

Does QM Embedded in 5th Dimensional Embedding Allow for Classical Black Hole Ideas Only in Early Universe, Whereas Corda Special Relativity Plus QM May Eliminate Event Horizons for Black Holes after Big Bang?

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Abstract

We first look at the possibility that the ideas of event horizons for black holes may have their application only in early universe conditions whereas Corda's ground breaking work rejecting event horizons may be due to the formation of quantum mechanics free of an embedding in 5 dimensions allowing for a simpler more direct approach, which rejects the idea of a firewall. First, we present the idea of classical black hole physics applied only once as for the early universe, whereas in such a setting, there may be a way to present NLED and structure formation due to an initial entropy approach as outlined. Then the ideas of Corda's breakthrough are presented for the reasons he illuminated in his recent work, due to QM being fully formed separate from higher dimensional embedding after the initial evolution of the universe.

Keywords

QM, Black Hole Ideas, Special Relativity

1. Introduction and Summary as of the Ideas of This Document

We first present an outrageous early universe model involving mimicking early universe conditions, via more traditional black hole physics and state without reservation that after the creation of a universe that we are following Corda's break through [1] which eliminates completely the idea of a firewall, *i.e.* when QM is not embedded in a semi deterministic setting. Furthermore in order to take benefit of an effective firewall occurring ONCE at the beginning of creation, we also explore NLED cosmology physics [2].

The author then links to gravity due to adopting the fifth force formalism of Fishbach *et al.*, [3] which shows up in a (1988) Rencontres De Moriond 5th force—Neutrino physics school. A further talk by Fishbach in (2015) Rencontres De Moriond gives motivation to using Unnishkan's linkage of classical gravity with magnetism in a way which the author extends to the problem of not only gravity, but gravitons (normally thought of as usually QM) with E and M forces. Then there is a derivation of a linkage between the number of gravitons, a minimum grid size, and the time evolution of Hubbles parameter, to ascertain a minimum number, n, of initial gravitons produced, which in turns of Ng's infinite quantum statistics can be then a measure of entropy. This "count" of gravitons is compared with String theory versions of entropy, initially, as well as comments as to how to avoid having zero entropy initially. As to structure formation, we find that the stronger an early universe magnetic field is, the greater the likelihood of production of about 20 new domains of size 1/H, with *H* early universe Hubble's constant, per Planck time interval in evolution.

In doing so in the NLED section, we state that prior to the production of Corda non firewall black holes [1] that NLED processes create an enormous vacuum energy [2], for reasons which are part of our discussion. The author will then, after discussing Corda's black hole [1] revolutionary papers findings commence based on his own work, state that there is reason to believe that the cosmological constant, separate from Vacuum energy, will be associated for the DE problem,. As separate from the vacuum energy.

After this structure formation is formed, we state we are in the regime of physics as to a no firewall treatment of black hole physics as brought up by Dr. Corda [1].

2. Starting off with a Classical Black Hole Treatment of the Early Universe. This Would Be the Only Time When an Event Horizon Would Ever Be Entertained or Discussed

When initial radius $R_{\text{initial}} \rightarrow 0$ if Stoica [4] actually derived Einstein equations in a formalism which remove the big bang singularity pathology, then the reason for Planck length no longer holds. We present entanglement entropy in the early universe with a shrinking scale factor, due to Muller and Lousto [5], and show that there are consequences due to initial entangled $S_{\text{Entropy}} = 0.3r_H^2/a^2$ for a time dependent horizon radius r_H in cosmology, with (flat space conditions) $r_H = \eta$ for conformal time. Even if the 3 dimensional spatial length goes to zero, this construction preserves a minimum non zero Λ vacuum energy, and in doing so keep the bits, for computational bits cosmological evolution even if $R_{\text{initial}} \rightarrow 0$. We state that the presence of computational bits is necessary for

cosmological evolution to commence.

This article is to investigate what happens physically if there is a non pathological singularity in terms of Einsteins equations at the start of space-time. This eliminates the necessity of having then put in the Planck length since then ther would be no reason to have a minimum non zero length. The reasons for such a proposal come from [4] by Stoica who may have removed the reason for the development of Planck's length as a minimum safety net to remove what appears to be unadvoidable pathologies at the start of applying the Einstein equations at a space-time singularity, and are commented upon in this article.

 $\rho \sim H^2/G \Leftrightarrow H \approx a^{-1}$ in particular is remarked upon. The idea is that entanglement entropy will help generate bits, due to the presence of a vacuum energy, as derived at the end of the article, and the presence of a vacuum energy non zero value, is necessary for comsological evolution. Before we get to that creation of what is a necessary creation of vacuum energy conditions we refer to constructions leading to extremely pathological problems which [4] could lead to minus the presence of initial non zero vacuum energy. [6] also adds more elaboration on this.

Note a change in entropy formula given by Lee [7] about the inter relationship between energy, entropy and temperature as given by

$$m \cdot c^{2} = \Delta E = T_{U} \cdot \Delta S = \frac{\hbar \cdot a}{2\pi \cdot c \cdot k_{B}} \cdot \Delta S$$
(1)

As a reviewer has asked about Equation (1) and the inter relationship of a mass m, and acceleration, the key point of this review is to look at if gravitons have a mass, m, in the beginning, and if Equation (1) is used, which the mass of a graviton is proportional to the following

$$m = \frac{\Delta E}{c^2} = \frac{T_U \cdot \Delta S}{c^2} = \frac{\hbar \cdot a}{2\pi \cdot c^4 \cdot k_B} \cdot \Delta S$$
(1a)

The reason why the mass of a graviton is stated as given by Equation (1a) is to presume that the relationship given by Lee [7], as to any mass, is given by Equation (1) and Equation (1a) so we can relate any presumed mass linked to gravitons to change in entropy. As to acceleration appearing, the acceleration, $a \cong \frac{c^2}{\Delta x}$ was part of the formula given by Equation (1) and by default Equation (1a). and also by thermodynamic reasoning the generalized temperature

$$F_U = \frac{\hbar \cdot a}{2\pi \cdot c^2 \cdot k_B} \tag{1b}$$

If we assume, in the onset of expansion of the universe, that Equation (1b) holds, then we can review the application of Equation (1a) to graviton mass, *m*, as $m = \frac{\Delta E}{c^2} = \frac{T_U \cdot \Delta S}{c^2}$, and to have acceleration, given by $a \cong \frac{c^2}{\Delta x}$ as part of a definition of generalized temperature, given by Equation (1b).

Note that temperature is, in this presentation by Lee [7] presumably a constant initially, *i.e.* very hot, so then we are really in this presentation, assuming that the acceleration as given by $a \cong \frac{c^2}{\Delta x}$ is a constant, so in fact what we are actually reviewing through Equation (1a) is a direct relationship of mass as proportional to entropy, *i.e.* as

$$n \sim \Delta S$$
 (1c)

i.e. the mass of a graviton is presumed to be proportional to entropy. *i.e.* in choosing Equation (1c) we are leading up to one of the themes of this document which is that if entropy is proportional to information and note that later, we will be relating entropy, as given, to a numerical count factor. *i.e.* then in fact, this will lead to a re write of Equation (1c) to read as, if N(count) is a numerical count proportional to the change in Entropy, that [8]

$$m \sim \Delta S \sim N(\text{count}) \Rightarrow m_{\text{graviton}} \sim \frac{\Delta S}{N(\text{count})}$$
 (1d)

This assumes, that we are evaluating Equation (1b) as a constant. *i.e.* that the temperature be fixed, which is leading to the acceleration, which the referee was so concerned about, as a constant, *i.e.* via the relationship of looking at $a \cong \frac{c^2}{\Delta x}$ as an acceleration factor, and presumably that the delta *x* factor in acceleration is of the interval of Planck length.

Lee's formula is crucial for what we will bring up in the latter part of this document. Namely that changes in initial energy could effectively vanish if [4] is right, *i.e.* Stoica removing the non pathological nature of a big bang singularity. That is, unless entanglement entropy is used.

If the mass *m*, *i.e.* for gravitons is set by acceleration (of the net universe) and a change in entropy $\Delta S \sim 10^{38}$ between the electroweak regime and the final entropy value of, if $a \cong \frac{c^2}{\Delta x}$ for acceleration is used, so then we obtain

$$S_{\text{Today}} \sim 10^{88} \tag{2}$$

Then we are really forced to look at (1) as a paring between gravitons (today) and gravitinos (electro weak) in the sense of preservation of information.

Having said this note by extention $\rho \sim H^2/G \Leftrightarrow H \approx a^{-1}$. As ρ changes due to $\rho \sim H^2/G$ and $R_{\text{initial}} \sim \frac{1}{\#} \ell_{\text{Ng}} < l_{\text{Planck}}$, t hen *a* is also altered *i.e.* goes to zero.

What will determine the answer to this question is if $\Delta E_{\text{initial}}$ goes to zero if $R_{\text{initial}} \rightarrow 0$ which happens if there is no minimum distance mandated to avoid the pathology of singularity behavior at the heart of the Einstein equations. In doing this, we avoid using the energy $E \rightarrow 0^+$ situation, *i.e.* of vanishing initial space-time energy, and instead refer to a nonzero energy, with $\Delta E_{\text{initial}}$ instead vanishing. In particular, the Entanglement entropy concept as presented by Muller and Lousto [5] is presented as a partial resolution of some of the pathologies brought up in this article before the entanglement entropy section. No

matter how small the length gets, S_{entropy} if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, t, rescaled, does not go to zero. This preserves a minimum non zero Λ vacuum energy, and in doing so keep non zero amounts of initial bits, for computational bits cosmological evolution even if $R_{\text{initial}} \rightarrow 0$.

I think that the common confusion here, is that $R_{\text{initial}} \rightarrow 0$ refers to initial RADII and not to curvature, which was also one of the questions raised by the referee. $R_{\text{initial}} \sim \frac{1}{\#} \ell_{\text{Ng}} < l_{\text{Planck}}$ is a minimum radii and has nothing to do with curvature. This formula, which evidently confused referees, *i.e.* if # refers to a computational bits value which will show up in our manuscript, then our statement is that we have an initial radii of less than Planck Length. As given by

$$R_{\text{initial}} \sim \frac{1}{\#} \ell_{\text{Ng}} < l_{\text{Planck}}$$
(2a)

Is part of the build-up of information seen in Equation (3) and should be read by readers so as to understand the significance of what is in this Equation (2a). *i.e.* $l_{Ng} < l_{Planck}$ does not hold, in general, and we get Equation (2a) only if the # value is used which refers to a computational bits value.

We also need to review the ideas as given in [6] and [7].

Before doing that, we review Ng [8] and his quantum foam hypothesis to give conceptual underpinnings as to why we later even review the implications of entanglement.entropy.

We state unequivocally here, that Equation (2a) has # referring to a computational bits value which is Equation (3) and will be part of treating entropy and its evolution.

Note that this evaluation is preformed in the Planck time interval, and is the basis of evaluation by our paper.

i.e. the concept of bits and computations is brought up because of applying energy uncertainty, as given by [8] and the Margolis theorem appears to indicate that the universe could not possibly evolve if [1] is applied, in a 4 dimensional closed universe. This bottle neck as indicated by Ng's [5] formalism is even more striking in the author's end of article proof of the necessity of using entanglement entropy in lieu of the conclusion involving entanglement entropy, which can be non zero, even if $R_{initial} \rightarrow 0$ provided there is a minimum non zero time length.

1) Review of Ng, [8] with comments.

First of all, Ng refers to the Margolus-Levitin theorem with the rate of operations $\langle E/\hbar \Rightarrow \#$ operations $\langle E/\hbar \times \text{time} = \frac{Mc^2}{\hbar} \cdot \frac{l}{c}$. Ng wishes to avoid blackhole formation $\Rightarrow M \leq \frac{lc^2}{G}$. This last step is not important to our view point, but we refer to it to keep fidelity to what Ng brought up in his presentation. Later on, Ng refers to the # operations $\leq (R_H/l_P)^2 \sim 10^{123}$ with R_H the Hubble radius. Next Ng refers to the #bits $\propto [\text{#operations}]^{3/4}$. Each bit energy is $1/R_H$ with $R_H \sim l_P \cdot 10^{123/2}$.

The key point as seen by Ng [8] and the author is in

#bits
$$\sim \left[\frac{E}{\hbar} \cdot \frac{l}{c}\right]^{3/4} \approx \left[\frac{Mc^2}{\hbar} \cdot \frac{l}{c}\right]^{3/4}$$
 (3)

Assuming that the initial energy *E* of the universe is not set equal to zero, which the author views as impossible, the above equation says that the number of available bits goes down dramatically if one sets $R_{\text{initial}} \sim \frac{1}{\#} \ell_{\text{Ng}} < l_{\text{Planck}}$? Also Ng writes entropy *S* as proportional to a particle count via *N*.

$$S \sim N \cong \left[R_H / l_P \right]^2 \tag{4}$$

We rescale R_H to be

$$R_H\Big|_{\text{rescale}} \sim \frac{l_{\text{Ng}}}{\#} \cdot 10^{123/2} \tag{5}$$

The upshot is that the entropy, in terms of the number of available particles drops dramatically if # becomes larger.

So, as
$$R_{\text{initial}} \sim \frac{1}{\#} \ell_{\text{Ng}} < l_{\text{Planck}}$$
 grows smaller, as $\#$ becomes larger

a) The initial entropy drops.

b) The number of bits initially available also drops.

This directly ties in with the ideas of reference [6] which need to be seriously considered.

2) We state specifically that if we are doing such a derivation which is extremely complex that we are by necessity involving a re do of the basic uncertainty principle, *i.e.* see this

Begin with the starting point of [9] [10] and then the ideas of modifying the uncertainty principle as seen in [11] [12]

$$\Delta l \cdot \Delta p \ge \frac{\hbar}{2} \tag{6}$$

We will be using the approximation given by Unruh [11] [12],

$$(\Delta l)_{ij} = \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2}$$

$$(\Delta p)_{ij} = \Delta T_{ij} \cdot \delta t \cdot \Delta A$$

$$(7)$$

If we use the following, from the Roberson-Walker metric [13],

$$g_{tt} = 1$$

$$g_{rr} = \frac{-a^{2}(t)}{1 - k \cdot r^{2}}$$

$$g_{\theta\theta} = -a^{2}(t) \cdot r^{2}$$

$$g_{\phi\phi} = -a^{2}(t) \cdot \sin^{2}\theta \cdot d\phi^{2}$$
(8)

Following Unruh [11] [12], write then, an uncertainty of metric tensor as, with the following inputs

$$a^{2}(t) \sim 10^{-110}, r \equiv l_{P} \sim 10^{-35} \text{ meters}$$
 (9)

Then, the surviving version of Equation (6) and Equation (7) is, then, if $\Delta T_u \sim \Delta \rho$

$$V^{(4)} = \delta t \cdot \Delta A \cdot r$$

$$\delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \ge \frac{\hbar}{2}$$

$$\Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \ge \frac{\hbar}{V^{(4)}}$$
(10)

This Equation (10) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time [13] for the stress energy tensor as given in Equation (11) below.

$$T_{ii} = diag(\rho, -p, -p, -p)$$
(11)

Then

$$\Delta T_{tt} \sim \Delta \rho \sim \frac{\Delta E}{V^{(3)}} \tag{12}$$

Then, Equation (10) and Equation (11) and Equation (12) together yield

$$\delta t \Delta E \ge \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2}$$
Unless $\delta g_{tt} \sim O(1)$
(13)

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early universe will keep

$$\delta g_{tt} \neq g_{tt} = 1 \tag{14}$$

In fact, we have that from Giovannini [14], that if ϕ is a scalar function, and $a^2(t) \sim 10^{-110}$, then if

$$\delta g_{tt} \sim a^2 \left(t \right) \cdot \phi \ll 1 \tag{15}$$

Then, there is no way that Equation (15) is going to come close to $\delta t \Delta E \ge \frac{h}{2}$. Hence, the Mukhanov suggestion as will be discussed toward the end of this ar-

Hence, the Mukhanov suggestion as will be discussed toward the end of this ticle, is not feasible.

3. How We Can Justifying Writing Very Small $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$ Values

To begin this process, we will break it down into the following co ordinates

In the $rr, \theta\theta$ and $\phi\phi$ coordinates, we will use the Fluid approximation, $T_{ii} = diag(\rho, -p, -p, -p)$ [13] with

$$\delta g_{rr} T_{rr} \ge -\left| \frac{\hbar \cdot a^{2}(t) \cdot r^{2}}{V^{(4)}} \right|_{a \to 0} \to 0$$

$$\delta g_{\theta\theta} T_{\theta\theta} \ge -\left| \frac{\hbar \cdot a^{2}(t)}{V^{(4)}(1 - k \cdot r^{2})} \right|_{a \to 0} \to 0 \qquad (16)$$

$$\delta g_{\phi\phi} T_{\phi\phi} \ge -\left| \frac{\hbar \cdot a^{2}(t) \cdot \sin^{2} \theta \cdot d\phi^{2}}{V^{(4)}} \right|_{a \to 0} \to 0$$

If as an example, we have negative pressure, with T_{rr} , $T_{\theta\theta}$ and $T_{\phi\phi} < 0$, and $p = -\rho$, then the only choice we have, then is to set $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$, since there is no way that $p = -\rho$ is zero valued.

i.e. this is a semi classical embedding via a use of the modification of the HUP as given, as to how we could have a semi classical embedding of QM within a "higher dimensional" structure. Within all that we can then, ONLY, consider at the foundations of space-time consider an NLED structure for initial space-time

4. Introduction as to NLED Ideas If We Start off with a Semi Classical Treatment of Initial Conditions

We start off with a description of both the Fifth force hypothesis of Fishbach [1] [2] [3] [15] [16] [17] as well as what Unnishkan brought up in Rencontres De Moriond [4] [5] [9] [10] with one of the predictions dove tailing closely with use of Gravitons as produced by early universe phase transition behaviour, leading to how QM relates to a semi classical approximation for E and M and other physical processes. For the Fifth force used, we use the following from Fishbach [15], namely what is admittedly an oversimplified model, as

$$V(r) = -\frac{G_{\infty} \cdot m_i \cdot m_j}{r} \pm \frac{Q_i \cdot Q_j}{r} \cdot \exp(-r/\lambda)$$
(17)

This second term in the potential is going to have, here $Q_i \& Q_j$ fifth force charges we will outline as

$$Q_i \cdot Q_j / G_{\infty} \cdot m_i \cdot m_j \approx 10^{-1} - 10^{-3}$$
 (18)

We have that Unnishkan shared in Rencontres Du Moriond [4] [5] [9] [10] which is an extension of what he did in [5] [10], *i.e.* looking at, if $i_1 \& i_2$ are currents in electricity and magnetism, and $i_{1g} \& i_{2g} = m_1 v_1 \& m_2 v_2$ are the "Newtonian" "gravity" equivalent expressions, with m_1 mass 1, m_2 mass 2, and v_1 and v_2 velocities of the particles in question so that the following, up to a point holds

$$\left[\frac{i_{1} \cdot i_{2}}{r^{2}} = k \cdot \frac{(q_{1} \cdot v_{1})(q_{2}v_{2})}{r^{2}}\right]_{\text{E\&M}} \sim \left[\frac{G}{c^{2}} \cdot \frac{i_{1g} \cdot i_{2g}}{r^{2}} = \frac{G}{c^{2}} \cdot \frac{(m_{1} \cdot v_{1})(m_{2}v_{2})}{r^{2}}\right]_{\text{Gravity}}$$
(19)

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\Phi_N}{c^2} \cdot \frac{\mathrm{d}v_i}{\mathrm{d}t}$$
(20)

The above relationship with its focus upon interexchange relations between gravity and magnetism is in a word focused upon looking at, if *A*, the nominal

vector potential used to define the magnetic field as in the Maxwell equation, the relationship we will be using at the beginning of the expansion of the universe, is a variation of the quantized Hall effect, *i.e.* from Barrett [6] [18], the current I about a loop with regards to electronic energy U, of a loop with the A electromagnetic vector potential going through the loop is given by, if L is a unit spatial length, and we approximate the beginning of the universe as having some of the same characteristics as a quantized Hall effect, then, if n is a particle count of some sort, then

$$I(\text{current}) = (c/L) \cdot \frac{\partial U}{\partial A} \Leftrightarrow A = n \cdot \hbar \cdot c/e \cdot L$$
(21)

We will be taking the right hand side of the A field, in the above, and approximate Equation (20) as given by

$$\frac{\mathrm{d}A}{\mathrm{d}t} \approx \frac{\mathrm{d}n}{\mathrm{d}t} \cdot \left(\hbar \cdot c/e \cdot L\right) \tag{22}$$

Then, we have an approximation for writing [9] [10]

$$\frac{\mathrm{d}A}{\mathrm{d}t} \approx \frac{\mathrm{d}n}{\mathrm{d}t} \cdot \left(\hbar \cdot c/e \cdot L\right) \equiv \frac{\Phi_N}{c^2} \cdot \frac{\mathrm{d}v_i}{\mathrm{d}t}$$

$$\Leftrightarrow \Phi_N \approx \frac{\mathrm{d}n}{\mathrm{d}t} \cdot \left(\hbar \cdot c^3/e \cdot L\right) / \frac{\mathrm{d}v_i}{\mathrm{d}t}$$
(23)

This also involves use of [6] [19].

Equation (23) needs to be interpolated, up to a point. *i.e.* in this case, we will conflate the *n*, here as a "graviton" count, initially, *i.e.* the number of early universe gravitons, then assume that $\frac{dv_i}{dt}$ is a net acceleration term which will be linked to the beginning of inflation, *i.e.* that we look then at Ng's "infinite" quantum statistics [7] [8], with entropy given as, initially a count of gravitons, with \mathbb{N} a generalized count. Then, if $\mathbb{N} \doteq n$ (particles), and we refer to the *n* of Equation (21) to Equation (23) as being the same as \mathbb{N} , keeping in mind some pitfalls of entropy in spacetime considerations as given in [8]

$$S \sim \mathbb{N} \approx \mathbb{N}_{\text{Graviton-count}} (\inf)$$
 (24)

We will elaborate upon this treatment of entropy in our derivations, as well as tie it in with some issues as to the uncertainty principle first elucidated in [20] in our minimization of energy and its tie in to presumed graviton physics. We should though link our work above to near singular physical spacetime and for that we will reference.

5. Entropy, Its Spatial Configuration near a Singularity and How We Use This Definition to Work in Effects of Non Linear Electrodynamics

The usual treatment of entropy, if there is the equivalent of an event horizon is, that (Padmanabhan) [10] [21] with r_{critial} to be set at the end of the article, with suggestions for future work. And *L* in Equation (23) is of the order of magnitude

proportional to L_p . *i.e.* also to be set at the end of this article, *i.e.* we will suggest a formal relationship between L and L_p . Here we leave this as to be a determined parameter

$$S(\text{classical entropy}) = \frac{1}{4L_P^2} \cdot \left(4\pi r_{\text{critial}}^2\right) \Leftrightarrow \text{Energy} \equiv \frac{c^4}{2G} \cdot r_{\text{critial}}$$
(25)

If so, then we have that from first principles, (and here we also will set $\frac{dr_{critical}}{dt}$ formally at the end of the paper, with suggested updates as far as an investigation)

$$\frac{\mathrm{d}n}{\mathrm{d}t} \sim 2\pi L_p^{-1} r_{\mathrm{critical}} \cdot \frac{\mathrm{d}r_{\mathrm{critical}}}{\mathrm{d}t}$$
(26)

Then Equation (23) is re written in terms of [4] [5] [9] [10] adopted formulation as given by

$$\Phi_{N} \approx \frac{dn}{dt} \cdot \left(\hbar \cdot c^{3}/e \cdot L\right) / \frac{dv_{i}}{dt} \propto 2\pi \frac{r_{\text{critical}}}{L_{P}} \cdot \frac{dr_{\text{critical}}}{dt} \cdot \left(\frac{dv_{i}}{dt}\right)^{-1} \left(\hbar \cdot c^{3}/e \cdot L\right)$$
(27)

The following parameters will be identified, *i.e.* what is $\frac{dv_i}{dt}$, what is *L*, and what is r_{critical} . These values will be set toward the end of the manuscript, with the consequences of the choices made discussed in this document as suggested new areas of inquiry. However, Equation (27) will be linkable to re writing Equation (20) as

$$\frac{\mathrm{d}A}{\mathrm{d}t} \sim 2\pi \frac{r_{\mathrm{critical}}}{L_P} \cdot \frac{\mathrm{d}r_{\mathrm{critical}}}{\mathrm{d}t} \cdot \left(\hbar \cdot c^2 / e \cdot L\right) \tag{28}$$

If $\frac{dr_{\text{critical}}}{dt}$ is ALMOST time independent, as we will assert in the end of our paper, Equation (28) will then lead to a primordial value of the magnitude of the *A* vector field as

$$A \sim t \cdot \left[2\pi \frac{r_{\text{critical}}}{L_P} \cdot \frac{\mathrm{d}r_{\text{critical}}}{\mathrm{d}t} \cdot \left(\hbar \cdot c^2 / e \cdot L \right) \right] + \text{H.O.T.}$$
(29)

If so, then the *E* field up to a point will be

$$E \sim -\nabla \phi - c^{-1} \cdot \partial_t A$$

$$\sim -c^{-1} \cdot \left[\frac{2\pi}{L_p} \cdot \frac{\mathrm{d}r_{\mathrm{critical}}}{\mathrm{d}t} \cdot \left(\hbar \cdot c^2 / e \cdot L \right) \cdot \left(r_{\mathrm{critical}} + t \cdot \frac{\mathrm{d}r_{\mathrm{critical}}}{\mathrm{d}t} \right) \right] - \nabla \phi$$
(30)

To reconstruct ϕ we have that we will use

$$\nabla \cdot A = -c^{-1} \cdot \frac{\partial \phi}{\partial t} \tag{31}$$

Then

$$\phi \sim -t^2 \cdot \left[\frac{\pi}{L_P} \cdot \frac{\mathrm{d} r_{\mathrm{critical}}}{\mathrm{d} t} \cdot \left(\hbar \cdot c^2 / e \cdot L \right) \right]$$
(32)

If so, then in Equation (30) becomes

$$E \sim -c^{-1} \cdot \left[\frac{2\pi}{L_P} \cdot \frac{\mathrm{d}r_{\mathrm{critical}}}{\mathrm{d}t} \cdot \left(\hbar \cdot c^2 / e \cdot L \right) \cdot \left(r_{\mathrm{critical}} + t \cdot \frac{\mathrm{d}r_{\mathrm{critical}}}{\mathrm{d}t} \right) \right]$$
(33)

The density, then is read as

$$\rho = -\frac{1}{4\pi c^2} \cdot \frac{\partial^2 \phi}{\partial t^2} \sim \frac{1}{2L_P} \cdot \frac{\mathrm{d}r_{\mathrm{critical}}}{\mathrm{d}t} \cdot \left(\hbar \cdot c^2 / e \cdot L\right) \tag{34}$$

The current we will work with, is also then linkable to, by order of magnitude similar to Equation (34) of

$$J = \frac{1}{4\pi c} \cdot \frac{\partial^2 A}{\partial t^2} \sim \frac{2}{L_P} \cdot \left(\frac{\mathrm{d}r_{\mathrm{critical}}}{\mathrm{d}t}\right)^2 \cdot \left(\hbar \cdot c/e \cdot L\right)$$
(35)

We also need to look at [11] [22].

Then we get an effective magnetic field, based upon the NLED approximation given by Corda *et al.* [12] [23] of

$$\rho_{\gamma} = \frac{16}{3} \cdot c_{1} \cdot B^{4} \sim \frac{1}{2L_{P}} \cdot \frac{dr_{\text{critical}}}{dt} \cdot \left(\hbar \cdot c^{2}/e \cdot L\right)$$

$$\Leftrightarrow B^{4} \sim \frac{3}{32L_{P} \cdot c_{1}} \cdot \frac{dr_{\text{critical}}}{dt} \cdot \left(\hbar \cdot c^{2}/e \cdot L\right)$$

$$\Leftrightarrow B_{\text{initial}} \sim \left(\frac{3}{32L_{P} \cdot c_{1}} \cdot \frac{dr_{\text{critical}}}{dt} \cdot \left(\hbar \cdot c^{2}/e \cdot L\right)\right)^{1/4}$$
(36)

Then we can also talk about an effective charge of the form, given by applying Gauss's law to Equation (34) of the form

$$Q = \varepsilon_0 \oint_S E \cdot n \cdot da$$

$$= \int_V \rho_\gamma dV \sim \frac{2\pi r_{\text{critical}}^3}{3L_P} \cdot \frac{dr_{\text{critical}}}{dt} \cdot \left(\hbar \cdot c^2 / e \cdot L\right)$$
(37)

This charge, Q, so presented, will be part of the effective 5th force [15] [16] [17], as to linking E and M and gravity, of Equation (17) which we will relate to our further derivational work done in this paper. Furthermore, the critical value of r_{critical} which will be made explicit in this paper, as well as L, and $\frac{dr_{\text{critical}}}{dt}$ as well as

Energy ~
$$\rho_{\gamma} \cdot r_{\text{critial}}^3 = \frac{16}{3} \cdot c_1 \cdot r_{\text{critial}}^3 \cdot B^4 \sim \frac{r_{\text{critial}}^3}{2L_p} \cdot \frac{\mathrm{d}r_{\text{critical}}}{\mathrm{d}t} \cdot \left(\hbar \cdot c^2/e \cdot L\right) \sim \frac{c^4}{2G} \cdot r_{\text{critial}}$$
(38)

This will lead to an evaluation of r_{critial} as

$$r_{\text{critial}} \sim L_p \cdot \frac{c \cdot e \cdot L}{\sqrt{\frac{\mathrm{d}r_{\text{critical}}}{\mathrm{d}t} \cdot (\hbar \cdot G)}} \propto L_p$$
(39)

The value of $\frac{dr_{\text{critical}}}{dt} \sim c$ (speed of light), and by Padmabhan [21], $G\hbar = L_p^2 c^3$, so then most likely

$$L \sim L_{P}$$

$$\Phi_{N} \approx \frac{2\pi}{L_{P}} \cdot \left(\frac{\mathrm{d}v_{i}}{\mathrm{d}t}\right)^{-1} \left(\hbar \cdot c^{4}/e\right)$$

$$S\left(\text{initial entropy}\right) = \frac{1}{L_{P}^{2}} \cdot \left(\pi L_{P}^{2}\right) \sim n_{\text{initial}} \neq 0$$

$$\Leftrightarrow 3 < n_{\text{initial}} < 4(?)$$

$$Q \sim \frac{2\pi L_{P}}{3} \cdot \left(\hbar \cdot c^{3}/e\right) \neq 0$$

$$\Leftrightarrow r_{\text{critial}}^{2} \sim n_{\text{initial}} \frac{L_{P}^{2}}{\pi} \sim L_{P}^{2} \qquad (40)$$

$$\Leftrightarrow E_{\text{initial}} \equiv \frac{c^{4}}{2G} \cdot r_{\text{critial}} \sim \frac{c^{4} L_{P}}{2G} \sqrt{\frac{n_{\text{initial}}}{\pi}}$$

This also will involve [21] [22] [23] [24].

These values of Equation (40) will up to a point be used to identify fillers into Equation (36) and Equation (37) of this document.

6. Gravitons, and All That

Equation (40), which has the influence of NLED in it, will be useful when ascertaining what would be a way to determine necessary and sufficient conditions for a massive graviton to exist. To do so, we will look first at Linde (Les Houches, 2013), whom wrote of the probability of creation of a closed universe as given by [11] [22]

$$P(\text{probability}) \sim \exp(-24\pi^2/V(\text{potential}))$$

$$\Leftrightarrow V(\text{potential}) \sim \text{Energy}(\text{Planck})$$
(41)

The potential energy, so identified in Equation (41) is none other than the one used by Padmanbhan [21] in which the H so identified is the Hubble "constant" parameter, which actually changes over time. In this case, the potential so identified in Equation (41) is given by

$$V \sim 3H^2 M_{\text{Planck}} \cdot \left(1 + \left(\dot{H}/3H^2\right)\right) \tag{42}$$

Here, if N is an integer number for dimensionality of space-time, and [21]

$$H = \dot{a}(t)/a(t) \& a(t) \sim t^{N}$$

$$\Leftrightarrow V \sim 2M_{\text{Planck}} \cdot N^{2}/t^{N}$$
(43)

If so, then if we have V as proportional to an energy E, then we can by the Heisenberg uncertainty principle be looking at a minimum uncertainty principle situation of [24]

$$\Delta E \Delta t = \hbar \tag{44}$$

Then, if
$$\Delta t = t$$
 (minimum), and $\Delta E = E_{\text{initial}} \equiv \frac{c^4}{2G} \cdot r_{\text{critial}} \sim \frac{c^4 L_P}{2G} \sqrt{\frac{n_{\text{initial}}}{\pi}}$

$$\Delta t = \hbar \left/ \frac{c^4 L_P}{2G} \sqrt{\frac{n_{\text{initial}}}{\pi}} = t_{\min} \right.$$
(45)

Now, by Valev, [13] [25] at the start of inflation, and this is before massive red shifting

$$m_{\text{graviton}} \sim \frac{\hbar H}{c^2} \sim \frac{\hbar N}{c^2 t_{\min}} \sim \frac{2GN}{c^6 L_P^2 \sqrt{\frac{n_{\text{initial}}}{\pi}}} \sim 10^{-61} \text{ grams}$$

$$\lambda_{\text{graviton}} \sim \frac{c}{H} \sim \frac{c \cdot t_{\min}}{N} \sim \frac{10}{N} \cdot L_p \sim \frac{1.61}{N} \times 10^{-34} \text{ meters} \qquad (46)$$

$$f (\text{frequency})_{\text{graviton}} \sim \frac{1.8 \times 10^{36}}{N} \text{ Hertz}$$

Inflation would reduce the frequency by 26 orders or so of magnitude (massive red shifting) [14] [26]

$$f(\text{frequency})_{\text{eraviton}} [\text{after-inf}] \sim 10^{10} \text{ Hertz}$$
 (47)

The difference in red shifted frequencies (a huge 26 order of magnitude reduction in frequency) due to inflation would be in tandem with what we will be identifying as structure formation issues, which are highlighted below.

7. Formation of Structure Due to NLED Formalism

This paper has several routes as to identifying NLED phenomenon pertinent to cosmology structure formation. First we look at what Mukhanov [15] [27] writes as far as structure formation. Mainly that there is a formulation of what is called self reproduction of inhomogeneity in terms of early universe conditions [15] [27]. In this, the starting point is if one used the meme of chaotic inflation, *i.e.* inflation generated by a potential of the form as given by Guth [26] [14] as well as Mukhanov [27] [15]

$$V(\text{potential}) \sim \phi^2 \tag{48}$$

In this, Mukhanov [15] [27] write that one can look at a scalar field at the end of (chaotic) inflation, with an amplitude given by, with ϕ_i for the initial value of the inflaton such that (where *m* will be determined by NLED inputs to be brought up later)

$$\delta_{\phi}^{Max} \sim m \cdot \phi_i^2 \tag{49}$$

In terms of the initial inflaton, inhomogenities do not form if the initial inflaton is bounded [15] [27] as given by

$$n^{-1} > \phi_i > m^{-1/2} \tag{50}$$

This leads to (low?) inhomogeneity in the space-time generated by inflation. Inflation is eternal [15] [27] if there is only the inequality

$$\phi_i > m^{-1/2}$$
 (51)

8. NLED Applied to Equation (51) Plus Details of Structure Formation Added

What we will do is to look at the following treatment of mass, and this will be

our starting point. *i.e.* we will be looking at, if l_p is Planck length, and $\alpha > 0$, then

$$m \sim 10^{\alpha} \cdot l_p^3 \cdot \rho(\text{density})$$
 (52)

Then we can consider the following formulation of density given below.

If we do not wish to consider a rotating universe, then Camara *et al.*, [16] [28] has an expression as to density, with a B field contribution to density, and we also can used the Weinberg result [4] of scaling density with one over the fourth power of a scale factor, which we will remark upon in the general section, as well the Corda and Questa result of [12] [23] for density of (note reference [12] [23] is for a star, whereas[16] [28] is for a universe).

In addition, Corda, and others in [12] [23] use quintessential density to falsify the null energy condition of a Penrose theorem cited in [29], Further details of what Penrose was trying to do as to this issue of GR, can be seen in [29], and to answer how to violate the null energy condition, one should go to [29] for quintessential density defined, Then in both the massive star and the early universe, the density result below is applicable [12] [23].

$$\rho_{\gamma} = \frac{16}{3} \cdot c_1 \cdot B^4 \tag{53}$$

Keeping in mind what was said as to choices of what to do about density, and its relationship to Equation (52) above, we then can reference what Mukhanov [27] says about structure formation as follows, namely look at how a Hubble parameter changes with respect to cosmic evolution. It changes with respect to H_{today} being the Hubble parameter in the recent era, and the scale factor *a*, with this scale factor being directly responsive to changes in density according to [17] [30], *i.e.*

$$o \sim a^{-4} \tag{54}$$

In the next section, we will examine how [3] suggests how to vary the scale factor cited in Equation (54), and we will in this section take note of what the scale factor does to the Hubble parameter given in Equation (55) below, and then in the section afterwards review a possible reconciliation of what Equation (53) and Equation (54) say about defining early universe parameters. But to know why we are doing it, we should take into consideration what happens to the Hubble parameter, as given below [27]

$$H \sim H_{\rm today} / a^{3/2} \tag{55}$$

According to [15] [27] inhomogeneous patches of space time appear in a causal region of space time for which [15] [27]

Causal domain ~
$$H^{-1}$$
~ $1/(H_{today}/a^{3/2})$ (56)

Furthermore, [15] [27] states that about 20 such domains are created in a Hubble time interval $\Delta t_H \propto H^{-1}$ *i.e.* As a function of say 10^{α} times Planck time, for a domain size given by Equation (56) above and that this requires then

a clear statement as to how the scale factor changes, due to considerations given by [3] and reconciling the density expression given in Equation (53) and Equation (54) above.

9. Showing a Non Zero Initial Radius of the Universe Due to Non Linear Space-Time E&M

What we are asserting is, in [16] [28] there exists a scaled parameter λ , and a parameter a_0 which is paired with α_0 . For the sake of argument, we will set the $a_0 \propto \sqrt{t_{\text{Planck}}}$, with $t_{\text{Planck}} \sim 10^{44}$ seconds. Also, Λ is a cosmological "constant" parameter which is described later, as in quintessence, via reference [17] [29], and is in [16] [28] via:

$$\alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}} B_0 \tag{57}$$

$$\lambda = \Lambda c^2 / 3 \tag{58}$$

Then if, initially, Equation (58) is large, due to a very large Λ the time, given in Equation (53) of [15] is such that we can write, most likely, that even though there is an expanding and contracting universe, that the key time parameter may be set, due to very large Λ as

$$t_{\rm min} \approx t_0 \equiv t_{\rm Planck} \sim 10^{-44} \, \rm s \tag{59}$$

Whenever one sees the coefficient like the magnetic field, with the small 0 coefficient, for large values of Λ , this should be the initial coefficient at the beginning of space-time which helps us make sense of the nonzero but tiny minimum scale factor [16] [28]

$$a_{\min} = a_0 \cdot \left[\frac{\alpha_0}{2\lambda} \left(\sqrt{\alpha_0^2 + 32\lambda\mu_0 \omega B_0^2} - \alpha_0 \right) \right]^{1/4}$$
(60)

The minimum time, as referenced in Equation (59) most likely means, due to large Λ that Equation (60) is of the order of about 10⁻⁵⁵, *i.e.* 33 orders of magnitude smaller than the square root of Planck time, in magnitude. We next will be justifying the relative size of the Λ .

10. Showing How to Obtain a Varying Λ with a Large Initial Value and Its Relationship to Obtaining a Scale Factor Value for the Early Universe via NLED Methods

Non withstanding the temperature variation in reference [17] [29] for the cosmological Hubble parameter, we also can reference what is done in reference [15] [28] namely due to

$$\Lambda(t) \sim \left(H_{\text{inflation}}\right)^2 \tag{61}$$

1) In short, what we obtain, via looking at due to [8] [31], that Equation (61) is also equivalent to

$$\Lambda_{Max} \sim c_2 \cdot T^{\beta}_{\text{temperature}} \tag{62}$$

Comparing Equation (61) and Equation (62) above, leads to the following constraints, *i.e.*

$$\left(\rho \sim a^{-4}\right)^{-1} \sim a^{4} \sim \frac{16}{3} \cdot c_{1}^{-1} \cdot B^{-4} \sim a_{0}^{4} \left[\frac{\alpha_{0}}{2\lambda} \left(\sqrt{\alpha_{0}^{2} + 32\lambda\mu_{0}\omega B_{0}^{2}} - \alpha_{0} \right) \right]$$
(63)

The above relationship will argue in favor of a large value for Equation (62) and Equation (63) B field and also the cosmological "constant" parameterized in Equation (61) and Equation (62), *i.e.* once fully worked out, the allowed values of B, for initial conditions will be large but tightly constrained, and this in turn will allow for Equation (63) having initially extremely small inhomogeneity behavior, in line with being proportional to the inverse of an allowed Hubble parameter based upon Equation (65) later on. Note that from [18] [32] we have

$$\frac{\Delta H}{H} \sim \Omega_m^2 h^2 \Delta_{\Re} \sim 10^{-5} \tag{64}$$

Here, we have that if there is a flat universe, that according to Guth [19] [33] and taking note of

$$H^2 = \frac{8\pi}{3} \cdot \rho \tag{65}$$

Roughly put, what we are predicting is, that if we use what Lloyd wrote, namely [20] [34] as well as use the magnetic field relations to density brought up in Equation (53). This is also in part related to the number of gravitons which could be expected as given by Peebles [21] [35], *i.e.* if one has a density related to energy via $\rho \propto \hbar \cdot \omega_{\text{Graviton}} \cdot V^{-1}$ (Volume) $\Leftrightarrow \hbar \cdot \omega_{\text{Graviton}} \sim \rho \cdot V^{+1}$ (Volume). Then one can write, say by using the approximation given by Peebles [21] [35]

$$\mathbb{N}_{\text{graviton}} = \text{graviton} \# \sim \left(\left(\exp\left[\hbar \cdot \omega_{\text{graviton}} / k_B T \right] \right) - 1 \right)^{-1} \\ \propto \left(\left(\exp\left[\rho \cdot V_{\text{Volume}} \cdot a_{\text{initial}} \left(t \right) / k_B \right] \right) - 1 \right)^{-1}$$
(66)

If we have such a treatment of information as given by Lloyd [20] [34], plus the above, we can estimate that there is a fluctuation due to early universe cosmology along the lines of, if we have a base line number for initial (expansion) value of the Hubble parameter, we call $H_{\text{base line}}$, as a starting point for an expanding universe, and with #operations, as given by Lloyd [34] as a function of entropy, initially. So then, in terms of what may be generated and show up in the CMBR we may see

$$\Delta H (\text{thermal}) \sim H_{\text{base line}} \cdot (\# \text{operations})^{1/4} \cdot 10^{-5} \cdot \sqrt{t/t_{\text{Planck}}}$$
(67)

The number of gravitons, as given by Equation (66) is significant, since we have, if we look at say what constitutes a contribution from $\rho \cdot V_{\text{Volume}}$, and from there, given a value of $H_{\text{base line}}$ according to the following procedure

$$H_{\text{initial}} \sim \sqrt{\frac{8\pi k_B}{12 \cdot V_{\text{Volume}} \cdot a_{\text{initial}}}} \cdot \left(\log \left[1 + \frac{1}{\mathbb{N}_{\text{graviton}} \left(\text{initial} \right)} \right] \right)$$
(68)

For the sake of simplicity, we will have, then

$$H_{\text{initial}} \approx H_{\text{base line}} \Rightarrow H_{\text{base line}} \sim \sqrt{\frac{8\pi k_B}{12 \cdot V_{\text{Volume}} \cdot a_{\text{initial}}}} \cdot \left(\log\left[1 + \frac{1}{\mathbb{N}_{\text{graviton}}}\left(\text{initial}\right)\right]\right)$$

& $\Delta H (\text{thermal}) \sim \sqrt{\frac{8\pi k_B}{12 \cdot V_{\text{Volume}} \cdot a_{\text{initial}}}} \cdot \left(\log\left[1 + \frac{1}{\mathbb{N}_{\text{graviton}}}\left(\text{initial}\right)\right]\right)$
 $\cdot (\# \text{operations})^{1/4} \cdot 10^{-5} \cdot \sqrt{t/t_{\text{Planck}}}$ (69)
& $\# \text{operations} \sim \left(\mathbb{N}_{\text{graviton}}\left(\text{initial}\right)\right)^{4/3}$
 $\Rightarrow \Delta H (\text{thermal}) \sim \sqrt{\frac{8\pi k_B}{12 \cdot V_{\text{Volume}} \cdot a_{\text{initial}}}} \cdot \left(\log\left[1 + \frac{1}{\mathbb{N}_{\text{graviton}}}\left(\text{initial}\right)\right]\right)$

 $\cdot \left(\mathbb{N}_{\text{graviton}}\left(\text{initial}\right)\right)^{1/3} \cdot 10^{-5} \cdot \sqrt{t/t_{\text{Planck}}}$ The upshot of Equation (68) is that if Equation (63) is commensurate with a

minimum value of the scale factor, *i.e.* so long as $a_{\text{initial}} \neq 0$ due to [16]

$$a_{\text{initial}} \approx a_{\min} \sim a_0 \cdot \left[\frac{\alpha_0}{2\lambda} \left(\sqrt{\alpha_0^2 + 32\lambda\mu_0 \omega B_0^2} - \alpha_0 \right) \right]^{1/4} \\ \propto \left[\frac{\alpha_0}{2\lambda} \left(\sqrt{\alpha_0^2 + 32\lambda\mu_0 \omega B_0^2} - \alpha_0 \right) \right]^{1/4}$$
(70)

Then the shift in the change in the Hubble parameter, in expansion to first order can be delineated as

$$\Delta H (\text{thermal}) \sim \sqrt{\frac{8\pi k_B}{12 \cdot V_{\text{Volume}}}} \cdot \frac{\log \left[1 + \frac{1}{\mathbb{N}_{\text{graviton}} (\text{initial})}\right]}{\left[\frac{\alpha_0}{2\lambda} \left(\sqrt{\alpha_0^2 + 32\lambda\mu_0 \omega B_0^2} - \alpha_0\right)\right]^{1/8}} \quad (71)$$
$$\cdot \left(\mathbb{N}_{\text{graviton}} (\text{initial})\right)^{1/3} \cdot 10^{-5} \cdot \sqrt{t/t_{\text{Planck}}}$$

_

By necessity to get non pathological values of the change in ΔH (thermal), we need to have [8]

$$\mathbb{N}_{\text{graviton}} (\text{initial}) \neq 0$$

$$\mathbb{N}_{\text{graviton}} (\text{initial}) \neq \infty$$

$$B_0 \equiv B_{\text{initial}} \neq 0$$

$$B_0 \equiv B_{\text{initial}} \neq \infty$$

$$\lambda \neq 0$$

$$\lambda \neq \infty$$
(72)

The initial volume would be at a minimum the cube of Planck's length, say 10^{-33} centimeters, cubed, leading to an enormous value for Equation (70), whereas we would be considering if we had an initial time step close to Planck time, and $0 < N_{\text{graviton}}$ (initial) < 10^5 , and

$$\Delta H (\text{thermal}) \sim \sqrt{\frac{8\pi k_B}{12 \cdot V_{\text{Volume}}}} \cdot \frac{\frac{1}{\mathbb{N}_{\text{graviton}}(\text{initial})} - \frac{1}{2\mathbb{N}_{\text{graviton}}^2(\text{initial})}}{\left[8\mu_0 \omega B_0^2\right]^{1/8}}$$
(73)

$$\cdot \left(\mathbb{N}_{\text{graviton}}(\text{initial})\right)^{1/3} \cdot 10^{-5} + \text{H.O.T}$$

This places an absolute requirement upon having the initial magnetic field not equal to zero.

As well as having a nonzero initial graviton production number, and also non zero initial volume.

With both these requirements in place, if $m_{\text{graviton}} \sim \frac{\hbar H}{c^2} \sim 10^{-61} \text{ grams}$, and we

set in a Planck time interval

$$m_{\text{graviton}} \sim \frac{\hbar H}{c^2} \sim 10^{-61} \text{ grams}$$

$$\sim \frac{\hbar}{c^2} \cdot \sqrt{\frac{8\pi k_B}{12 \cdot V_{\text{Volume}}}} \cdot \frac{\frac{1}{\mathbb{N}_{\text{graviton}}(\text{initial})} - \frac{1}{2\mathbb{N}_{\text{graviton}}^2(\text{initial})} \cdot \left(\mathbb{N}_{\text{graviton}}(\text{initial})\right)^{1/3} \cdot 10^{-5}$$
(74)

And that Equation (73) may give some insight as to the fluctuations which show up in figure 2, of [10].

11. Does the Existence of Tightly Constrained but Very Large Magnetic Fields Allow for Inhomogeneous Patches Due to NLED Showing up in CMBR: Relevance to Bicep 2 Dispute?

We then get an inter relationship between $\mathbb{N}_{\text{graviton}}$ (initial), the initial Volume, and the initial magnetic field to consider. Moreover, what we have also shown, is that NLED. Appearing initially, that it is very probable that if one uses infinite quantum statistics as given by Ng [8]

$$\mathbb{N}_{\text{graviton}}\left(\text{initial}\right) \approx S\left(\text{initial entropy}\right) \neq 0 \tag{75}$$

Note that in usual treatment of entropy, and entropy density we usually assume a fourth order dependence upon temperature for entropy density. Here we say that this entropy is most likely independent of Temperature, by Infinite quantum statistics, as given by Ng [8]. But we also will be talking about a necessary bound of quantum fluctuations which will be given below. *i.e.* consider if we have the following restrictions in fluctuations due to quantum effects which we give as follows.

What we will mention, is that co current with Equation (73), Equation (74) and Equation (75) that there is a situation for which, as given by Mukhanov [27] there are conditions in which a quantum fluctuation would spoil initial homogeneity if there exist quantum fluctuations exceeding

$$\begin{aligned} \lambda_{\text{Critical value}} &\sim (\Delta H)^{-1} \exp\left(m_{\text{graviton}}^{-1}\right) \\ &\sim \sqrt{\frac{12 \cdot V_{\text{Volume}}}{8\pi k_B}} \cdot \frac{\left[8\mu_0 \omega B_0^2\right]^{1/8} \left(\mathbb{N}_{\text{graviton}}\left(\text{initial}\right)\right)^{-1/3} \cdot 10^5}{\frac{1}{\mathbb{N}_{\text{graviton}}\left(\text{initial}\right)} - \frac{1}{2\mathbb{N}_{\text{graviton}}^2\left(\text{initial}\right)}} \end{aligned}$$
(76)
$$\cdot \exp\left(\frac{c^2}{\hbar} \cdot \sqrt{\frac{12 \cdot V_{\text{Volume}}}{8\pi k_B}} \cdot \frac{\left[8\mu_0 \omega B_0^2\right]^{1/8} \left(\mathbb{N}_{\text{graviton}}\left(\text{initial}\right)\right)^{-1/3} \cdot 10^5}{\frac{1}{\mathbb{N}_{\text{graviton}}\left(\text{initial}\right)} - \frac{1}{2\mathbb{N}_{\text{graviton}}^2\left(\text{initial}\right)}}\right) \end{aligned}$$

The quantum uncertainty in position which will be referred to is of the form $\Delta x \Delta p \equiv \lambda_{\rm OM} \cdot m \cdot c \approx \hbar$

$$\Leftrightarrow \lambda_{\text{QM-graviton}} \approx \frac{\hbar}{m_{\text{graviton}} \cdot c} \sim c \cdot \sqrt{\frac{12 \cdot V_{\text{Volume}}}{8\pi k_B}} \cdot \frac{\left[8\mu_0 \omega B_0^2\right]^{1/8} \left(\mathbb{N}_{\text{graviton}}\left(\text{initial}\right)\right)^{-1/3} \cdot 10^5}{\frac{1}{\mathbb{N}_{\text{graviton}}\left(\text{initial}\right)} - \frac{1}{2\mathbb{N}_{\text{graviton}}^2\left(\text{initial}\right)}}$$
(77)

When the wavelength function of Equation (76) and Equation (77) are about the same value, one has the destruction of inhomogeneity, in early universe conditions, which puts restrictions on the value of graviton mass, of presumed entropy, as given by Ng's infinite quantum statistics, and more. The details of such will be elaborated upon in further publications. Furthermore, it also puts constraints upon the magnetic fields which may be present in early universe conditions. In any case the expected mass of the graviton would be of the order of about 10^{-62} grans, and the entropy would be here about [8]

$$l < \mathbb{N}_{\text{graviton}} (\text{initial}) \sim S_{\text{initial}} < 10^{5}$$
 (78)

This also refers to [34] [35].

The implications of Equation (75) to Equation (77) need to be considered and evaluated fully. We hope that in due time, Equation (55) to Equation (77) will allow for evaluating the apparent falsification of inflationary results first reported by [36] which was discussed at length in Rencontres De Moriond, Cosmology in both 2014 and 2015, which the author views as of paramount importance in constructing a gravitational astronomy initiative. As well as making sense of the Mukhanov based [27] criteria as to the formation of structure during the Dark ages, just before the turn on of the CMBR at z (redshift) ~1100

Structure
$$\propto 1 / \sqrt{\frac{8\pi k_B}{12 \cdot V_{\text{Volume}}}} \cdot \frac{\frac{1}{\mathbb{N}_{\text{graviton}}(\text{initial})} - \frac{1}{2\mathbb{N}_{\text{graviton}}^2(\text{initial})}}{\left[8\mu_0 \omega B_0^2\right]^{1/8}} \cdot \left(\mathbb{N}_{\text{graviton}}(\text{initial})\right)^{1/3} \cdot 10^{-5}$$
(79)

Equation (79) has to be commensurate with Equation (75) and Equation (76) which will take some serious work. We also state that Equation (79) in itself may be enough to falsify the results of [34] [35], in line with work presented in [35] which gave extremely specific magnetic field strengths for early universe cosmology.

12. Bringing Up Then the Use of Corda Treatment of Black Holes, Plus Work Done by the Author as to Formation of Present Day Cosmological Constant as a Result of Black Hole Formation

Our idea is to set up conditions after modeling BHs as BEC (boson Einstein condensates) to set up how to incorporate the insights of [1] in our modeling But to do this we need to do some initial works.

From [37] we will posit the following to consider as a creation of black holes. We then would have by [38] the following to consider

$$m \to m_g \approx \frac{M_P}{\sqrt{N_{\text{graviton}}}} \Rightarrow N_{\text{graviton}} \approx 10^{122}$$
 (80)

In addition the radius of the universe as a giant black hole "particle" would be of the form given by

$$R \to R_{\text{universe}} \approx \sqrt{N_{\text{graviton}}} \cdot \ell_P \approx 10^{61} \cdot \ell_P \tag{81}$$

Also the overall mass M would scale as

$$M \to M_{\text{universe}} \approx \sqrt{N_{\text{graviton}}} \cdot M_P \approx 10^{61} \cdot M_P$$
 (82)

Whereas the entropy

$$S \to S_{\text{universe}} (\text{gravitons}) \approx k_B \cdot 10^{122} \xrightarrow{k_B \to 1} 10^{122}$$
(83)

And the final temperature

$$T \to T_{\text{universe}} \left(\text{gravitons} \right) \approx \frac{T_P}{\sqrt{N_{\text{graviton}}}} \approx 10^{-61} \cdot T_P$$
 (84)

We should use [37] [38] [39] [40] to gain background on this particular set up of the Universe as a black hole.

In this case, we have that the mass of the graviton, allowing for this scaling is given by [37] [41] [42]

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \tag{85}$$

This treatment of graviton mass, as given by Equation (85) sets us up to ask how one could have formed the parameter Λ .

To begin with, we consider, that the expansion we have that for a scale factor expansion of the universe, that

$$a(t) = a_0 \left\{ \frac{1}{2\Omega_{\Lambda}} \cdot \left[\cosh\left(\sqrt{3\Lambda t}\right) - 1 \right] \right\}^{1/3} \xrightarrow[t \to \text{Large}]{} \exp\left(\sqrt{\frac{\Lambda}{3}}t\right)$$
(86)

Roughly speaking we will by running backwards ascertain if an initial value of scale factor can actually go to zero and what would stop that from happening.

Here, Equation (80) will be by [37] [38] [39]

$$a(t) = a_{\text{initial}} t^{\nu}$$

$$\Rightarrow \phi = \ln \left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t \right)^{\sqrt{16\pi G}}$$

$$\Rightarrow \dot{\phi} = \sqrt{\frac{\nu}{4\pi G}} \cdot t^{-1}$$

$$\Rightarrow \frac{H^2}{\dot{\phi}} \approx \sqrt{\frac{4\pi G}{\nu}} \cdot t \cdot T^4 \cdot \frac{1.66^2 \cdot g_*}{m_P^2} \approx 10^{-5}$$
(87)

Table 1. From [37] [38] assuming Penrose recycling of the Universe as stated in that document.

End of Prior Universe time frame	Mass (black hole): super massive end of time BH 1.98910 ⁴¹ to about 10 ⁴⁴ grams	Number (black holes) 10 ⁶ to 10 ⁹ of them usually from center of galaxies
Planck era Black hole formation Assuming start of merging of micro black hole pairs	Mass (black hole) 10^{-5} to 10^{-4} grams (an order of magnitude of the Planck mass value)	Number (black holes) 10 ⁴⁰ to about 10 ⁴⁵ , assuming that there was not too much destruction of matter-energy from the Pre Planck conditions to Planck conditions
Post Planck era black holes with the possibility of using Equation (1) to have say 10 ¹⁰ gravitons/second released per black hole	Mass (black hole) 10 grams to say 10 ⁶ grams per black hole	Number (black holes) Due to repeated Black hole pair forming a single black hole multiple time. 10^{20} to at most 10^{25}

This use of **Table 1** is such that it would lead to an expansion parameter, a Hubble constant as valued with respect to a temperature T as given in [37] [39].

This of course makes uses of [40]

$$H = 1.66\sqrt{g_*} \cdot \frac{T_{\text{temperature}}^2}{m_P}$$
(88)

Now let us reconstruct the idea of a traditional cosmological constant from all of this [37] [38] [39] [40] [41] [43].

13. And Now the Question of the Cosmological Constant, *i.e.* Where Could It Be Formed?

First of all is the old standby namely in the onset of inflation, there would be a huge speed of inflationary expansion with the coefficient of Equation (87) for scale factor given as [37] [38]

$$\nu \xrightarrow{\text{Planck normalization}} 4\pi \times (\omega_{gw})^{12} \times \frac{\varsigma^4}{\tilde{\beta}^2}$$
(89)

This is all defined in [37] in an article written by the author for Intech, for our convenience.

If so, by Novello [44] we then have a bridge to the cosmological constant as given by

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \tag{90}$$

Consider first the relationship between vacuum energy and the cosmological constant. Namely $\rho_{\Lambda} \approx \hbar k_{\text{max}}^4$ where we have that [45]

$$\rho_{\Lambda} \approx \hbar k_{\text{max}}^4 \approx \left(10^{18} \,\text{GeV}\right)^4 \xrightarrow[\text{reduced}]{} \left(10^{-12} \,\text{GeV}\right)^4$$
(91)

Where we define the mass of a graviton as in the numerator given by Equation (90), and then we can also use the following.

This is useful in terms of determining conditions for a cosmological constant [15] [37]

$$\rho_{\Lambda}c^{2} = \int_{0}^{E_{\text{Plank}}/c} \frac{4\pi p^{2} dp}{(2\pi\hbar)^{3}} \cdot \left(\frac{1}{2} \cdot \sqrt{p^{2}c^{2} + m^{2}c^{4}}\right) \approx \frac{(3 \times 10^{19} \text{ GeV})^{4}}{(2\pi\hbar)^{3}}$$

$$\xrightarrow{(2.5 \times 10^{-11} \text{ GeV})^{4}} (92)$$

This means shifting the energy level of the Equation (91) downward by 10^{-30} , *i.e.* the top value energy becomes a down scale of Planck energy times 10^{-30} .

14. And Now How to Tie in the Cosmological Constant from Black Holes as Far as the NLED Discussion of a Vacuum Energy Given Earlier?

We claim that the NLED treatment of a quintessence varying cosmological constant is separate from the DE treatment of a contribution of the cosmological constant as given by Equation (92), *i.e.* Equation (92) will be formed by black holes, which obey [1] of Christian Corda, as well as the scaling given in [37] for BEC condensates. *i.e.* we have two separate processes.

15. And Now How to Tie in the NLED Treatment of an Initial Starting Point for the Cosmological Expansion with the GUP Given by Beckwith in Section III?

What we are going to do is to, in the initial variation of the GUP is to look hard at the initial idea given in Equation (13) is to make the following treatment at the start of expansion of the Universe

 $\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1$ Goes to become effectively almost ZERO. (93)

If this is effectively almost zero, the effect would be to embedd Quantum mechanics within a 5 dimensional structure, and that the treatment of BHs as given in [1] is a direct consequence of having quantum mechanics rid of this deterministic structure completely. *i.e.* this deterministic embedding is in part in spirit similar to what is given by Wesson [46].

16. Conclusion

Initial configuration of space time affected by the dynamics of section XV, with QM embedded in a deterministic structure initially, allowing for the Corda treatment of black holes in [1] as a direct consequence of Equation (93) not being almost zero when one is away from the situation where Equation (93) is almost zero.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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