{3}_{\mu\nu}$ is one of them.)

4.3. Derivation of Charge

If we apply ∂_μ to both sides in (3-2), we get:

$$\partial_\lambda \partial^\lambda \partial_\mu \Phi^{\mu\nu} + e\partial_\mu \Phi^{\mu\nu} = 0 \tag{4-7}$$

According to (2-10), since $\partial_\mu \Phi^{\mu\nu} = 0$, both terms on the left side are 0. The first term becomes:

$$\partial_\lambda \partial^\lambda \partial_\mu \Phi^{\mu\nu} = 0 \tag{4-8}$$

This always holds since $\partial_\mu \Phi^{\mu\nu} = 0$. According to (3-1),

$$\partial_\lambda \partial^\lambda \partial_\mu \left(h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu} h \right) = 0 \tag{4-9}$$

The two terms on the left side of (4-9) can be defined as j^ν, respectively, as shown in (4-10):

$$\partial_\lambda \partial^\lambda \partial_\mu h^{\mu\nu} = \partial_\lambda \partial^\lambda \partial_\mu \left(\frac{1}{2}\eta'^{\mu\nu} h \right) \equiv j^\nu \tag{4-10}$$

The difference of the two terms becomes 0 because they are equal, which implies the pair annihilation of particle and antiparticle.

Be careful that $\partial_\lambda \partial^\lambda$ is not four-dimensional but two-dimensional, and it is not strictly an electromagnetic equation. However, it is analogous to the four-vector potential of an electromagnetic field. Let us define $A^\nu = (\Phi, A)$, which is similar to the electromagnetic 4-potential [10], and we adopt this formalism in 2-dimensions (4-11), (4-12), and (4-13):

$$\Box A = \mu_0 J \tag{4-11}$$

$$\Box \Phi = -\frac{\rho}{\varepsilon_0} \tag{4-12}$$

$$\partial_\lambda \partial^\lambda A^\nu = \Box(\Phi, A) = \left(-\frac{\rho}{\varepsilon_0}, \mu_0 J \right) = 0 \tag{4-13}$$

where $\Box = \partial_\lambda \partial^\lambda = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$.

Here, $\Box$ is the 2-dimensional d'Alembertian operator. Comparing Equation (4-8) with (4-13), it is as if there is no charge source.

4.4. Derivation of the Metric Tensor

If we know $\Phi^{\mu\nu}$ as in Equation (3-15), we can obtain $h^{\mu\nu}$ in the linearly approximated space-time, and we can also obtain the metric tensor $g^{\mu\nu}$ as given in Equation (4-14) [4] (pp. 176-179).

$$g^{\mu\nu} = \eta^{\mu\nu} + kh^{\mu\nu} \tag{4-14}$$

where k is an infinitesimally small value. Assuming $h^{\mu\nu}$ as in Equation (4-15), we can obtain $\Phi^{\mu\nu}$ as shown in Equation (3-15).

$$h^{\mu\nu}=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \tag{4-15}$$

where a and b are certain constants. According to Equation (3-1),

$$\Phi^{\mu\nu}=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}-\frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}(-a+b) \tag{4-16}$$

$$=\begin{pmatrix} \frac{a+b}{2} & 0 \\ 0 & \frac{a+b}{2} \end{pmatrix}=\frac{1}{k}\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} \tag{4-17}$$

The parameter α in Equation (3-15) and (3-16) is $1/k$ [4] (pp. 176-179). That is, (3-15) and (4-17) are the same because $(a+b)/2$ is constant. Therefore, a choice such as Equation (4-15) is appropriate. Using (4-15), we can obtain (4-18).

$$g^{\mu\nu}=\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}+k\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}=\begin{pmatrix} -1+ka & 0 \\ 0 & 1+kb \end{pmatrix} \tag{4-18}$$

where $k \ll 1$. $g^{\mu\nu}$ is the metric tensor for the potential well. Although we have used a 2-dimensional representation (2 × 2 matrix) in (4-15) to (4-18) for convenience, they are part of the 4-dimensional representation and can have 3 variants in a 4 × 4 representation, as shown in Table 5.

5. Reinterpretation of Existing Phenomenon Using New Particle

5.1. Physical Meaning of the New Potential

In the previous chapters, we have identified the mass, charge, and spin of the new particle, which suggests that it could be either a gluon or a photon in the standard model, as depicted in Figure 2 [11].

Figure 3 shows the force lines of gluons (up) and photons (down) [12].

Table 5. All kinds of $h^{\mu\nu}$, $\eta'^{\mu\nu}$, and $g^{\mu\nu}$.

$h_1^{\mu\nu}=\begin{pmatrix} a&0&0&0\\0&b&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}$	$h_2^{\mu\nu}=\begin{pmatrix} a&0&0&0\\0&0&0&0\\0&0&b&0\\0&0&0&0 \end{pmatrix}$	$h_3^{\mu\nu}=\begin{pmatrix} a&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&b \end{pmatrix}$
$\eta_1'^{\mu\nu}=\begin{pmatrix} -1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}$	$\eta_2'^{\mu\nu}=\begin{pmatrix} -1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0 \end{pmatrix}$	$\eta_3'^{\mu\nu}=\begin{pmatrix} -1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1 \end{pmatrix}$
$g_1^{\mu\nu}=\begin{pmatrix} -1+ka&0&0&0\\0&1+kb&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}$	$g_2^{\mu\nu}=\begin{pmatrix} -1+ka&0&0&0\\0&0&0&0\\0&0&1+kb&0\\0&0&0&0 \end{pmatrix}$	$g_3^{\mu\nu}=\begin{pmatrix} -1+ka&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1+kb \end{pmatrix}$

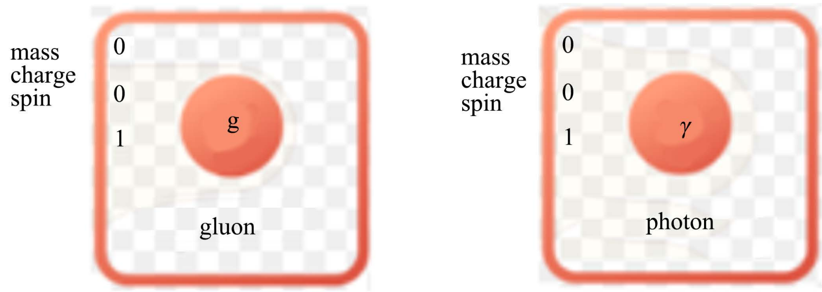

Figure 2. Mass: 0, Charge: 0, Spin: 1 particle among elementary particles.

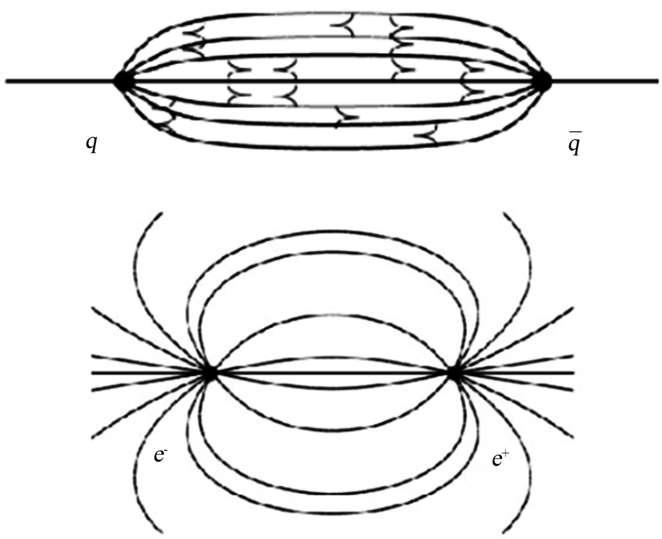

Figure 3. Force lines of gluons (up) and photons (down).

Among the two candidates, as shown in Figure 3, it is reasonable to interpret the new particle as a gluon, rather than a photon with radial force, since the trace of the tensor is 2, implying that the force acting is one-dimensional spatial force, as depicted in the upper part of Figure 3.

We have obtained a local gauge using linear approximated gravitational equations [1] and differentiated the strong force from it. The differentiation can be justified by comparing it with cosmological phenomena, which suggest that there was differentiation of force during the period of linear expansion after cosmic inflation following the Big Bang. Nature is made up of four-dimensional space-time, and there is an action to create three-dimensional symmetry spatially. However, as the trace value remains at 2, a new pattern emerges. In the next chapter, we will give a qualitative interpretation of this, followed by a quantitative one.

5.2. Reason for Three Quarks in Three Dimensions

In this section, we qualitatively discuss how and why three quarks can be stably gathered when the new potential is considered. Figure 4 and Figure 5 illustrate why the case with three quarks is stable. According to Table 5, force lines can be created from any angle in 3D, allowing many quarks to stick together. However, we observe that only three quarks always come together. The reason for this is related to the default space-time being trace 2.

In Figure 4 (left), there are initially 3 quarks. When 2 quarks are far enough to reach the limit of confinement, a force line (red line) is generated. It should be noted that, according to (9-1), only one force line can exist to satisfy the condition of trace 2. Even though the quarks connected by the red force line become closer, no quarks move farther apart than the initial distance. In Figure 4 (right), a becomes a', and b becomes b', but the distance between each quark becomes closer.

Now, let's consider the case where four quarks are inside and one tries to go out. It shows that the outgoing quark cannot be captured because only one force line is allowed. Figure 5 shows the transition from 4 quarks to 3. We assume that initially, 4 quarks exist within the range of confinement. The force line is a single strand, so the 2nd quark cannot simultaneously attract the 1st and 3rd quarks. Consequently, if the 2nd quark is pulling the 3rd quark (Figure 4, left), it cannot pull the 1st quark. As the 2nd and 3rd quarks move into a stable position (Figure 4, right), the distance between the 1st and 2nd quarks is already beyond the confinement. So it has no force between them. In this way, the 1st quark becomes disconnected from the rest of the three-quark ensemble. New force lines can only be generated according to the distances between pairs of the 2nd, 3rd, and 4th quarks. The 1st quark might disappear after meeting an anti-quark or might form a new nucleon if it acquires sufficient energy. Thus, when quarks are collected in groups of 4, 5, or more, they eventually fall off and result in 3 quarks.

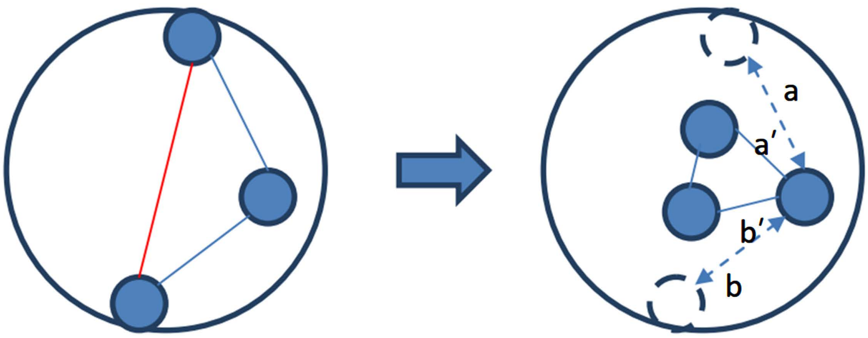

Figure 4. Change from 4 quarks to 3.

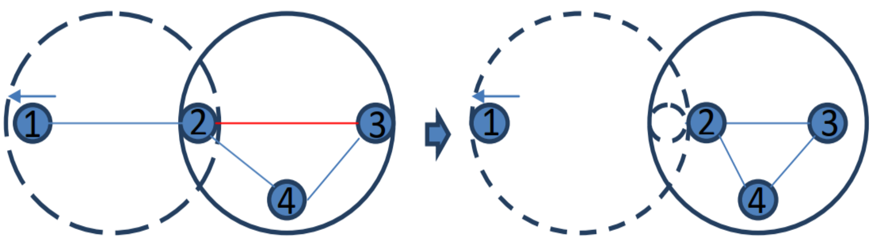

Figure 5. The reason why 3 quarks are more stable than others.

Previously, this phenomenon was explained using the color charge model to explain why the three quarks gather, as shown in **Figure 6** (top). Each quark has one charge among R, G, and B, and when they come together, they neutralize and become stable [13] [14]. This is not to say that the existing model is incorrect, but both the existing model and our model independently explain the same phenomenon. According to the existing theory, gluons themselves have two color charges. They are composed of particles and antiparticles, where one has a color among R, G, B, and the other has a color charge with its complementary color. In our case, as shown in **Figure 6** (bottom) and according to (2-13), $\Phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta'^{\mu\nu}h$, the two potentials are always bundled together. From (4-13), the charges of the two are opposite, and the mass is the same, indicating a particle-antiparticle relationship. This result aligns with the existing analysis.

5.3. Analogy to 8-Fold Way Using 3 Kinds of Tensor Replacing Color Charges

Now, we would like to quantitatively describe the reason for the gathering of three quarks. The existing Gluon eight-fold way is shown on the left of **Figure 7** [15]. Similarly, you can see the eight-fold way claimed in this paper on the right. In Chapter 10, we said that $h^{\mu\nu}$ and $-\frac{1}{2}\eta'^{\mu\nu}h$ are particle and antiparticle relationships. Therefore, the logic shown in **Figure 7** can be made.

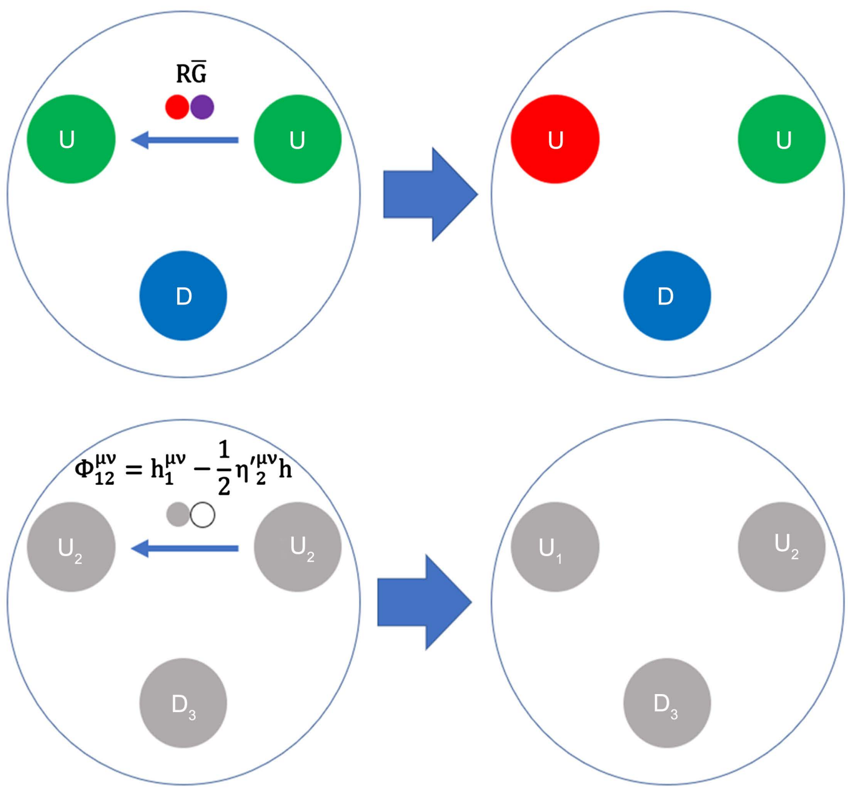

Figure 6. Existing proton model (top) another model in this paper (bottom).

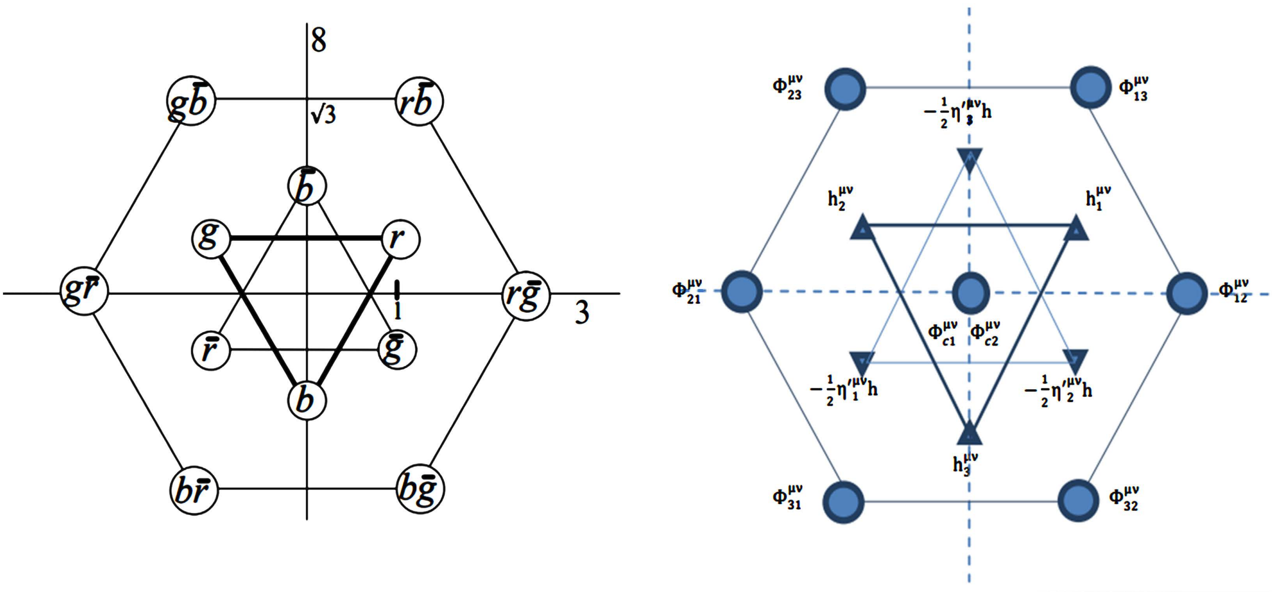

Figure 7. Comparison of the existing gluon eight-fold way (left) and this paper's eight-fold way (right).

The potential of each position at the eight-fold way in **Figure 7** is as follows. $h^{\mu\nu}$ and $-\frac{1}{2}\eta'^{\mu\nu}h$ can be expressed for each component through **Table 2** and **Table 4**.

$$\Phi_{12}^{\mu\nu} = h_1^{\mu\nu} - \frac{1}{2}\eta_2'^{\mu\nu}h = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a-b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -a+b & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a+b}{2} & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & \frac{a-b}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{5-1}$$

$$\Phi_{13}^{\mu\nu} = h_1^{\mu\nu} - \frac{1}{2}\eta_3'^{\mu\nu}h = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a-b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a+b \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a+b}{2} & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{a-b}{2} \end{pmatrix} \tag{5-2}$$

$$\Phi_{21}^{\mu\nu}=h_2^{\mu\nu}-\frac{1}{2}\eta_1'^{\mu\lambda}h=\begin{pmatrix}a&0&0&0\\0&0&0&0\\0&0&b&0\\0&0&0&0\end{pmatrix}-\frac{1}{2}\begin{pmatrix}a-b&0&0&0\\0&-a+b&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$=\begin{pmatrix}\frac{a+b}{2}&0&0&0\\0&\frac{a-b}{2}&0&0\\0&0&b&0\\b&0&0&0\end{pmatrix}\tag{5-3}$$

$$\Phi_{23}^{\mu\nu}=h_2^{\mu\nu}-\frac{1}{2}\eta_3'^{\mu\nu}h=\begin{pmatrix}a&0&0&0\\0&0&0&0\\0&0&b&0\\0&0&0&0\end{pmatrix}-\frac{1}{2}\begin{pmatrix}a-b&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&-a+b\end{pmatrix}$$

$$=\begin{pmatrix}\frac{a+b}{2}&0&0&0\\0&0&0&0\\0&0&b&0\\b&0&0&\frac{a-b}{2}\end{pmatrix}\tag{5-4}$$

$$\Phi_{31}^{\mu\nu}=h_3^{\mu\nu}-\frac{1}{2}\eta_1'^{\mu\nu}h=\begin{pmatrix}a&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&b\end{pmatrix}-\frac{1}{2}\begin{pmatrix}a-b&0&0&0\\0&-a+b&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$=\begin{pmatrix}\frac{a+b}{2}&0&0&0\\0&\frac{a-b}{2}&0&0\\0&0&0&0\\0&0&0&b\end{pmatrix}\tag{5-5}$$

$$\Phi_{32}^{\mu\nu}=h_3^{\mu\nu}-\frac{1}{2}\eta_2'^{\mu\nu}h=\begin{pmatrix}a&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&b\end{pmatrix}-\frac{1}{2}\begin{pmatrix}a-b&0&0&0\\0&0&0&0\\0&0&-a+b&0\\0&0&0&0\end{pmatrix}$$

$$=\begin{pmatrix}\frac{a+b}{2}&0&0&0\\0&0&0&0\\0&0&\frac{a-b}{2}&0\\0&0&0&b\end{pmatrix}\tag{5-6}$$

$$\Phi_{11}^{\mu\nu}=\begin{pmatrix}a&0&0&0\\0&b&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}-\frac{1}{2}\begin{pmatrix}a-b&0&0&0\\0&-a+b&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}=\begin{pmatrix}\frac{a+b}{2}&0&0&0\\0&\frac{a+b}{2}&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\tag{5-7}$$

$$\Phi_{22}^{\mu\nu}=\begin{pmatrix}a&0&0&0\\0&0&0&0\\0&0&b&0\\0&0&0&0\end{pmatrix}-\frac{1}{2}\begin{pmatrix}a-b&0&0&0\\0&0&0&0\\0&0&-a+b&0\\0&0&0&0\end{pmatrix}=\begin{pmatrix}\frac{a+b}{2}&0&0&0\\0&0&0&0\\0&0&\frac{a+b}{2}&0\\0&0&0&0\end{pmatrix} \tag{5-8}$$

$$\Phi_{33}^{\mu\nu}=\begin{pmatrix}a&0&0&0\\0&0&0&0\\0&0&b&0\\0&0&0&0\end{pmatrix}-\frac{1}{2}\begin{pmatrix}a-b&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&-a+b\end{pmatrix}=\begin{pmatrix}\frac{a+b}{2}&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&\frac{a+b}{2}\end{pmatrix} \tag{5-9}$$

$$\Phi_{c1}^{\mu\nu}=\Phi_{11}^{\mu\nu}-\Phi_{22}^{\mu\nu}=\begin{pmatrix}0&0&0&0\\0&a+b&0&0\\0&0&-a-b&0\\0&0&0&0\end{pmatrix} \tag{5-10}$$

$$\Phi_{c2}^{\mu\nu}=\frac{1}{\sqrt{3}}\left(\Phi_{11}^{\mu\nu}+\Phi_{22}^{\mu\nu}-2\Phi_{33}^{\mu\nu}\right)=\frac{1}{\sqrt{3}}\begin{pmatrix}0&0&0&0\\0&\frac{a+b}{2}&0&0\\0&0&\frac{a+b}{2}&0\\0&0&0&-a-b\end{pmatrix} \tag{5-11}$$

At the eight-fold way, the outer 6 ($\Phi_{12}^{\mu\nu}$, $\Phi_{13}^{\mu\nu}$, $\Phi_{21}^{\mu\nu}$, $\Phi_{23}^{\mu\nu}$, $\Phi_{31}^{\mu\nu}$, $\Phi_{32}^{\mu\nu}$) have a diagonal sum of $a+b$, and the middle two ($\Phi_{c1}^{\mu\nu}$, $\Phi_{c2}^{\mu\nu}$) are 0. The six outer potentials represent the attraction between quarks when they move away from each other by a certain distance. When there is nothing to attract, the two potentials in the middle hide the force but retain energy by setting the sum of the diagonals to 0.

6. Conclusions

In this paper, we demonstrate that when satisfying the scalar gauge symmetry and conservation law in general rank-2 linear gravitational equation, a new potential is generated with zero mass and spatially one-dimensional attributes. We summarize the characteristics of this newly proposed potential as follows:

- The force line is one-dimensional in space because the trace of its metric tensor is 2.
- This force has a spin of 1, mass of 0, and charge of 0, and it is composed of particles and antiparticles. As a result, this force is short-lived and disappears immediately, exerting its influence only over a limited distance. It is considered a very short-range force.
- As the potential has the shape of a well, the force is confined to a very narrow space, and it behaves like a free particle inside and outside this region.
- In a 3-dimensional space with a trace of 4, a force can exist in any direction. However, in a space where the trace is 2, the force acts like a one-dimensional

strand. As a result, it is impossible to bind multiple quarks together using this force.

- When more than four quarks are gathered, the force line is unable to attract them all simultaneously, making it impossible to create a stable state. Thus, only three quarks can create a stable system. In the event that a nucleon loses a quark during a collision, it is compelled to form three quarks again, which suggests that removing a quark from a nucleon may be an insurmountable challenge.

Quantum chromodynamics (QCD) describes the interaction between quarks that carry color charges and gluons, which are SU (3) gauge particles. While the results of this study are consistent with previous findings, a new interpretation is presented from a unique perspective.

Building on the gravitational equation as the foundation, this study demonstrates the unification of all forces. Previous research has already shown the unification of gravity and electromagnetic forces, and this study expands the range to include the strong force. Through a linear approximation of the equation, it is found that when the symmetry of the scalar gauge is made and the symmetry of the vector gauge is broken, a new potential is induced from the gravitational potential, which exhibits the main characteristics of the strong interaction force.

Traditionally, the Higgs mechanism has been proposed to explain the mechanism of gaining mass, but this study introduces a new mechanism. With two types of gauges, breaking the symmetry of only one can lead to the conservation of energy-momentum while gaining mass. Further exploration of this study may reveal additional similarities between the proposed mechanism and the existing standard model. In summary, this study offers a significant contribution by providing a new perspective on the same phenomenon analyzed in the standard model.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Yun, Y.H. and Jang, K. (2021) Proposal of a New Scalar Gauge in Relativity Field Equation. *Journal of High Energy Physics, Gravitation and Cosmology*, **7**, 1524-1534. https://doi.org/10.4236/jhepgc.2021.74093

[2] Noether, E. (1918) Invariante Variations Problem. Nachrichten von der Königliche Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, 235-257.

[3] Noether, E. (1971) Invariant Variation Problems. *Transport Theory and Statistical Physics*, **1**, 186-207. arXiv:physics/0503066 https://doi.org/10.1080/00411457108231446

[4] Ohanian, R. (1994) Gravitation and Spacetime. 2nd Edition, W.W. Norton & Company, New York, London, 134-146.

[5] Yun, Y.H., Jang, K. and Sung, Y.K. (2021) Unification of Gravitational and Electromagnetic Fields Using Gauge Symmetry. *Journal of High Energy Physics*, *Gravitation and Cosmology*, **7**, 344-351. https://doi.org/10.4236/jhepgc.2021.71018

[6] Yun, Kiho Jang (2021) Unification of Gravitational and Electromagnetic Fields in Curved Space-Time Using Gauge Symmetry of Bianchi Identities. *Journal of High Energy Physics*, *Gravitation and Cosmology*, **7**, 1202-1212. https://doi.org/10.4236/jhepgc.2021.73071

[7] (2001) Klein-Gordon Equation. In: *Encyclopedia of Mathematics*, Springer-Verlag, Berlin. https://encyclopediaofmath.org/wiki/Klein-Gordon_equation

[8] Gross, D.J. and Wilczek, F. (1973) Ultraviolet Behavior of Non-Abelian Gauge Theories. *Physical Review Letters*, **30**, 1343-1346. https://doi.org/10.1103/PhysRevLett.30.1343

[9] https://physics.stackexchange.com/questions/479602/why-arent-gravitons-spin-1

[10] Jackson, J.D. (1998) Classical Electrodynamics. Wiley, Hoboken, 549.

[11] https://en.wikipedia.org/wiki/Elementary_particle

[12] https://universe-review.ca/R15-04-confinement.htm

[13] Griffiths, D.J. (1987) Introduction to Elementary Particles. John Wiley & Sons, New York.

[14] Howard, G. (1999) Lie Algebras in Particle Physics. Perseus Books Group, New York.

[15] Traxler, C.T., Mosel, U. and Biró, T.S. (1999) Hadronization of a Quark-Gluon Plasma in the Chromodielectric Model. *Physical Review C*, **59**, 1620. https://doi.org/10.1103/PhysRevC.59.1620