Scientific Research Publishing

Journal of High Energy Physics, Gravitation and Cosmology, 2024, 10, 386-397
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

# The Extremal Universe Exact Solution from Einstein's Field Equation Gives the Cosmological Constant Directly

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**How to cite this paper:** Haug, E.G. (2024) The Extremal Universe Exact Solution from Einstein's Field Equation Gives the Cosmological Constant Directly. *Journal of High Energy Physics, Gravitation and Cosmology*, **10**, 386-397.
https://doi.org/10.4236/jhepgc.2024.101027

**Received:** November 25, 2023
**Accepted:** January 28, 2024
**Published:** January 31, 2024



## Abstract

Einstein's field equation is a highly general equation consisting of sixteen equations. However, the equation itself provides limited information about the universe unless it is solved with different boundary conditions. Multiple solutions have been utilized to predict cosmic scales, and among them, the Friedmann-Lemaître-Robertson-Walker solution that is the back-bone of the development into today standard model of modern cosmology: The Λ-CDM model. However, this is naturally not the only solution to Einstein's field equation. We will investigate the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics. Interestingly, in their extremal cases, these solutions yield identical predictions for horizons and escape velocity. These solutions can be employed to formulate a new cosmological model that resembles the Friedmann equation. However, a significant distinction arises in the extremal universe solution, which does not necessitate the ad hoc insertion of the cosmological constant; instead, it emerges naturally from the derivation itself. To the best of our knowledge, all other solutions relying on the cosmological constant do so by initially ad hoc inserting it into Einstein's field equation. This clarification unveils the true nature of the cosmological constant, suggesting that it serves as a correction factor for strong gravitational fields, accurately predicting real-world cosmological phenomena only within the extremal solutions of the discussed metrics, all derived strictly from Einstein's field equation.

## Keywords

General Relativity Theory, Cosmological Constant Extremal Solution, Reissner-Nordström, Kerr, Kerr-Newman

## 1. Extremal Solutions to Einsteins Field Equation

The Reissner-Nordström [1] [2] metric for a spherical charged gravitational object is an exact solution to Einsteins [3] field equation and is given by:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{c^2 r} + \frac{r_Q^2}{r^2}\right) c^2 \mathrm{d}t^2 - \left(1 - \frac{2GM}{c^2 r} + \frac{r_Q^2}{r^2}\right)^{-1} \mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \quad (1)$$

Here, in SI units, we have $r_Q^2 = k_e q q \frac{G}{c^4}$, where $k_e$ is the Coulomb constant and $q$ is the charge. The special case when $r_Q = \frac{GM}{c^2}$ is well known as the extremal solution of the Reissner-Nordström metric, seen for example Zee [4] and Aretakis [5]. Furthermore, the Kerr [6] metric is given by:

$$\mathrm{d}s^2 = \frac{\Delta}{\Sigma}\left(c\mathrm{d}t - a\sin^2\theta \mathrm{d}\phi\right)^2 - \frac{\Sigma}{\Delta}\mathrm{d}r^2 - \Sigma \mathrm{d}\theta^2 - \frac{\sin^2\theta}{\Sigma}\left(a\mathrm{d}t - \left(r^2 + a^2\right)\mathrm{d}\phi\right)^2 \quad (2)$$

where $\Delta = r^2 - r_s r + a^2$ and $\Sigma = r^2 + a^2\cos^2\theta$, and $r_s = \frac{2GM}{c^2}$. The Kerr metric also has an extremal solution when $a = \frac{GM}{c^2}$.

The Kerr-Newman [7] [8] metric extends the Kerr metric to include charge, and it is given by:

$$\mathrm{d}s^2 = \frac{\Delta}{\Sigma}\left(c\mathrm{d}t - a\sin^2\theta \mathrm{d}\phi\right)^2 - \frac{\Sigma}{\Delta}\mathrm{d}r^2 - \Sigma \mathrm{d}\theta^2 - \frac{\sin^2\theta}{\Sigma}\left(a\mathrm{d}t - \left(r^2 + a^2\right)\mathrm{d}\phi\right)^2 \quad (3)$$

Here, $\Delta = r^2 - r_s r + a^2 + r_Q^2$ and $\Sigma = r^2 + a^2\cos^2\theta$, where $r_s = \frac{2GM}{c^2}$. In the special case of $a = 0$, it simplifies to the Reissner-Nordström metric and in the special case of $r_Q = 0$, it simplifies to the Kerr metric.

The extremal solutions of the Reissner-Nordström metric ($r_Q = GM/c^2$), as well as the extremal solution of the Kerr metric ($a = GM/c^2$) and the extremal solution of the Kerr-Newman metric ($a^2 + r_Q^2 = G^2M^2/c^4$) (see [4] [9]), all have one and the same horizon given by:

$$r_h = \frac{GM}{c^2} \quad (4)$$

This is half the Schwarzschild radius. In the Schwarzschild [10] metric, the escape velocity is $v_e = \sqrt{\frac{2GM}{r}}$, see [11]. However, in the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics, the escape velocity is given by:

$$v_e = \sqrt{\frac{2GM}{r} - \frac{G^2M^2}{c^2r^2}} \quad (5)$$

We will soon see that this higher-order term $\frac{G^2M^2}{c^2r^2}$, which differentiates it from, for example, the Schwarzschild metric, could play a critical role in understanding the cosmos.

Table 1 summarizes the known key results from the extremal solutions. However, relatively few people have shown interest in these extremal solutions, resulting in a limited number of predictions being discussed based on them.

## 2. Cosmological Model

The horizon and escape velocity play a central role in predicting black holes. Interestingly, the Hubble sphere also exhibits several mathematical aspects of a black hole, including a horizon known as the Hubble radius ( $r_H = \frac{c}{H_0}$ ) as we now will discuss. The Friedmann critical mass-equivalent for the universe is given by $M_c = \frac{c^3}{2GH_0}$. When considering a Hubble sphere with the Friedmann critical mass, the Schwarzschild radius is given by

$$r_s = \frac{2GM_c}{c^2} = \frac{2G\frac{c^3}{2GH_0}}{c^2} = \frac{c}{H_0} \tag{6}$$

This implies that the Schwarzschild radius is exactly identical to the Hubble radius if the Hubble sphere were filled with the Friedmann critical mass-energy. The mathematical similarities between Hubble spheres and black holes have led several researchers in prominent journals such as Nature to suggest and speculate that the observable universe could be inside a black hole (see Pathria [12] and Stuckey [13]).

It is important to note that our intention is not to claim that we live inside a black hole, but rather to highlight the mathematical properties shared by the Hubble sphere and black holes. However, in the Λ-CMD model, the universe has expanded well beyond the Hubble radius due to the assumption of space expansion, including an accelerating expansion attributed to dark energy. The hypothesis of dark energy appeared necessary to reconcile the model with high-redshift supernova observations.

**Table 1.** The table summarizes the extremal solutions of the metrics we will be looking at.

| | Reissner-Nordström | Kerr | Kerr-Newman |
|---|---|---|---|
| Horizon | $r_h = \frac{GM}{c^2} \pm \sqrt{\frac{G^2M^2}{c^4} - r_Q^2}$ | $r_h = \frac{GM}{c^2} \pm \sqrt{\frac{G^2M^2}{c^4} - a^2}$ | $r_h = \frac{GM}{c^2} \pm \sqrt{\frac{G^2M^2}{c^4} - r_Q^2 - a^2}$ |
| Extremal solution | $r_Q = \frac{GM}{c^2}$ | $a = \frac{GM}{c^2}$ | $r_Q^2 + a^2 = \frac{G^2M^2}{c^4}$ |
| Extremal solution horizon | $r_h = \frac{GM}{c^2}$ | $r_h = \frac{GM}{c^2}$ | $r_h = \frac{GM}{c^2}$ |
| Escape velocity extremal solution | $v_e = \sqrt{\frac{2GM}{r} - \frac{G^2M^2}{c^2r^2}}$ | $v_e = \sqrt{\frac{2GM}{r} - \frac{G^2M^2}{c^2r^2}}$ | $v_e = \sqrt{\frac{2GM}{r} - \frac{G^2M^2}{c^2r^2}}$ |

The escape velocity and the horizon derived from the metric of interest have practical applications, even in cosmology. For instance, Schutz [14] derived the critical density of the universe based on simply the escape velocity formula from the Schwarzschild metric, resulting in an equation that depends on the Hubble constant. However, in this discussion, we will not explore cosmology through the escape velocity derived from the Schwarzschild metric. Instead, we will focus on the predictions of the escape velocity using the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics.

Subsequently, we will derive an equation analogous to the Friedmann [15] equation from the escape velocity in the extremal solutions. Our interest lies initially in the special case where the escape velocity is equal to the speed of light ($c$). Let us rephrase the escape velocity formula as follows:

$$\sqrt{\frac{2GM}{r}-\frac{G^2M^2}{c^2r}}=c$$

$$\frac{2GM}{r}-\frac{G^2M^2}{c^2r^2}=c^2$$

$$\frac{8\pi G\rho r^2}{3}-\frac{G^2M^2}{c^2r^2}=c^2$$

$$\frac{8\pi G\rho}{3}-\frac{G^2M^2}{c^2r^4}=\frac{c^2}{r^2}$$

$$\frac{8\pi G\rho-3\frac{G^2M^2}{c^2r^4}}{3}=\frac{c^2}{r^2} \tag{7}$$

Here, $\rho=\frac{M}{V}=\frac{M}{\frac{4}{3}\pi r^3}$ represents the volumetric mass density of a sphere with radius $r$. Next, we replace $r$ with the Hubble radius $r_H=\frac{c}{H_0}$, and furthermore, it must be equal to $r_h=\frac{GM}{c^2}$ since the escape velocity is $c$ at the horizon $r_h=\frac{GM}{c^2}$. Therefore, we assume that

$$r_H=r_h=\frac{GM_u}{c^2}=\frac{c}{H_0} \tag{8}$$

Here, $M_u$ is the mass equivalent of all mass and energy in the universe we are considering. Solving for $M_u$ gives $M_u=\frac{c^3}{GH_0}\approx 1.77\times 10^{53}\text{ kg}$ (when assuming $H_0=70\text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$), which means the mass is exactly twice that of the critical mass (mass equivalent) of the Friedmann universe, which is $M_c=\frac{c^3}{2GH_0}$. Next, let's replace $r=r_H=\frac{c}{H_0}$ in Equation (7), and we obtain:

$$\frac{8\pi G\rho-3\frac{G^2M_u^2}{c^2r_H^4}}{3}=\frac{c^2}{r_H^2}$$

$$\frac{8\pi G\rho - 3\frac{G^2 M_u^2}{c^4 r_H^4}c^2}{3} = \frac{c^2}{r_H^2}$$

$$\frac{8\pi G\rho - \frac{3c^2}{r_H^2}}{3} = \frac{c^2}{\frac{c^2}{H_0^2}}$$

$$\frac{8\pi G\rho - \frac{3c^2}{r_H^2}}{3} = H_0^2$$

$$\frac{8\pi G\rho - 3\left(\frac{H_0}{c}\right)^2 c^2}{3} = H_0^2 \tag{9}$$

Next, it's important to note that the ad hoc inserted cosmological constant in general relativity theory is identical to $\Lambda = 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda$. When $\Omega_\Lambda = 1$, we have $\Lambda = 3\left(\frac{H_0}{c}\right)^2$, which means we can rewrite the equation as follows:

$$H_0^2 = \frac{8\pi G\rho - \Lambda c^2}{3} \tag{10}$$

This equation is very similar to the Friedmann equation for homogeneous, isotropic universe, except our cosmological constant has been derived from the extremal solutions of Einstein's field equations rather than being ad hoc inserted in the field equation (or later). Also, our cosmological constant has the opposite sign in the Friedmann solution. That means we have a negative cosmological constant, which is still actively debated to this day in a series of papers, see [16]-[22] for more information. To our knowledge, the extremal solutions have not been previously utilized to construct a cosmological model. In 1917, Einstein [23] ad hoc inserted a cosmological constant into his field equation, which he referred to as an extended field equation. It was based on sound reasoning and was actually aimed at achieving a steady-state universe model. However, after Hubble's discovery of cosmological redshift in 1929, Einstein removed the cosmological constant, allegedly referring to it as his biggest blunder (although this statement is uncertain, as it comes from a single source, Gamow [24]). Later, in 1998, a astrophysicists team led by Saul Perlmutter [25] and another led by Brian Schmidt and Adam Riess [26] that observed high-redshift supernovae that did not conform to the model, the cosmological constant was again reintroduced and praised along with the hypothesis of dark energy. However, for the first time, we have a cosmological model that is similar to the Friedmann model, except the cosmological constant is derived and likely carries a considerably different interpretation than it is traditionally given.

If we use energy density rather than mass density we get

$$H_0^2 = \frac{8\pi G\rho_E - \Lambda c^4}{3} \tag{11}$$

where: $\rho_E = \frac{E}{V} = \frac{E}{\frac{4}{3}\pi r_H^3} = \frac{Mc^2}{\frac{4}{3}\pi r_H^3}$, represents the volumetric energy density of a sphere with radius equal to the Hubble radius.

## 3. Cosmological Redshift from Einstein's Extremal Universe

The cosmological redshift in the extremal solution could be as follow:

$$z = \frac{\lambda_1 - \lambda_2}{\lambda_2} = \frac{\sqrt{1 - \frac{2GM_u}{r_1 c^2} + \frac{G^2 M_u^2}{c^4 r_1^2}}}{\sqrt{1 - \frac{2GM_u}{r_2 c^2} + \frac{G^2 M_u^2}{c^4 r_2^2}}} - 1 \tag{12}$$

Here, $r_1$ and $r_2$ represent the distance from the emitter (for example a supernova or galaxy) to the Hubble sphere horizon and $r_2$ the distance from the Hubble sphere horizon to the observer (in our case basically Earth based observatories). The first term of the Taylor series expansion is given by:

$$z \approx \frac{(r_1 - r_2)GM_u}{c^2 r_1 r_2} \tag{13}$$

If $r_1 = r_H = \frac{GM_u}{c^2}$, which corresponds to the observer's distance to the Hubble sphere horizon, then we have:

$$z \approx \frac{(r_H - r_2)GM_u}{c^2 r_H r_2}$$

Furthermore, since $M_u = \frac{c^3}{GH_0}$, we can substitute $\frac{GM_u}{c^2}$ with $r_H$, resulting in:

$$z \approx \frac{r_H - r_2}{r_2} \tag{14}$$

Moreover, when the object emitting the photons (galaxies, quasars[1], supernovas) is not too far away from us, we can approximate also $r_2$ in the denominator as $r_H$. Substituting this approximation into the denominator, we obtain:

$$z \approx \frac{r_H - r_2}{\frac{c}{H_0}}$$

$$z \approx \frac{(r_H - r_2)H_0}{c} \tag{15}$$

We define the distance $d$ as the difference between $r_H$ and $r_2$. This distance represents the distance from us to the object that emits the photons, such as stars, galaxies, supernovas, and quasars. Consequently, the expression becomes:

[1]Quasars are assumed to be early forming glaxies.

$$z \approx \frac{dH_0}{c} \tag{16}$$

This corresponds to the well-known prediction of the cosmological redshift approximation, which is also used in the standard model (see, for example, [14] [27] [28]). However, in the extremal solutions of Einstein's field equations that we have derived, this redshift does not necessarily seem to be related to the expansion of space. Instead, it appears to be a pure gravitational redshift caused by the mass (energy) within the entire Hubble sphere. However, multiple interpretations could exist here, and one should naturally carefully investigate this before any firm conclusions are made.

## 4. A Closer Look at the Mass (Energy) of the Universe

We can derive a new, more general formula for the mass (or more precisely, the mass-equivalent since we do not distinguish between mass and energy in this context (as we naturally have $M = E/c^2$ ) of the universe by solving the universe equation, $H_0^2 = \frac{8\pi G\rho_E - \Lambda c^4}{3}$, for *M*. This yields:

$$M = \frac{c^3\left(3 + \Lambda r_H^2\right)}{6GH_0} \tag{17}$$

Since $\Lambda = 3\left(\frac{H_0}{c}\right)^2 = \frac{3}{r_H^2}$, we can simplify further:

$$M = \frac{c^3\left(3 + \frac{3}{r_H^2} r_H^2\right)}{6GH_0} = \frac{c^3}{GH_0} \tag{18}$$

However, if we set the cosmological constant to zero ( $\Lambda = 0$ ), we obtain $M = \frac{c^3}{2GH_0}$, which corresponds to the critical mass of the universe in the Friedmann model. Nevertheless, in the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics, the cosmological constant automatically emerges as an additional term. This suggests that the mass (energy) of the universe may be exactly twice that given by the critical mass in the Friedmann universe. At the very least, we believe that more researchers should carefully investigate this alternative model of the universe, which is an exact solution to Einstein's field equations. What is important here is not that the critical Friedmann universe mass is what it is when the cosmological constant is set to zero, but rather that a different mass emerges where the cosmological constant instead directly arises from the extremal solutions of Einstein's field equations.

We can also rewrite the mass as:

$$M = \frac{c^3}{2GH_0} + \frac{c^3\Lambda r_H^2}{6GH_0} = \frac{c^3}{2GH_0} + \frac{c^3}{2GH_0} \tag{19}$$

The first part of this equation now corresponds to the critical mass in the

Friedmann universe, and the last part, $\frac{c^3\Lambda r_H^2}{6GH_0}=\frac{c^3}{2GH_0}$, is exactly half of the total mass in the universe. This last part is due to relativistic effects not taken into account in the Friedmann model. The second part could even be coined as "dark energy", as it arises from relativistic effects that likely only impact gravity. It can only be observed as a gravitational effect and cannot be detected as baryonic matter and in this sense it is dark, not detectable except from gravity observations.

## 5. Deeper Philosophical Aspects of the Extremal Solutions

The extremal solutions have received relatively little attention, especially regarding their predictions in cosmology. Although all three metrics studied yield the same horizon and escape velocity (in the extremal solutions), they differ in their interpretation. The extremal solution of the Reissner-Nordström metric lacks rotation but possesses charge, while the extremal solution of the Kerr metric has rotation but no charge. The Kerr-Newman metric's extremal solution possesses both charge and rotation. Nonetheless, all of these solutions lead to the same cosmological equation, as shown in Equation (11). Still from a deeper philosophical aspect they have different interpretations.

It is also considered a mystery why the electromagnetic force is enormous compared to the gravitational force. If we compare the Coulomb force between a proton and an electron to the theoretical gravitational force between a proton and an electron, we obtain:

$$\frac{|F_c|}{|F_G|}=\frac{k_e\frac{|e||e|}{r^2}}{G\frac{M_{pr}m_e}{r^2}}\approx 2.26\times 10^{39} \tag{20}$$

where $F_c$ represents the Coulomb force [29], $F_G$ denotes the Newtonian gravitational force [30], and $e$ represents the elementary charge. Additionally, $M_{pr}$ and $m_e$ respectively refer to the proton and electron masses. The significant disparity in strength between the electrostatic and gravitational forces is well-documented in the literature. However, despite this knowledge, the gravitational force between a proton and an electron has never been measured. Thus, there is still clearly room for us to gain a deeper understanding of gravity at the atomic and subatomic scales.

On the other hand for two Planck [31] [32] masses ( $m_p=\sqrt{\frac{\hbar c}{G}}$ ) the electrostatic force is identical to the gravitational force as we have

$$\frac{|F_c|}{|F_G|}=\frac{k_e\frac{|q_p||q_p|}{r^2}}{G\frac{m_p m_p}{r^2}}=1 \tag{21}$$

Here, $q_p$ represents the Planck charge: $q_p=\frac{e}{\sqrt{\alpha}}$. The fact that these forces are equal at the Planck scale indicates the potential unification of electromagnetic and gravitational forces at the Planck scale as expected by multiple researchers. However, this is based on Newton's theory, and we need to move beyond it. In the extremal solution of the Reissner-Nordström metric, we have:

$$r_Q^2 = nk_e \left|q_p\right|\left|q_p\right| \frac{G}{c^4} = nGm_p m_p \frac{G}{c^4} \tag{22}$$

Here, $n$ represents the number of Planck masses in the large gravitational mass $M$, so we have $n = \frac{M}{m_p}$. The extremal solution of Reissner-Nordström is consistent with the electrostatic force being identical to the gravitational force at the Planck scale. The term $\frac{G}{c^4}$ is identical to part of Einsteins gravitational constant and is needed to convert the units to the right form needed for predicting gravity phenomena.

If gravity is ultimately caused at the Planck scale as first suggested by Eddington [33] in 1918 and assumed by most researchers working on quantum gravity theory today (see for example [34] [35] [36] [37] [38]), then the extremal solution could be the only truly valid exact solution for real phenomena. This suggests that the extremal solution of the Reissner-Nordström metric could be the most realistic model for the universe. This possibility could explain why no ad hoc inserted constants are needed in this specific solution to fit cosmological observations. Naturally, this hypothesis needs to be carefully investigated and, at this stage, can be seen as plausible.

## 6. Implications

We will now shortly summarize some of the most important implications in terms of cosmological predictions of the extremal universe:

- The cosmological constant does not need to be ad hoc inserted as done today; it automatically arises from the extremal solutions of Einsteins field equation and is given by $\Lambda = 3\left(\frac{H_0}{c}\right)^2$.
- The cosmological redshift prediction in the extremal solutions will give different predictions of cosmological redshift for objects very far away, *i.e.*, those significantly close to the Hubble sphere horizon (the Hubble horizon). This could potentially change the interpretation of Hubble red-shift and be seen as an alternative model to other cosmological models such as the Λ-CDM model. As a minimum this should be carefully investigated.
- The amount of energy (mass) in the Hubble sphere is twice the Friedmann critical mass. Therefore, it is $M_u = \frac{c^3}{GH_0}$ instead of $M_c = \frac{c^3}{2GH_0}$. Half of this mass arises from relativistic effects and is likely detectable only through gravitational phenomena, thus bearing resemblance to dark energy, as briefly discussed in Section 4.

## 7. Conclusion

We have demonstrated that the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics yield the same cosmological model, which is analogous to the Friedmann equation. However, a significant distinction exists in that the cosmological constant is now derived rather than being ad hoc inserted into the field equation. Furthermore, the cosmological redshift observed in the extremal solution appears to be a specific instance of gravitational redshift caused by the mass and energy within the Hubble sphere. Naturally, this notion necessitates careful study and investigation. Nevertheless, considering that this new cosmological model is an exact solution to Einstein's field equation and the only known solution where the cosmological constants emerge automatically without the need for ad hoc insertion, we believe that it merits thorough consideration by the research community over an extended period, during which it can be compared to other cosmological models that are also compatible with Einstein's field equation.

## Acknowledgements

Thanks to (AB-anonymous) for useful comments.

## Data Availability Statement

All data used in this study is in the paper.

## Conflicts of Interest

The author declares no conflict of interest.

## References

[1] Reissner, H. (1916) Über die eigengravitation des elektrischen feldes nach der einsteinschen theorie. *Annalen der Physik*, **355**, 106-120. https://doi.org/10.1002/andp.19163550905

[2] Nordström, G. (1918) On the Energy of the Gravitation Field in Einstein's Theory. *Koninklijke Nederlandsche Akademie van Wetenschappen Proceedings*, **20**, 1238.

[3] Einstein, A. (1916) Die grundlage der allgemeinen relativitätstheorie. *Annalen der Physics*, **354**, 769-822. https://doi.org/10.1002/andp.19163540702

[4] Zee, A. (2013) Einstein Gravity in a Nutshell. Princeton University Press, Princeton.

[5] Aretakis, S. (2018) Dynamics of Extremal Black Holes. Springer Verlag, Berlin. https://doi.org/10.1007/978-3-319-95183-6

[6] Kerr, R.P. (1963) Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics. *Physical Review Letters*, **11**, 237-238. https://doi.org/10.1103/PhysRevLett.11.237

[7] Newman, E.T. and Janis, A.I. (1965) Note on the Kerr Spinning-Particle Metric. *Journal of Mathematical Physics*, **6**, 915-917. https://doi.org/10.1063/1.1704350

[8] Newman, E., Couch, E., Chinnapared, K., Exton, A., Prakash, A. and Torrence, R. (1965) Metric of a Rotating, Charged Mass. *Journal of Mathematical Physics*, **6**, 918-919. https://doi.org/10.1063/1.1704351

[9] Guidry, M. (2019) Modern General Relativity. Cambridge University Press, Cambridge. https://doi.org/10.1017/9781108181938

[10] Schwarzschild, K. (1916) Über das gravitationsfeld einer kugel aus inkompressibler flussigkeit nach der Einsteinschen theorie. Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik, 424.

[11] Augousti, A.T. and Radosz, A. (2006) An Observation on the Congruence of the Escape Velocity in Classical Mechanics and General Relativity in a Schwarzschild Metric. *European Journal of Physics*, **376**, 331-335. https://doi.org/10.1088/0143-0807/27/2/015

[12] Pathria, R.K. (1972) The Universe as a Black Hole. *Nature*, **240**, 298-299. https://doi.org/10.1038/240298a0

[13] Stuckey, W.M. (1994) The Observable Universe inside a Black Hole. *American Journal of Physics*, **62**, 788-795. https://doi.org/10.1119/1.17460

[14] Schutz, B. (2003) Gravity from the Ground up. Cambridge University Press, Cambridge. https://doi.org/10.1017/CBO9780511807800

[15] Friedmann, A. (1922) Über die krüng des Raumes. *Zeitschrift für Physik*, **10**, 377. https://doi.org/10.1007/BF01332580

[16] Monerat, G.A., Filho, F.L.G., Silva, E.V.C., *et al.* (2010) The Planck Era with a Negative Cosmological Constant and Cosmic Strings. *Physics Letters A*, **374**, 4741-4745. https://doi.org/10.1016/j.physleta.2010.09.067

[17] Prokopec, T. (2011) Negative Energy Cosmology and the Cosmological Constant.

[18] Maeda, K. and Ohta, N. (2014) Cosmic Acceleration with a Negative Cosmological Constant in Higher Dimensions. *Journal of High Energy Physics*, 2014, Article No. 95. https://doi.org/10.1007/JHEP06(2014)095

[19] Visinelli, L., Vagnozzi, S. and Danielsson, U. (2019) Revisiting a Negative Cosmological Constant from Low-Redshift Data. *Symmetry*, **11**, Article No. 1035. https://doi.org/10.3390/sym11081035

[20] Calderón, R., Gannouji, R., L'Huillier, B. and Polarski, D. (2021) Negative Cosmological Constant in the Dark Sector? *Physical Review D*, **103**, Article ID: 023526. https://doi.org/10.1103/PhysRevD.103.023526

[21] Benizri, L. and Troost, J. (2023) More on Pure Gravity with a Negative Cosmological Constant. *Journal of High Energy Physics*, **2023**, Article No. 93. https://doi.org/10.1007/JHEP09(2023)093

[22] Sen, A.A., Adil, S.A. and Sen, S. (2023) Do Cosmological Observations Allow a Negative $\lambda$? *Monthly Notices of the Royal Astronomical Society*, **518**, 1098-1105. https://doi.org/10.1093/mnras/stac2796

[23] Einstein, A. (1917) Cosmological Considerations in the General Theory of Relativity. Sitzungsber. Preuss. Akad. Wiss, Berlin (Math. Phys.), 142.

[24] Gamow, G. (1970) My World Line: An Informal Autobiography. *Physics Today*, **24**, 51-52. https://doi.org/10.1063/1.3022626

[25] Perlmutter, S., *et al.* (1999) Measurements of $\omega$ and $\lambda$ from 42 High-Redshift Supernovae. *The Astrophysical Journal*, **517**, Article No. 565. https://doi.org/10.1086/307221

[26] Reiss, A.G., *et al.* (1998) Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal*, **116**, 1009-1038.

[27] Wald, R.M. (1984) General Relativity. The University of Chicago Press, Chicago.

https://doi.org/10.7208/chicago/9780226870373.001.0001

[28] Schneider, P. (2015) Extragalactic Astronomy and Cosmology. Springer, Berlin. https://doi.org/10.1007/978-3-642-54083-7

[29] Coulomb, C.A. (1785) Premier mémoire sur l'électricité et le magnétisme. Histoire de Académie Royale des Sciences, 569-577.

[30] Newton, I. (1686) Philosophiae Naturalis Principia Mathematica. Jussu Societatis Regiae ac Typis Josephi Streater, London.

[31] Planck, M. (1899) Natuerliche Masseinheiten. Der Königlich Preussischen Akademie Der Wissenschaften, Berlin. https://www.biodiversitylibrary.org/item/93034#page/7/mode/1up

[32] Planck, M. (1906) Vorlesungen über die Theorie der Wärmestrahlung. J.A. Barth, Leipzig, 163.

[33] Eddington, A.S. (1918) Report on the Relativity Theory of Gravitation. The Physical Society of London, Fleetway Press, London.

[34] Adler, S.L. (2010) Six Easy Roads to the Planck Scale. *American Journal of Physics*, **78**, 925-932. https://doi.org/10.1119/1.3439650

[35] Hossenfelder, S. (2012) Can We Measure Structures to a Precision Better than the Planck Length? *Classical and Quantum Gravity*, **29**, Article ID: 115011. https://doi.org/10.1088/0264-9381/29/11/115011

[36] Hossenfelder, S. (2013) Minimal Length Scale Scenarios for Quantum Gravity. *Living Reviews in Relativity*, **16**, Article No. 2. https://doi.org/10.12942/lrr-2013-2

[37] Haug, E.G. (2023) Different Mass Definitions and Their Pluses and Minuses Related to Gravity. *Foundations*, **3**, 199-219. https://doi.org/10.3390/foundations3020017

[38] Haug, E.G. (2023) Quantized Newton and General Relativity Theory. https://doi.org/10.20944/preprints202309.1399.v1