

Using Model of a Universe as Similar to a Black Hole, Ask If We Have to Have Singularities, If We Are Looking at Initial Time Step and Entropy, from the Beginning

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Abstract

Based on the idea of cyclic conformal cosmology, we formulate entropy and quantum number n , and then utilize the minimum uncertainty principle, where ΔE times Δt equals \hbar , to actualize a prototype Δt time step in the breakup of supermassive black holes into countless Planck mass-sized black holes. This helps to link entropy, time step, and primordial conditions and define when the cosmological constant may form and the initial inflationary expansion “speed”. All this is used to obtain a model of if a singularity, initially is needed.

Keywords

Inflation, Fifth force, Gravitational Waves, Gravitons, Hubble Parameter

1. Introduction—First of All Model the Universe as Acting Like a Giant Black Hole

We then would have by [1] the following to consider

$$m \rightarrow m_g \approx \frac{M_p}{\sqrt{N_{\text{graviton}}}} \Rightarrow N_{\text{graviton}} \approx 10^{122} \quad (1)$$

In addition the radius of the universe as a giant black hole “particle” would be of the form given by

$$R \rightarrow R_{\text{universe}} \approx \sqrt{N_{\text{graviton}}} \cdot \ell_p \approx 10^{61} \cdot \ell_p \quad (2)$$

Also the overall mass M would scale as

$$M \rightarrow M_{\text{universe}} \approx \sqrt{N_{\text{graviton}}} \cdot M_P \approx 10^{61} \cdot M_P \tag{3}$$

whereas the entropy

$$S \rightarrow S_{\text{universe}} (\text{gravitons}) \approx k_B \cdot 10^{122} \xrightarrow{k_B \rightarrow 1} 10^{122} \tag{4}$$

And the final temperature

$$T \rightarrow T_{\text{universe}} (\text{gravitons}) \approx \frac{T_P}{\sqrt{N_{\text{graviton}}}} \approx 10^{-61} \cdot T_P \tag{5}$$

We should use [1] [2] [3] and [4] to gain background on this particular set up of the Universe as a black hole.

In this case, we have that the mass of the graviton, allowing for this scaling is given by [5] [6]

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \tag{6}$$

This treatment of graviton mass, as given by Equation (6) sets us up to ask how one could have.

Formed the parameter Λ .

To begin with, we consider, that the expansion.

We have that for a scale factor expansion of the universe, that

$$a(t) = a_0 \left\{ \frac{1}{2\Omega_\Lambda} \cdot [\cosh(\sqrt{3\Lambda}t) - 1] \right\}^{1/3} \xrightarrow{t \rightarrow \text{Large}} \exp\left(\sqrt{\frac{\Lambda}{3}}t\right) \tag{7}$$

Roughly speaking we will by running backwards ascertain if an initial value of scale factor can actually go to zero and what would stop that from happening.

Table 1 from [1] assuming Penrose recycling of the Universe as stated in that document.

Here, Equation (1) will be by [1] [2] [3]

Table 1. Penrose recycling of the universe.

End of Prior Universe time frame	Mass (black hole): super massive end of time BH 1.98910 ⁺⁴¹ to about 10 ⁴⁴ grams	Number (black holes) 10 ⁶ to 10 ⁹ of them usually from center of galaxies
Planck era Black hole formation Assuming start of merging of micro black hole pairs	Mass (black hole) 10 ⁻⁵ to 10 ⁻⁴ grams (an order of magnitude of the Planck mass value)	Number (black holes) 10 ⁴⁰ to about 10 ⁴⁵ , assuming that there was not too much destruction of matter-energy from the Pre Planck conditions to Planck conditions
Post Planck era black holes with the possibility of using Equation (1) to have say 10 ¹⁰ gravitons/second released per black hole	Mass (black hole) 10 grams to say 10 ⁶ grams per black hole	Number (black holes) Due to repeated Black hole pair forming a single black hole multiple time 10 ²⁰ to at most 10 ²⁵

$$\begin{aligned}
 a(t) &= a_{\text{initial}} t^\nu \\
 \Rightarrow \phi &= \ln \left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t \right)^{\sqrt{\frac{\nu}{16\pi G}}} \\
 \Rightarrow \dot{\phi} &= \sqrt{\frac{\nu}{4\pi G}} \cdot t^{-1} \\
 \Rightarrow \frac{H^2}{\dot{\phi}} &\approx \sqrt{\frac{4\pi G}{\nu}} \cdot t \cdot T^4 \cdot \frac{1.66^2 \cdot g_*}{m_p^2} \approx 10^{-5}
 \end{aligned}
 \tag{8}$$

This would lead to an expansion parameter, a Hubble constant as valuated as [2].

This of course makes uses of [3]

$$H = 1.66 \sqrt{g_*} \cdot \frac{T_{\text{temperature}}^2}{m_p}
 \tag{9}$$

Now for the sake of primordial black holes.

The formula which is for Luminosity from a black hole and in page 16 of reference [3] the text states that the two black holes emit GW with a wave frequency 2 times the rotation frequency of the orbit of the two black holes to each other.

If we assume that we are still using this approximation above, from [4] we can see support for our choice of Planck length as the minimum separation distance between the two black holes via using Plank units normalized to 1 as yielding [4]

$$\begin{aligned}
 R(\text{separation}) &\approx r_g^{\text{eff}} = \frac{(M_1 + M_2)}{(M_{\text{Planck}})^2} \\
 \xrightarrow[M_{\text{Planck}=1}]{M_1=M_2=M_{\text{Planck}}} &\rightarrow 1 \equiv R(\text{Planck length})
 \end{aligned}
 \tag{10}$$

i.e. this means that the primordial black holes, presumably of Planck size would be separated about 1 Planck length from each other, that their recombination would be quick and that the frequency range would likely be of the magnitude of about 10^{25} Hz in terms of Gravitational Waves (GW) which would then be massively red shifted downward to about 1 Hz in an Earth bound detector system.

i.e. a huge downward red shifting from 10^{25} Hz to about 1 Hz value in Earth orbit.

We will use this in order to formulate that what will be the behavior of primordial back holes in the formulation of entropy & an initial time step and what this has to say about the relationship of time and entropy in cosmology.

2. Formulation of Entropy per Primordial Black Hole, Using the Model of BEC Condensates

We will be using the ideas given in when we have the Gravitational wave frequency specified by [5] with r in the following equation of the order of Planck length, more or less [1].

$$\begin{aligned}\omega_{gw}^6 &\approx c^7 \times \frac{\tilde{\beta}}{2m_p r} \cdot \sqrt{\frac{v}{\pi G}} \times \frac{1}{Gc \cdot (M_{\text{mass}})^2 \langle r^2 \rangle^2} \\ \Rightarrow \omega_{gw} &\approx \left(\sqrt{\frac{v}{4\pi G}} \times \frac{\tilde{\beta} \cdot c^6}{G \cdot (M_{\text{mass}})^2 m_p r \cdot \langle r^2 \rangle^2} \right)^{1/6}\end{aligned}\quad (11)$$

The idea of a fifth force contribution makes its way via the argument given in [1] to the effect that the power of a signal of GW generation in the primordial GW sense would be given by

$$\begin{aligned}P_{GW} &\approx \frac{Gc \cdot (M_{\text{mass}})^2 \omega_{gw}^6 \langle r^2 \rangle^2}{c^6} \\ &\approx c \times |F_{\text{5th-force}}| = \left| -c \times \frac{\tilde{\beta} \cdot (\vec{\nabla} \phi)}{m_p} \right| \approx c \times \frac{\tilde{\beta}}{2m_p r} \cdot \sqrt{\frac{v}{\pi G}}\end{aligned}\quad (12)$$

Having said that we are now ready to discuss the role of individual black holes. In order to do this we use [7] namely

$$t = \frac{r}{\varpi c} \quad (13)$$

The term of ϖ is a dimensionless value less than or at most equal to the value 1, and never negative.

If so, then Equation (12) will yield a radial force component which we will write as [1]

$$F_{\text{5th-force}} = -\frac{\tilde{\beta} \cdot (\vec{\nabla} \phi)}{m_p} \approx -\frac{\tilde{\beta}}{2m_p r} \cdot \sqrt{\frac{v}{\pi G}} \quad (14)$$

And then the BEC condensate given by [1] [6],

$$\begin{aligned}m &\approx \frac{M_P}{\sqrt{N_{\text{gravitons}}}} \\ M_{BH} &\approx \sqrt{N_{\text{gravitons}}} \cdot M_P \\ R_{BH} &\approx \sqrt{N_{\text{gravitons}}} \cdot l_P \\ S_{BH} &\approx k_B \cdot N_{\text{gravitons}} \\ T_{BH} &\approx \frac{T_P}{\sqrt{N_{\text{gravitons}}}}\end{aligned}\quad (15)$$

Here, the first term, m , is in the effective mass of a graviton. This is my take as to how to make all this commensurate as to special relativity.

$$m \approx \frac{m_g}{\sqrt{1 - \left(\frac{v_g}{c}\right)^2}} \approx \frac{M_P}{\sqrt{N_{\text{gravitons}}}} \approx 10^{-10} \text{ grams} \quad (16)$$

If this is done the effect would have been 10^5 gravitons per Planck mass black hole, and if the primordial black holes went up to 1 gram, we would then see an upper bound of say per primordial black hole of effective graviton mass defined

due to Equation (16) of say

$$\therefore N_{\text{gravitons}} \approx 10^{10} \tag{17}$$

3. Shorthand for a Working Hypothesis for the Start of a Wave Function as a Start to Understanding the Significance of Equation (17) per Black Hole

The simplest way to do that is as follows: By [7] we can do the following, namely look at page 79 which has at a start

$$\begin{aligned} \Delta\Psi + \tilde{k}^2\Psi &= 0 \\ \Rightarrow \Psi &= \Psi_{\text{initial}} \exp(i\tilde{k}r) \\ \tilde{k} &= (2mE)^{1/2} \\ E &= \hbar\omega_{\text{graviton}} \end{aligned} \tag{18}$$

In doing so we have that we will have to make some assumptions, some of them seemingly arbitrary due to this one [8].

And so due to [8] we have this to consider.

In 1931, the Austrian logician Kurt Gödel published his incompleteness theorem, a result widely considered one of the greatest intellectual achievements of modern times. The theorem states that in any reasonable mathematical system there will always be true statements that cannot be proved.

Having said this, what is considered here as arbitrary?

First, the idea of mass of a graviton, *i.e.* heavy Gravity [9].

Secondly, is the idea of precursors of the frequency which is what our article is trying to fill-in.

Third in terms of Ψ_{initial} . Here, we will make a candidate description of just this problem at the end of the paper. But to do that at all, we need to get a handle on Entropy which is the next section of our document.

4. Considerations as to Entropy Which Will Be Used at the End of the Article Is to Make an Assumption about Initial Wave Functions Terms of Ψ_{initial}

Now having done this and let us assume we work with a Planck mass black hole, we make the following approximation of the quantum number, entropy and energy of a Planck sized black hole being given by Mukhanov [10] with a boson model of a black hole,

$$S_{BH} = \frac{A(\text{area})}{4} \approx k_B \cdot N_{\text{effective graviton}} \propto (n_{\text{quantum}} - 1) \ln 2 \tag{19}$$

Here, for a Planck mass sized black hole, we will set an entropy per black hole approximately as

$$N_{\text{effective graviton}} \Big|_{\text{Planck mass black hole}} \approx 10^5 \tag{20}$$

If so then, we have if we do Planck normalization of $\hbar = k_B = c = 1$, then Equation (19) and Equation (20) yield a quantum number of for a Planck mass sized

black hole of

$$\begin{aligned} N_{\text{effective graviton}} &\propto (n_{\text{quantum}} - 1) \ln 2 \\ \Rightarrow n_{\text{quantum}} &\approx 1 + \frac{N_{\text{effective graviton}}}{\ln 2} \approx 1 + \frac{10^5}{\ln 2} \end{aligned} \quad (21)$$

We do not assume fractional quantum numbers per black hole, so we take the round off of Equation (21) to a whole number.

This appeal to quantum states will be essential for our end of Ψ_{initial} .

5. Using This Quantum Number n , per Black Hole to Tie in with Dr. Corda's Outstanding Work on Black Holes

Also then we will be assuming then using these Planck units that approximately we use the Corda result of per black hole of bound state energy as given by [11]

$$E_{Bh} = -\frac{n_{\text{quantum}}}{2} \quad (22)$$

If we say that this is for black hole, as induced by the Penrose model and Figure 1, due to [1] we can write a minimum uncertainty of

$$\begin{aligned} \left(\Delta E \approx \left| E_{Bh} = -\frac{n_{\text{quantum}}}{2} \right| \right) \times \Delta t &\approx 1 \\ \Rightarrow \Delta t &\approx \frac{2}{\left| E_{Bh} = -\frac{n_{\text{quantum}}}{2} \right|} \approx 10^{-5} \end{aligned} \quad (23)$$

i.e. we would have for a Planck sized black hole, initial time step of 10^{-5} times Planck time, which is incredible, whereas we would have entropy of 10^5 , Per Planck sized black hole.

This would be right due to the black hole figures given in Table 1, which are commensurate with [1].

6. Having the Onset of Time, as Given in Equation (23) Can We Make a Reference to the " Ψ_{initial} "? As Well as Quantum Number n ?

To do this look at [12], *i.e.* in a word we would be to first order look at [12]'s Equation (2.5) which after we parse it, would be acting like primordial atom. As given in page 15 of the document [12] we have this caution put in.

Quote

we would like first to see more explicitly how it can be that the Standard Model particles, with all their quantum numbers, can leave their imprint on the out-going particles exclusively through their momentum distribution, which we describe in terms of spherical partial waves. This part of our proposed mechanism has not yet been described well: how do we transform the information of all fields of Standard Model particles, as well as the perturbative gravitons, in terms of momentum distribution functions only?

End of quote

i.e. what we are doing is to designate a future work in progress as far as Eq. 2.5 of reference [12], but this also should have some similarities as to orbitals as seen in the hydrogen atom, as seen in [13].

Having specified this as an important project to do, and for future reference, we next will go to a way to look at the cosmological constant, as given based upon the information we are acquiring as to initial black holes.

7. And Now the Question of the Cosmological Constant, *i.e.* Where Could It Be Formed?

First of all is the old standby namely in the onset of inflation, there would be a huge speed of inflationary expansion with the coefficient of Equation (8) for scale factor given as [1]

$$v \xrightarrow{\text{Planck normalization}} 4\pi \times (\omega_{gw})^{12} \times \frac{G^4}{\beta^2} \tag{24}$$

This is all defined in [1] in an article written by the author for Intech, for our convenience.

If so, by Novello [14] we then have a bridge to the cosmological constant as given by

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \tag{25}$$

Consider first the relationship between vacuum energy and the cosmological constant. Namely $\rho_\Lambda \approx \hbar k_{\max}^4$ where we have that [15]

$$\rho_\Lambda \approx \hbar k_{\max}^4 \approx (10^{18} \text{ GeV})^4 \xrightarrow{\text{reduced}} (10^{-12} \text{ GeV})^4 \tag{26}$$

Where we define the mass of a graviton as in the numerator given by Equation (16), and then we can also use the following.

This is useful in terms of determining conditions for a cosmological constant [1] [15]

$$\rho_\Lambda c^2 = \int_0^{E_{\text{Plank}}/c} \frac{4\pi p^2 dp}{(2\pi\hbar)^3} \cdot \left(\frac{1}{2} \cdot \sqrt{p^2 c^2 + m^2 c^4} \right) \approx \frac{(3 \times 10^{19} \text{ GeV})^4}{(2\pi\hbar)^3} \tag{27}$$

$$\xrightarrow{E_{\text{Plank}}/c \rightarrow 10^{-30}} \frac{(2.5 \times 10^{-11} \text{ GeV})^4}{(2\pi\hbar)^3}$$

This means shifting the energy level of the Equation (26) downward by 10^{-30} , *i.e.* the top value energy becomes a down scale of Planck energy times 10^{-30} .

8. How We Change the Energy in Equation (27). Reference Black Hole Energy Values as Given by Dr. Corda, as Subtracted from the Rest Energy of a Planck Sized Black Hole for Equation (27)

Now what is the energy in the integration pertinent to Equation (27) coming from?

If we look at the idea of a Planck mass black hole, and look at the Corda formula as given in Equation (21) we have that there is indeed a down step implied by the quantum number n_{quantum} we could come up with the following argument as to reduction of the top end of the integration given in Equation (27).

i.e. what we will rewrite the top end of the integration is, as follows, for a Planck mass sized black hole

$$\frac{\Delta E}{c} = 10^{18} \text{ GeV} - \frac{n_{\text{quantum}}}{2c} \approx 10^{-12} \text{ GeV} \tag{28}$$

i.e. the cosmological constant energy range would be established near the area of Primordial black holes.

This limiting value of Equation (26) and Equation (28) for calculation of Equation (27) would be by [1] necessary to reset the vacuum energy to be the cosmological constant due to the use of Corda’s energy value in the vicinity of the Primordial Planck mass sized black holes.

9. Future Research Questions to Look at for Additional Follow Up

The suppositions which we have are follows:

- 1) Equation (28) for calculating the defacto energy available for a cosmological constant vacuum energy calculation centered about relic micro black holes, and using Corda’s formulation of available black hole energy.
- 2) The use of the absolute value of black hole energy to obtain entropy, and entropy and minimum time step in this case even smaller that Planck time, as a starting contribution due to relic micro black holes as given in **Table 1**.
- 3) The task in future research would be to combine these two and to extrapolate as to an example contribution to CMB in future research endeavors.

As to the cosmological considerations independent of the black holes the following would be used.

We will make the following calculation [1] [2]

$$V_0 = \left(\frac{0.022}{\sqrt{qN_{\text{efolds}}}} \right)^4 = \frac{\nu(\nu-1)\lambda^2}{8\pi Gm_p^2} \tag{29}$$

“ λ ” as a dimensionless parameter. From [1] we have a Chameleon mechanism for fifth force as [1] [16]

$$F_{\text{5th-force}} = -\frac{\tilde{\beta} \cdot (\vec{\nabla} \phi)}{m_p} \tag{30}$$

We use here in Pre Planckian conditions

$$t = \frac{r}{\varpi c} \tag{31}$$

First, r is almost Planck in length, if so then

$$\omega_{\text{gw}} \approx \frac{c^{7/6} \tilde{\beta}^{1/6}}{(2m_p r)^{1/6}} \cdot \left(\frac{\nu}{\pi G} \right)^{1/12} \cdot \frac{1}{\left(GcM_{\text{Mass}}^2 \cdot \langle r^2 \rangle \right)^{1/6}} \text{ so if } G = m_p = \ell_p = 1 \tag{32}$$

Using this instead of the ω_{gw}^6 expression, then write the rest of it as follows which would have a minimum value as

$$\begin{aligned}
 \omega_{gw}^6 &\approx c^7 \times \frac{\tilde{\beta}}{2m_p r} \cdot \sqrt{\frac{v}{\pi G}} \times \frac{1}{Gc \cdot (M_{\text{mass}})^2 \langle r^2 \rangle^2} \\
 \Rightarrow \omega_{gw} &\approx G, m_p, r \approx \ell_p \xrightarrow{\text{Planck normalization}} 1 \\
 M_{\text{mass}} &\approx \zeta \cdot m_p \xrightarrow{\text{Planck normalization}} \zeta \\
 \langle r^2 \rangle^2 &\approx \ell_p^4 \xrightarrow{\text{Planck normalization}} 1 \\
 \therefore \omega_{gw} &\xrightarrow{\text{Planck normalization}} \left(\sqrt{\frac{v}{4\pi}} \times \frac{\tilde{\beta}}{\zeta^2} \right)^{1/6}
 \end{aligned} \tag{33}$$

when doing this, in line with our goal to an initial wave function for the black hole, updating [12] and [13] in terms of contributions. Difficult? Yes, but doable.

10. Conclusion with a Prospectus Which Will Create More Geometrical Insights

In conclusion, Assuming Planck length as a minimum separation distance between two primordial black holes, their quick recombination may be analyzed. A huge downward red shift appears then from 10^{25} Hz to about 1 Hz in an Earth bound detector system. By using this, the behavior of primordial black holes in the formulation of entropy and an initial time step as well as the relationship of time and entropy in the context of cyclic conformal cosmology are formulated. There are initial time step of 10^{-5} times Planck time and the entropy of 10^5 , Per Planck sized black hole. Further, mass of graviton in the numerator may be defined by considering the relationship between cosmological constant and vacuum energy which is useful for determining conditions for cosmological constant. The cosmological constant energy range is established near the area of Primordial black holes. The top value energy becomes a down scale of Planck energy times 10^{-30} .

In doing this we also found that [17] is useful as to giving background for future consideration.

This is first order summary as to results so concluded. But in addition our findings lead to potentially resolving one very important issue brought up in [18], page 84-85.

In the section given in [18] corresponding to section 11.1 we have the following.

“We wrap one dimension of our D1 brane to T^5 in order to give us a zero dimensional black hole like object”.

On page 85 the following is written out, namely for [18],

“For a D1 brane, we have equation 21.2

$$\begin{aligned}
 ds_{\text{string}}^2 &= H_1^{-1/2} (-dt^2 + dy^2) + H_1^{1/2} (dx_1^2 + \dots + dx_8^2) \\
 H_1 &= 1 + \frac{Q}{r^6}
 \end{aligned} \tag{34}$$

And a dimension of $\Delta = 6$ is for the dimension of the D-1 brane.”

If one uses a dilaton of

$$\exp(-2\phi_{10}) = H_1(r)^{-1} \tag{35}$$

The metric is rescaled by Equation (35) so we get a metric in the Einstein frame as

$$ds_E^2 = H_1^{-2/3} \cdot (dt^2) + H_1^{1/3} (dr^2 + r^2 d\Omega_{(3)}^2) \tag{36}$$

which according to P 85 of [18] is a metric for a (1 + 4) dimensional uncompactified space. We then get an area as

$$A = 2\pi^2 \lim_{r \rightarrow 0} \left(r^3 \sqrt{\frac{Q_1}{r^2}} \right) \rightarrow 0 \tag{37}$$

This is held on page 85 of [18] that this is $\frac{1}{2}$ BPS object, And that we need to further “reduce” symmetry.

If our results are right as far as a minimum grid size created, we do not have the radial component of our space-time going to zero. Hence the simple result we are looking for, may be a way to simplify, even assuming 5 dimensional result of the Einstein Frame, Equation (36), and that we avoid the problem of Equation (37). This also depends upon if our emergent space time picture is correct, and if we have a correct eventual working out say of a generalized uncertainty principle for 5 dimensions to work with.

This issue can be investigated in future work.

Finally but not least is the issue of if our results tie into creation of Dark Energy. In [3] page 404 we have that for a scale factor expansion of the universe, that

$$a(t) = a_0 \left\{ \frac{1}{2\Omega_\Lambda} \cdot [\cosh(\sqrt{3\Lambda}t) - 1] \right\}^{1/3} \xrightarrow{t \rightarrow \text{Large}} \exp\left(\sqrt{\frac{\Lambda}{3}}t\right) \tag{38}$$

If we do not have an unchanging Λ value we can assert that when the cosmological constant is formed and if it is when Primordial black holes to the creation of the cosmological constant we have effectively created Dark Energy (DE). This is the simplest DE model, at least conceptually speaking.

If Dark Energy (DE) is instead defined by Quintessence, where its value changes over time, then no we have NOT created DE. Reference [19], summarizes this very well as well [20] arguing clearly that DE is due to an invariant cosmological constant.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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