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Noncommutative-Geometry Wormholes Based on the Casimir Effect

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Abstract

While wormholes are as good a prediction of Einstein's theory as black holes, they are subject to severe restrictions from quantum field theory. In particular, holding a wormhole open requires a violation of the null energy condition, calling for the existence of exotic matter. The Casimir effect has shown that this physical requirement can be met on a small scale, thereby solving a key conceptual problem. The Casimir effect does not, however, guarantee that the small-scale violation is sufficient for supporting a macroscopic wormhole. The purpose of this paper is to connect the Casimir effect to noncommutative geometry, which also aims to accommodate small-scale effects, the difference being that these can now be viewed as intrinsic properties of spacetime. As a result, the noncommutative effects can be implemented by modifying only the energy momentum tensor in the Einstein field equations, while leaving the Einstein tensor unchanged. The wormhole can therefore be macroscopic in spite of the small Casimir effect.

Keywords

Traversable Wormholes, Noncommutative Geometry, Casimir Effect

1. Introduction

Wormholes are handles or tunnels in spacetime connecting widely separated regions of our Universe or different universes in a multiverse. In many ways, wormholes are as good a prediction of Einstein's theory as black holes, but they are subject to some severe restrictions. In particular, a wormhole can only be held open by violating the null energy condition, leading to what is called "exotic matter" in Ref. [1]. This outcome is not a conceptual problem in the sense that exotic matter can be made in the laboratory by means of the Caimir effect [2]. Being a rather small effect, it is not clear whether this is sufficient for supporting a macroscopic traversable wormhole.

Another area dealing with small effects is noncommutative geometry, an offshoot of string theory. Here point-like particles are replaced by smeared objects, an effect that is often modeled using a Gaussian distribution of minimal length $\sqrt{\beta}$. Even though the existence of the small parameter β seems to echo the Casimir effect, the noncommutative-geometry background does not seem to prevent a wormhole from being macroscopic.

The purpose of this paper is to examine the possibility of combining these two approaches.

2. Background

2.1. The Casimir Effect

To describe the Casimir effect, one normally starts with two closely spaced parallel metallic plates in a vacuum. The plates can be replaced by two concentric spherical shells to preserve the spherical symmetry preferred in a wormhole setting. If a is the separation between the shells, then the pressure p as a function of the separation is [3]

$$p(a) = -3\frac{\hbar c \pi^2}{720 a^4} \tag{1}$$

and the density is

$$\rho_C(a) = -\frac{\hbar c \pi^2}{720 a^4}. \tag{2}$$

Here $\hbar$ is Planck's constant and c is the speed of light. Observe that the equation of state has the form $p = \omega\rho$ with $\omega = 3$.

2.2. Noncommutatice Geometry

In recent years, string theory has become ever more influential, as exemplified by the realization that coordinates may become noncommutative operators on a D-brane [4] [5]. The main idea is that noncommutativity replaces point-like particles by smeared objects [6] [7] [8], thereby eliminating the divergences that normally appear in general relativity. As a result, spacetime can be encoded in the commutator $\left[\mathbf{x}^\mu, \mathbf{x}^\nu\right] = i\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is an antisymmetric matrix that determines the fundamental cell discretization of spacetime in the same way that Planck's constant $\hbar$ discretizes phase space [7]. According to Refs. [7] [9] [10] [11], the smearing can be modeled using a Gaussian distribution of minimal length $\sqrt{\beta}$ instead of the Dirac delta function. It is shown in Refs. [12] [13] that an equally effective way is to assume that the energy density of a static and spherically symmetric and particle-like gravitational source is given by

$$\rho(r) = \frac{m\sqrt{\beta}}{\pi^2\left(r^2+\beta\right)^2}. \tag{3}$$

The usual interpretation is that the gravitational source causes the mass m of a particle to be diffused throughout the region of linear dimension $\sqrt{\beta}$ due to the uncertainty. In the next section, we are going to be concerned with a smeared spherical surface (referred to as the throat of a wormhole). So the smeared particle is replaced by a smeared surface. According to Ref. [14], the energy density ρ_s is given by

$$\rho_s\left(r-r_0\right)=\frac{\mu\sqrt{\beta}}{\pi^2\left[\left(r-r_0\right)^2+\beta\right]^2}, \tag{4}$$

where μ now denotes the mass of the surface.

3. Possible Macroscopic Wormholes

Our first task in this section is to recall the basic wormhole model proposed by Morris and Thorne [1],

$$\mathrm{d}s^2=-\mathrm{e}^{2\Phi(r)}\mathrm{d}t^2+\frac{\mathrm{d}r^2}{1-b(r)/r}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right), \tag{5}$$

using units in which $c=G=1$. Here $b=b(r)$ is called the *shape function* and $\Phi=\Phi(r)$ is called the *redshift function*, which must be everywhere finite to prevent the occurrence of an event horizon. The spherical surface $r=r_0$ is called the *throat* of the wormhole, where $b(r_0)=r_0$. The shape function must also meet the requirement $b'(r_0)<1$, called the *flare-out condition*, while $b(r)<r$ for $r>r_0$. We also require that $b'(r_0)>0$.

Returning now to Equation (3), the energy density ρ as a function of the separation $r=a$ is

$$\rho(a)=\frac{m\sqrt{\beta}}{\pi^2\left(a^2+\beta\right)^2}. \tag{6}$$

According to Equation (4), in the vicinity of the throat, *i.e.*, whenever $r-r_0=a$, we get

$$\rho_s(a)=\frac{\mu\sqrt{\beta}}{\pi^2\left(a^2+\beta\right)^2}. \tag{7}$$

So by Equation (2),

$$\frac{\mu\sqrt{\beta}}{\pi^2\left(a^2+\beta\right)^2}=\left|\rho_C(a)\right|=\frac{\hbar c\pi^2}{720a^4}, \tag{8}$$

thereby connecting the Casimir effect to the noncommutative-geometry background. Now, while the parameter *a* may be small, it is still macroscopic. So we can assume that $\beta=\left(\sqrt{\beta}\right)^2\ll a^2$. Being an additive constant, β in the denominator of Equation (8) becomes negligible. So

$$\sqrt{\beta}=\frac{\hbar c\pi^4}{720\mu}. \tag{9}$$

Since $\hbar=1.0546\times10^{-34}\ \mathrm{J\cdot s}$,

$$\sqrt{\beta} = \frac{4.28 \times 10^{-27}}{\mu}. \tag{10}$$

Now recall that μ is the mass of the throat $r = r_0$, a spherical surface of negligible thickness. This is an idealization that is hard to quantify. In practice, of course, we are dealing with a thin shell that has a definite mass μ, thereby defining $\sqrt{\beta}$ in Equation (10). An alternative approach is to give a direct physical interpretation to the smearing effect by letting $\sqrt{\beta} = a$. Then Equation (8) yields

$$a = \frac{\hbar c \pi^4}{180\mu} \tag{11}$$

and we can write

$$a\mu = \frac{\hbar c \pi^4}{180}, \tag{12}$$

a fixed quantity. So there are many possible choices for a and μ.

Connecting the Casimir effect to noncommutative geometry has some important consequences. In particular, the noncommutative effects can be implemented in the Einstein field equations $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ by modifying only the energy momentum tensor, while leaving the Einstein tensor intact. The reasons for this are discussed in Ref. [7]: A metric field is a geometric structure defined over an underlying manifold whose strength is measured by its curvature. But the curvature is nothing more than response to the presence of a mass-energy distribution. Here it is emphasized in Ref. [7] that noncommutativity is an intrinsic property of spacetime rather than a superimposed geometric structure. So it naturally affects the mass-energy and momentum distributions, which, in turn, determines the spacetime curvature, thereby explaining why the Einstein tensor can be left unchanged. As a consequence, the length scales can be macroscopic.

A final issue to be addressed is the large radial tension at the throat of a Morris-Thorne wormhole. First we need to recall that the radial tension $\tau(r)$ is the negative of the radial pressure. According to Ref. [1], the Einstein field equations can be rearranged to yield

$$\tau(r) = \frac{b(r)/r - 2\left[r - b(r)\right]\Phi'(r)}{8\pi G c^{-4} r^2}. \tag{13}$$

It follows that the radial tension at the throat is

$$\tau(r_0) = \frac{1}{8\pi G c^{-4} r_0^2} \approx 5 \times 10^{41} \frac{\text{dyn}}{\text{cm}^2} \left(\frac{10\ \text{m}}{r_0}\right)^2. \tag{14}$$

In particular, for $r_0 = 3\ \text{km}$, τ has the same magnitude as the pressure at the center of a massive neutron star. This is rather hard to explain if we are not actually dealing with neutron matter. It is shown in Ref. [14], however, that a noncommutative-geometry background can indeed account for the large radial tension.

4. Conclusion

While the possible existence of wormholes is a consequence of Einstein's theory, such wormholes would be subject to severe restrictions from quantum field theory. In particular, holding a macroscopic wormhole open requires a violation of the null energy condition, calling for the existence of exotic matter in the vicinity of the throat. The Casimir effect has shown that such a violation can be produced in the laboratory, thereby eliminating a major conceptual problem. So the real question is whether enough exotic matter could be produced to sustain a macroscopic wormhole. The purpose of this paper is to connect the Casimir effect to noncommutative geometry, which also involves small effects thanks to the parameter β in Equation (4). Here the gravitational source causes the mass μ of a surface to become diffused due to the uncertainty. The result is a wormhole whose throat is a smeared surface. The subsequent connection to the Casimir effect yields a physical interpretation of the length $\sqrt{\beta}$, which is a measure of the uncertainty. Apart from this, the noncommutative effects can be implemented in the Einstein field equations by modifying only the energy momentum tensor, while leaving the Einstein tensor unchanged. We conclude that the wormhole can indeed be macroscopic in spite of the small Casimir effect. Finally, the noncommutative-geometry background can account for the enormous radial tension that is characteristic of moderately-sized Morris-Thorne wormholes.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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