

# A Cosmological Model for the Early Universe: The Formation of Fundamental Particles

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## Abstract

A cosmological model for the very early universe is proposed which may modify the present point of view of physicists and astrophysicists, concerning the very early universe at a miniscule fraction of a second, approximately  $10^{-60}$  seconds after the Big Bang. The model proposes the presence of a primordial *s-particle* that, following the Big Bang, was violently ejected in all spatial directions together with extremely high-frequency radiation that dominates this era. The proposed *s-particles* underwent two geometrical phase transitions in space-time that led to the formation of the known fundamental particles (*i.e.*, dark matter, quarks, electrons, neutrinos, etc.). Furthermore, in the model, the four fundamental forces may be accommodated within one structural framework. It shows that the electronic charge is not a fundamental quantity (intrinsic property of the particle), but rather that it can be derived from the tangential velocity of the *s-particle*. Moreover, it appears that the masses of the fundamental particles are proportional to the curvature of the path of the *s-particle*.

## Keywords

Cosmology, Nongalactic Astrophysics, Particle Physics, High Energy Physics, Atomic Physics

## 1. Introduction

In the past decades, different theories and models concerning the very early universe [1] [2] [3] have been suggested, pertaining to the inflation period that started around  $10^{-36}$  seconds after the Big Bang. In this article, a cosmological model is proposed that might shed a light on the timespan before the inflation period, starting at  $10^{-60}$  seconds after the Big Bang. The model postulates that it was within this timespan that fundamental particles started to be formed, the tem-

perature of the universe still being extremely high, of the order of about  $10^{38}$  K, and the mass-energy density more than  $10^{30}$  g/cm<sup>3</sup>.

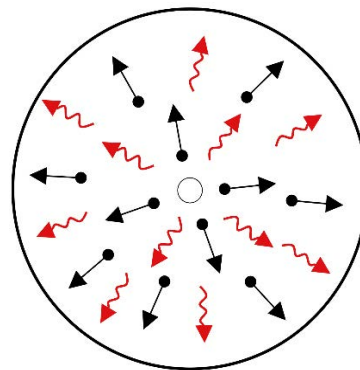
The Big Bang is considered as the initial state of the universe (the initial singularity) at  $t = 0$ , where a violent explosion occurred that was followed by the ejection of particles with extremely high linear and angular momentum, together with extremely high-frequency radiation in all spatial directions. The amount of mass-energy density of the universe around the singular point is considered as having (nearly) infinite value and therefore the temperature and the density of the universe were tremendously high at the beginning of the explosion. The structure of the space-time mesh was created simultaneously with the Big Bang while the ejection of particles and radiation was spherically symmetric. The universe at that time was perfectly homogenous and isotropic, consistent with the cosmological principle.

In a previous publication [4] dealing with the shape of the electron, a particle was postulated to be involved in the formation of the electron—in the present article referred to as the *s-particle*—which was ejected at the point of singularity during the Big Bang explosion. It is proposed that during the expansion and cooling of the universe in the periods that followed, the *s-particle* was subjected to geometrical phase transitions that altered its dynamics of motion and fundamental particles started to be formed.

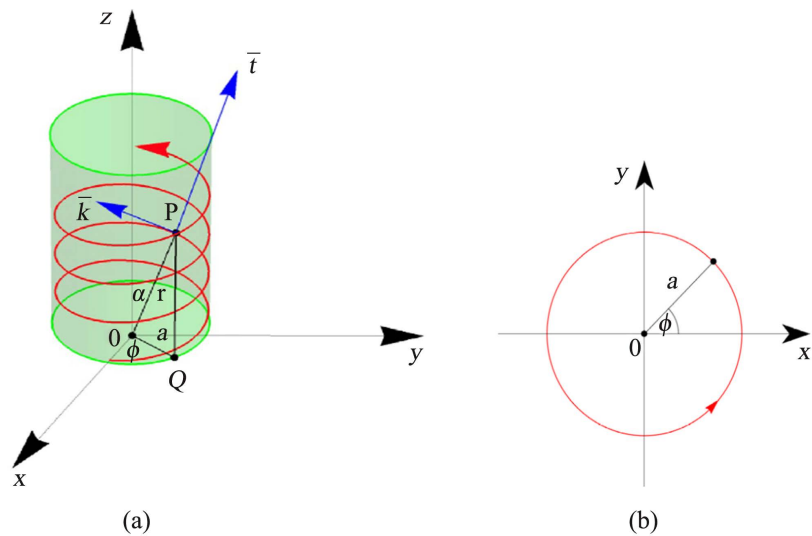
## 2. The “*s-Particle* Model”

The Big Bang explosion was followed by the violent ejection of particles and radiation in a perfect spherically symmetric shape around the point of singularity (see, **Figure 1**).

Soon afterwards, the expansion of the universe slowed down until at about  $10^{-60}$  seconds the temperature and the density were reduced to values allowing for the first geometrical phase transition to occur. In this transition the angular velocity of the *s-particle*, that was inherently present from the beginning, started to manifest as the particle started to follow a curved path defined by a symmetric cylindrical helix (see, **Figure 2**).



**Figure 1.** Violent ejection of particles and radiation in a spherically symmetric shape around the singular point.



**Figure 2.** The *s-particle* is moving along a helix on a right circular cylinder (a). The projection of the curved path of the *s-particle* on the *xy*-plane (b).

The projection of the curved path of the particle on a plane normal to the axis of the cylinder, appears as a circle with a radius equal to the radius of the cylinder. In this model it is assumed that the only force existing in this period was gravitational, induced by the *s-particle* with its angular motion; therefore, the *s-particle* in this context appears in the form of dark matter. The gravitational force which was exerted by the dark matter is very strong since it is proportional to the curvature of the helix and the projected circle (the radius of curvature is extremely small in this period, in accordance with the radius of the circle, possibly around  $10^{-35}$  cm). In conclusion, in the first phase transition at a critical temperature of about  $10^{38}$  K, the spherically symmetric geometrical shape of the universe was broken around  $10^{-60}$  seconds after the Big Bang to a less symmetric shape due to *s-particle* aggregations moving along cylindrical helices, because of the strong attractive gravitational forces exerted by each *s-particle* upon surrounding particles.

The expansion of the universe continued and the subsequent reduction in temperature and density, led to an era of another geometrical phase transition at about  $10^{-43}$  seconds, where some fraction of the *s-particles* changed their dynamics of motion and started following helical paths on right circular cones of revolution. The projection of the helical paths on a plane perpendicular to the axis of the cone appears as logarithmic spirals. Due to the curved path, centrifugal and tangential forces were generated by the *s-particles* along their motion on the circular helices and their projected logarithmic spirals. These forces led the primordial *s-particles* to appear as other fundamental particles, namely quarks, electrons, and neutrinos together with their antiparticles. According to the proposed model, the first of the fundamental particles to be formed along the *s-particle's* helical path on the cone would have been the unstable quark particles, where three of them attach to a dark matter, thus forming a stable proton.

Electrons and neutrinos will be formed later, on the helical *s-particle's* path. The generation of the strong nuclear force, the electromagnetic force and the weak nuclear force could be explained through the application of the unifying “*s-particle* model” as well, and the model could play a significant role in the unification of these three fundamental forces together with gravity.

After the above qualitative discussion of the “*s-particle* model”, we proceed to a more quantitative description in the next section.

### 3. Mathematical Framework

In this section, we assume that after the Big Bang the universe was in thermal equilibrium between *s-particles* and radiation. The radiation continuously decayed to form *s-particles* and their anti-particles while the continuous annihilation of *s-particles* and anti-particles by random collision, in turn, formed radiation, leading to a constant ratio between particles and radiation. The difference between the *s-particles* and their anti-particles is the direction of rotation, where the former follow a right-hand rotation whereas the latter follow a left-hand rotation.

The first phase transition at about  $10^{-60}$  seconds, due to the expansion and cooling of the universe, transformed the spherically symmetric universe to a cylindrically symmetric one where the *s-particles* together with their anti-particles took curved helical paths around right circular cylinders. The curved motion of the *s-particles* together with their projection on a plane normal to the axis of the cylinder (**Figure 2**) manifests as the dark matter.

Let the vector  $\vec{r}$  be the position vector of the *s-particle* at the point P on the helix, given by [5] [6]

$$\vec{r} = (a \cos \varphi, a \sin \varphi, b\varphi), \quad (1)$$

where  $a = r \sin \alpha$  and  $b = a \cot \alpha$ .

The curvature vector at the point P is given by [6],

$$\vec{K} = -\left[ a / (a^2 + b^2) \right] [(\cos \varphi) \vec{e}_x + (\sin \varphi) \vec{e}_y + 0 \vec{e}_z], \quad (2)$$

where the curvature amplitude is,

$$k = |\vec{K}| = a / (a^2 + b^2),$$

and the radius of curvature is given by,

$$\rho = 1/k = (a^2 + b^2) / a. \quad (3)$$

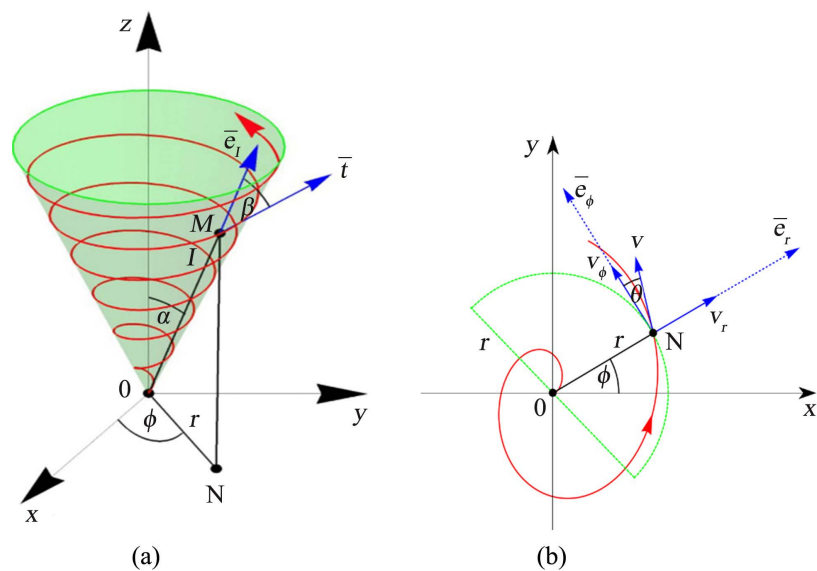
For  $b \ll a$  where “*b*” approaches zero and the value of  $\rho$  becomes the value of the radius of the projected circle, *i.e.*, the radius of the cylinder, which is justified by the slow rate of expansion. The curvature has a constant value on the cylinder, *i.e.*, it has the same value for every point on the curved path of the *s-particles*.

The only force that is exerted by the *s-particle* due to its circular motion in this phase is thus, the centripetal force. This force is opposing the force exerted on the *s-particles* during the expansion and hence, for the stability of the

*s-particle* (dark matter) in this phase, the angular velocity of the particles should increase with the increased force of expansion, for compensation and keeping the radius at fixed value.

As mentioned in the previous section, with the expansion and cooling of the universe the *s-particles* gather in clusters under their gravitational attractive forces, which is asserted nowadays in suggestions that galaxies are embedded in haloes of cold dark matter made of non-baryonic particles [3]. The discovery that stars in spiral galaxies orbit far from the galaxy's centre with larger speeds than those much nearer to the centre, leading to the conclusion that much greater gravitational forces are provided by unseen matter (dark matter) which resides in the haloes of the galaxies [7]. Moreover, nowadays conclusive evidence supports the suggestions that invisible dark matter would be responsible for the large-scale structure of the universe through gathering together the ordinary observable matter to form stars and galaxies [8]. Advanced measurements and calculations indicate that the universe contains about six times as much dark matter as visible matter (protons, neutrons, and electrons) [8]. The main difference between dark matter and visible matter is that the latter interacts through the four fundamental forces (electromagnetic force, strong and weak nuclear forces, gravitational force), while the former interacts only through gravitational force.

Further expansion of the universe with a much higher rate of expansion, together with the fast reduction in temperature and density, led to an era of a second geometrical phase transition around about  $10^{-43}$  seconds. In this transitional period a fraction of the cylindrically symmetric *s-particles*, follow helical paths on right circular cones of revolution with a semi-vertical angle  $\alpha$ . The projection of the helical path on a plane perpendicular to its axis forms a logarithmic spiral (see, **Figure 3**).



**Figure 3.** Helical motion of the *s-particle* on a right circular cone (a). The projection of the helical path of the *s-particle* on a plane perpendicular to the axis of the cone showing the logarithmic spiral shape.

The forces that are generated by the curved motion in this transition are the radial centrifugal force  $\bar{F}_c$  and the perpendicular tangential force  $\bar{F}_t$ , which act on the *s-particles*. These forces affect the motion of the particles on the conical helices and therefore the projected motion on the logarithmic spirals, leading to the formation of quarks, electrons, neutrinos, and their anti-particles.

Let  $\bar{l}$  be a position vector of a particle at point M on the helical path, given by,

$$\bar{l} = (r \cos \varphi, r \sin \varphi, r \cot \alpha), \tag{4}$$

where  $r = l \sin \alpha$ .

The arc-rate of rotation of the position vector along the helix gives the tangent vector  $\bar{t}$ , given by [5] [6],

$$\bar{t} = (\sin \alpha \cos \beta \cos \varphi - \sin \beta \sin \varphi, \sin \alpha \cos \beta \sin \varphi + \sin \beta \cos \varphi, \cos \alpha \cos \beta). \tag{5}$$

The arc-rate of rotation of the tangent vector  $\bar{t}$  gives the curvature vector  $\bar{K}$ , where the radius of curvature  $\rho$  is the reciprocal of the magnitude of curvature vector, given by,

$$k = |\bar{K}| = (\sin \beta / r) \left[ (\sin^2 \alpha \cos^2 \beta + \sin^2 \beta)^{1/2} \right], \tag{6}$$

where  $\beta$  is the angle between the tangent vector  $\bar{t}$  and the direction of the position vector  $\bar{e}_l$ , and it is constant having the same value on the right circular cone. The projection of the helical path on the xy-plane gives the projected spiral shape having a radius  $r$  given by [6],

$$r = \sin \alpha \cos \beta e^{\varphi \sin \alpha \cot \beta}. \tag{7}$$

Now, consider the projected motion of the *s-particle* on the spiral (**Figure 3(b)**) and let us assume a general logarithmic spiral with a radial equation in the form:

$$r = r_0 e^{a\varphi}, \tag{8}$$

where  $(r_0, a)$  are arbitrary constants. The velocity of the particle on the spiral at the point N is,

$$\bar{v} = (dr/dt)\bar{e}_r + r(d\varphi/dt)\bar{e}_\varphi, \tag{9}$$

where,  $(dr/dt) = \dot{r}$  and  $(d\varphi/dt) = \dot{\varphi}$ .

The components of  $\bar{v}$  along the radial direction,  $\bar{v}_r = \dot{r}\bar{e}_r$  and along the tangent on the polar circle,  $\bar{v}_\varphi = r\dot{\varphi}\bar{e}_\varphi$ . The angle  $\theta$  between the spiral tangent at N and that corresponding to the polar circle is called the angle of the polar slope, where  $\tan \theta$  is the polar slope. For the spiral described by Equation (8), the polar slope is given by,

$$\tan \theta = v_r / v_\varphi = a r \dot{\varphi} / r \dot{\varphi} = a. \tag{10}$$

If the force  $\bar{F}$  exerted by the particle at the point N on the spiral has the two components  $\bar{F}_r \bar{e}_r$  and  $\bar{F}_t \bar{e}_\varphi$  are aligned in the same directions of the velocity components, then the polar slope will have the value,

$$\tan \theta = F_r/F_t = m(\ddot{r} - r\dot{\varphi}^2)/m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}), \quad (11)$$

substituting Equation (10) in Equation (11) and solving for the value of  $a$ , we get,  $a = \pm\sqrt{-1}$ , and hence, the path of the  $s$ -particle will take the shape :

$$r = r_0 e^{\pm i\varphi} = r_0 (\cos \varphi \pm i \sin \varphi). \quad (12)$$

Thus, the  $s$ -particle takes a stable circular path with a radius of curvature  $r_0$ , forming the free electron [4].

We conclude here that during the expansion the  $s$ -particle was detached from the helical motion and formed a stable electron with a radius  $r_0$ . However, if the stable circular motion of the  $s$ -particle (the electron) is subjected to an external force, it will accelerate or decelerate and move along the projected spiral shape in (Figure 2 in Ref. [4]), where the polar angle for the  $s$ -particle, say  $\theta_s$ , is given by [4],

$$\theta_s = \arctan(v_r/v_\varphi) = \arctan(1/2\alpha) (= 89.16 \text{ degrees}), \quad (13)$$

where  $\alpha$  is the fine structure constant ( $\alpha = 1/137$ ).

During the acceleration of the  $s$ -particle along the spiral path (just before the formation of the stable electron) and its rotation about the spiral's axis at a distance  $r$ , a torque  $\tau$  is generated.

The torque is given by,

$$\tau = I\tilde{\alpha}, \quad (14)$$

where  $\tilde{\alpha}$  is the angular acceleration and  $I$  is the moment of inertia of the  $s$ -particle given by,

$$I = L/\omega, \quad (15)$$

where,  $L$  is the angular momentum of the particle and  $\omega$  is its angular velocity. For a constant angular momentum, the decrease of  $\omega$  corresponds to the increase of  $r$  and vice versa.

The torque  $\tau$ , which is the rate of change of the angular momentum, is then given by,  $mr^2\tilde{\alpha}$ . For the electron the torque is  $mr_0^2\tilde{\alpha}$  and hence the moment of the torque ( $r_0 \times \tau a$ ) is thus given by  $r_0 L a / v_t$ , where  $L$  is the angular momentum of the  $s$ -particle ( $=\hbar$ , Planck's constant divided by  $2\pi$ ) and,  $v_t$  &  $a_t$  are the tangential velocity and the tangential acceleration of the  $s$ -particle respectively. If  $e^2$  (the square of the electron charge) equals to the moment of the torque, we get

$$e^2 = \hbar v_t, \quad (16)$$

where,  $v_t$  is the tangential velocity of the  $s$ -particle within the electron, given by  $v_t = \alpha c$ , (see, [4]). Substitute the value of  $v_t$  into Equation (16), we get

$$e^2 = \hbar \alpha c \quad \text{or} \quad \alpha = e^2 / \hbar c \quad (17)$$

The electron charge appears only when an external applied force is applied on the free electron.

When a neutral atom—say the hydrogen atom—is formed, the free electron is bound to the proton and circles around it in stable atomic orbits. In these orbits there will be a decrease in the free electron radius with an increase in the binding energy until  $r_0$  (the free electron radius) reaches the minimum value in the

ground state of the atom [4]. The relations of electromagnetic fields  $E$  and  $B$  with  $F_{cr}$  and  $F_t$  were discussed in (Equation (13), [4]).

On the other hand, before reaching the point of stability on the helical motion where the electron was formed (**Figure 3(a)**), the  $s$ -particle passed through a period of instability while the radius of the path was still growing. In this period three of the unstable  $s$ -particles were detached from the helical path and attached to a dark matter by its strong gravitational force, to form a stable proton (these particles are the unstable quarks). This strong gravitational force, which might be generated from the extremely fast rotation of the dark matter, and which attracted the unstable quarks, should be studied, and treated in a way completely different from the gravitational force in the Newtonian gravitational theory. This classical treatment does not contradict the most widely accepted quantum theory of “quark confinement” [9], but rather it supports it since the force of attraction between the three quarks will increase with the increasing distance of the quarks from the dark matter. If the three  $s$ -particles (unstable quarks) approach the dark matter under its strong gravitational force, the radius of each attracted particle will decrease, while its angular velocity will increase and hence, they will appear as free particles, *i.e.*, far from each other. The unstable  $s$ -particles (quarks in this phase), if not attached to the dark matter and therefore kept on the helical path, will eventually reach a point where they become stable electrons.

Consider the spiral given by Equation (8) as a golden spiral, *i.e.*, a logarithmic spiral the expansion or growth factor of which is the golden ratio  $\phi$  ( $\approx 1.618$ ) [10]. This spiral becomes wider or further from the origin by a factor  $\phi$  for every quarter turn it makes, when the angle  $\varphi$  is equal to  $n\pi/2$ , where  $n = 1, 2, 3, \dots, \infty$ . The growth factor is then,  $\phi = e^{a\varphi}$  and the numerical value of “ $a$ ” will be,

$$a = \ln \phi / \varphi, \quad (18)$$

where for  $\varphi = \pi/2$  and from Equation (10) we can calculate the polar angle  $\theta$  for the golden spiral to be approximately 17 degrees. This angle might be the angle at which the unstable quarks were formed due to the motion of the  $s$ -particles along the spiral.

Furthermore, if the  $s$ -particles keep moving on the helical path (**Figure 3**) reaching larger radii of rotation, the path will be a nearly straight line with about zero curvature. In this situation the motion is constant and stable neutrinos are formed with nearly zero masses with respect to the other fundamental particles (few eV, about  $10^4$  times smaller than the mass of the electron). Consequently, these particles travel with speeds approaching the speed of light while interacting only with weak nuclear forces, the interaction with gravitational forces being negligible. It is apparent from Equation (6), that the curvature of the path of the  $s$ -particle is proportional to  $1/r$ , where  $r$  is the distance of the particle from the origin of the motion, *i.e.*, for small  $r$  the curvature is large and vice versa (see, **Figure 3**). Consequently, on the path of the  $s$ -particle the quark, the heaviest fundamental particle, appears first at high curvature followed by the electron



and then the neutrino, the lightest particle. Finally, we can conclude that during the very early expansion of the universe at about  $10^{-60}$  seconds after the Big Bang, fundamental particles started to be formed due to the change in the motion of a primordial *s-particle* along helical paths. First dark matter was formed, followed by quarks, electrons and finally neutrinos. There is substantial evidence that the masses of the particles are proportional to the curvature along the helical path. The anti-particles were formed by reversing the direction of rotation along the helical path.

On basis of the above discussion, we infer that the dynamical motion of the unifying *s-particle* is manifested in the appearance of the different fundamental particles, namely dark matter, quarks, electrons, and neutrinos.

In a qualitative discussion of the inflationary models [2], there were suggestions that in the universe above  $10^{27}$  K, a symmetry existed among the three fundamental particle forces of interaction with matter (strong nuclear, weak nuclear and electromagnetic forces), in accordance with the predictions of the Grand Unified Theories, and that below this temperature the symmetry was broken. In this breaking of symmetry, the strong nuclear force was separated from the electro-weak force and started to differentiate, while symmetry was still maintained between the electromagnetic and weak nuclear forces that both continued to behave in a similar manner. This was the case until the universe further cooled down and another critical temperature was reached at  $3 \times 10^{15}$  K, where according to the unified electroweak theory by Weinberg – Salam, a phase transition occurred and below the critical temperature the symmetry between electromagnetic and nuclear weak interactions was broken and each force appeared with a different identity.

In view of the above discussion of the “*s-particle* model”, in the time before  $10^{-60}$  seconds and a temperature above  $10^{38}$  K, the universe was completely symmetric, and below this critical temperature the first geometrical phase transition led to the appearance of the gravitational force. In the second geometrical phase transition at about  $10^{-43}$  seconds, the motion of some of the *s-particles* was transformed from path motions on cylindrical helices to the motion on conical helices, leading to the appearance of the other fundamental forces.

In the next section we propose three experiments to make the model testable.

#### 4. Proposed Experiments

To confirm and support the validity of the “*s-particle* model”, the following experiments are proposed:

- 1) The equipment used in ultrafast laser spectroscopy [11], for the study of the dynamical processes in atoms at extremely short time scales (a temporal scale of attoseconds), can be used in the study of the shape of the electron. The temporal width duration of the light pulses must be on the same scale as the dynamics of the *s-particles* (for example the rotation dynamics of the *s-particles*) or even shorter. Let a sequence of ultra-short pulses of light with attosecond or sub-attosecond

duration from an attosecond laser, irradiate nearly free electrons (say, electrons in the Rydberg states of highly excited heavy noble gases). The laser pulses will induce dynamical changes in the  $s$ -particle motion within the electron, together with these initial pulses one can follow the motion by pulses from another laser to probe and analyze the ultra-fast changes in the motion of the  $s$ -particles and to confirm the model and to verify that the electron can be considered as extended object having a structure.

2) If X-ray free-electron laser pulses, XFEL, (which are nearly coherent radiation in the hard X-ray region of the electromagnetic spectrum with photon energies of the order of ten thousands of eV) [12], strike a slowly moving beam of protons from neutral hydrogen atoms after ionization according to the  $s$ -particle model the quarks could be liberated from the proposed dark matter and appears as electrons on an electron detector. The liberated quarks will relax from the binding force to the dark matter and reach the stable electron state discussed in the previous section. A coincident circuit could be designed to measure the liberated electrons in coincidence with the striking beam of the coherent X-ray pulses. This experiment will confirm the way the protons were formed.

An alternative experiment that might approach the structure of the proton from the point of view of the  $s$ -particle model, consists of subjecting the slow beam of protons to strong laser fields in an ultra-short time domain (attosecond light pulses). In this case the attosecond laser source should emit an ultra-intense photon beam with peak intensity of the order of few Peta watts per  $\text{cm}^2$ , in the X-ray spectral range. The technique used in this experiment should be like the one applied in the study of solid-state materials under the interaction with strong light fields [13].

## 5. Conclusions

Following the previous discussions, one should consider the idea of a “unified particle theory” based on the presence of a primordial “ $s$ -particle” on equal footing with the search for a unified field theory (the “theory of everything”). Both theories may complement each other, in essence being two sides of the same coin as endeavours to present a model that could encompass all laws of nature deriving from a common source.

The assumption of the  $s$ -particle as a binding thread through which all elementary particles take form, may provide a novel model allowing for gravitational force to be accommodated into the same structural framework as other forces, something that on the basis of field theories could not yet be solved.

Furthermore, the model shows that the electronic charge is not a fundamental quantity (intrinsic property of the particle), but rather it can be derived from the tangential velocity of the  $s$ -particle.

It appears from the model that the masses of the fundamental particles are proportional to the curvature of the path of the  $s$ -particle.

Physicists in the past recognized the duality between particles and waves

through De Broglie's equation. A similar duality could exist between the electromagnetic fields (E, B) and the fictitious forces ( $F_{cs}$ ,  $F_{tr}$ ) on the *s-particles* (see Equation (13), [4]).

### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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