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The Theory of Gravitons in the Expansion of the Universe

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Abstract

The theory that gravitons lose energy thru gravitational redshift while traveling in a gravitational field is applied to the universe. It is proposed that a co-moving volume element is required for the luminosity distance relation because the gravitational field acts simultaneously in three dimensions rather than just along a geodesic curve. With only a relatively small baryonic mass density the curve fit of the novel luminosity distance relation to Type Ia supernovae distance data is of the same quality as for the standard Lambda Cold Dark Matter model.

Keywords

Gravitons, Gravitational Redshift, Hubble Law, Luminosity Distance, Supernova

1. Introduction

This paper describes a theory of gravitons acting in the expansion of the universe from the paper [1]. We will explore in more detail the ways the graviton gravitational redshift affects the properties of the universe.

We assume that gravitons have both wave and particle properties. Here we treat gravitons as particles which travel at constant speed c in vacuum, where c equals the speed of light. Gravitons travelling in a gravitational field of a source mass M, modeled by the equivalence principle as an accelerating system, should experience an average energy loss of $\delta\xi$ due to motion in that field, over a short time period $\delta t = \delta r/c$, where the acceleration a at a point r in the field is given by $a = -GM/r^2$. The energy loss is expressed differentially as

$$\delta\xi = -\left(pm_g c^2\right)\frac{\delta u}{c} = -\left(pm_g c\right)a\delta t = -\left(\frac{pGMm_g}{r^2}\right)\delta r, \tag{1}$$

where p is the probability of transmission of the graviton, $m_g = m/n$ is the average relativistic graviton mass, n is the number of gravitons, m is the test mass, δu is the change in velocity of the system observed from an inertial reference frame, G is Newton's gravitational constant, M is the baryonic mass of the field source, r is the distance between the center of the source and the location of the moving gravitons, δt is the short travel time of the gravitons at speed c over distance δr. The energy change is a loss (negative) because δu is in the same direction as the motion of the gravitons, so that for an inertial observer moving with velocity δu, the energy of the graviton is redshifted. The graviton transmission probability $p = p(r)$ is defined as a logic function

$$p(r) = \left(\left|\alpha U(r)\right| \geq \left|V(r)\right|\right) = \left(\alpha c H_0 \geq \frac{GM(r)}{r^2}\right), \tag{2}$$

where $V(r) = -GM/r$ is the gravitational potential, $U(r) = -cH_0 r$ is defined as *the graviton induced potential* from (12), H_0 is Hubble's constant and α is a dimensionless parameter which is galaxy dependent. Then the probability $q = q(r)$ that the graviton will be reflected at location r toward a position less than r is defined by

$$q(r) = 1 - p(r). \tag{3}$$

Outside of galaxies and clusters of galaxies, where the distance r is far from the center of mass M, the graviton transmission probability $p = 1$.

Gravitons travel in a gravitational field, which is an accelerating system. Assume that the total graviton energy for a system of two masses is expressed by

$$\Xi = \frac{GMm}{r}, \tag{4}$$

where $m = nm_g$ is the total graviton mass associated with the test mass m, where m_g is the average graviton mass and n is the number of gravitons. The total graviton energy decrease $\delta\Xi$ due to its freefall in the gravitational field of mass M, when viewed from an inertial system, is expressed by

$$\delta\Xi = -p\Xi\frac{\delta u}{c} = -\left(\frac{pGMnm_g}{r}\right)\frac{\delta u}{c}, \tag{5}$$

where p is the probability of transmission of the graviton and δu is the velocity increase in the accelerated reference frame equivalent, according to the principle of equivalence, to the gravitational field of mass M at the position r. Multiplying (1) by the number of gravitons n and equating the result to (5) gives,

$$\delta\Xi = n\delta\xi = -\left(\frac{pGMnm_g}{r^2}\right)\delta r = -\left(\frac{pGMnm_g}{r}\right)\frac{\delta u}{c}, \tag{6}$$

which simplifies to,

$$\frac{\delta r}{r} = \frac{\delta u}{c}. \tag{7}$$

Integrating (7) from r_1 to r, where

$$r_1 = r / \exp(rH_0/c), \tag{8}$$

and $r \le c/H_0$, where H_0 is Hubble's constant, we obtain,

$$\int_{r_1}^{r} \frac{\mathrm{d}r}{r} = \ln\left(\frac{r}{r_1}\right) = \frac{H_0 r}{c} = \int_0^u \frac{\delta u}{c} = \frac{u}{c}, \tag{9}$$

where we used the fact that $\ln(r/r_1) = \ln\left(r/\left(r/\exp(rH_0/c)\right)\right) = rH_0/c$. Since $\exp(rH_0/c) \ge 1$, then $r_1 \le r$. Simplifying (9) we get

$$u = H_0 r, \tag{10}$$

which we recognize as the form of Hubble's law [2], where in this case *u* is the instantaneous free fall speed in the source gravitational field and *r* is the distance of the graviton from the source. To be clear, in (10), *u* is the velocity of free fall relative to the equivalent accelerating reference frame, and not the peculiar velocity of the mass *m* relative to mass *M*. Considering two galaxies in free fall, separated by distance *r*, when an observer in one of the galaxies measures the light from a molecular substance in the other galaxy, the observer will find a redshifting of the molecular spectrum due to the effect of the free fall velocity $u = H_0 r$ between the galaxies. In this way, we realize that Hubble's law operates within galaxies and between galaxies.

Differentiating (10) with respect to time *t* we obtain the acceleration a_g,

$$a_g = \frac{\mathrm{d}u}{\mathrm{d}t} = H_0 \frac{\mathrm{d}r}{\mathrm{d}t} = cH_0, \tag{11}$$

where the graviton speed is $\mathrm{d}r/\mathrm{d}t = c$. The acceleration a_g is not the acceleration due to the source mass *M*, but is instead the rate of change of the free fall velocity field *u* relative to the traveling gravitons. The acceleration a_g acting over a distance *r* is a potential function $U(r)$ which we call the *graviton induced potential*, defined by integrating (11) over distance *r*,

$$U(r) = -\int_0^r cH_0 \mathrm{d}r = -cH_0 r. \tag{12}$$

2. Gravitons Acting as Dark Matter and Dark Energy

Consider the universe as a sphere of interior mass *M* with a thin spherical shell of mass *m*. The masses *M* and *m* are constants. The thin shell has a radius $r(t)$ at time *t*. Only the mass interior to the shell has an effect on the shell. The total graviton energy $\Xi(t)$ within the shell at time *t* is given by (4), where $r = r(t)$. Assume a uniform baryonic mass density of ρ_b. Then the mass $M(r)$ at radius *r* is given by,

$$M(r) = \frac{4\pi\rho_b r^3}{3}. \tag{13}$$

2.1. Graviton Energy Loss as Apparent Dark Matter

Gravitons lose energy due to gravitational redshift as they travel in the gravitational field between masses. This energy loss $\delta\xi$ is given by

$$\delta\xi = -\Xi\left(\frac{\delta v}{c}\right) = \frac{-GMm}{r}\left(\frac{GM\,\delta t}{cr^2}\right) = \frac{-16\pi^2}{9}\left(mG^2\rho_b^2 r^6\right)\left(\frac{\delta r}{c^2 r^3}\right) = \frac{-16\pi^2}{9}\left(\frac{mG^2\rho_b^2 r^3}{c^2}\right)\delta r, \tag{14}$$

where $\delta t = \delta r / c$ is the time traveled by the gravitons in freefall in the gravitational field and we substituted for M from (13). Integrating (14) gives the energy loss as if from an apparent dark matter,

$$\Delta\Xi_{dm} = \int \delta\xi = \int_0^r \frac{-16\pi^2}{9}\left(\frac{mG^2\rho_b^2 r^3}{c^2}\right)\delta r = \frac{-4\pi^2}{9}\left(\frac{mG^2\rho_b^2 r^4}{c^2}\right). \tag{15}$$

2.2. Graviton Energy Loss Due to an Expanding Universe

Gravitons traveling at speed c in the vacuum of the expanding universe undergo cosmological redshift in three dimensions on the way to interaction with the masses. We express this redshift by applying the 3-D velocity differential ($\delta v_x \delta v_y \delta v_z / c^3$) to the total graviton energy Ξ from (4), given in the form,

$$\delta\xi = -\frac{GM}{r}\left(\frac{\delta v_x \delta v_y \delta v_z}{c^3}\right), \tag{16}$$

where the negative sign is applied because the motion of the gravitons is in the same direction as the freefall in the field. We can convert the 3-D velocity differential to a ratio of 3-D volume differentials by the construction,

$$\frac{\delta v_x \delta v_y \delta v_z}{c^3} = \left(\frac{\delta x}{c\delta t_x}\right)\left(\frac{\delta y}{c\delta t_y}\right)\left(\frac{\delta z}{c\delta t_z}\right) = \frac{\delta x \delta y \delta z}{c^3 \delta t_x \delta t_y \delta t_z}, \tag{17}$$

where x, y and z are Cartesian co-ordinates, t_x, t_y and t_z are independent times and where $\delta v_x = \delta x/\delta t_x$, $\delta v_y = \delta y/\delta t_y$ and $\delta v_z = \delta z/\delta t_z$. Further more, we convert the volume differential in Cartesian co-ordinates to radial co-ordinates, in the form

$$\delta x \delta y \delta z = 4\pi r^2 \delta r. \tag{18}$$

Now, applying the transformations (17) and (18) to (16), while also moving the volume differential $c^3 \delta t_x \delta t_y \delta t_z$ to the left hand side of the equation we get,

$$\delta\xi\left(c^3 \delta t_x \delta t_y \delta t_z\right) = -\frac{GM}{r}\left(4\pi r^2 \delta r\right). \tag{19}$$

The left hand side of (19) is a quadruple differential whilst the right hand side is a single differential. Integrating both sides of (19) yields,

$$\left(\frac{\bar{a}^3}{\sigma}\right)\Delta\Xi = \int_0^T\int_0^T\int_0^T\int_0^{\Delta\Xi} \delta\xi\left(c^3\delta t_x \delta t_y \delta t_z\right) = \int_0^r \left(\frac{-4\pi m G\rho_b r^3}{3r}\right)4\pi r^2 \delta r = \int_0^r \frac{-16\pi^2 m G\rho_b r^4}{3}\delta r = \frac{-16\pi^2 m G \rho_b r^5}{15}, \tag{20}$$

where time $T = \bar{a}/c\sqrt[3]{\sigma}$, where $\bar{a}$ is the present radius of the universe and σ is a dimensionless constant and where we substituted for M from (13). Rearranging (20) we get the graviton energy loss due to the expansion of the universe as if from an apparent dark energy,

$$\Delta\Xi_{de} = \frac{-16\pi^2\sigma m G\rho_b r^5}{15\overline{a}^3}. \tag{21}$$

3. Equation of the Expanding Universe

The total energy of the shell of baryonic mass m, having kinetic energy, gravitational potential energy, apparent dark matter energy loss (15) and the cosmological graviton energy loss (21) due to the expansion, is expressed by

$$\begin{aligned} &\frac{1}{2}mv^2 - \frac{GM_b m}{r} + \Delta\Xi_{dm} + \Delta\Xi_{de} \\ &= \frac{1}{2}mv^2 - \frac{GM_b m}{r} - \frac{4\pi^2 mG^2\rho_b^2 r^4}{9c^2} - \frac{16\pi^2\sigma mG\rho_b r^5}{15\overline{a}^3} \\ &= -\frac{1}{2}mc^2 k\overline{a}^2, \end{aligned} \tag{22}$$

where the term on the far right is the total energy, where k is a constant (curvature) with dimensionality [length]$^{-2}$, $\overline{a}$ is the present radius of the universe and M_b is the total mass (baryonic) of the universe. The baryon mass density at the present epoch of time t_0 is given by

$$\rho_b = \frac{3M_b}{4\pi\overline{a}^3}, \tag{23}$$

where at the present epoch $r(t_0) = \overline{a}$. Similarly, the interior mass M_b (baryon, non-relativistic mass) is given by

$$M_b = \left(\frac{4\pi r^3(t)}{3}\right)\rho_m(t), \tag{24}$$

where $\rho_m(t)$ is the mass density. Since the mass M_b is constant, this implies that $\rho_m(t) \propto r^{-3}(t)$. Substituting (23) for ρ_b and (24) for M_b and multiplying (22) by $2/mr^2$ and simplifying, we get the expression for the expansion of the shell,

$$\frac{v^2}{r^2} = \frac{8\pi G\rho_m}{3} + \frac{8\pi^2 G^2\rho_b^2 r^2}{9c^2} + \frac{32\pi^2\sigma G\rho_b r^3}{15\overline{a}^3} - \frac{kc^2\overline{a}^2}{r^2}. \tag{25}$$

We remark that for this anaysis, the shell mass m is an arbitrarily negligible fraction of the universe total mass M_b.

Define the distance r by

$$r = \overline{a}a, \tag{26}$$

where the time varying scale factor a is dimensionless with $0 < a \le 1$. Using (26), the velocity v takes the form

$$v = \frac{\mathrm{d}r}{\mathrm{d}t} = \overline{a}\frac{\mathrm{d}a}{\mathrm{d}t}. \tag{27}$$

Substituting (27) into (25) our expansion equation takes the form,

$$\left(\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 = \frac{8\pi G}{3}\left(\rho_m(a) + \rho_{dm}(a) + \rho_{de}(a) + \rho_k(a)\right), \tag{28}$$

where

$$\rho_m(a) = \frac{\rho_b}{a^3} \tag{29}$$

is the baryonic mass density,

$$\rho_{dm}(a) = \frac{\pi G \rho_b^2 \bar{a}^{-2} a^2}{3c^2} \tag{30}$$

is the dark matter mass density ,

$$\rho_{de}(a) = \frac{4\pi\sigma\rho_b a^3}{5} \tag{31}$$

is the dark energy mass density and

$$\rho_k(a) = \frac{-3kc^2}{8\pi G a^2} \tag{32}$$

is the curvature mass density.

Define the Hubble parameter $H(t)$ by

$$H(t) = \frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}t}, \tag{33}$$

where, by (26), $r = \bar{a}a$. Equation (33) can also be written as

$$v = \frac{\mathrm{d}r}{\mathrm{d}t} = H(t)r(t), \tag{34}$$

which is identical to (10) where $H_0 = H(t_0)$ where t_0 is the present epoch of cosmic time. Thus, $H(t)$ defined by (33) is the general form of Hubble's law. Substituting $H(t)$ for $\mathrm{d}a/a\mathrm{d}t$ in (28), with some manipulation, we get

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G} = \rho_m(t) + \rho_{dm}(t) + \rho_{de}(t) + \rho_k(t), \tag{35}$$

where $\rho_c(t)$ is called the critical mass density at time *t*. Dividing (35) by $\rho_c(t)$ yields the parametric equation

$$\frac{\rho_c(t)}{\rho_c(t)} = \Omega_c = 1 = \Omega_m(t) + \Omega_{dm}(t) + \Omega_{de}(t) + \Omega_k(t), \tag{36}$$

where $\Omega_m(t) = \rho_m(t)/\rho_c(t)$, $\Omega_{dm}(t) = \rho_{dm}(t)/\rho_c(t)$, $\Omega_{de}(t) = \rho_{de}(t)/\rho_c(t)$ and $\Omega_k(t) = \rho_k(t)/\rho_c(t)$. At the present epoch t_0, the mass density parameter

$$\Omega_m(t_0) = \Omega_b, \tag{37}$$

where Ω_b is the baryon mass density parameter, and assuming that the universe radius $\bar{a}$ is the Hubble length $\bar{a} = c/H_0$, then the dark matter mass density parameter is given by

$$\Omega_{dm}(t_0) = \frac{\Omega_b^2}{8}, \tag{38}$$

the dark energy mass density parameter is given by

$$\Omega_{de}(t_0) = \frac{4\pi\sigma\Omega_b}{5} \tag{39}$$

and the curvature density parameter is given by

$$\Omega_k(t_0) = -\frac{kc^2}{H_0^2}. \tag{40}$$

Assuming that the curvature $k = 0$, so that $\Omega_k = 0$, then from (36) we have for the present epoch,

$$1 = \Omega_b + \frac{\Omega_b^2}{8} + \frac{4\pi\sigma\Omega_b}{5}. \tag{41}$$

From (41) we obtain,

$$\sigma = \frac{5\left(1 - \Omega_b - \Omega_b^2/8\right)}{4\pi\Omega_b}. \tag{42}$$

4. The Universe of General Relativity with Graviton Interaction

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric [3] [4] [5] [6] in terms of the scale factor $a(t)$ is given by

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}^2\theta + r^2\sin^2(\theta)\mathrm{d}^2\phi\right), \tag{43}$$

where the scale factor $0 \le a(t) \le 1$ and the curvature k has units of $[\text{length}]^{-2}$ where $k < 0$, $k > 0$ or $k = 0$. The Einstein equations [7] [8] in trace reverse form is given by,

$$R_{\mu\nu} = \kappa\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right), \tag{44}$$

where $R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu}$ is the energy-momentum tensor, T is the contracted energy-momentum tensor and $g_{\mu\nu}$ is the metric tensor. Using the metric (43), the metric tensor $g_{\mu\nu}$ in spherical coordinates (ct, r, θ, ϕ) is given by,

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2/\left(1-kr^2\right) & 0 & 0 \\ 0 & 0 & a^2r^2 & 0 \\ 0 & 0 & 0 & a^2r^2\sin^2(\theta) \end{pmatrix}. \tag{45}$$

With $g_{\mu\nu}$ defined by (45), the Ricci tensor is given by,

$$R_{\mu\nu} = \begin{pmatrix} -3\ddot{a}/c^2a & 0 & 0 & 0 \\ 0 & f/\left(1-kr^2\right) & 0 & 0 \\ 0 & 0 & fr^2 & 0 \\ 0 & 0 & 0 & fr^2\sin^2(\theta) \end{pmatrix}, \tag{46}$$

where

$$f = a\ddot{a} + 2\dot{a}^2 + kc^2. \tag{47}$$

Define the energy-momentum tensor $T_{\mu\nu}$ of a perfect fluid,

$$T_{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}, \tag{48}$$

where, referring to (29) to (32) for the mass densities, the total mass density ρ is given by

$$\rho = \rho_m + \rho_{dm} + \rho_{de} + \rho_k, \tag{49}$$

where

$$\rho_m = \frac{\rho_b \bar{a}^3}{r^3}, \tag{50}$$

$$\rho_{dm} = \frac{\pi G \rho_b^2 r^2}{3c^2}, \tag{51}$$

$$\rho_{de} = \frac{4\pi\sigma\rho_b r^3}{5\bar{a}^3}, \tag{52}$$

$$\rho_k = \frac{-3k\bar{a}^2 c^2}{8\pi G r^2}, \tag{53}$$

and where the current baryon density is given by,

$$\rho_b = \rho_c \Omega_b, \tag{54}$$

where $\rho_c = 3H_0^2/8\pi G$ is the critical mass density and Ω_b is the baryon mass density parameter. We have assumed the equation of state for the relativistic particles $p_i = \omega_i \rho c^2$, where $\omega_{dm} = -5/3$ and $\omega_{de} = -2$.

Solving the Einstein Equations (44) given the metric tensor (45), the Ricci tensor (46) and the mass-energy tensor (48), yields the equations,

$$-3\frac{\ddot{a}}{a} = 4\pi G\left(\rho + 3\frac{p}{c^2}\right), \tag{55}$$

and

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} = 4\pi G\left(\rho - \frac{p}{c^2}\right). \tag{56}$$

The acceleration of the scale factor $\ddot{a}$ can be eliminated between (55) and (56), yielding,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{57}$$

Assuming $k = 0$ for no curvature, and the total mass density given by (49)-(54), the Hubble parameter $H(a)$ is given by,

$$H(a) = H_0\sqrt{\frac{\Omega_b}{a^3} + \frac{\Omega_b^2 a^2}{8} + \frac{4\pi\sigma\Omega_b a^3}{5}}, \tag{58}$$

where $\Omega_b = 8\pi G\rho_b/3H_0^2$. Since

$$\frac{\mathrm{d}a}{H(a)} = \frac{a\mathrm{d}a}{\mathrm{d}a/\mathrm{d}t} = a\mathrm{d}t = \frac{\mathrm{d}D}{c}, \tag{59}$$

where D is the proper distance from the free falling observer to an emitting source, then integrating (59) from the scale factor of emission a to the present scale factor a_0, yields,

$$\int_a^{a_0} \frac{\mathrm{d}a}{H(a)} = \frac{1}{c}\int_0^D \mathrm{d}D = \frac{D}{c}. \tag{60}$$

5. Fits to Type Ia Supernova Data and Comparison with the Standard Model

The flux Φ_0 from a distant light source at redshift z is defined in terms of the observed luminosity $L_o = L/(1+z)^2$, where L is the luminosity of the emitting source,

$$\Phi_0 = \frac{L}{4\pi(1+z)^2 d_p^2}, \tag{61}$$

where d_p is the proper distance. The luminosity distance, from (61) is given by,

$$(1+z)d_p = \sqrt{\frac{L}{4\pi\Phi_0}}. \tag{62}$$

Using D from (60) for the proper distance in (62), the luminosity distance is given by,

$$D_L(z) = (1+z)D = c(1+z)\int_0^z \frac{\mathrm{d}z}{(1+z)^2 H(z)}, \tag{63}$$

where we transformed $\mathrm{d}a$ in (60) in terms of $a = 1/(1+z)$, where $\mathrm{d}a = -\mathrm{d}z/(1+z)^2$,

$$H(z) = H_0\sqrt{\Omega_b(1+z)^3 + \frac{\Omega_b^2}{8(1+z)^2} + \frac{4\pi\sigma\Omega_b}{5(1+z)^3}}, \tag{64}$$

and we changed the negative sign to positive by inverting the limits of integration. Our model for the magnitude is defined, in the standard way,

$$Mu(z) = 5\log(D_L(z)) - \mu_B + a_{off}, \tag{65}$$

where μ_B is the source magnitude and a_{off} is an offset. Generally, the source magnitude is combined into a_{off}.

We applied (65) in a fit to 580 Type Ia supernovae (SNe Ia) magnitude data from the Supernova Cosmology Project Union 2.1 data set [9]. A best fit was obtained for a value

$$\Omega_b = 0.00841 \tag{66}$$

for the baryon density parameter and $a_{off} = 0.78$, producing a two parameter $\chi^2 = 0.9761$.

The luminosity distance relation for the Lambda Cold Dark Matter (LCDM) model is expressed by,

$$D_{L\lambda cdm} = c(1+z)\int_0^z \frac{\mathrm{d}z}{H_{\lambda cdm}(z)}, \tag{67}$$

where the Hubble parameter $H_{\lambda cdm}(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$. Likewise, the magnitude is defined in the same way as for the graviton model, $Mu_{\lambda cdm}(z) = 5\log\left(D_{L\lambda cdm}(z)\right) - \mu_B + a_{\lambda off}$. With densities $\Omega_m = 0.271$, $\Omega_\Lambda = 0.729$ [9] and offset $a_{\lambda off} = 0.78$, the fit of the LCDM model to the Sne Ia data set obtained a two parameter $\chi^2_{\lambda cdm} = 0.9769$. The error between the models

$$Err = \left(\frac{1}{N-1}\right)\Sigma_1^N\left(Mu(z_i) - Mu_{\lambda cdm}(z_i)\right)^2 \approx 1.273\times10^{-4}, \tag{68}$$

where $N = 580$. Thus, the fits are vitually identical. **Figure 1** shows the fit made by our graviton model where the LCDM fit would essentially overlay it. The LCDM fit is shown in **Figure 2** along with our graviton model fits for $\Omega_b = 0.00841$ and $\Omega_b = 0.049$.

6. Evidence for $\Omega_b \approx 0.008$

Aside from the fact that the standard model supports a mass of $\Omega_m \approx 0.3$ with 20% baryonic mass and 80% dark matter and a dark energy mass of $\Omega_\Lambda \approx 0.7$, the main issue is with big bang nucleosynthes (BBN) and the baryon to photon number density ratio $\eta = n_b/n_\gamma$, which requires the range $5.8\times10^{-10} \le \eta \le 6.6\times10^{-10}$ [10] to explain the abundances of the light elements H, D, ^{3}He, ^{4}He and ^{7}Li. This implies that the baryon mass density parameter takes the range $0.021h^{-2} \le \Omega_b \le 0.024h^{-2}$, where $h = H_0\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}/100\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. For our graviton model, assuming $H_0 = 70\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, we get a value $n_b = \rho_b/m_p = 4.628\times10^{-8}\ \text{cm}^{-3}$, where m_p is the proton mass. Given the measured photon density $\eta_\gamma = 410\ \text{cm}^{-3}$, we get a value $\eta = 1.129\times10^{-10}$, which is only about 20% of the required amount. A future endeavor would be to apply graviton energy loss to BBN to see if it improves this result in our favor.

To some extent there is physical evidence which correlates with the results of our graviton model. Consider results from measurements of the presense of visible baryons in galaxies and the intergalactic medium, and other forms of matter by [11] which found that the amount of visible stars in galaxies is estimated to be $\Omega_b^{stars} \approx 0.002$ and the amount of gas in clusters and groups of galaxies is estimated to be $\Omega_b^{gas} \approx 0.001$ for a total $\Omega_b \approx 0.003$, which is about 36% of our fitted value (66) of $\Omega_b = 0.00841$. In another report of [12] the tallied mass for interstellar plasma, main sequence stars, white dwarfs, neutron stars, black holes, substellar objects, HI and HeI gas and molecular gas amounts to $\Omega_b \approx 0.00525$ which is about 62% of our fitted value.

7. Accelerated Expansion and the Transition Redshift

To obtain the acceleration of the expansion, set $k = 0$, take the time derivative of (28) or (57) and simplify the result to obtain

$$\frac{1}{a}\frac{\mathrm{d}^2a}{\mathrm{d}t^2} = \frac{8\pi G}{6}\frac{\rho_b}{a^3}\left(-1 + \frac{4\pi G\rho_b\bar{a}^2a^5}{3c^2} + 4\pi\sigma a^6\right). \tag{69}$$

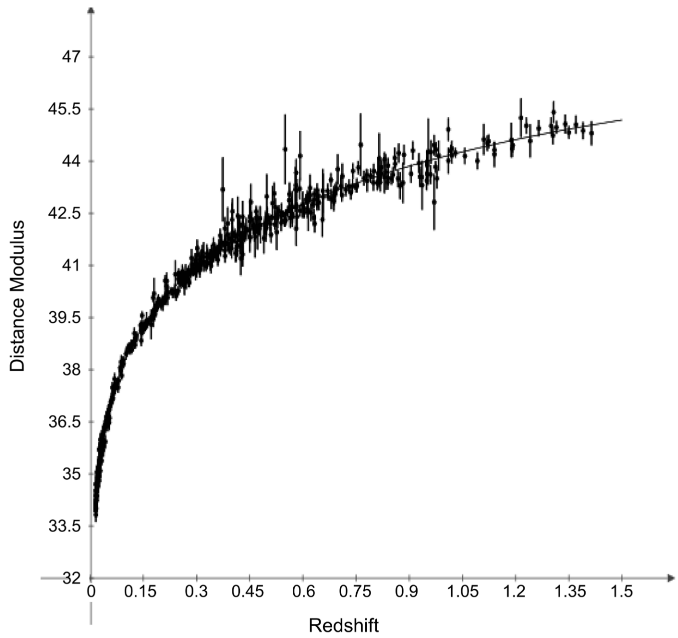

Figure 1. Supernova Cosmology Project Union 2.1 SNe magnitude vs redshift data points and error bars. The solid line is from the fit for the graviton gravitational redshift model with $\Omega_b = 0.00841$. The two parameter (Ω_b and A_{off}) $\chi^2 = 0.9761$.

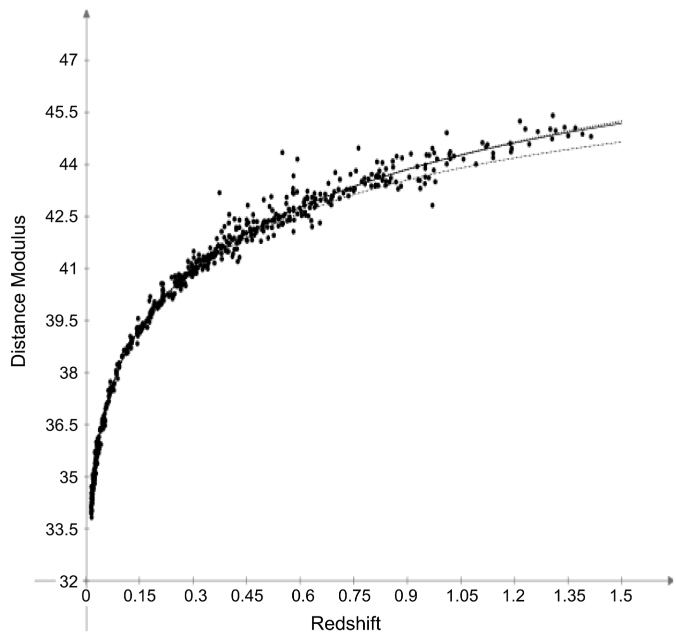

Figure 2. Supernova Cosmology Project Union 2.1 SNe magnitude vs redshift data points without error bars. The solid line is the fit for the graviton model with $\Omega_b = 0.00841$, with a two parameter $\chi^2 = 0.9761$. The dotted line is the fit for the LCDM model with $\Omega_m = 0.271$ and $\Omega_\Lambda = 0.729$ which has a two parameter $\chi^2 = 0.9769$. The dash-dot line (lowest curve) is for the graviton model with $\Omega_b = 0.049$, which has a two parameter $\chi^2 = 1.4019$.

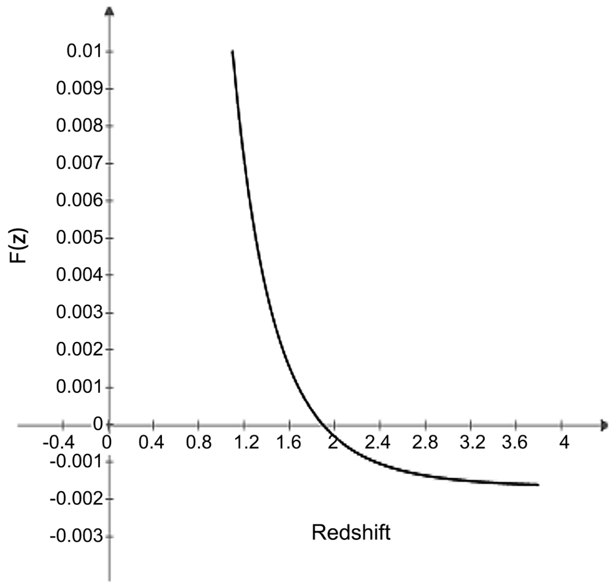

Figure 3. Plot of acceleration transition function $F(a(z))$ given by (70) in terms of redshift *z*. For $\Omega_b = 0.00841$ the transition from decelerating to acccelerating expansion occurs at a redshift of $z = 1.896$.

By definition, the scale factor has the range $0 < a(t) \le 1$. The scale factor *a* for which the expansion transitions from decelerating to accelerating is when (69) becomes 0, given by the transition function

$$F(a) = -1 + \frac{\Omega_b a^5}{2} + 4\pi\sigma a^6 = 0. \tag{70}$$

This is a sixth order polynomial, easily solved iteratively for the zero point transition. Solving (70) for the transition scale factor at $\Omega_b = 0.00841$ with σ given by (42) gives the value $a_t = 0.345345$. The cosmological redshift z_t related to this scale factor is,

$$z_t = \frac{1}{0.345345} - 1 = 1.896. \tag{71}$$

Figure 3 shows the transition function $F(a(z))$. For the standard LCDM model, with mass density $\Omega_m = 0.271$ and vacuum density $\Omega_{de} = 0.729$, the transition scale factor is given by $a = \sqrt[3]{\Omega_m / 2\Omega_{de}} = 0.571$ and the transition redshift is given by $z = 0.752$. The transition redshift from SNe Ia observations [13] in the redshift range $0.2 < z < 1.6$ was $z = 0.46 \pm 0.13$. Obviously the LCDM prediction is more than one sigma from the observed transition and our prediction is outside the range of the study, implying that analyses of SNe Ia at higher redshifts are required to resolve this issue.

8. Conclusions

The graviton model is a method to account for the effect of the gravitational field in free fall, where we have used the concept of gravitons in free fall losing energy thru gravitational redshift, without emitting any radiation, to account for the entire gravitational field energy loss thru gravitational redshift. In order to fit the SNe Ia data, it was necessary to employ a novel form for the luminosity distance (63). Assuming a mass density of $\Omega_b = 0.049$ with our model D_L does not produce a good fit to the more distant SNe Ia data, with a $\chi^2 = 1.4019$, as shown in **Figure 2**. However, using a density of $\Omega_b = 0.00841$ fits the data with the least error of $\chi^2 = 0.9761$. We think that this is telling us that the standard model use of the co-moving distance (63), which is constant in free fall along a geodesic path, does not take into consideration the loss of field energy in free fall nor of the three dimensional property of the gravitational field. This may be the reason that the standard model requires amounts of unknown dark matter and dark energy to fit the SNe Ia data.

The distance measure we use, (59), can be shown to be proportional to a volume element, and a co-moving volume element better describes the gravitational field which is responsible for the geometry in three dimensions, not just one dimension as for the co-moving distance $D_C = r$. Consider the relationship that the distance element $ca\mathrm{d}t$ has with the volume element $\mathrm{d}V$ defined by,

$$ca\mathrm{d}t = a^2\left(\frac{c\mathrm{d}t}{a}\right) = 4\pi A r^2 \mathrm{d}r = A\mathrm{d}V, \tag{72}$$

where $\mathrm{d}r = c\mathrm{d}t/a$ for light traveling in flat space, $r = ca/H_0$ and $A = H_0^2/4\pi c^2$. This relationship (also see (20)) may be the reason that the graviton model of $H(z)$ reacts well to the luminosity distance defined in (63) and is able to fit the data very well.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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