Scientific Research Publishing

Journal of High Energy Physics, Gravitation and Cosmology, 2024, 10, 185-196
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

Unified Description of the Three Stable Particles in Self-Action Allows Determination of Their Relative Masses

Yair Goldin Halfon

Mexico City, Mexico
Email: goldin@comunidad.unam.mx

How to cite this paper: Halfon, Y.G. (2024) Unified Description of the Three Stable Particles in Self-Action Allows Determination of Their Relative Masses. *Journal of High Energy Physics, Gravitation and Cosmology*, **10**, 185-196.
https://doi.org/10.4236/jhepgc.2024.101017

Received: September 14, 2023
Accepted: January 22, 2024
Published: January 25, 2024

Abstract

The Dirac equation $\gamma_\mu\left(\partial_\mu - eA_\mu\right)\Psi = mc^2\Psi$ describes the bound states of the electron under the action of external potentials, A_μ. We assumed that the fundamental form of the Dirac equation $\gamma_\mu\left(\partial_\mu - S_\mu\right)\Psi = 0$ should describe the stable particles (the electron, the proton and the dark-matter-particle (dmp)) bound to themselves under the action of their own potentials S_μ. The new equation reveals that self energy is consequence of self action, it also reveals that the spin angular momentum is consequence of the dynamic structure of the stable particles. The quantitative results are the determination of their relative masses as well as the determination of the electromagnetic coupling constant.

Keywords

Electron in Self Action, Electron-Dark-Matter Particle Mass Ratio, Analytic Description Dark-Matter-Particle

1. Receptive Charge Instead of Electric Charge

Let's start by pointing out that the usual assertion $\nabla \cdot r^{-3}\boldsymbol{r} = \delta(r)$ would imply that the integral of the divergence of the Coulomb field namely

$$4\pi\int_0^a \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{r^2}\right)r^2 \mathrm{d}r \tag{1}$$

must return 4π. However, the integral returns zero. Textbooks invoke the divergence theorem trying to convince the readership that the integral above equals the flux of the Coulomb field through any surface enclosing the singularity [1].

Yes, the flux is 4π, but textbooks never mention that said theorem does not hold for radial fields diverging at the origin as, or more rapidly than, r^{-2}. The proof is quite simple: consider the radial vector field $r^{-2+\eta}\hat{\boldsymbol{r}}$. The integral of its divergence over a spherical volume of radius a minus the flux of the field over the corresponding surface takes the form:

$$4\pi\int_0^a \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{r^{2-\eta}}\right)r^2\mathrm{d}r - \int r^{-2+\eta}\hat{\boldsymbol{r}}\cdot\mathrm{d}\boldsymbol{A} = -4\pi 0^{\eta} \tag{2}$$

The term $-4\pi 0^{\eta}$ at the right would vanish only if η is a positive number. Consequently one cannot prove that the integral (1) gives 4π; it gives zero.

Deep seated misconceptions can confuse anyone, even luminaries. In his famous lectures on Physics Richard Feynman explains how to solve the Laplace equation for the radial potential ϕ namely $r^{-1}\mathrm{d}^2(r\phi)/\mathrm{d}r^2 = 0$ [2]. He argues that the function $\mathrm{d}(r\phi)/\mathrm{d}r$ must be a constant, say c_1. Integrating again he gets $\phi = c_1 + c_2/r$. Since the result is at variance with the prevailing notion that a point charge resides at the singularity of the Coulomb potential, he ends exclaiming that his own straightforward procedure is not exactly correct because $r\phi$ gives 0/0 at $r = 0$ and the quotient is not defined.

To avoid fictitious arguments, it is necessary to define that the quotient of two identical continuous functions which vanish only at certain point x_0 yields 1 throughout the real line. With this definition we will never get contradictions or inconsistency which is all that really matters. For instance, the term x/x^2 at $x = 0$ gives 0/0. Also the term x^2/x gives 0/0, but the terms are very different. With the definition we have just mentioned the first term reduces to 1/0 whereas the second term reduces to 0/1. In other words, this definition prevents confusion and right away shows that the divergence of the Coulomb field yields zero.

Physicists, in general, pay no attention to the fact that the Dirac delta $\delta[r]$ *perse* is merely a symbol that represents a family of parametric functions with certain properties. To work with the Dirac delta one must specify the parametric representation $\delta(r,\varepsilon)$. That is, Gauss's equation must be written as shown:

$$\nabla\cdot\boldsymbol{E} = \delta(r,\varepsilon) \tag{3}$$

The length-parameter ε can be dealt with as an indefinite small positive number. It cannot be replaced by zero as is usually done after the integration of a parametric representation. The parameter ε is always present in the solution, hence the Coulomb field cannot possibly be solution of Gauss's equation. To give an example consider the simplest 1D continuous representation $\delta(x,\varepsilon) = (1/\varepsilon)\times\exp(-x/e)$. The corresponding 3D representation is obtained by dividing the 1D representation by r^2, thus $\delta(r,\varepsilon) = (1/r^2\varepsilon)\times\exp(-r/e)$. The solution of Gauss' equation $\nabla\cdot\boldsymbol{E} = (1/r^2\varepsilon)\times\exp(-r/e)$ would be $\boldsymbol{E} = -(\boldsymbol{r}/r^3)\times\exp(-r/e)$. The result makes no physical sense because the source is not confined within a finite volume:

$$\int_{R\varepsilon}^{\infty}\delta(r,\varepsilon)r^2\mathrm{d}r = \exp(-R) \neq 0 \tag{4}$$

where R is any positive number R.

To cope with the fact the Coulomb potential is not functionally related with any source the electric charge density must be replaced by a new concept we have named receptive charge density. The electron is made up of receptive charge upon which the Coulomb potential and a gauge-invariant potential, both inherent to the electron, act to provide this particle with rest energy.

Thus the main premise of the new approach is that each stable particle has a characteristic set of self potentials S_μ and a distinctive kind of receptive charge density over which only the characteristic set of self potentials can act. The analytic form of the receptive charge density is given by a quadratic expression of the wave functions that satisfy fundamental form of the Dirac equation,

$$\gamma_\mu\left(\partial_\mu - S_\mu\right)\Psi = 0, \tag{5}$$

In this manuscript, however, we are going to restrict attention to the study of the dmp and the electron in self-action. Nevertheless it is important to mention in advance that for the study of the proton we will need Equation (5) and an additional scalar wave equation which takes into account the strong action of the proton upon itself. The theorists of QCD are getting increasingly frustrated after decades of futile efforts to produce a consistent mathematical description of the proton based on the notion of three quarks exchanging gluons [3]. The prevailing notion that the proton can be studied with no relation whatsoever with the study of electron, or that the dmp can be studied with no relation with any other particle right away denies the possibility to interrelate their masses and hardly makes sense since in the universe everything is interrelated.

Naturally the Dirac and the Schroedinger free particle equations would be meaningful exclusively in context with problems involving well delimited potential-free regions where the solutions must necessarily satisfy boundary conditions. The customary free particle solutions per se, harmonic wave functions uniformly extending throughout space, would be superfluous and should be discarded.

To complement the discussion lets recall that the parameter of relative velocity in the Lorentz transformations is defined in relation to two free particles, A and B, that obey the principle of inertia, that is, the covariance of physical theories depends on the existence of systems of reference with respect to which free particles remain at rest.

The founders of quantum mechanics had no alternative but to propose philosophical interpretations to the fact the customary wave equations cannot describe the free electron according to the principle of inertia, but now we have the equation in which we can insert the Coulomb potential for the description of the free electron. This way we can start visualizing the particle at rest around the singularity of the potential. In other words, by considering self actions we are actually establishing a real connection between the particle and the inertial system of reference; indeed the singularity of the potential is also the origin of coordinates. That connection is missing in the Schroedinger free particle equa-

tion, hence the particle appears dispersed throughout space. About 75 years ago, during the development of quantum electrodynamics, physicists wanted to visualize the free electron in self action emitting and absorbing virtual photons [4]. It is generally said that the QED approach failed because it yields divergent self energy with no remedy. In my understanding, however, the QED approach was doomed because in that context the electron cannot be described according to the first principle of physics, the principle of inertia.

2. The Dark Matter Particle (DMP)

2.1. Self-Potentials

The potentials of the stable particles are obtained by a covariant procedure which consists in the combination of the 4-velocity of the singularity (u_k) and the zero-length 4-vector $r_k=(r,-\boldsymbol{r})$, ($r_k r_k=0$). The latter indicates that physical information travels from the singularity to the observation point with the speed of light. The potentials we need for the description of the dmp are:

- The gauge invariant potential,

$$G_k=-\nabla_k\left(r_i u_i\right)^{-1},\ \rightarrow\left(0,r^{-3}\boldsymbol{r}\right). \tag{6}$$

the expression at the right hold for the system of reference where the particle is at rest. Soon we will see this potential makes the wave functions and all their derivatives vanish at the origin of coordinates. Its corresponding coupling constant would be $\lambda_0=\hbar/m_x c$. Where m_x denotes the mass of the dmp. The coupling constant is a new fundamental constant with dimension of length (or mass). It will serve to interrelate the masses of the stable particles.

- We also need the potential

$$\Delta_k=r_k\left(r_i u_i\right)^{-2}\rightarrow\left(r^{-1},\boldsymbol{r}r^{-2}\right) \tag{7}$$

Differentiation of retarded potentials show that the corresponding antisymmetric tensor, $\Delta_{ij}=\partial_i\Delta_j-\partial_j\Delta_i$, does not contain radiation fields (fields that decay as r^{-1}). The fields in Δ_{ij} decay as r^{-2} even when the singularity is undergoing acceleration (that is the big difference with the electron's electromagnetic fields). Hence the dmp does not radiate or absorbs electromagnetic energy. The fields in Δ_{ij} are a new type of electromagnetic fields which do not interact over electrons and protons. Since the electric charge is not a real concept we will handle the electromagnetic coupling constant α as a dimensionless parameter whose real nature we will discover later on. In other words, in the new approach we cannot consider α as $e^2/\hbar c$.

2.2. System of Differential Equations for the Radial Functions

From Section 2.1 it follows that Equation (5) takes the form:

$$\gamma_\mu\left(\nabla_\mu+i\alpha\left[\frac{1}{r},\frac{\boldsymbol{r}}{r^2}\right]-\frac{\hbar}{m_x c}\left[0,\frac{\boldsymbol{r}}{r^3}\right]\right)\Psi=0 \tag{8}$$

Taking for reference the solution of the hydrogen atom [5] it is easy to infer

that the substitution of the wave functions,

$$\begin{aligned}
\Psi_1 &= \mathrm{e}^{-iEt/\hbar}\left(\frac{j+1-m}{2(j+1)}\right)^{1/2} Y_{j+\frac{1}{2},m-\frac{1}{2}}\widehat{F(r)} \to \mathrm{e}^{-iEt/\hbar}\sqrt{\frac{1}{3}}Y_{1,0}\widehat{F(r)} \\
\Psi_2 &= -\mathrm{e}^{-iEt/\hbar}\left(\frac{j+1+m}{2(j+1)}\right)^{1/2} Y_{j+\frac{1}{2},m+\frac{1}{2}}\widehat{F(r)} \to -\mathrm{e}^{-iEt/\hbar}\sqrt{\frac{2}{3}}Y_{1,1}\widehat{F(r)} \\
\Psi_3 &= i\mathrm{e}^{-iEt/\hbar}\left(\frac{j+m}{2j}\right)^{1/2} Y_{j-\frac{1}{2},m-\frac{1}{2}}\widehat{G(r)} \to i\mathrm{e}^{-iEt/\hbar}Y_{0,0}\widehat{G(r)} \\
\Psi_4 &= i\mathrm{e}^{-iEt/\hbar}\left(\frac{j-m}{2j}\right)^{1/2} Y_{j-\frac{1}{2},m+\frac{1}{2}}\widehat{G(r)} \to 0
\end{aligned} \tag{9}$$

into Equation (8) gives the following system of differential equations for a particle with total angular momentum $j=1/2$ and z-component $m=1/2$

$$\frac{1}{r^2}\frac{\partial\left(r^2\hat{F}\right)}{\partial r}+i\alpha\frac{\hat{F}}{r}-\frac{\hbar}{m_x c}\frac{\hat{F}}{r^2}=\left(\frac{E}{\hbar c}-\frac{\alpha}{r}\right)\hat{G} \tag{10}$$

$$\frac{\partial\hat{G}}{\partial r}+i\alpha\frac{\hat{G}}{r}-\frac{\hbar}{m_x c}\frac{\hat{G}}{r^2}=-\left(\frac{E}{\hbar c}-\frac{\alpha}{r}\right)\hat{F} \tag{11}$$

The substitutions

$$\left(\hat{F},\hat{G}\right)=(F,G)\exp\left(-\frac{\hbar}{m_x c}\times\frac{1}{r}\right)\exp\left(-i\alpha\times\ln(r)\right) \tag{12}$$

give the following system of differential equations

$$\frac{1}{s^2}\frac{\partial\left(s^2F\right)}{\partial s}=\left(1-\frac{1}{s}\right)\alpha G \tag{13}$$

$$\frac{\partial G}{\partial s}=-\left(1-\frac{1}{s}\right)\alpha F \tag{14}$$

in the variable

$$s=\frac{E}{\alpha\hbar c}r \tag{15}$$

2.3. The First Radial Solution

The first independent solution of system (13, 14) can be written as

$$F=\sum_{n=0}^{\infty}\alpha^{2n+1}F_n(s)=\alpha F_0(s)+\alpha^3F_1(s)+\alpha^5F_2(s)+\cdots \tag{16}$$

$$G=\sum_{n=0}^{\infty}\alpha^{2n}G_n=1+\alpha^2G_1(s)+\alpha^4G_2(s)+\cdots \tag{17}$$

where all the functions F_n and G_{n+1} (starting with $n=0$), as well as their first derivative vanish at $s=1$. The method of solution is iterative and it is based on the comparison of terms with equal powers of α. The iteration process starts with the insertion of the generator $G_0 = 1$ into Equation (13) (the generator is the function that allows the left side of Equation (14) vanish.

So we get:

$$\frac{1}{s^2}\frac{\partial\left(s^2F_0\right)}{\partial s}=\left(1-\frac{1}{s}\right)G_0$$

$$\Rightarrow F_0=\frac{1}{6}\left(s^{-2}-3+2s\right) \tag{18}$$

afterwards we insert the function F_0 into Equation (14):

$$\frac{\partial G_1}{\partial s}=-\left(1-\frac{1}{s}\right)F_0 \tag{19}$$

$$\Rightarrow G_1=-\frac{1}{12}\left(s^{-2}-2s^{-1}+6\ln\left(s\right)+9-10s+2s^2\right)$$

And so on with no end.

We need smoothly continuous solutions throughout space but at the same time we need to confine the receptive charge close around the singularity of the potentials. How is that possible? The first step is to consider that Equations (13) and (14) are valid for $s\le 1$. To obtain the solution for $s>1$ the fundamental Equation (4) involves the momentum 4-vector:

$$\gamma_\mu\left(\nabla_\mu+i\frac{m_xc}{\hbar}[1,\mathbf{0}]-\frac{\hbar}{m_xc}\left[0,\frac{\boldsymbol{r}}{r^3}\right]\right)\Psi=0, \tag{20}$$

Thus the counterpart of Equations (13) and (14) would be

$$\frac{1}{r^2}\frac{\partial\left(r^2F\right)}{\partial r}=\left(\frac{E}{\hbar c}-\frac{m_ec}{\hbar}\right)G, \tag{21}$$

$$\frac{\partial G}{\partial r}=-\left(\frac{E}{\hbar c}-\frac{m_ec}{\hbar}\right)F \tag{22}$$

Letting $E=m_xc^2$ we get the solution

$$F=0,\quad G=1 \tag{23}$$

Thus the radial functions $\hat{F}$ and $\hat{G}$ are smoothly continuous throughout space when the interaction energy E equals the rest energy of the particle! (see Figure 1).

2.4. Second Radial Solution

The second independent solution of system (13) and (14), denoted as (f, g) can be written as

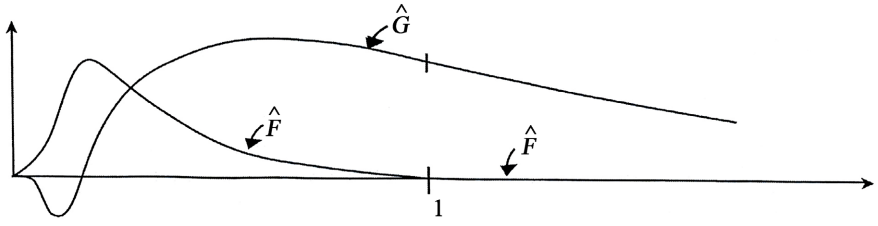

Figure 1. Sketch of the first radial solution.

$$f = s^{-2} + \alpha^2 f_1(s) + \alpha^4 f_2(s) + \cdots \tag{24}$$

$$g = \alpha^1 g_0(s) + \alpha^3 g_1(s) + \alpha^5 g_2(s) + \cdots \tag{25}$$

where all the functions f_{n+1} and g_n (starting with $n=0$) as well as their first derivatives vanish at $s=1$. The second independent solution starts with the generator $f_0 = s^{-2}$ into Equation (14):

$$\frac{\partial g_0}{\partial s} = -\left(1-\frac{1}{s}\right)s^{-2} \Rightarrow g_0 = -(1-s)^2 s^{-2} \tag{26}$$

afterwards we obtain f_1 inserting g_0 into Equation (13):

$$\frac{1}{s^2}\frac{\partial\left(s^2 f_1\right)}{\partial s} = \left(1-s^{-1}\right)g_0 \tag{27}$$

$$\Rightarrow f_1 = \frac{1}{12}\left(11s^{-2} + 6s^{-2}\ln s - 18s^{-1} + 9 - 2s\right)$$

And so on with no end.

The solutions (24) and (25) are valid for $s \le 1$. The functions (f, g) for $s > 1$ are obtained with the second solution of system (21) and (22):

$$f = s^{-2}, \quad g = 0 \tag{28}$$

Thus the functions $\hat{f}$ *and* $\hat{g}$ *are smoothly continuous throughout space* (see Figure 2).

$$\gamma_\mu\left(\nabla_\mu + i\alpha\left[\frac{1}{r}, \frac{\boldsymbol{r}}{r^2}\right] - \frac{\hbar}{m_x c}\left[0, \frac{\boldsymbol{r}}{r^3}\right]\right)\Psi = 0$$

$$\gamma_\mu\left(\nabla_\mu + i\frac{\hbar}{m_x c}[1, \boldsymbol{0}] - \frac{\hbar}{m_x c}\left[0, \frac{\boldsymbol{r}}{r^3}\right]\right)\Psi = 0$$

2.5. Product of Radial Solutions Confines the Receptive Charge of the DMP

- Let Ψ represent the wave functions corresponding to the first radial solution (F, G) arranged in a 1 × 4 matrix.
- Let Ψ^* represent the complex conjugated wave functions corresponding to the second radial solution (f, g) arranged in 4 × 1 matrix.

Consider the standard form of the Dirac equation [5]

$$\gamma_0\partial_{ct}\Psi + \gamma_1\partial_x\Psi + \gamma_2\partial_y\Psi + \gamma_3\partial_z\Psi = \gamma_m\left(mc/\hbar\right)\Psi$$

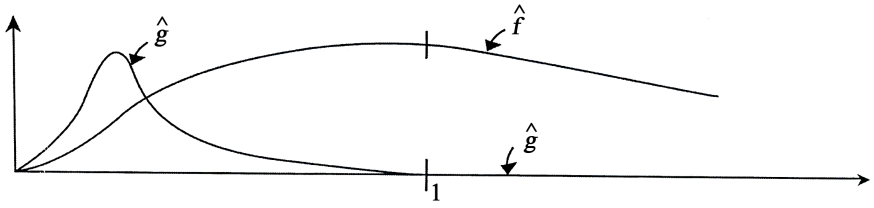

Figure 2. Sketch of the second radial solution.

where $\gamma_0 = diag(1,1,1,1)$ and $\gamma_m = diag(1,1,-1,-1)$. Also consider the 4-velocity of the singularity $u_k = (1,0,0,0)$. We can form the following invariant density involving only two components of the wave function:

$$\begin{aligned}\mathcal{I}_{dmp} &= \frac{1}{2}\left(u_\mu \Psi \gamma_\mu \Psi^* + \Psi \gamma_m \Psi^*\right) \\ &= \Psi_1 \Psi_1^* + \Psi_2 \Psi_2^* \\ &= \exp\left(-\frac{2}{\alpha s}\right) \times \left[\frac{1}{3}\left|Y_{1,0}\right|^2 + \frac{2}{3}\left|Y_{1,1}\right|^2\right] \times Ff \\ &= (4\pi)^{-1} \exp\left(-\frac{2}{\alpha s}\right) \times \left[\alpha F_0 f_0 + \alpha^3 \left(F_0 f_1 + F_1 f_0\right) + \cdots\right]\end{aligned} \tag{29}$$

The first approximation of $\mathcal{I}_{dmp}$, denoted as $\mathcal{I}_{dmp}^{(1)}$, takes the explicit form

$$\mathcal{I}_{dmp}^{(1)} = (4\pi)^{-1} \exp\left(-\frac{2}{\alpha s}\right) \times \left[\frac{\alpha}{6}\left(s^{-2} - 3 + 2s\right)\left(s^{-2}\right)\right] \tag{30}$$

The presence of the normalized spherical harmonics,

$$Y_{1,0} = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta \quad Y_{1,1} = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta e^{i\phi}$$

in the third line of Equation (29) suggests that two fractions of receptive charge, containing 1/3 and 2/3 of the whole, are orbiting the singularity of the potentials with the corresponding angular momentum. We have called these fractions new quarks (n-quarks). The percentage of receptive charge between $s = 0$ and $s = s_0$ should be defined as (**Figure 3**)

$$\left(\int_0^{s_0} \hat{F}\hat{f}s^2 \mathrm{d}s\right) \times \left(\int_0^1 \hat{F}\hat{f}s^2 \mathrm{d}s\right)^{-1} \tag{31}$$

Figure 4, the dmp is like a minute planetary system with no star. Two n-quarks orbit the singularity of the potentials. This way the dmp acquires spin angular momentum.

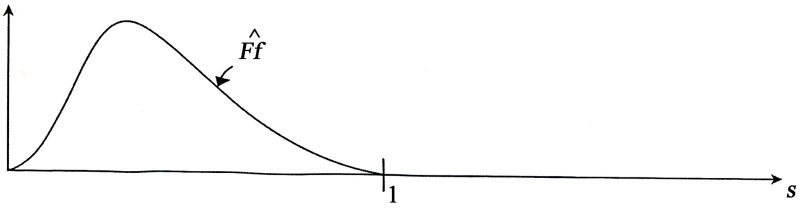

Figure 3. Sketch of the receptive charge density: The product Ff.

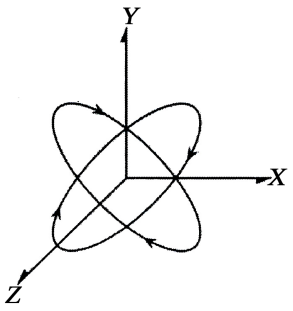

Figure 4. Internal structure of the dmp.

3. The Electron-Positron Solution

Since the Coulomb potential is not functionally related with any source it follows that the Lienard-Wiechert potentials of an electron in arbitrary motion,

$$A_k = \frac{u_k}{r_i u_i} \to \left(r^{-1}, \mathbf{0}\right)$$

cannot possibly be obtained from Maxwell's equations. Nevertheless Maxwell's equations apply well provided the density of electrons is represented with a differentiable function

$$\nabla^2 \phi = \text{differentable density of electrons}$$

The density of electrons can be named electric charge density but the conceptual structure of Maxwell's electromagnetic theory doesn't work for a single electron. That said we can write down Equation (5) for the electron case. We are going the insert the L-W potentials and the gauge invariant G_k in Equation (5), so we get

$$\gamma_\mu \left(\nabla_\mu + i\alpha \left[\frac{1}{r}, \mathbf{0} \right] - \frac{\hbar}{m_x c} \left[0, \frac{\boldsymbol{r}}{r^3} \right] \right) \Psi = 0 \tag{32}$$

Comparing Equation (8) and Equation (32) and noticing that the imaginary vector potential in Equation (8) has no influence on the fundamental invariant of the dmp in Equations (29) and (30) (its contribution to the dmp wave functions is the factor $\exp\left(-i\alpha \times \ln s\right)$) it is easy to infer the fundamental invariant for the electron solution namely

$$\mathcal{I}_e^{(1)} = \left(4\pi\right)^{-1} \exp\left(-\frac{2}{\alpha s} \frac{m_e}{m_x} \right) \times \left[\frac{\alpha}{6} \left(s^{-2} - 3 + 2s \right) \left(s^{-2} \right) \right] \tag{33}$$

where

$$s = \frac{E_e}{\alpha \hbar c} r \tag{34}$$

The invariant $\mathcal{I}_e^{(1)}$ differs from $\mathcal{I}_{dmp}^{(1)}$ in that the exponent now involves the mass ratio m_e/m_x. We are going to take advantage of this circumstance to determine m_e/m_x. But before we do that it is very important to know that the fundamental wave equation has a second general solution (the positron) in which the order of the wave functions is different (compare Equations (9) with Equations (35)).

$$\begin{aligned}
&\Psi_1 = i \exp\left(-iEt/\hbar\right) Y_{0,0} G\left(r\right) && \Psi_1^* = -i \exp\left(-iEt/\hbar\right) Y_{0,0}^* g\left(r\right) \\
&\Psi_2 = 0 && \Psi_2^* = 0 \\
&\Psi_3 = \exp\left(-iEt/\hbar\right) \sqrt{\frac{1}{3}} Y_{1,0} F\left(r\right) && \Psi_3^* = \exp\left(-iEt/\hbar\right) \sqrt{\frac{1}{3}} Y_{1,0}^* f\left(r\right) \\
&\Psi_4 = \exp\left(-iEt/\hbar\right) \sqrt{\frac{2}{3}} Y_{1,1} F\left(r\right) && \Psi_4^* = \exp\left(-iEt/\hbar\right) \sqrt{\frac{2}{3}} Y_{1,1}^* f\left(r\right)
\end{aligned} \tag{35}$$

Lets recall that Ψ corresponds to the first radial solution and Ψ^* corresponds to the transposed complex conjugate of the second radial solution.

The solutions are identical for the electron and for the positron. Both particles have positive rest energy: the Dirac interpretation to his positron with negative rest energy is false. Nevertheless there is an important physical difference between the electron and the positron solutions:

The density

$$S_z = \Psi_1\Psi_1^* - \Psi_2\Psi_2^* + \Psi_3\Psi_3^* - \Psi_4\Psi_4^*, \tag{36}$$

gives the exact same expression for the wave function in Equation (9) and Equation (35). But the density:

$$M_z = \Psi_1\Psi_1^* - \Psi_2\Psi_2^* - \Psi_3\Psi_3^* + \Psi_4\Psi_4^*: \tag{37}$$

gives opposite expressions (one is the negative of the other).

The density (36) corresponds to the z component of the spin pseudo 4-vector whereas the density (37) corresponds to z component to the magnetization in an antisymmetric tensor [6]. In other words, if the electron and the positron spins are parallel, then their magnetic moments are anti-parallel.

4. Determination(s) of the Electromagnetic Coupling Constant and the Relative Masses of the Stable Particles

Let me start this section by quoting some lines of the Feynman's conjecture in this regard [7]: "*There is a most profound and beautiful question associated with the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to −0.08542455.* (*My physicist friends won't recognize this number, because they like to remember it as the reciprocal of its square: about 137.03597 with an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to π or perhaps to the base of natural logarithms. Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the hand of God wrote that number and we don't know how He pushed his pencil…*"

Professor Feynman was not precisely right because the fundamental invariant (33) clearly suggests how God pushed his pencil to write that number: The fundamental invariant (33) was determined up to a multiplicative constant which we now chose as $4\pi\alpha^{-k-1}$. We have two factors: 4π and α^{-k-1}. The first factor cancels out the angular normalization factor $(4\pi)^{-1}$ which should not be involved in the determination of α in the next paragraphs. The factor α^{-k-1} is the simplest expression allowing the ratio m_e/m_x acquires an extreme value:

We first express m_e/m_x as a function of α. How? Integrating the natural logarithm of the fundamental invariant (33):

$$\int_{space} \ln\left[\alpha^{-k-1}\exp\left(-\frac{2}{\alpha s}\frac{m_e}{m_x}\right)\times(1/6)\left(s^{-2}-3+2s\right)\left(s^{-2}\right)\right]\mathrm{d}v = 0 \tag{38}$$

Thus

$$3m_e/m_x = -k\alpha \ln \alpha - 3.222487\alpha \tag{39}$$

Therefore, the condition,

$$\frac{\partial}{\partial \alpha}\left(\frac{m_e}{m_d}\right) = 0 \tag{40}$$

gives

$$k \ln \alpha + 3.222487 + k = 0 \tag{41}$$

Adding Equations (39) and (41) we get

$$k = 3m_e/\alpha m_x \tag{42}$$

We can make the "expectation value" of the energy $\mathcal{E} = \hbar c \times \left(r_k u_k\right)^{-1} \to \hbar c r^{-1}$ acquire fundamental meaning when we let it be equivalent to the electron and the dmp rest energy:

$$\int_{space} \mathcal{E} \times \mathcal{I}_e^{(1)} \mathrm{d}v = \left\langle m_e c^2 + m_x c^2 \right\rangle \int_{space} \mathcal{I}_e^{(1)} \mathrm{d}v \tag{43}$$

From Equation (33) and the equations above we get:

$$\int_0^1 \left[\frac{1}{s} + \frac{3\left(1+\ln\alpha\right)}{3.222487} - \alpha\right]\left[\exp\left(\frac{6.444974}{3+3\ln\alpha} \times \frac{1}{s}\right)\right]\left[\frac{1}{s^2} - 3 + 2s\right] \mathrm{d}s = 0 \tag{44}$$

Numerical integration gives the first approximation for the value of α:

$$\alpha = 1/137.51 \tag{45}$$

From Equation (41) we get $k = 0.822129$ and from Equation (42) we finally get

$$m_x = 502.26 m_e \tag{46}$$

The parameter α is the coupling constant of the potential r^{-1} in the electron solution and in the dmp solution and it will be the coupling constant in the proton solution too. It is natural to assume that α interrelates the three particles. Since m_x is the reference with respect to which m_e is measured, we assume that it is also the reference with respect to which the proton mass m_p is measured. And to treat the electron and the proton on equal footing we have:

$$\frac{m_p}{m_x}\frac{m_e}{m_x} = \alpha \tag{47}$$

Thus

$$m_p/m_e = 1834.76 \tag{48}$$

The number 1834.76 above follows from the values of α and m_e/m_x we obtained in first approximation from Equations (45) and (46). The exact ratio $m_e/m_x = 501.6164033$ is obtained substituting the experimental values: $\alpha = 1/137.0359486$ and $m_p = 1836.1526734 m_e$ into Equation (47).

Since the relative masses of the stable particles do not follow from the analysis of any stage in the history of the universe, we wonder whether the criterion that we followed to determine the electromagnetic coupling constant is part of the

metaphysical design of the universe. We believe the universe was designed even before it began to exist by the metaphysical might that resides in the 4D space in which our 3D universe began to emerge as a spherical hyper surface [6]. The creation of the universe, starting with no universe at all, thus becomes an ironic manifestation of conservation of energy as follows: while the universe is expanding mass-energy is generated at the exact same rate that negative gravitational energy accumulates due to the presence of mass-energy, $1 - 1 = 0$.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Jackson, J.D. (1973) Classical Electrodynamics (First Chapter). John Wiley and Sons, N.Y.

[2] Feynman, R.P. (1964) The Feynman Lectures on Physics. Vol. II, Chapter 20-4, Addison Wesley, Boston.

[3] Wood, C. (2005) What Goes on in a Proton. *Quanta Magazine*, **1**, 1-6.

[4] Feynman, R.P. (1949) Space-Time Approach to Quantum Electrodynamics. *Physical Review Journals Archive*, **76**, 769. https://doi.org/10.1103/PhysRev.76.769

[5] Hill, E.L. and Landshoff, R. (1938) The Dirac Electron Theory. *Reviews of Modern Physics*, **10**, 87, https://doi.org/10.1103/RevModPhys.10.87

[6] Halfon, Y.G. (2023) Unified Description of the Three Stable Particles in Self-Action Allows Determination of Their Relative Masses. https://www.researchgate.net/publication/376308847_Unified_description_of_the_three_stable_particles_in_self-action_allows_determination_of_their_relative_masses?channel=doi&linkId=657235fcea5f7f02054ced8a&showFulltext=true

[7] Halfon, Y.G. (2023) Formal Inclusion of God in the Realm of Cosmology. Reseachgate.