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Curved Space-Time at the Planck Scale

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Abstract

This paper presents a physically plausible and somewhat illuminating first step in extending the fundamental principles of mechanical stress and strain to space-time. Here the geometry of space-time, encoded in the metric tensor, is considered to be made up of a dynamic lattice of extremely small, localized fields that form a perfectly elastic Lorentz symmetric space-time at the global (macroscopic) scale. This theoretical model of space-time at the Planck scale leads to a somewhat surprising result in which matter waves in curved space-time radiate thermal gravitational energy, as well as an equally intriguing relationship for the anomalous dispersion of light in a gravitational field.

Keywords

Schwarzschild Space-Time, Continuum Mechanics, Planck Lattice, Gravitational Radiation

1. Introduction

In Einstein's theory of gravitation, matter and its dynamical interactions are based on the notion of an intrinsic geometric structure of the space-time continuum. The ideal aspiration of this theory is to describe space and time in terms of a four-dimensional manifold endowed with a certain intrinsic geometric structure subject to certain inherent purely geometrical laws that allow it to be an adequate model or picture of the real world around us, with all that it contains and its total behavior.

Yet for all its remarkable success and profound insight into the subtle nature of space and time, the formal description of space-time laid bare by general relativity does not address the mechanical properties (e.g., modulus of elasticity, strain, etc.) of space-time itself [1] [2]. This study is not intended to replace relativity theory, but rather to explore the dynamics of curved space-time at the

Planck scale within the theoretical framework of continuum mechanics, as well as evaluate the corresponding effects of gravity on the wave-particle duality of energy [3] [4] which lies at the heart of quantum mechanics.

Continuum mechanics is a branch of mechanics that deals with the mechanical behavior of materials modeled as a continuous mass rather than as discrete particles. Here, the concept of continuum is justified by the assumption that curved (non-Euclidean) space-time is composed of sufficiently closely spaced localized fields, so that its descriptive functions can be considered continuous and differentiable. In particular, we can define the stress at a given point in a gravitational field, thereby enabling us to apply calculus to the study of forces within a region of curved space-time. This definition and the subsequent application of calculus are associated with the work of August in-Louis Cauchy in the first half of the nineteenth century. Instead of studying atomic forces among individual particles, he introduced the concepts of stress and strain in a continuum, resulting in equations associated with the theory of elasticity.

This paper is organized as follows. In Section 2, based on Newton's gravitational potential and Hooke's law, we formulate the mechanical properties of Schwarzschild space-time at the Planck scale. In Section 3, we evaluate the wave-particle duality of energy in Schwarzschild space-time and deduce the de Broglie wavelength for a parcel of energy in non-Euclidean space-time. In closing, a brief discussion of the physical significance of the conceptual ideas and results derived in this paper are presented in Section 4. It is hoped that the ideas presented herein may spark the imagination of others working in this particular area of research [5] and ultimately lead to novel experiments with momentous observations that bring about a deeper and more rigorous understanding of the foundations of quantum space-time dynamics [6] [7].

2. A Mechanical Description of Schwarzschild Space-Time[1] at the Planck Scale

In the presence of a parcel or distribution of energy, the equilibrium or zero-point position of the oscillations of an event node [8] undergoes a linear elastic displacement from the center of the Planck unit cell in the opposite direction to the gravitational field line (or strength), as shown in **Figure 1**. It can be shown that the zero-point or gravitational displacement of an event node is proportional to the dilation of time. Event nodes located at distance R from the center of a parcel or spherical distribution of energy have equal zero-point displacements. Hence, a space-like surface containing the equilibrium position of the event nodes at a distance R from the center of a parcel or spherical distribution of energy is called a time-dilation surface. By virtue of the equivalence principle of relativity theory, the dilation of time corresponds to relative motion. In this case, the relative motion is produced by a field of force; therefore, the relative motion

[1]Empty space outside a static spherically symmetric parcel or distribution of energy whose radius is much larger than its Schwarzschild radius.

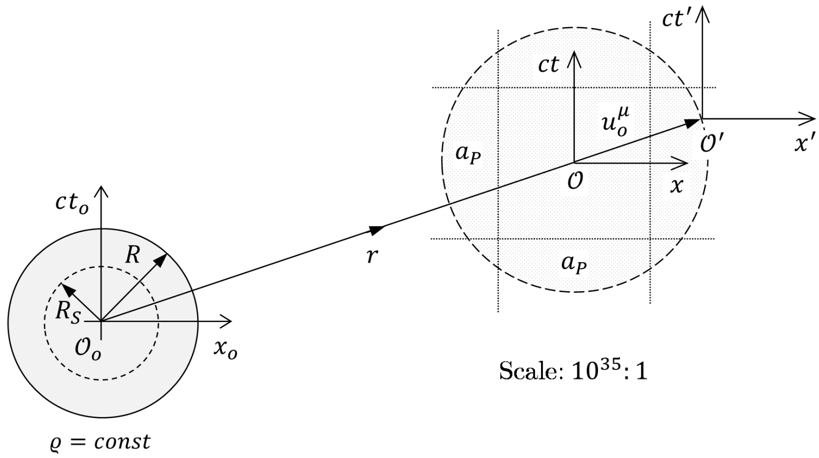

Figure 1. The zero-point displacement field (dotted region) of an event node passing through a point $\mathcal{O}$ in empty space a distance r outside a distribution of energy, of constant density ϱ. The dotted grid lines represent the Planck unit cell of side $a_P \approx 10^{-33}$ cm (the Planck length).

is one of acceleration. The force field which points in the direction of increasing time dilation or zero-point displacement corresponds to decreasing time-dilation surface areas[2]. Therefore, the field of force produced by the Planck lattice falls off as $1/R^2$ which is consistent with Newton's law of gravity. Hence, the restoring field of force produced by increasing zero-point deformations of the Planck lattice corresponds to Newton's gravitational field.

The constitutive equations for a linear elastic solid are related to the stress and strain tensors using the following expression:

$$\sigma_{\mu\nu} = C_{\mu\nu\lambda\rho}\varepsilon^{\lambda\rho},$$

which is known as generalized Hooke's law [9]. Hereafter, the summation from 0 to 3 is made, for the four dimensions of space-time, according to the repeated Greek indices with index 0 representing time and indices 1 through 3 representing the dimensions of space. The tensor of the elastic constant $C_{\mu\nu\lambda\rho}$ has 256 components. However, owing to the symmetry of both the stress $\sigma_{\mu\nu}$ and strain $\varepsilon_{\lambda\rho}$ tensors, at most 100 distinct elastic constants exist. The zero state of strain energy may be chosen arbitrarily, as the stress must vanish with the strains. The simplest form of the strain energy function for the Planck lattice that leads to a linear stress-strain relation has a quadratic form [9]

$$w_P = \frac{1}{2}C_{\mu\nu\lambda\rho}\varepsilon^{\mu\nu}\varepsilon^{\lambda\rho} = \frac{1}{2}\sigma_{\mu\nu}\varepsilon^{\mu\nu} = \frac{1}{2}\sigma_P\varepsilon_P,$$

where σ_P and ε_P are the stress and strain of the Planck lattice deformation, respectively. In general, for small oscillations of the Planck lattice that obey Hooke's law, the modulus of elasticity E_P of the Planck lattice and energy density ϱ of the oscillations can be readily shown to be related to the speed of the

[2]In theory, zero-point deformations of the Planck lattice are greatest near a parcel or distribution of energy.

corresponding waves[3] using the following equation [10]:

$$c = \sqrt{E_P/\varrho}\ .$$

In accordance with Hooke's law of elasticity, the stress-strain relationship of the Planck lattice is given by the well-known relation:

$$\sigma_P = E_P \varepsilon_P\ .$$

Thus, the strain energy density of the Planck lattice takes the form

$$w_p = \frac{1}{2}\varrho \varepsilon_P^2 c^2\ .$$

As previously postulated, the nature of the force produced by zero-point deformations of the Planck lattice is attractive; therefore, the strain potential of the Planck lattice is considered to be negative, $\Phi_P < 0$. In general, the potential is equal to the potential energy divided by the source of the force field (or charge), which is equivalent to the strain energy density w_P divided by the oscillation energy density. From these assertions, we can readily deduce that the strain potential energy density and strain potential of the Planck lattice are related by the following equation:

$$\Phi_P(\varepsilon_P) = -\frac{1}{\varrho} w_P = -\frac{1}{2}\varepsilon_P^2 c^2 \quad \text{for} \quad 0 \le \varepsilon_P < 1\ .$$[4]

We now suppose that the Planck strain potential of empty space-time outside a spherically symmetric parcel or distribution of energy of mass M and radius $R \gg R_S$ is equal to the Newtonian gravitational potential. Thus

$$\Phi_N(r) = -GM/r = -\frac{1}{2}\varepsilon_P^2 c^2\ ,$$

which gives

$$\varepsilon_P = \varepsilon_P(r) = \sqrt{R_S/r}\ ,$$

where $R_S = 2GM/c^2$ is the Schwarzschild radius and r is the distance between the center of mass of a parcel or distribution of energy and the center of a given Planck unit cell, as shown in **Figure 1** and G is Newton's universal gravitational constant. In general, the Planck strain is related to the zero-point displacement δ_o of an event node, as follows:

$$\varepsilon_P \equiv \delta_o/a_P\ ,$$

where $a_P \approx 10^{-33}$ cm is the Planck length. Hence, the zero-point displacement of the Planck lattice outside a static (non-rotating) spherically symmetric mass distribution M of radius R is given by

$$\delta_o = \delta_o(r) = \sqrt{\frac{R_S}{r}} a_P\ .$$

[3]Because the Planck lattice has zero rest mass, Planck lattice waves travel at the speed of light.

[4]This condition represents the elastic limit of the Planck lattice (or space-time) and can be considered a consequence of the causal structure of space-time, as prescribed by relativity theory [11].

It is worth noting here that

$$\delta_o = t_P \sqrt{2\left|\Phi_N\right|} = \mathrm{v}_e t_P ,$$

with $\mathrm{v}_e = \sqrt{2gr}$ the escape velocity of the mass distribution at a distance r from its center and $\mathrm{g} = GM/r^2$ is its Newtonian gravity. This relation resembles Newton's familiar equation for the change in position of a particle in uniform motion with constant velocity v_e over the Planck time $t_P = a_P/c$.

Now let us compute the gradient of the zero-point deformations of the Planck lattice near Earth, as follows:

$$-\nabla\Phi_N(r) = GM/r^2 = \frac{\delta_o}{t_P^2}\left(\frac{\partial\delta_o}{\partial r}\right)$$

$$\left(\frac{\partial\delta_o}{\partial r}\right)_{r=R_E} = t_P\sqrt{\frac{\mathrm{g}_E}{2R_E}} \sim 10^{-47} ,$$

where $\mathrm{g}_E = GM_E/R_E^2$ is Earth's gravity, M_E and R_E are the mass and radius of Earth, respectively. The extremely small zero-point deformation gradient of the Planck lattice near Earth is consistent with the remarkably high bending (or warping) stiffness of space-time predicted by the theory of general relativity[5].

At first sight, Einstein's law of gravitation does not resemble Newton's. To see a similarity, we must consider the components of the metric tensor $\mathrm{g}_{\mu\nu}$ as potentials describing the gravitational field. There are ten of them, instead of just one potential of the Newtonian theory. They describe not only the gravitational field, but also the system of coordinates. The gravitational field and the system of coordinates are inextricably mixed in Einstein's theory, as one cannot be described without the other. Because the Newtonian gravitational potential can be written in terms of the zero-point deformations of the Planck lattice to arrive at the Planck lattice potential, the metric tensor is a function of the gravitational potential and is thereby related to the Planck lattice potential, as well. Hence, in terms of the zero-point deformations of the Planck lattice, the Schwarzschild metric solution [12] of Einstein's field equation can be written as

$$\mathrm{g}_{\mu\nu} = \mathrm{diag}\left(-\left(1-\left(\delta_o/a_P\right)^2\right),\left(1-\left(\delta_o/a_P\right)^2\right)^{-1},r^2,r^2\sin^2\theta\right).$$

Here, we imagine that a curved space-time geometry, as defined by the metric tensor of relativity theory, is made up of event nodes displaced from the center of their corresponding Planck unit cells.

Because of the spherical symmetry of the zero-point (gravitational) displacement field in four dimensions, the magnitudes of the space and time components of the gravitational displacement field are considered to be equal, forming a four-dimensional Planck null hypersphere (or subspace), which implies that the deformation field of the Planck lattice is indeed light-like. Thus, for a zero-point displacement along the direction given by the unit vector $\boldsymbol{n}$ in space,

[5]The constant appearing in Einstein's field equation can be interpreted as the modulus of elasticity of the Planck lattice, that is $E_P = c^4/8\pi G \sim 10^{43}\ \mathrm{N/m^2}$, which represents the force per unit area required to give space-time unit curvature.

the four-vector for the gravitational displacement field is given by

$$u_o^{\mu} = \delta_o n^{\mu},$$

where $n^{\mu} n_{\mu} = 0$ so that n^{μ} is light-like, and hence $n^{\mu} = (1, \boldsymbol{n})$.

In accordance with the constitutive equations of continuum mechanics for a linear elastic solid, we compute the Eulerian strain tensor [10] for infinitesimal strains of the Planck lattice in Schwarzschild space-time

$$\varepsilon_{\mu\nu} = \frac{1}{2}\left(\eta_{\mu\alpha}\frac{\partial u_o^{\alpha}}{\partial x^{\nu}} + \eta_{\nu\beta}\frac{\partial u_o^{\beta}}{\partial x^{\mu}}\right) = \eta_{\mu\nu}\delta_o / r,$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ is the Minkowski metric. At the Planck scale, the geometry of space-time is flat (Euclidean); therefore, the Minkowski metric readily accommodates the geometry of space-time at the Planck length scale[6]. It immediately follows that the shear strains of Schwarzschild space-time are zero $\varepsilon_{\mu\nu} = 0$ for $\mu \neq \nu$. Therefore, in theory, for small zero-point deformations (where $\delta_o \ll a_P$) in Schwarzschild space-time, the Planck lattice experiences only normal strains $\varepsilon_{\mu\mu} \neq 0$.

Analogous to the stress-strain equations for an isotropic elastic material, the corresponding tensor form of the stress-strain equations for the Planck lattice is

$$\varepsilon_{\mu\nu} = \frac{1}{E_P}\left[(1+\nu_P)\eta_{\mu\lambda}\eta_{\nu\rho} - \nu_P\eta_{\mu\nu}\eta_{\lambda\rho}\right]\sigma^{\lambda\rho} = S_{\mu\nu\lambda\rho}\sigma^{\lambda\rho},$$

where $S_{\mu\nu\lambda\rho} = \frac{1}{E_P}\left[(1+\nu_P)\eta_{\mu\lambda}\eta_{\nu\rho} - \nu_P\eta_{\mu\nu}\eta_{\lambda\rho}\right]$ is the Planck lattice compliance tensor, which can be viewed as the space-time compliance tensor. For $r \gg R_S$, the tensor equation above can be written in matrix form as follows:

$$\begin{bmatrix}\epsilon_{00}\\ \epsilon_{11}\\ \epsilon_{22}\\ \epsilon_{33}\end{bmatrix} = \frac{1}{E_P}\begin{bmatrix}1 & -\nu_P & -\nu_P & -\nu_P\\ & 1 & -\nu_P & -\nu_P\\ & symm & 1 & -\nu_P\\ & & & 1\end{bmatrix}\begin{bmatrix}\sigma_{00}\\ \sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\end{bmatrix},$$

where $\varepsilon_{\mu\nu} = \sigma_{\mu\nu} = 0$ for $\mu \neq \nu$.

Of course, this matrix equation can be readily inverted to obtain the generalized stiffness form of Hooke's law for the Planck lattice, for $r \gg R_S$

$$\begin{bmatrix}\sigma_{00}\\ \sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\end{bmatrix} = \frac{E_P}{(1+\nu_P)(1-3\nu_P)}\begin{bmatrix}1-2\nu_P & \nu_P & \nu_P & \nu_P\\ & 1-2\nu_P & \nu_P & \nu_P\\ & symm & 1-2\nu_P & \nu_P\\ & & & 1-2\nu_P\end{bmatrix}\begin{bmatrix}\epsilon_{00}\\ \epsilon_{11}\\ \epsilon_{22}\\ \epsilon_{33}\end{bmatrix}.$$

This form of Hooke's law for the Planck lattice can be written in tensor form, as follows:

$$\sigma_{\mu\nu} = \frac{E_P}{1+\nu_P}\left[\eta_{\mu\lambda}\eta_{\nu\rho} + \frac{\nu_P}{1-3\nu_P}\eta_{\mu\nu}\eta_{\lambda\rho}\right]\varepsilon^{\lambda\rho} = C_{\mu\nu\lambda\rho}\varepsilon^{\lambda\rho},$$

[6]Curvature of spacetime geometry has no meaning at the fundamental (or Planck) length scale.

where $C_{\mu\nu\lambda\rho} = \frac{E_P}{1+\nu_P}\left[\eta_{\mu\lambda}\eta_{\nu\rho} + \frac{\nu_P}{1-3\nu_P}\eta_{\mu\nu}\eta_{\lambda\rho}\right]$ is the elastic modulus (or stiffness) tensor of the Planck lattice or space-time.

For a simple uniaxial state of stress[7] in the x^1 direction, the Poisson's ratio[8] of the Planck lattice can be determined using the following relationship

$$\varepsilon_{22} = \varepsilon_{33} = \varepsilon_{02} = \varepsilon_{03} = -\nu_P \varepsilon_{11} .$$

Since the total strain energy is constant, $\varepsilon_{ii} = -\sum_{j\neq i}\varepsilon_{\mu j}$ which gives $\nu_P = 1/4$.

Outside a parcel or distribution of energy, the gravitational displacement fields of the Planck lattice macroscopically form a perfectly elastic non-Euclidean space-time. Therefore, the shear modulus G_P is related to the corresponding Young's or elastic modulus E_P by the relation [10]

$$G_P = \frac{2}{5}E_P .$$

3. Compton Waves[9] in Schwarzschild Space-Time

The zero-point displacement of an event node outside a parcel or distribution of energy causes the amplitude of its vacuum or zero-point oscillations to increase. Because the energy of a wave is proportional to the amplitude [14], the oscillation energy of the event nodes, outside a parcel or distribution of energy, increases with increasing zero-point displacement. This agrees with the requirement that the ground state oscillation energy of the vacuum[10] corresponds to the minimum oscillation energy state of the Planck lattice. Thus, the zero-point oscillation frequency of the event nodes increases near a parcel or distribution of energy, which means time intervals in the rest frame of the gravitational source are increased (stretched). The dilation of time intervals in the presence of a gravitational field [15] is consistent with both relativity theory and experimental results.

Because clocks run at different rates when situated at different positions in a gravitational field, one may compare the local (proper) time interval $d\tau$ and the coordinate time interval dt and discover that they are related by the following equation:

$$d\tau(r) = 1 + \frac{\Phi(r)}{c^2}dt .$$

The potential $\Phi(r)$ corresponds to the familiar Newtonian gravitational potential

$$\Phi(r) = \Phi_N(r) = -GM/r ,$$

[7]Time is coupled to each dimension of space; therefore, in general, we expect $\varepsilon_{00} = \varepsilon_{ii}$ for uniaxial states of stress of the Planck lattice.

[8]Because the Planck lattice is imagined to be a "perfectly" elastic isotropic medium for which the Lame elastic constants λ and μ are equal, its Poisson's ratio is 0.25; like that of a Poisson (perfect) solid [13].

[9]Virtual event wave packets produced by coherent harmonic oscillations of the event nodes of the Planck lattice for a given parcel or distribution of energy [8].

[10]Empty flat space-time far away from any parcels or distributions of energy.

with r the radial position of a parcel of energy relative to the center of the gravitational source. It immediately follows from the geometric approach defined by the Schwarzschild solution determined from general relativity that the intrinsic coordinate (proper) time t_o for an observer near the source of a gravitational field $r > R_S$ and the coordinate time t for an observer at an arbitrarily large distance from the gravitational source $r' \gg r$ are related by the following equation:

$$t_o = t\sqrt{1 - \frac{R_S}{r}}.$$

We do not restrict generality by taking the direction of motion of a nonrelativistic[11] parcel of energy as the x-axis. The phase coherence of the oscillations of the event nodes and corresponding Compton wave [8] is imposed by the following principle of phase coherence[12]

$$\phi_o = \phi \to \nu_o t_o = \nu\left(t - \frac{x}{\mathrm{V}}\right),$$

with $\phi_o = \nu_o t_o$ the phase of the oscillations of the event nodes, $\phi = \nu\left(t - x/\mathrm{V}\right)$ the phase of the Compton wave, and V the phase velocity of the Compton wave. In accordance with the principle of phase coherence, we can now evaluate the phase of the Compton wave of a parcel of energy in relative motion in a gravitational field as follows:

$$\nu_o t_o = \nu_o t\sqrt{1 - \frac{R_S}{r} - \beta^2} = \frac{\nu_o}{\sqrt{1 - \frac{R_S}{r} - \beta^2}}\left[t - \left(\frac{R_S}{r} + \beta^2\right)t\right] = \nu\left(t - \frac{x}{\mathrm{V}}\right),$$

where $\beta = \mathrm{v}/c$ and the v is the speed of the particle relative to an observer at rest relative to the source of the gravitational field. Hence, the frequency ν of the particle's Compton wave, as observed in the reference frame of the gravitational source, is

$$\nu = \frac{\nu_o}{\sqrt{1 - \frac{R_S}{r} - \beta^2}},$$

and the phase velocity V of the particle's Compton wave for $r > R_S$ is

$$\left(\frac{R_S}{r} + \beta^2\right)t = \left(\frac{R_S}{r} + \beta^2\right)\frac{x}{\mathrm{v}} = \frac{x}{\mathrm{V}}$$

$$\Rightarrow \mathrm{V} = \frac{\mathrm{v}}{R_S/r + \beta^2},$$

where $x = \mathrm{v}t$.

According to relativity theory, the Compton wave of a parcel of energy is permitted to transport energy when the phase velocity is less than or equal to the speed of light $\mathrm{V} \le c$. Hence, it follows that Compton waves propagating in curved space-time can transport energy provided they satisfy the following condition:

[11]As used here this term is in reference to speeds slower than light $\mathrm{v} < c$.

[12]This condition is what Louis de Broglie (1924) referred to as "the theorem of phase harmony" [16].

$$R_S/r+\beta^2 \ge \beta .$$

Owing to the inherent oscillatory nature of Compton waves (or event nodes), the energy radiated is taken to be thermal gravitational radiation. As shown in Figure 2, a parcel or distribution of energy at rest or in uniform motion in a gravitational field radiates thermal gravitational energy except in certain instances where $r > 4R_S$.

Because of the dynamic behavior of the Planck lattice, which preserves the observed Lorentz symmetry of space-time, the phase velocity of the event node oscillations in vacuum $R_S/r \approx 0$ is equal to or less than the speed of light. Thus, empty space outside a parcel or distribution of energy radiates thermal gravitational energy [17]. In general, the gravitational charge of thermal gravitational radiation increases the zero-point displacement of neighboring event nodes, which increases the energy of the thermal gravitational radiation, further increasing the zero-point displacement of the neighboring event nodes ad infinitum. The nonlinear nature of this process leads to infinite (divergent) gravitational energy, similar to the ultraviolet catastrophe of the theory of electromagnetic blackbody radiation, first posited by Paul Ehrenfest in 1911. However, before the gravitational energy becomes infinite, the Schwarzschild limit is exceeded, and a black hole is formed. This undesired feature of divergent thermal gravitational radiation is remedied by Planck's radiation law [18], which describes the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature. Here, the radiation is gravitational and is produced by oscillating event nodes in motion relative to a parcel or distribution of energy. From this perspective, gravitational radiation in thermal equilibrium with the Planck lattice at a given temperature or oscillation energy is gravitational black-body radiation [19]. The Compton wave of a parcel of energy is composed of a localized group of superpositioned virtual event waves, where the phase speed of the Compton wave corresponds to the speed of the virtual event waves within its envelope, which propagates at the speed of the particle or group velocity.

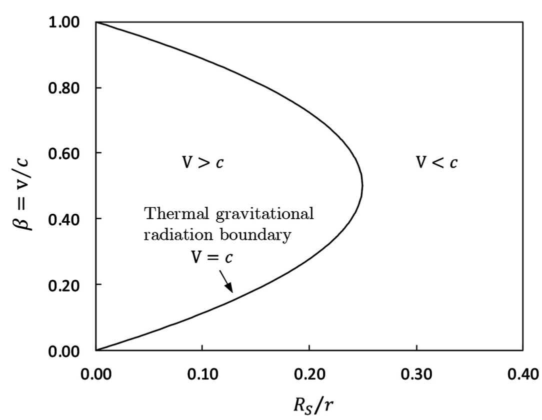

Figure 2. The phase velocity transition curve of the Compton wave defines the boundary for the emission of thermal gravitational radiation by a parcel or distribution of energy in Schwarzschild (or non-Euclidean) space-time.

Based on an understanding of the relationship between frequency and energy implied by the Planck-Einstein relationship

$$E = h\nu = mc^2,$$

the relativistic momentum of a parcel of energy with rest mass m_o is

$$p = \frac{m_o \mathrm{v}}{\sqrt{1-\beta^2}} = \frac{h\nu_o}{c}\left(\frac{\beta}{\sqrt{1-\beta^2}}\right),$$

with $m = m_o \big/ \sqrt{1-\beta^2}$ the relativistic mass of the particle. The relation for the phase velocity of a Compton wave in terms of its phase frequency and wavelength is given by $\mathrm{V} = \lambda\nu$. Thus, the wavelength of the Compton wave of a nonrelativistic particle in non-Euclidean space-time for $r > R_S$ has the form:

$$\lambda = \frac{h}{p}\left(\frac{\beta^2}{R_S/r+\beta^2}\right)\sqrt{1-\frac{R_S}{r\left(1-\beta^2\right)}}.$$

The general formula established for the de Broglie wavelength of a nonrelativistic parcel of energy in Schwarzschild space-time can be equally applied to corpuscles of light known as photons, which have zero rest mass. As the relative speed of a parcel of energy approaches the speed of light ($\mathrm{v} \approx c$) the phase velocity V for $r > R_S$ becomes

$$\mathrm{V} = \frac{c}{1+R_S/r}.$$

Thus, the relativistic momentum of the photon is given by

$$p_\gamma = m_\gamma c = \frac{h\nu_\gamma}{c} = \frac{h\mathrm{V}}{c\lambda_\gamma}$$

$$\Rightarrow \lambda_\gamma = \frac{h}{p_\gamma}\left(\frac{1}{1+R_S/r}\right),$$

where $m_\gamma c^2 = h\nu_\gamma$ is the relativistic energy of the photon and $\mathrm{V} = \lambda_\gamma \nu_\gamma$ is the relation for the phase velocity of the photon in terms of the phase frequency and wavelength of its associated Compton wave.

As shown in **Figure 3**, the phase velocity of a photon's Compton wave is never superluminal $\mathrm{v} \le c$; thus, in general, light quanta transmit both electromagnetic and gravitational energy while traveling in a gravitational field. In accordance with the customary preservation of the conservation of energy, the fraction of electromagnetic energy carried by a photon in a gravitational field is presumed to be proportional to the ratio of the phase velocity to the speed of light (or group velocity) V/c. Hence, as light approaches the event horizon of a Schwarzschild singularity, the amount of gravitational energy carried by light increases, whereas the amount of electromagnetic energy carried by light decreases. However, at large distances from the event horizon, where the gravitational field is small, almost all the energy transported by light is electromagnetic, which agrees with empirical observations.

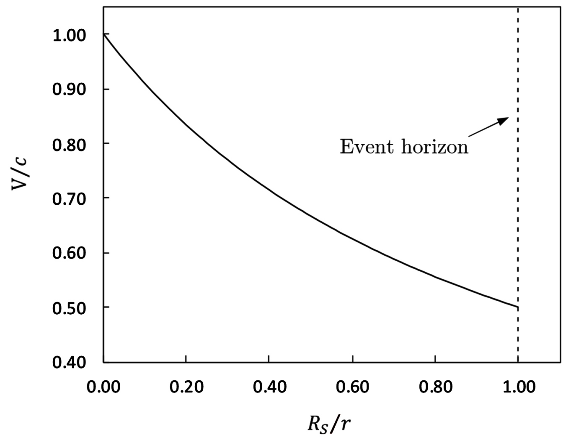

Figure 3. The anomalous dispersion[13] curve for light in a gravitational field. The phase velocity of the Compton wave of a photon in a gravitational field is subluminal $\text{V} < c$, especially near the event horizon of a black hole. Such an effect may play a significant role in the observed amplification of gravitational lensing [20] produced by galaxy clusters.

4. Conclusions

By describing Schwarzschild space-time in terms of a Planck lattice with unit cells and oscillating event nodes, the constitutive mechanical properties of space-time, in the form of the stress and strain tensors of the Planck lattice, were observed to follow directly from the underlying mechanical description of the gravitational displacement field. Furthermore, displacement of an event node from the origin of a Planck unit cell results in a restoring force field acting in the direction of increasing zero-point deformation of the Planck lattice. Thus, outside a parcel or distribution of energy, where $r \gg R_S$, Newtonian gravity naturally emerges from the dynamics of the Planck lattice.

For infinitesimal strains of the Planck lattice in Schwarzschild space-time, the gravitational displacement field was assumed to have a pure normal state of stress. Therefore, for weak static gravitational fields, no shear stress components are present in the Planck lattice stress tensor. However, for gravitational waves, dynamic (rotating) gravitational fields, as in the case of Kerr space-time [21], and or near gravitationally compact objects, such as the event horizon of a black hole or collapsing star, the adjacent gravitational displacement fields of the Planck lattice overlap $\delta_o > a_P/2$, in which case the shear components of the Planck lattice stress tensor no longer vanish $\sigma_{\mu\nu} \neq 0$ for $\mu \neq \nu$.

Although in this study we presumed that the gravitational displacement field of an event node possessed a continuous displacement spectrum, it should be noted that there is no fundamental law of physics that prevents it from having a discrete displacement spectrum. This mode of thinking leads to the usual quantum or nonclassical terms, such as the ground state and excited states, when referring to the quantum state of the gravitational displacement field of the Planck lattice. Moreover, a discrete zero-point displacement spectrum gives rise to a ra-

[13]When the phase velocity is less than the group velocity V < c anomalous dispersion occurs which results in the refraction or bending of light.

ther straightforward mechanism[14] for the absorption and emission of gravity quanta, which resembles that used for the absorption and emission of light quanta. Thus, in accordance with our Planck lattice conception of space-time, one may imagine that at each point in space surrounding a parcel or distribution of energy, there exist gravitons (which are hypothetical massless quanta of the gravitational field) coupled to the harmonic oscillators of the Planck lattice. In theory, the thermal gravitational radiation of the Planck lattice has a temperature that can be explicitly evaluated using a quantum statistical description of the Planck lattice oscillators. Therefore, as in the case of oscillating electrons in a metal radiating electromagnetic energy, oscillating gravitons may exist in curved space-time radiating gravitational energy in a similar fashion.

Conflicts of Interest

The author declares that they have no known competing financial interests or personal relationships that could have influenced the work reported in this study.

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