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# Gravitational Term in Semi Empirical Mass Formula

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## Abstract

A new term was added to the well-known semi-empirical mass formula to account for the changes due to gravitational attraction between nucleons in the liquid drop, as well as, accommodates for the necessary corrections in the binding energy of a nucleus. The results of our calculations show a straight forward evidence that the gravitational attraction bears a reasonable contribution to the binding energy. On the other hand, employing the gravitational term in the semi empirical mass formula was led to the calculation of gravitational constant at subnuclear level.

## Keywords

Liquid Drop, Binding Energy, *Odd-A* Nuclei, Gravitation, Weak Interaction

## 1. Introduction

In the resulting liquid-drop model [1] [2], the nucleus has an energy which arises partly from some aspects e.g., surface tension, electrical repulsion, etc. The liquid-drop model is able to reproduce many features of nuclei, including the general trend of binding energy, as well as the nuclear fission. A basic property of a nucleus is the mass defect [1] [3] which implies that the mass of the nucleus is less than the sum of the masses of its constituent nucleons:

$$\Delta M(A,Z) = (Zm_p + Nm_n) - M(A,Z)$$

where $m_p$ and $m_n$ are the masses of proton and neutron, respectively and $\Delta Mc^2$ is now termed the binding energy (*BE*) of the nucleus [4] [5]. The formula accounts for the binding energy of the nucleus was developed by Weizsacker [2] under assumption that the nucleus is considered as a droplet of incompressible matter which is maintained by the strong nuclear interaction that exists between nucleons. The binding energy is expressed by a relation containing few terms, e.g., five terms formula [6] is:

$$BE = E_v - E_s - E_c - E_a \pm E_p$$

namely, volume energy, surface energy, Coulomb energy, asymmetry energy, and pairing energy, respectively [7] [8]. Although the formula contains a number of constants that have to be extracted by fitting with data. The theoretical part arises from two major properties common to all nuclei: The interior mass densities are approximately equal, and that the total binding energies are approximately proportional to the masses. The common expression for the binding energy can have the following form [9] [10] [11] [12]

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm a_p A^{\lambda} \tag{1}$$

where $a_p$ with the polarity either positive for *e-e* nuclei, negative for *o-o* nuclei, or zero for *odd-A* nuclei, with the value $\lambda$ was assumed to be −3/4, but recent evaluations indicate a value of −1/2 for convenience [13] [14] [15] [16].

In this work, we wish to propose a new term through the semi-empirical mass formula accounts for the gravitational attraction between nucleons.

## 2. Theory and Approach

Gravity is the most significant interaction between objects at the macroscopic scale, its influence also exists at subnuclear level [17]. The gravitational force has an infinite range, although its effects become weaker as objects get farther away. In a liquid drop, the effect of gravity between particles cannot be simply ignored. For a spherical body of uniform density, the gravitational binding energy $E_g$ is given by classical expression [18] [19] of the form

$$E_g \propto -\frac{GM^2}{R} \tag{2}$$

where $G$ is the universal gravitational constant, $M$ is the mass of the sphere, and $R$ its radius. However, expression (2) is not guaranteed to be valid for subnuclear particles, where the gravitational effects is still incomplete [20], therefore we generally consider the gravitational attraction between subnuclear particle of the proportional form

$$E_g = -a_g \frac{A(A-1)}{A^{1/3}} \tag{3}$$

where we have used the empirical radius $R = r_0 A^{1/3}$ and the mass of the form $M^2 = A(A-1)$, this reflects the fact that gravitational attraction will appear only if there are more than single particle, and the proportionality constant $a_g$ needs to be determined from fitting the data. For convenience, we use Equation (1) to calculate the binding energy of *odd-A* nuclei, with vanishing asymmetry term $a_p = 0$. In this context, the semi-empirical mass formula given by Equation (1) may take the following form:

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} - a_g \frac{A(A-1)}{A^{1/3}} \tag{4}$$

Equation (4) is our fundamental expression and will be used throughout our calculations.

## 3. Results and Comparisons

We tabulate hereunder the results of different approaches for the purpose of comparisons (**Table 1**; **Table 2**).

**Table 1.** A list of the results in chronological order of various sets of calculated coefficient as cited in ref. [21] [22].

| Coefficients [MeV] | Years | $a_v$ | $a_s$ | $a_c$ | $a_a$ | $a_p$ |
|---|---|---|---|---|---|---|
| Benzaid *et al.* [22] | 2019 | 14.64 | 14.08 | 0.64 | 21.07 | 11.54 |
| Mavrodiev *et al.* [22] | 2018 | 19.12 | 18.19 | 0.52 | 12.54 | 28.99 |
| Kirson [22] | 2007 | 15.36 | 16.43 | 0.69 | 22.54 | – |
| Chowdhury *et al.* [22] | 2005 | 15.78 | 18.34 | 0.71 | 23.21 | 12 |
| Samanta & Adhikuri [22] | 2004 | 15.77 | 18.34 | 0.71 | 23.21 | 12 |
| Myers & Myers [22] | 1996 | 16.24 | 18.63 | – | – | – |
| Wapstra [22] | 1958 | 15.84 | 18.33 | 0.18 | 23.2 | 11.2 |
| Evans [21] | 1955 | 14.1 ± 0.2 | 13 ± 0.1 | 0.595 ± 0.02 | 19.0 ± 0.9 | – |
| Green [21] | 1954 | 15.75 | 17.8 | 0.71 | 23.7 | – |
| Metropolis & Reitweisner [21] | 1950 | 14.0 | 13.0 | 0.583 | 19.3 | $33.5A^{-3/4}$ |
| Friedlander & Kennedy [21] | 1949 | 14.1 | 13.1 | 0.585 | 18.1 | $132A^{-1}$ |
| Rosenfeld [21] | 1949 | 14.66 | 15.4 | 0.602 | 20.54 | – |
| Feenberg [21] | 1947 | 14.1 | 13.1 | 0.585 | 18.1 | $33.5A^{-3/4}$ |
| Fowler [21] | 1947 | 15.3 | 16.7 | 0.69 | 22.6 | – |
| Fermi [21] | 1945 | 14.0 | 13.0 | 0.583 | 19.3 | $33.5A^{-3/4}$ |
| Can. Nat. Res. Council [21] | 1945 | 14.05 | 14.0 | 0.61 | 19.6 | – |
| Mattauch & Flugge [21] | 1942 | 14.66 | 15.4 | 0.602 | 20.5 | – |
| Feenberg [21] | 1939 | – | 13.3 | 0.62 | – | – |
| Bethe & Bacher [21] | 1936 | 13.86 | 13.2 | 0.58 | 19.5 | – |

**Table 2.** Showing the effect of adding a new thermal term $a_T$ to the semi-empirical mass formula, by Khadri and others, as it discussed in ref. [23].

| Coefficients [MeV] | Year | $a_v$ | $a_s$ | $a_c$ | $a_a$ | $a_p$ | $a_T$ |
|---|---|---|---|---|---|---|---|
| Khdari *et al.* | 2020 | 15.829 | 17.992 | 0.739 | 20.89 | 0 | 0.631 |
| Khdari *et al.* | 2020 | 15.832 | 18.399 | 0.705 | 24.172 | 23.489 | 0.440 |

**Table 3.** A sample of our results of *odd-A* nuclei compared with nuclei suggested in ref. [21].

| Coefficients [MeV] | Year | $a_v$ | $a_s$ | $a_c$ | $a_a$ | $a_g$ |
|---|---|---|---|---|---|---|
| Present Work | 2022 | 15.587 | 17.649 | 0.699 | 23.458 | $1.192 \times 10^{-3}$ |
| José *et al.* | 2016 | 15.593 | 17.345 | 0.694 | 23.601 | – |

**Table 4.** A sample of our results of *odd-A* nuclei compared with ref. [24].

| Coefficients [MeV] | Year | $a_v$ | $a_s$ | $a_c$ | $a_a$ | $a_g$ |
|---|---|---|---|---|---|---|
| Present Work | 2022 | 15.531 | 17.503 | 0.681 | 24.899 | $1.192 \times 10^{-3}$ |
| Mirzaei *et al.* | 2017 | 15.519 | 17.746 | 0.674 | 24.576 | – |

In **Table 3** and **Table 4**, we compare the results of our calculations with samples of selected approaches focused on *odd-A* nuclei.

On the other hand, the obtained value of gravitational constant $a_g$ is further employed to determine the renormalized gravitation constant [25], associated with interactions at subnuclear scale. From Equations (2) and (3), we write

$$a_g = \Gamma \frac{m_N^2}{r_0} \tag{5}$$

where $m_N$ is the mass of a nucleon and $\Gamma$ is the subnuclear gravitational constant, which absorbing Newton gravitational constant *G*, giving

$$\Gamma = 1.23 \times 10^{33} G = 8.02 \times 10^{28}\ \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2} \tag{6}$$

This value agrees with the one suggested by Onofrio [26] [27], and also falls within the range of weak interactions as also suggested in ref. [28].

## 4. Conclusion

It is known that the gravitational effect at subnuclear scale is still under considerations, we thus encouraged to add a new term to the semi-empirical mass formula to account for any deviation in binding energy due to gravitational effects between subnuclear particles. The added gravitational term is consistent, hence the semi-empirical mass formula shows agreement compared with earlier studies. On the other hand, we could extract a new constant representing the gravitational constant at subnuclear scale, which bears an excellent agreement compared with available studies concerning gravitational interaction at subnuclear scale.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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