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# Does a Fine Tuning of a Quartic Potential Allow for an Invariant Cosmological Constant? How This Supposition Could Lead to a Macro Model of Pressure in the Start of Inflation?

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## Abstract

Our assumptions in this paper are for a more traditional cosmological constant, not varying over time, which has the result of forming a much simpler entropy expression. But in doing so, we have to make note of how a quartic phase transition initially may impose a level of precision in parameters impossible to verify with present experimental equipment. We also have the conundrum of how this may be linked to macroscopic interpretation of pressure in the initial phases of the universe, using black hole physics in Pre Planckian to Planckian physics domains.

## Keywords

High-Frequency Gravitational Waves (HFGW), Symmetry, Causal Discontinuity

## 1. Working with an Invariant Cosmological Constant

Beginning first with the case where there is just one value for the cosmological constant, this involves [1]-[7]. We acknowledge that Glinka, in [3] pursued this idea in 2007. Our approach is different from his, as we specify the mass of a graviton as $10^{-62}$ grams as given in [8]. Ng "infinite quantum statistics" as given by [8] allows, $S$ (entropy) as $\sim N$ (counting number), and we specify $N$, via first of all an inflation mass of m given by the following formula and giving

$$m \sim \left( \frac{\lambda}{\sqrt{2\pi^3}} T_{\text{Planck}}^{3/2} t_{\text{Planck}}^{\sqrt{50/32}} \right)^{2/5} \sim N_{\text{graviton}} \cdot m_{\text{graviton}}$$
$$\Rightarrow N_{\text{graviton}} \sim S(\text{Initial entropy}) \sim \left( \frac{\lambda}{\sqrt{2\pi^3}} T_{\text{Planck}}^{3/2} t_{\text{Planck}}^{\sqrt{50/32}} \right)^{2/5} \Big/ m_{\text{graviton}} \quad (1)$$

What we find is that we have a much simpler entropy expression, which is commensurate with say the formation of many massive gravitons being emitted initially and forming micro sized black holes. There is though a problem that the approach may require fine tuning of $\lambda$. In doing this, we are making use of [4] as a non quintessent model of a cosmological constant in [4]. In addition [5] [6] allow for a bose Einstein condensate for early universe black holes. If so, by Novello [7]

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \tag{2}$$

As well as verifying [5] [6]

$$\begin{aligned} m &\approx \frac{m_P}{\sqrt{N_{\text{gravitons}}}} \\ M_{BH} &\approx \sqrt{N_{\text{gravitons}}} \cdot m_P \\ R_{BH} &\approx \sqrt{N_{\text{gravitons}}} \cdot l_P \\ S_{BH} &\approx k_B \cdot N_{\text{gravitons}} \\ T_{BH} &\approx \frac{T_P}{\sqrt{N_{\text{gravitons}}}} \end{aligned} \tag{3}$$

And by [2]

$$\phi(r(t)) \sim \phi(t) \approx \sqrt{(50/32) \cdot m_{Pl}} \cdot \ln(t) \tag{4}$$

Then according to [3], we should look at the spontaneous symmetry breaking potential, given by

$$U(\phi) \sim -m^2\phi^2 + \lambda\phi^4 \tag{5}$$

Setting the temperature, $T$, and the time, $t$ and specifying we are adhering to Equation (4) leads to a spontaneous symmetry breaking potential of the form with $\lambda$

$$\begin{aligned} &\left(2.4 \times 10^{-11}\,\text{GeV}/c^2\right)^4 \cdot \left(1.2009^2 \times 10^{38}\,(\text{GeV})^2/c^4\right) \\ &\sim \frac{4\pi \cdot \lambda^{3/5}}{\left(\sqrt{2\pi^3}\right)^{8/5}} \cdot \left(T_{\text{Planck}}^{3/2} \cdot t_{\text{Planck}}^{\sqrt{50/32}}\right)^{8/5} \end{aligned} \tag{6}$$

This is in tandem with mass of a black hole given by

$$\begin{aligned} M_{BH}\big|_{\text{Planck}} &\approx \sqrt{N_{\text{graviton}}\big|_{\text{Black hole}}}\, m_{\text{Planck}} \\ &\approx \left(\left(\frac{\lambda}{\sqrt{2\pi^3}} T_{\text{Planck}}^{3/2} t_{\text{Planck}}^{\sqrt{50/32}}\right)^{1/5} \Big/ \sqrt{m_{\text{graviton}}}\right) \cdot m_{\text{Planck}} \\ &\equiv \left(\frac{\lambda}{\sqrt{2\pi^3}} T_{\text{Planck}}^{3/2} t_{\text{Planck}}^{\sqrt{50/32}}\right)^{1/5} 10^{65/2} \cdot \sqrt{m_{\text{Planck}}} \end{aligned} \tag{7}$$

And a cosmological black hole generation given by (**Table 1**)

**Table 1.** From [9] assuming penrose recycling of the Universe as stated in that document.

| | | |
|---|---|---|
| End of Prior Universe time frame | Mass (black hole): super massive end of time BH 1.98910^+41 to about 10^44 grams | Number (black holes) 10^6 to 10^9 of them usually from center of galaxies |
| Planck era Black hole formation Assuming start of merging of micro black hole pairs | Mass (black hole) 10^−5 to 10^−4 grams | Number (black holes) 10^22 to about 10^21, |
| Post Planck era black holes just before the CMBR formation | Mass (black hole) Up to 10^6 grams per black hole | Number (black holes) 10^18 to at most 10^20 |

As well as [10]

$$\rho_{\Lambda}c^2 \approx \int_0^{E_{\text{Planck}}/c} \frac{4\pi p^2 \mathrm{d}p}{(2\pi\hbar)^{3/2}} \cdot \frac{1}{2} \cdot \sqrt{p^2c^2 + m^2c^4} \approx \frac{(3\times10^{19}\ \text{GeV})^4}{(2\pi\hbar)^{3/2}} \xrightarrow[E_{\text{Planck}}/c \to 10^{-30}\cdot E_{\text{Planck}}/c]{} \frac{(3\times10^{-11}\ \text{GeV})^4}{(2\pi\hbar)^{3/2}} \tag{8}$$

This means reducing the upper limit of the integral in Equation (8) by $10^{-30}$ to obtain the observed value of the cosmological constant. And this ties into observational inflation as seen below:

[11] was the basis of [1] and possibly [12].

"*The standard inflationary cosmology involves a scalar field φ which obeys a standard wave equation. The potential is this function which I diagram 'above'. The scalar field starts and rolls down the slope until it reaches a value of φ where the potential is $V(\varphi) \sim \varphi^2$.*"

*Adding in a quartic potential may be necessary if we wish to have a derivation of an invariant cosmological constant*!

Equation (9) may rely upon fine tuning as seen here

$$\begin{aligned} &M_{BH}\big|_{\text{Planck}} \approx \left(\frac{\lambda}{\sqrt{2\pi}}\cdot\right)^{1/5} \cdot 10^{65/2}\sqrt{m_P} \approx 100 \text{ Planck units} \\ &\lambda \approx \sqrt{2\pi^3} \times 10^{-305/2} \\ &\rho_{\text{Vaccum}} \sim \left(m^4/2\lambda\right) \sim \frac{\lambda^{8/5}}{2\lambda\cdot\left(\sqrt{2\pi^3}\right)^{8/5}} \cdot \left(T_P^{3/2}\cdot t_P^{\sqrt{50/32}}\right)^{8/5} \end{aligned} \tag{9}$$

## 2. My Simple Quantum Pressure Model

[13] on page 109 has a simple model based on GR for a change in "mass" leading to pressure It is

$$\dot{M}c^2 = -4\pi p(t) \times \left[r(t)\right]^2 \dot{r}(t) \tag{10}$$

Here we are assuming as a simplification that $p(t) \xrightarrow{\text{initial conditions}} P_0$ for pressure. This is going to lead us to re interpret the mass term $\dot{M}$ Now using [13] we can assume if the surviving effective "mass" is represented in **Table 1**, with surviving mass in the Pre Planckian to Planckian regime massive gravitons that according to [13], there would be an effective velocity of if $\hbar = c = 1$ on page 89 of [13]

$$v(\omega) = \sqrt{1 - \frac{m^2}{\omega^2}} \tag{11}$$

In terms of gravitons if they have a mass of $10^{-65}$ grams, and a frequency of say $10^9$ Ghz this means we can write [13] [14]

$$v(\omega) = \sqrt{1 - \frac{m^2}{\omega^2}} \approx 1 - \xi^+ \tag{11a}$$

Now make the following substitution

$$\dot{M} = \frac{\Delta M}{\Delta t} \tag{12}$$

Use a minimum uncertainty principle of

$$\Delta E \Delta t = \hbar \xrightarrow[\text{Planck Units}]{} \Delta\omega\Delta t = 1 \tag{13}$$

Then

$$\dot{M} = \frac{\Delta M}{\Delta t} \approx \Delta M \cdot \omega = -4\pi \cdot P_0 \times \left[r(t)\right]^2 \cdot \left(1 - \xi^+\right) \tag{14}$$

Then to first order we can write a minimum uncertainty (QM) as

$$P_0 \approx -\frac{\Delta M \cdot \omega}{4\pi\left[r(t)\right]^2} \cdot \left(1 + \xi^+\right) \approx -\frac{\Delta M \cdot \omega}{4\pi\left[\ell_P\right]^2} \cdot \left(1 + \xi^+\right) \tag{15}$$

The radius would be, $r(t)$ approximately Planck length, whereas the change in $\Delta M$ would be a decrease from the mass, $M$ of the black holes, Pre Planck (very large) as seen in **Table 1**, to the tiny micro sized black holes of the order of Planck mass, *i.e.* a down grading of mass from about $10^{60}$ grams to say $10^5$ grams with millions of micro black holes, Plank mass or above.

The details of the change in $\Delta M$ would be a decrease from the mass, M of the black holes, Pre Planck (very large) as seen in **Table 1**, to the tiny micro sized black holes of the order of Planck mass, await further investigation. We should before ending state that we do not expect r in Equation (10) to ever go to zero, Point of fact, if there is, no conservation of energy due to **Table 1** I then do not see a Penrose singularity [15] [16]. Also we argue that this may imply very significant fine tuning in the quartic potential for inflation physics which is in Equation (5). After stating this we should note that.

Whereas there is an inflaton mass $m$

$$m \cong \frac{\lambda^{2/5}}{\left(\sqrt{2\pi^3}\right)^{2/5}} \cdot \left(T_P^{3/2} \cdot t_P^{\sqrt{50/32}}\right)^{2/5} \tag{16}$$

Dolgov, in [17] has an emergent value of the vacuum energy density which he gives as follows with our subsequent valuation.

Our next idea will reference the inspiration given in [18]. *I.e.* of a "quantum pressure".

Here we will NOT make a linkage between pressure $P_0$ and temperature $T$ via applying treatment of a "graviton gas" as an ideal gas law. *I.e.* we state emphatically

$$P_0 \neq \frac{n\Re T}{V_{\text{volume}}} \tag{17}$$

But in doing this, we first of all need to consider how to obtain the vacuum energy which is of the order of $10^{-30}$. We will derive arguments for an enormous pressure value, but this is commensurate if we wish to isolate out a vacuum energy scaled to a value of about $10^{-30}$ or so, to isolate out the value of the cosmological constant which is of the order of $10^{-120}$ or so, which is fine tuning on steroids. Doing this means looking at: initially

$$\begin{aligned} \rho_{\text{Vaccum}} &\sim \left(m^4/2\lambda\right) \sim \frac{\lambda^{3/5}}{2\cdot\left(\sqrt{2\pi^3}\right)^{8/5}}\cdot\left(T_P^{3/2}\cdot t_P^{\sqrt{50/32}}\right)^{8/5} \\ &\xrightarrow[\text{Planck Units}]{} \frac{\lambda^{3/5}}{2\cdot\left(\sqrt{2\pi^3}\right)^{8/5}} \end{aligned} \tag{18}$$

If we do this with the fine-tuned version of the $\lambda$ value with its incredibly small value we will then look at [17] with a partition function as rendered below. And right during the Planck era we also can look at an effective temperature. Starting with the

$$\begin{aligned} Z &= \iint \wp\phi\cdot\exp\left(-S(\phi)\right) \\ S(\phi) &= \frac{1}{T_{eff}}\cdot\iiint \mathrm{d}^3x\cdot\left(\frac{(\nabla\phi)^2 - 2\mu^2\cdot\exp\left(\phi(x)\right)}{2}\right) \end{aligned} \tag{19}$$

Then, if $T$ is a temperature, and $z$ is the fugacity, and $m$ is the inflaton mass, which we will decompose:

$$\begin{aligned} T_{eff} &= 4\pi G m^2\cdot T^{-1} \\ \mu^2 &= \sqrt{\frac{2}{\pi}}\cdot z\cdot G\cdot m^{7/2}\cdot\sqrt{T} \end{aligned} \tag{20}$$

The key element which we will be working with is, a particle density expression of [17] as [18]

$$\left\langle\rho(r)\right\rangle = \mu^2 T_{eff}^{-1}\cdot\left\langle\exp\left(\phi(r)\right)\right\rangle \tag{21}$$

If we use the following from Padmanabhan, [19], using the approximation of $a(t)\approx t^{\bar{n}}$, then

$$\begin{aligned} \left(\phi(r)\right) &\approx \ln t^{\sqrt{2\bar{n}\cdot m_P}} \\ \left\langle\rho(r)\right\rangle &= \mu^2 T_{eff}^{-1}\cdot\left\langle\exp\left(\phi(r)\right)\right\rangle \approx \mu^2 T_{eff}^{-1}\cdot t^{\sqrt{2\bar{n}}} \end{aligned} \tag{22}$$

We will be utilizing these five equations from Equation (3), next. See Equation (3) as to the Mass of a black hole, and its entropy. And use only the following for the production of initial black holes, *I.e.* the Mass and the Entropy for BHs in Equation (3).

We shall next come up with a value for the number of Gravitons initially where we set T in our initial configuration as set equal to Planck temperature.

## 3. Existence of Graviton Gas? Non Zero Initial Entropy? and Our Concluding Comparisons

We need to refer to [20] following up upon the Ng "infinite quantum statistics" as we then write, $S$ (entropy) as $\sim N$ (counting number), and we specify $N$, via Equation (1) we have that we get a massive graviton factored in.

The value of the initial graviton mass is specified as of being $10^{-62}$ grams. We hope that, if this is conclusively non zero, then it will enable CMBR style studies as well as looking at non zero vacuum energy as given by non linear electrodynamics as in [21] and also the issue of the nature of gravity as up by Corda [22], as far as future studies and investigations. Further developments should specifically investigate the symmetry breaking potential as written up as enabling the metric tensor approach given in [23].

$$N_{\text{graviton}} m_{\text{graviton}} \approx \frac{\lambda^{2/5}}{\left(\sqrt{2\pi^3}\right)^{2/5}} \cdot \left(T_P^{3/2} \cdot t_P^{\sqrt{50/32}}\right)^{2/5} \tag{23}$$

For black holes, we would write Equation (9) as given earlier, and we assume that we have micro sized black holes as given in Equation (7). If so and use Planck units as given in [24] [25] [26]

$$\begin{aligned} \ell_P &= \sqrt{\frac{\hbar G}{c^3}} \xrightarrow{\text{Plank unit renormalization}} 1 \\ t_P &= \sqrt{\frac{\hbar G}{c^5}} \xrightarrow{\text{Plank unit renormalization}} 1 \\ T_P &= \sqrt{\frac{\hbar c}{G}} \xrightarrow{\text{Plank unit renormalization}} 1 \end{aligned} \tag{24}$$

We also write

$$m_P = \sqrt{\frac{\hbar c}{G}} \xrightarrow{\text{Plank unit renormalization}} 1 \tag{25}$$

We can normalize all these to be 1 for which then we have Equation (7) again so then as noted in Equation (9) this has $\lambda \approx \sqrt{2\pi^3} \times 10^{-305/2}$, which means that we have for the existence of a Planck regime quartic potential a level of fine tuning for the quartic potential term which in turn leads to an incredibly small value, and bespeaks of an almost insane level of fine tuning for the effective potential in Equation (5) which is the symmetry breaking value. *I.e.* it is almost chaotic potential.

Meanwhile if the following holds for vacuum density which is to the 1/4 power of the Cosmological constant

$$\rho_{\text{Vaccum}} \sim \left(m^4/2\lambda\right) \sim \frac{\lambda^{3/5}}{2\cdot\left(\sqrt{2\pi^3}\right)^{8/5}}\cdot\left(T_P^{3/2}\cdot t_P^{\sqrt{50/32}}\right)^{8/5} \xrightarrow[\text{Planck Units}]{} \frac{\lambda^{3/5}}{2\cdot\left(\sqrt{2\pi^3}\right)^{8/5}} \approx \frac{\left(\lambda=\sqrt{2\pi^3}\times 10^{-305/2}\right)^{3/5}}{2\cdot\left(\sqrt{2\pi^3}\right)^{8/5}} \tag{26}$$

It means in this situation for a time interval that there is expansion to an almost infinite degree, which is commensurate with pressure due to black holes from the prior universe of about $10^{61}$ grams being cut to about $10^{6}$ grams, so as to make the resulting pressure $P_0$ almost inconceivably large. Also [2] [27] should be considered as well as the simpler approaches given in [28] [29].

Finally we need to refer to [30] to do more refinements on the crude pressure argument rendered and note the similarities of this work with the ideas presented in [31].

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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