

Late Time Behavior of the Cosmological Model in Modified Theory of Gravity

Sankarsan Tarai, Jagadish Kumar

Centre of High Energy and Condensed Matter Physics, Department of Physics, Utkal University, Vani Vihar, Bhubaneswar, India
Email: tsankarsan87@gmail.com, jagadish.physics@utkaluniversity.ac.in

How to cite this paper: Tarai, S. and Kumar, J. (2022) Late Time Behavior of the Cosmological Model in Modified Theory of Gravity. *Journal of High Energy Physics, Gravitation and Cosmology*, 8, 1019-1031. <https://doi.org/10.4236/jhepgc.2022.84072>

Received: July 5, 2022

Accepted: October 10, 2022

Published: October 13, 2022

Copyright © 2022 by author(s) and Scientific Research Publishing Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

We report a viable exponential gravity model for the accelerated expansion of the universe in Bianchi VI_h space-time. By considering the estimated physical parameters, the cosmological models are constructed and analyzed in detail. We found that the state parameter in both the models increases to a higher negative range in an early epoch of the phantom domain and it goes to the positive domain at a late phase of the evolution. The effective cosmological constant remains in a positive domain for both models, which is a good sign of accelerating expansion of the universe.

Keywords

$f(R, T)$ Gravity, Bianchi Type VI_h , Perfect Fluid

1. Introduction

The standard model of cosmology has been given more attention because of its simple theoretical structure. Also, this model has the ability to answer many complex observational related issues. Also, this model has the ability to answer many complex observational related issues. The two remarkable achievements of this model are: 1) to explain the abundances of observed light element from an analysis of nuclear processes which operates at high temperatures in the early universe, and 2) in the prediction of the relic Cosmic Microwave Background (CMB) [1]. As a result, cosmological theories have the ability to explain the presence of topological defects, inflation, extra dimensions, and relic non-baryonic dark matter candidates, etc. The main issue that remains in cosmology is the agreement on both theoretical and observational results. To fill the gap between theory and observations, a lot of new findings in cosmological data have been investigated in recent years. This also gives an insight into the underlying phe-

nomena associated with the very early universe. Over the past two decades, it has been reported with great evidence regarding the accelerating nature of the expansion of the universe by the Supernovae cosmology project and the high- z supernovae groups [2] [3] [4]. They have estimated the distances to the cosmological supernovae by considering the close correlation between intrinsic luminosity of type I_a supernovae and their decline rate from maximum brightness. These quantities can be measured independently. The combination of these measurements and supernovae red-shift data is useful to predict the accelerating universe. Results obtained by both these groups *i.e.* $\Omega_m \approx 0.3$, $\Omega_\Lambda \approx 0.7$ are strongly disagree with the traditional values $(\Omega_m, \Omega_\Lambda) = (1, 0)$ of the universe.

However, it has been assumed that this cosmic acceleration is due to the energy with negative pressure known as dark energy (DE). Its cosmological origin and its nature are mysteries to date. Varieties of theoretical models such as quintessence [5] [6], phantom field [7], k-essence [9] [10], tachyons [11], quintom [12] [13], etc. have been proposed to explain the nature of the DE and the accelerated expansion. The recent discovery of gravitational waves by LIGO (Laser Interferometer Gravitational Waves Observatory) [14] [15] experiments has greatly supported the prediction of general relativity. Moreover, the detectors for Gravitational Waves (GWs) will be important for a better knowledge of the Universe and for either confirming or ruling out the physical consistency of general relativity or any other theory of gravitation. In fact, a brief discussion about the detection of GWs, and how the frequency dependent response functions of interferometers for GWs arise from various theories of gravity, *i.e.* general relativity and extended theories of gravity, will be the definitive test for general relativity [16].

However, general relativity fails to resolve the theoretical challenge posed by late-time cosmic speed. So, modified gravity is an intriguing candidate to resolve the theoretical challenge and the mechanism behind the late-time cosmic speed. In recent times, to balance the mismatch between the theory and observations; some significant development has been proposed in the construction of the dark energy models by modification of the Einstein-Hilbert action. This phenomenological approach is called as the Modified Gravity: which can successfully explain the cosmological observations without use of the dark energy or Einstein's cosmological constant. By modifying the Einstein-Hilbert action, different modified theories have been proposed to address the cosmological and observational evidence of expanding universe without use of dark energy [17] [18] [19] [20]. Different issues related to cosmology and astrophysics in modified gravity theories have been investigated by many authors such as $f(R)$ gravity [21] [22], $f(T)$ gravity [23]-[29] and $f(G)$ gravity [30] [31] [32]. The $f(R)$ gravity deals with the most general function of the Ricci scalar R whereas the general version of teleparallel gravity is represented by $f(T)$ gravity. Dark energy is not required to explain late-time cosmic acceleration in the modified gravity models; this is being supported by modifying the underlying geometry. From the litera-

ture, it is found that an interesting framework has been proposed in the form of $f(R, T)$ gravity [33] to investigate the accelerating models. Here, the Einstein-Hilbert action contains a functional $f(R, T)$ in place of R , where the trace of the energy-momentum tensor is $T = g^{ij}T_{ij}$. With such a form, many noble authors [34] [35] [36] [37] have worked on explaining the accelerating expansion of the universe in the modified theory of gravity and gained a lot of applause. Bianchi Morphologies are more general to study the anisotropic nature of the universe in comparison with Friedmann models. However, some strong debate on the viability of models redeem that it can be useful to reduce the isotropic behaviour at late time with appropriate technique from the early inflationary phase.

The evolution of the universe in Bianchi type-V space-time has been investigated in $f(R, T)$ gravity in the presence of perfect fluid [38] [39]. Jamil *et al.* [40] have studied different cosmological models in presence of Chaplygin gas, scalar field in the form of $f(R, T)$ gravity using the higher order of Ricci scalar. Sharif and Zubair [41] have studied the massless scalar field using perfect fluid distribution in the Bianchi type-I universe, whereas the stability factor of newton stars have examined by Bhatti and Yousuf [42] in the context of Palatini form in $f(R, T)$ gravity. Tiwari *et al.* [43] have investigated the phase transition of the FRW universe in $f(R, T)$ gravity with a perfect fluid distribution. Mishra *et al.* [44] have investigated a quintessence bound nature Bianchi VI_h universe filled with perfect fluid in this framework. Moreover, Roy *et al.* [45] have studied the big rip and pseudo rip scenario in $f(R, T)$ gravity where the space-time is considered to be anisotropic and the matter field is a perfect fluid. Also, Mishra *et al.* [46]-[51] have studied the different aspects of Bianchi VI_h universe in the frame of $f(R, T)$ gravity using different matter fields. The field equations for $f(R, T)$ gravity are derived from Einstein-Hilbert variational principle as

$$S = \int \sqrt{-g} \left(\frac{1}{16\pi G} f(R, T) + L_m \right) d^4x \quad (1)$$

where L_m is the Lagrangian density of matter fields and the functional $f(R, T)$ are combination two arbitrary function of Ricci scalar R and the trace of the stress energy tensor T . The Equation (1) with respect to the metric tensor gives the Einstein field equation, where the force acting on the matter is defined as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta L_m}{\delta g^{ij}}, \quad (2)$$

However the matter Lagrangian depends only on the metric tensor component but not its derivatives. So, Equation (2) becomes

$$T_{ij} = g_{ij}L_m - 2 \frac{\partial L_m}{\partial g^{ij}}. \quad (3)$$

Following Harko *et al.* [33] work, we have taken the matter Lagrangian as $\mathcal{L}_m = -p$ where p is the pressure of the perfect cosmic fluid. The trace $T = g^{ij}T_{ij}$

of the energy-momentum tensor is obtained from

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}. \tag{4}$$

ρ is the rest energy density and $u^i = \delta_0^i$ is the four velocity vector. The algebraic $f(R, T)$ has been chosen from the modified Einstein-Hilbert action principle (1) such as a sum of two independent functions

$f(R, T) = f_1(R) + f_1(R)f_2(T)$ depends on the curvature R whereas $f_2(T)$ is the sum of energy momentum tensor T ([33]). Hence, the generalized EFE (4) yields

$$f_R R_{ij} - \frac{1}{2} f(R) g_{ij} + (g_{ij} \square - \nabla_i \nabla_j) f_R = 8\pi T_{ij} + f_T T_{ij} + \left[p f_T + \frac{1}{2} f(T) \right] g_{ij}. \tag{5}$$

Here, $f_R = \frac{\partial f(R)}{\partial R}$ and $f_T = \frac{\partial f(T)}{\partial T}$. For constructing the cosmological model, we consider the functional $f(R, T)$ in the form $f(R, T) = \lambda R + \lambda T$, then the field Equations (5), becomes

$$R_{ij} - \frac{1}{2} R g_{ij} = \left(1 + \frac{8\pi}{\lambda} \right) T_{ij} + \Lambda(T) g_{ij}. \tag{6}$$

where, $\Lambda(T) = p + \frac{1}{2} T$ is considered with cosmic time. We have studied two models for two different values of h i.e. $h = -1, 0$ considering a presumed exponential scale factor. The basic formalism of $f(R, T)$ gravity is included in the introduction section. The paper is structured as follow: in Section 2, we have derived the field equations and general mathematical scheme for the equation of state (EoS) parameter and effective cosmological constant (ECC) in term of Hubble parameter. The model dynamics have been studied in Section 3. Finally conclusions of the work presented in Section 4.

2. Basic Field Equations

In this section, we present briefly the field equations by the developed formalism of $f(R, T)$ gravity. We consider a Bianchi-type VI_h space-time in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{2hx} dz^2 \tag{7}$$

The field Equations (6) for the metric (7) and energy momentum tensor (4) can be obtained by the co-moving coordinate system as,

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{h}{A^2} = \left(\frac{16\pi + 3\lambda}{2\lambda} \right) \rho - \frac{\rho}{2} \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2} = \left(\frac{16\pi + 3\lambda}{2\lambda} \right) \rho - \frac{\rho}{2} \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \left(\frac{16\pi + 3\lambda}{2\lambda} \right) \rho - \frac{\rho}{2} \tag{10}$$

$$-\frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA} + \frac{1+h+h^2}{A^2} = \left(\frac{16\pi + 3\lambda}{2\lambda} \right) \rho - \frac{\rho}{2} \tag{11}$$

$$\frac{\dot{B}}{B} + h \frac{\dot{C}}{C} - (1+h) \frac{\dot{A}}{A} = 0 \quad (12)$$

where A, B, C are time dependent components. The important part of this space-time is the constant factor h which decide the behavior of the model. We express the denoting Hubble scales along with separate directions as $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$. The mean Hubble parameter becomes, $H = \frac{1}{3}(H_x + H_y + H_z)$. Now, the field equations for Bianchi type VI_h space-time with the matter field in term of perfect fluid can be expressed in form of the Hubble parameter as

$$\dot{H}_y + \dot{H}_z + H_y^2 + H_z^2 + H_y H_z - \frac{h}{A^2} = \gamma p - \frac{\rho}{2} \quad (13)$$

$$\dot{H}_x + \dot{H}_z + H_x^2 + H_z^2 + H_x H_z - \frac{h^2}{A^2} = \gamma p - \frac{\rho}{2} \quad (14)$$

$$\dot{H}_x + \dot{H}_y + H_x^2 + H_y^2 + H_x H_y - \frac{1}{A^2} = \gamma p - \frac{\rho}{2} \quad (15)$$

$$-H_x H_y - H_y H_z - H_x H_z + \frac{1+h+h^2}{A^2} = \gamma p - \frac{\rho}{2} \quad (16)$$

$$H_y + h H_z - (1+h) H_x = 0 \quad (17)$$

$$\text{Here } \gamma = \frac{16\pi + 3\lambda}{2\lambda}.$$

On solving the field Equations (13)-(17) the functional form of pressure p and rest energy density ρ and subsequently the equation of state (EoS) parameter w and effective cosmological constant (ECC) Λ can be obtained as

$$p = \frac{2}{4\gamma^2 - 1} \left[2\gamma\phi(H_x, H_y) - \psi(H_x, H_y, H_z, h) \right], \quad (18)$$

$$\rho = \frac{2}{4\gamma^2 - 1} \left[\phi(H_x, H_y) - 2\gamma\psi(H_x, H_y, H_z, h) \right]. \quad (19)$$

$$\omega = 2\gamma + \frac{(4\gamma^2 - 1)\psi(H_x, H_y, H_z, h)}{\phi(H_x, H_y) - 2\gamma\psi(H_x, H_y, H_z, h)} \quad (20)$$

$$\Lambda = -\frac{\phi(H_x, H_y) + \psi(H_x, H_y, H_z, h)}{2\gamma + 1} \quad (21)$$

where $\phi(H_x, H_y) = \dot{H}_x + \dot{H}_y + H_x^2 + H_y^2 + H_x H_y - \frac{1}{A^2}$ and

$$\psi(H_x, H_y, H_z, h) = H_x H_y + H_y H_z + H_x H_z - \frac{1+h+h^2}{A^2}.$$

It is interesting to study the cosmological models from the point of their existence with other cosmological and astrophysical data, as well as with the theory of cosmological perturbations, especially with the theory of scalar perturbations

in the late universe. Furthermore, the hydrodynamic approach is inadequate when inhomogeneities are already formed at the late stage of evolution. In this situation, the mechanical approach [52] [53] is more suitable. It is more appropriate inside the cell of uniformity [54] which provides a good tool for different cosmological models to investigate scalar perturbations. Therefore, it is worthy to study the cosmological models of the universe filled with perfect fluids which are having EoS parametrizations and study the existence of these models with the mechanical approach. So, Equations (18)-(21) will not be a perfect form to characterize the dynamical nature of the universe. Though, we need to analyse the nature of the universe from the (18)-(21), so we have to change the directional Hubble parameter to directional time from with the help of scale factor.

We have considered the exponential gravity in the form $\mathcal{R} = e^{m\tau}$ to check the reliability of the model, where m is an arbitrary constant to be determined from the background cosmology. With this consideration the radius scale factor is

$R = (ABC)^{\frac{1}{3}} = e^{\frac{m\tau}{3}}$. As we know from the Hubble Telescope, the universe is not only expanding, but also the rate at which it is expanding with change of time. The deceleration parameter is a way to judge the expansion of the universe. Recently, the research groups studying distant supernovae type I_a suggest that the universe appears to be accelerating at present, $q < 0$ [3]. We have obtained the deceleration parameter of the model considering the exponential scale factor as $q = -\frac{R\ddot{R}}{\dot{R}^2} = -1$. The obtained value of the deceleration parameter permits an accelerated expansion of the universe by recent discovery [3]. The Hubble parameter $H = \frac{\dot{R}}{R}$, and the scalar expansion $\theta = 3H$, and shear scalar

$$\sigma^2 = \frac{1}{2} \left[(H_x^2 + H_y^2 + H_z^2) - \frac{1}{3} \theta^2 \right],$$

and the average anisotropy parameter

$$\mathcal{A} = \frac{1}{3} \sum \left(\frac{\Delta H_i}{H} \right)^2$$

where $i = x, y, z$ of the model can be respectively obtained as

$$m, 3m, \frac{m^2}{3} - \frac{2km^2}{(k+2)^2} \text{ and } \frac{4}{3}.$$

The state finder analysis gives us an opportunity to study the geometric interpretation of dark energy through state finder pairs (r, s) [55]. The state finder pair $r = \frac{\ddot{a}}{aH^3}$ and $s = \frac{r-1}{3\left(q-\frac{1}{2}\right)}$ can be respectively

found to be 1 and 0, which is per to the study the cosmic speed of the universe.

3. Simplified Model Dynamics

In this part, we have studied the backdrop cosmology for each value of $h = -1, 0$. Two different cosmological models have been constructed for these values of h in the framework of $f(R, T)$ gravity. For $h = 1$, the model does not conform to present day observations. So, because of this reason, we have not considered

studying the physical nature of the model.

1) Model-I ($h = -1$)

For $h = -1$, Equation (14) yields $H_y = H_z$, where the integration constant is fixed as unity. The two functional $\phi(H_x, H_y)$ and $\psi(H_x, H_y, H_z, h)$ can be obtained with an assumed anisotropic relation between the directional Hubble parameters $H_x = kH_y$ as

$$\phi(H_x, H_y) = (k+1)\dot{H}_y + (k^2 + k + 1)H_y^2 - \frac{1}{A^2} \quad (22)$$

$$\psi(H_x, H_y, H_z, h) = (2k+1)H_y^2 - \frac{1}{A^2} \quad (23)$$

where k is an arbitrary positive constant. For exponential function, the directional Hubble rate becomes $H_x = \frac{km}{k+2}$, $H_y = H_z = \frac{m}{k+2}$. Consequently the directional scale factors are $A = e^{\frac{ikm}{k+2}}$, $B = C = e^{\frac{tm}{k+2}}$. Then Equations (22) and (23) becomes

$$\phi = \frac{m^2(k^2 + k + 1)}{(k+2)^2} - e^{-\frac{2ktm}{k+2}} \quad (24)$$

$$\psi = \frac{m^2(2k+1)}{(k+2)^2} - e^{-\frac{2ktm}{k+2}}. \quad (25)$$

The EoS parameter (ω) and ECC (Λ) for $h = -1$ can be obtained as

$$\omega = 2\gamma + \frac{(4\gamma^2 - 1) \left[(2k+1)m^2 - (k+2)^2 e^{-\frac{2ktm}{k+2}} \right]}{(3 - 4\gamma k - 2\gamma)m^2 + (2\gamma - 1)(k+2)^2 e^{-\frac{2ktm}{k+2}}} \quad (26)$$

$$\Lambda = \frac{-(k^2 + 3k + 2)m^2 + 2(k+2)^2 e^{-\frac{2ktm}{k+2}}}{(2\gamma + 1)(k+2)^2} \quad (27)$$

Since the Equations (18)-(21) are highly non-linear, an explicit relation between p and ρ could not be established. All the solutions are implicit in nature; hence it is difficult to study the role of the universe. Therefore, we have studied the physical nature first by determining a general relationship between p and ρ with the help of ω . By the representative values of the parameters, the model will reduce to different physical states. In **Figure 1**(top), we have shown the graph between the EoS parameter (ω) vs red-shift (z) for three different values of model parameter k i.e. $k = 1.2, 1.3$ and 1.5 . It is observed that, ω is asymptotically increasing with the evolution of cosmic time from a higher negative value. ω starts from the phantom region ($\omega < -1$) [7] [8] at the early phase of evolution, and then goes slowly to positive region at the late phase of evolution. In **Figure 1**(down), we have plotted the effective cosmological constant (Λ) vs red-shift (z) for three different values of model parameter k i.e. $k = 1.2, 1.3$ and 1.5 . The effective cosmological constant remains positive throughout the cosmic

evolution. At initial stage, the magnitude of Λ is more compared to the late phase. However, the cosmological constant decreases at later epoch with the growth of the cosmic time. Such behaviour is appropriate with the Λ CDM model to explain the cosmic evolution of the universe (Figure 2).

2) Model-II ($h = 0$)

For $h = 0$, Equation (14) yields $H_x = H_y$. By an anisotropic relation $H_z = lH_y$ (l is a additive constant), the respective Hubble rates in the exponential cosmology are $H_x = H_y = \frac{m}{l+2}$, $H_z = \frac{lm}{l+2}$. Consequently the directional scale factors are $A = B = e^{\frac{tm}{l+2}}$, $C = e^{\frac{tlm}{l+2}}$. Thus the functional ϕ and ψ for this model are

$$\phi = \frac{3m^2}{(l+2)^2} e^{-\frac{2tm}{l+2}} \tag{28}$$

$$\psi = \frac{m^2(2l+1)}{(l+2)^2} e^{-\frac{2tm}{l+2}} \tag{29}$$

The equation of state parameter and the effective cosmological constant for this model are obtained from (28) and (29) as (Figure 3 and Figure 4).

$$\omega = 2\gamma + \frac{(4\gamma^2 - 1) \left[m^2(2l+1) - (l+2)^2 e^{\frac{2tm}{l+2}} \right]}{(3 - 2\gamma - 4\gamma l)m^2 + (2\gamma - 1)(l+2)^2 e^{\frac{2tm}{l+2}}} \tag{30}$$

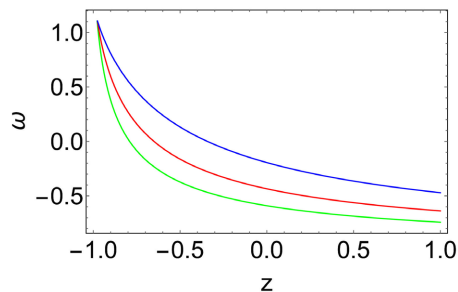


Figure 1. Graphical behaviour of EoS parameter in red-shift (z) with the representative values of $m = 0.45$, $\gamma = 0.51$ and $k = 1.2, 1.3, 1.5$.

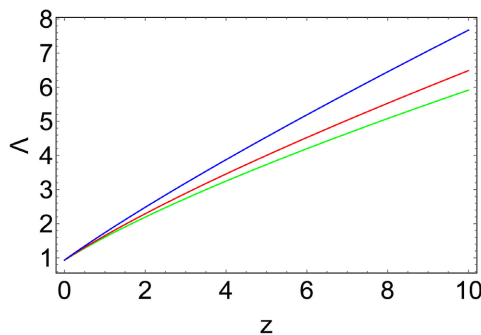


Figure 2. Graphical behaviour of effective cosmological constant in red-shift (z) with the representative values of $m = 0.45$, $\gamma = 0.51$ and $k = 1.2, 1.3, 1.5$.

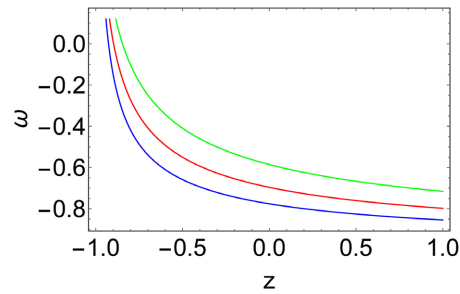


Figure 3. Graphical behaviour of EoS parameter in red-shift (z) with the representative values of $m = 0.45$, $\gamma = 0.51$ and $k = 0.9, 0.93, 0.95$.

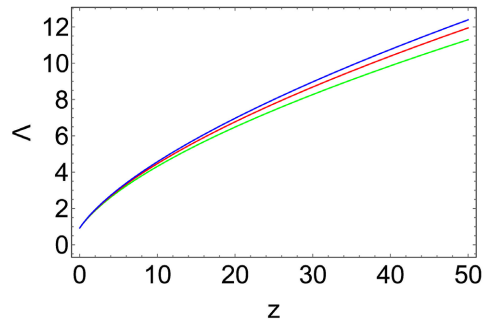


Figure 4. Graphical behaviour of effective cosmological constant in red-shift (z) with the representative values of $m = 0.45$, $\gamma = 0.51$ and $k = 0.9, 0.93, 0.95$.

$$\Lambda = \frac{-m^2 (4 + 2l) + 2(l + 2)^2 e^{\frac{-2m}{l+2}}}{(l + 2)^2 (2\gamma + 1)} \quad (31)$$

Though our study emphasizes cosmic acceleration, the equation (30) gives a clear idea of it. The equation of state parameter ω describes whether the model is accelerating or decelerating. In **Figure 2**(top), ω is plotted as a function of red-shift for three different values of the anisotropic parameter k *i.e.* $k = 0.9, 0.93, 0.95$. The other parameters are considered to be $m = 0.45$ and m is picked from the observationally constrained value of deceleration parameter $q = -0.5981$ [56] and $\gamma = 0.51$. With all considered values, ω remains in the phantom region [7] [8] at the early evolution of the universe and goes positive region in the late phase of the universe. We have plotted the cosmological constant in **Figure 2**(down), which is lying in the positive domain. It is clear from the above figure that at early phase of cosmic evolution Λ decreases from large positive values to small positive values and vanishes at the late phase of evolution.

4. Conclusions

This work describes the study of the background anisotropies in the Bianchi type VI_h universe in the presence of perfect fluid. The cosmological models are reconstructed with assumption of exponential function of the scale factor in the framework of $f(R, T)$ gravity. To reconstruct and study dynamical features of

the model, we chose the functional $f(R, T)$ as $f(R, T) = \Lambda R + \Lambda T$ to avoid the non-linearity. Additionally, two different models are investigated corresponding to two values of the metric parameter $h = -1, 0$ assuming a dimensional analysis method to constraint the model parameters. We calculated the expression of the equation of state parameter and the effective cosmological constant from the general expressions of the physical quantities, with assumed scale factor and studied the background cosmologies.

It is observed that the models start from the aggressive phantom region and finally approach the positive region. The EoS parameter of the derived models corresponds to phantom era of the universe which is a favourable sign to pilgrim DE conjecture which confirms the works of Yi-Fu Cai [57]. We have reported the anisotropic cosmological models in exponential expansion and the ECC is found positive throughout the model from small positive values at the beginning to large values at late times. The EoS parameter for both models remains negative (phantom domain) at the early phase of evolution and behaves like a cosmological constant at the late phase of evolution where with power law, the models remain in the negative zone and favour a quintessence phase ($-\frac{2}{3} \leq \omega \leq -\frac{1}{3}$).

The present study will definitely put some support towards cosmic acceleration in the context of the uncertainty prevailing happens in the studies of the late time cosmic phenomena using an exponential function.

Acknowledgements

Sankarsan Tarai acknowledges Rashtriya Uchchatar Shiksha Abhiyan (RUSA), Ministry of Human Resource Development, and Government of India for financial support.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

- [1] Penzias, A.A. and Wilson, R.W. (1965) A Measurement of Excess Antenna Temperature at 4080 m/s. *The Astrophysical Journal*, **142**, 419-421. <https://doi.org/10.1086/148307>
- [2] Schmidt, B.P., *et al.* (1998) The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Curvature of the Universe Using Type Ia Supernovae. *The Astronomical Journal*, **507**, 46-63. <https://doi.org/10.1086/306308>
- [3] Riess, A.G., *et al.* (1998) Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal*, **116**, 1009-1038. <https://doi.org/10.1086/300499>
- [4] Perlmutter, S., *et al.* (1999) Measurements of Omega and Lambda from 42 High-Redshift Supernovae. *The Astrophysical Journal*, **517**, 565-586.
- [5] Ratra, B. and Peebles, P.J.E. (1988) Cosmological Consequences of a Rolling Homogeneous Scalar Field. *Physical Review D*, **37**, 3406-3427. <https://doi.org/10.1103/PhysRevD.37.3406>

- [6] Sahni, V. and Starobinsky, A. (2000) The Case for a Positive Cosmological Lambda Term. *International Journal of Modern Physics D*, **9**, 373-444. <https://doi.org/10.1142/S0218271800000542>
- [7] Caldwell, R.R. (2002) A Phantom Menace? Cosmological Consequences of a Dark Energy Component with Super-Negative Equation of State. *Physics Letters B*, **545**, 23-29. [https://doi.org/10.1016/S0370-2693\(02\)02589-3](https://doi.org/10.1016/S0370-2693(02)02589-3)
- [8] Caldwell, R.R., Kamionkowski, M. and Weinberg, N.N. (2003) Phantom Energy and Cosmic Doomsday. *Physical Review Letters*, **91**, Article ID: 071301. <https://doi.org/10.1103/PhysRevLett.91.071301>
- [9] Armendariz-Picon, C., *et al.* (2000) Dynamical Solution to the Problem of a Small Cosmological Constant and Late-Time Cosmic Acceleration. *Physical Review Letters*, **85**, 4438-4441. <https://doi.org/10.1103/PhysRevLett.85.4438>
- [10] Armendariz-Picon, C., *et al.* (2001) Essentials of k-Essence. *Physical Review D*, **63**, Article ID: 103510. <https://doi.org/10.1103/PhysRevD.63.103510>
- [11] Sen, A. (2002) Rolling Tachyon. *Journal of High Energy Physics*, **204**, 48. <https://doi.org/10.1088/1126-6708/2002/04/048>
- [12] Feng, B., *et al.* (2005) Dark Energy Constraints from the Cosmic Age and Supernova. *Physics Letters B*, **607**, 35-41. <https://doi.org/10.1016/j.physletb.2004.12.071>
- [13] Guo, Z., *et al.* (2005) Parametrization of Quintessence and Its Potential. *Physical Review D*, **72**, Article ID: 023504. <https://doi.org/10.1103/PhysRevD.72.023504>
- [14] Abbott, B.P., *et al.* (2016) Observation of Gravitational Waves from a Binary Black Hole Merger (LIGO Collaboration). *Physical Review Letters*, **116**, Article ID: 061102.
- [15] Abbott, B.P., *et al.* (2016) Astrophysical Implications of the Binary Black-Hole Merger (LIGO Collaboration). *The Astrophysical Journal Letters*, **818**, L22.
- [16] Corda, C. (2009) Interferometric Detection of Gravitational Waves: The Definitive Test for General Relativity. *International Journal of Modern Physics D*, **18**, 2275-2282. <https://doi.org/10.1142/S0218271809015904>
- [17] Cappozziello, S. and De Laurentis, M. (2011) Extended Theories of Gravity. *Physics Reports*, **509**, 167-321. <https://doi.org/10.1016/j.physrep.2011.09.003>
- [18] Bamba, K., *et al.* (2012) Dark Energy Cosmology: The Equivalent Description via Different Theoretical Models and Cosmography Tests. *Astrophysics and Space Science*, **342**, 155-228. <https://doi.org/10.1007/s10509-012-1181-8>
- [19] Clifton, T., *et al.* (2012) Modified Gravity and Cosmology. *Physics Reports*, **513**, 1-189. <https://doi.org/10.1016/j.physrep.2012.01.001>
- [20] Sotiriou, T.P. and Faraoni, V. (2010) f(R) Theories of Gravity. *Reviews of Modern Physics*, **82**, 451-497. <https://doi.org/10.1103/RevModPhys.82.451>
- [21] Nojiri, S. and Odintsov, S.D. (2006) Modified f(R) Gravity Consistent with Realistic Cosmology: From a Matter Dominated Epoch to a Dark Energy Universe. *Physical Review D*, **74**, Article ID: 086005. <https://doi.org/10.1103/PhysRevD.74.086009>
- [22] Nojiri, S. and Odintsov, S.D. (2007) Introduction to Modified Gravity and Gravitational Alternative for Dark Energy. *International Journal of Geometric Methods in Modern Physics*, **4**, 115-146. <https://doi.org/10.1142/S0219887807001928>
- [23] Linder, E.V. (2010) Einstein's Other Gravity and the Acceleration of the Universe. *Physical Review D*, **82**, Article ID: 109902. <https://doi.org/10.1103/PhysRevD.82.109902>
- [24] Myrzakulov, R. (2011) Accelerating Universe from f(T) Gravity. *The European Physical Journal C*, **71**, Article No. 1752. <https://doi.org/10.1140/epjc/s10052-011-1752-9>

- [25] Sheng-Feng, Y., *et al.* (2020) Interpreting Cosmological Tensions from the Effective Field Theory of Torsional Gravity. *Physical Review D*, **101**, Article ID: 121301. <https://doi.org/10.1103/PhysRevD.101.121301>
- [26] Cai, Y.F., Khurshudyan, M. and Saridakis, E.N. (2020) Model-Independent Reconstruction of $f(T)$ Gravity from Gaussian Processes. *The Astrophysical Journal*, **888**, Article No. 62. <https://doi.org/10.3847/1538-4357/ab5a7f>
- [27] Cai, Y.F., *et al.* (2018) $f(T)$ Gravity after GW170817 and GRB170817A. *Physical Review D*, **97**, Article ID: 103513. <https://doi.org/10.1103/PhysRevD.97.103513>
- [28] Cai Y.F., *et al.* (2011) Matter Bounce Cosmology with the $f(T)$ Gravity. *Classical and Quantum Gravity*, **28**, Article ID: 215011. <https://doi.org/10.1088/0264-9381/28/21/215011>
- [29] Cai, Y.F., Capozziello, M. and De Laurentis, M. (2016) $f(T)$ Teleparallel Gravity and Cosmology. *Reports on Progress in Physics*, **79**, Article ID: 106901. <https://doi.org/10.1088/0034-4885/79/10/106901>
- [30] Nojiri, S. and Odintsov, S.D. (2005) Modified Gauss-Bonnet Theory as Gravitational Alternative for Dark Energy. *Physics Letters B*, **631**, 1-6. <https://doi.org/10.1016/j.physletb.2005.10.010>
- [31] Li, B., *et al.* (2007) Cosmology of Modified Gauss-Bonnet Gravity. *Physical Review D*, **76**, Article ID: 044027. <https://doi.org/10.1103/PhysRevD.76.044027>
- [32] Kofinas, G. and Saridakis, E.N. (2014) Teleparallel Equivalent of Gauss-Bonnet Gravity and Its Modifications. *Physical Review D*, 2014; **90**, Article ID: 084044. <https://doi.org/10.1103/PhysRevD.90.084044>
- [33] Harko, T., *et al.* (2011) $f(R,T)$ Gravity. *Physical Review D*, **84**, Article ID: 024020. <https://doi.org/10.1103/PhysRevD.84.024020>
- [34] Myrzakulov, R. (2012) FRW Cosmology in $f(R,T)$ Gravity. *The European Physical Journal C*, **72**, Article No. 2203. <https://doi.org/10.1140/epjc/s10052-012-2203-y>
- [35] Houndjo, M.J.S. and Piattella, O.F. (2012) Reconstructing $f(R,T)$ Gravity from Holographic Dark Energy. *International Journal of Modern Physics D*, **21**, Article ID: 1250024. <https://doi.org/10.1142/S0218271812500241>
- [36] Yousaf, Z., Bamba, K. and Bhatti, M.Z. (2016) Causes of Irregular Energy Density in $f(R,T)$ Gravity. *Physical Review D*, **93**, Article ID: 124048. <https://doi.org/10.1103/PhysRevD.93.064059>
- [37] Barrientos, J. and Rubilar, G.B. (2014) Comment on $f(R,T)$ Gravity. *Physical Review D*, **90**, Article ID: 028501. <https://doi.org/10.1103/PhysRevD.90.028501>
- [38] Ahmed, N. and Pradhan, A. (2014) Bianchi Type-V Cosmology in $f(R,T)$ Gravity with $\Lambda(T)$. *International Journal of Theoretical Physics*, **53**, 289-306. <https://doi.org/10.1007/s10773-013-1809-7>
- [39] Shamir, M.F. (2015) Exact Solutions of Bianchi Type V Spacetime in $f(R,T)$ Gravity. *International Journal of Theoretical Physics*, **54**, 1304-1315. <https://doi.org/10.1007/s10773-014-2328-x>
- [40] Jamil, M., *et al.* (2012) Reconstruction of Some Cosmological Models in $f(R,T)$ Cosmology. *The European Physical Journal C*, **72**, Article No. 1999. <https://doi.org/10.1140/epjc/s10052-012-1999-9>
- [41] Sharif, M. and Zubair, M. (2012) Anisotropic Universe Models with Perfect Fluid and Scalar Field in $f(R,T)$ Gravity. *Journal of the Physical Society of Japan*, **81**, Article ID: 114005. <https://doi.org/10.1143/JPSJ.81.114005>
- [42] Bhatti, M.Z. and Yousaf, Z. (2019) Stability Analysis of Neutron Stars in Palatini $f(R,T)$ Gravity. *General Relativity and Gravitation*, **51**, Article No. 144.

- <https://doi.org/10.1007/s10714-019-2631-1>
- [43] Tiwari, R.K., Sofuoglu, D. and Beesham, A. (2021) FRW Universe in $f(R,T)$ Gravity. *International Journal of Geometric Methods in Modern Physics*, **18**, Article ID: 2150104. <https://doi.org/10.1142/S0219887821501048>
- [44] Mishra, B., Tarai, S. and Tripathy, S.K. (2016) Dynamics of an Anisotropic Universe in $f(R,T)$ Theory. *Advances in High Energy Physics*, **2016**, Article ID: 8543560. <https://doi.org/10.1155/2016/8543560>
- [45] Ray, P.P., Tarai, S., Mishra, B. and Tripathy, S.K. (2021) Cosmological Models with Big Rip and Pseudo Rip Scenarios in Extended Theory of Gravity. *Fortschritte der Physik*, **69**, Article ID: 2100086. <https://doi.org/10.1002/prop.202100086>
- [46] Mishra, B., Tripathy, S.K. and Tarai, S. (2018) Cosmological Models with a Hybrid Scale Factor in an Extended Gravity Theory. *Modern Physics Letters A*, **33**, Article ID: 1850052. <https://doi.org/10.1142/S0217732318500529>
- [47] Mishra, B., Tarai, S. and Tripathy, S.K. (2018) Dynamical Features of an Anisotropic Cosmological Model. *Indian Journal of Physics*, **92**, 1199. <https://doi.org/10.1007/s12648-018-1194-4>
- [48] Mishra, B., Tarai, S. and Tripathy, S.K. (2018) Anisotropic Cosmological Reconstruction in $f(R,T)$ Gravity. *Modern Physics Letters A*, **33**, Article ID: 1850170. <https://doi.org/10.1142/S0217732318501705>
- [49] Mishra, B., Tarai, S. and Pacif, S.K.J. (2018) Dynamics of Bianchi V_h Universe with Bulk Viscous Fluid in Modified Gravity. *International Journal of Geometric Methods in Modern Physics*, **15**, Article ID: 1850036. <https://doi.org/10.1142/S0219887818500366>
- [50] Mishra, B., Ribeiro, G. and Moraes, P.H.R.S. (2019) De Sitter and Bounce Solutions from Anisotropy in Extended Gravity Cosmology. *Modern Physics Letters A*, **34**, Article ID: 1950321. <https://doi.org/10.1142/S0217732319503218>
- [51] Tripathy, S.K. and Mishra, B. (2020) Phantom Cosmology in an Extended Theory of Gravity. *Chinese Journal of Physics*, **63**, 448-458. <https://doi.org/10.1016/j.cjph.2019.12.022>
- [52] Eingorn, M. and Zhuk, A. (2012) Hubble Flows and Gravitational Potentials in Observable Universe. *Journal of Cosmology and Astroparticle Physics*, **9**, 26. <https://doi.org/10.1088/1475-7516/2012/09/026>
- [53] Eingorn, M., *et al.* (2013) Dynamics of Astrophysical Objects against the Cosmological Background. *Journal of Cosmology and Astroparticle Physics*, **4**, 10. <https://doi.org/10.1088/1475-7516/2013/04/010>
- [54] Eingorn, M. and Zhuk, A. (2014) Remarks on Mechanical Approach to Observable Universe. *Journal of Cosmology and Astroparticle Physics*, **5**, 24. <https://doi.org/10.1088/1475-7516/2014/05/024>
- [55] Sahni, V., *et al.* (2003) Statefinder—A New Geometrical Diagnostic of Dark Energy. *Journal of Experimental and Theoretical Physics*, **77**, 201-206. <https://doi.org/10.1134/1.1574831>
- [56] Montiel, A., Salzano, V. and Lazkoz, R. (2014) Observational Constraints on the Unified Dark Matter and Dark Energy Model Based on the Quark Bag Model. *Physics Letters B*, **733**, 209-216. <https://doi.org/10.1016/j.physletb.2014.04.048>
- [57] Cai, Y.-F. (2014) Exploring Bouncing Cosmologies with Cosmological Surveys. *Science China Physics, Mechanics & Astronomy*, **57**, 1414-1430. <https://doi.org/10.1007/s11433-014-5512-3>