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A Study in the Variation of *G*

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Abstract

There are indications that the Newtonian gravitational constant may not be a constant but may vary with respect to some other physical parameter. Various possible characterizations of Newton's gravitational parameter as a function of the cosmic scale parameter are proposed and studied within the framework of classical Newtonian cosmology. A number of toy cosmologies with varying Newtonian gravitational parameters are developed and analyzed. The numerical solutions to the temporal evolution of the universe from the Friedmann equation are examined and discussed as well as kinematic observables. Finally, other avenues of research are addressed.

Keywords

Newton's Gravitation Constant, Cosmology, Fundamental Constants

1. Introduction

Though familiar and fascinating, gravity is the feeblest of the fundamental forces known in physics today. The so-called Newton's gravitation constant, $G = \left(6.67430 \pm 0.00015\right) \times 10^{-11}\ \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$ [1], is a measure of the strength of this interaction.

There are a number of indications that Newton's gravitation constant may not be a constant but a parameter. The premise of a gravitational parameter that varies in relation to another is not novel. In the early 20th century, Weyl [2], Eddington [3], and Dirac [4] [5] [6] considered that there may be a connection between the fundamental quantum mechanical and large scale constants of the universe describing the underlining structure nature. This culminated in Dirac's so-called "Large Number Hypothesis", which proposed that taking the ratio of certain fundamental quantities leads to extremely large dimensionless numbers. Examples are

$$\frac{\text{size of the observable universe}}{\text{classical electron radius}} = \frac{\frac{c}{H_0}}{\frac{e^2}{m_e c^2}} \approx 10^{40}$$

and

$$\frac{\text{electromagnetic force of the proton and electron}}{\text{gravitational force of the proton and electron}} = \frac{e^2}{G m_e m_p} \approx 10^{40}$$

where H_0 is Hubble's parameter in the current epoch, c is the speed of light, m_e is the mass of the electron, m_p is the mass of the proton, G is the Newton gravitational parameter and e is the electrical charge of the electron. Dirac speculated that the ratio between these relatively enormous numbers which, is of the order of unity, was not by chance. Though it is commonly regarded as a form of numerology based on coincidences of nature, there may be a hint of physics lying underneath. Dirac assumed that the ratios of the large numbers would remain constant with time thus conjecturing it was not a coincidence. In order for this numerology to hold for all time, it required that some of the parameters must vary with respect to time. So as to prevent changing atomic physics, this led Dirac to a semi-quantitative argument that the gravitational parameter should vary with the inverse of time, t,

$$G \propto \frac{1}{t}. \tag{1}$$

This implies that the gravitational parameter was greater in the past and is weakening with the passage of time. Dirac's prediction of the gravitational parameter's temporal evolution leads to a variety of testable predictions.

Within the context of General Relativity, the time dependence of the gravitational parameter has been explored and various technical issues have arisen [7] [8] [9]. In General Relativity, the fundamental constants are inextricably linked to the equivalence principle. Energy conservation and how the Bianchi identity is handled become critical issues that must be calculated with caution [10]. In this study, the variation of the gravitational parameter is examined not on a local scale but on a cosmological scale. The relationship between the gravitational parameter and the cosmological constant becomes much richer with a time-dependence shared between the two parameters depending on how they are managed in energy momentum tensor [11]. Further complications are involved if the speed of light is also considered to change with respect to time [12].

Alternative theories such as Brans-Dicke models of gravity [13] [14] predict variations in the gravitational parameter. Typically, in this family of theories, the gravitational parameter is determined by inverse of a variable scalar field, ϕ,

$$G \sim \frac{1}{\phi}. \tag{2}$$

In these models of gravity, the scalar field couples gravity through some constant ω. Later the Brans-Dicke models were extended to more general sca-

lar-tensor theories [15] [16] [17] that include variable coupling free parameter that depends on the scalar field, $\omega(\phi)$, known as the coupling function.

Besides examining a time dependence on the gravitational parameter others have sought to explore a temperature dependence. These theories often involve introducing a coupling term to the Lagrangian between a scalar field and the scalar curvature [18] [19] [20]. Generally, the gravitational parameter takes form

$$G = \frac{G_0}{1-\alpha G_0 T^2} \tag{3}$$

where G_0 in this case is the so-called "zero temperature" value of the gravitational parameter in the laboratory, α typically depends upon some choice of coupling parameters, and T is the temperature dependence. In some ways, these models are the inheritors of an idea initially studied by P.E. Shaw [21] who conducted laboratory temperature dependent Cavendish-type torsion experiments (between 20˚C and 250˚C) in the early 20th century utilizing 20 cm diameter leads spheres with masses of 47 kg each. Initially, Shaw determined that the gravitational parameter had the temperature dependent form

$$G = G_0(1+\beta T) \tag{4}$$

where the fit parameter $\beta = 1.2\times 10^{-5\circ}\mathrm{C}^{-1}$. Upon improving the experimental apparatus (installing a more rigid support structure), he determined that there was no obvious effect and that the original measured dependency was probably due to mechanical defects that had been improved in the experimental design.

It is worth noting that experimentally measuring the gravitational constant is challenging due to the extreme weakness of the gravitational force, which may cause the relatively weak interaction signal to be masked by other phenomena. The gravitational force cannot be shielded from other background interactions as some other forces can be. Finally, the gravitational parameter is independent of other fundamental constants and can only be determined via the gravitational interaction and thus the density profiles and mass distributions of test masses must be well known. This all leads to the fact that the gravitational parameter is the least precisely known of the fundamental constants.

In this work, a number of toy matter-only cosmological theories will be considered. In these models, the gravitational parameter will be allowed to vary with respect to the Hubble scale parameter (to be described later). These types of model dependencies have been considered in the past [22] [23] [24] [25].

2. Various Models for the Gravitational Parameter

In the models under consideration, the characterization of the Newtonian gravitational parameter is of the form

$$G(a) = F(a)G_0 \tag{5}$$

where there is the dimensionless Hubble cosmic scale parameter, *a*, dependency. The current value of the gravitation parameter is given as G_0 and the functional scale dependency is described by the dimensionless modification function $F(a)$.

Three models of the scale dependency have been examined based on the logistic, hyperbolic tangent, and generalized algebraic sigmoid functions that are all bounded from above and below.

These models are not meant to replicate observations of the actual universe on the classical scale. Instead, they are toy models to begin exploring the possible effects of a certain characterization of gravity. Studying these toy models with their associated exaggerated results may provide insight into applying similarly minded ideas to more realistic models where the effects may be present but not as pronounced.

2.1. The Logistic Function

The logistic function, $f(a)$, is of the form

$$f(a)=\frac{1}{1+\mathrm{e}^{-k(a-a_0)}} \tag{6}$$

where the parameter k describes the scale that controls the width of the function and a_0 describes the offset of the inflection of the function (see **Figure 1**). This function is often used to model changes in the future relating to exponential growth with some sort of upper bound known as the carrying capacity. Some of the applications of the logistic function include modeling populations with limited resources [26] [27], the growth of tumors [28], and predictions of credit scores [29]. The appeal of this function for modeling the gravitational parameter is that it begins at zero then exponentially grows until reaching a value of unity resulting in the current value of the Newtonian gravitational parameter.

2.2. The Hyperbolic Tangent Function

The hyperbolic tangent function, $f(a)$, is of the form

$$f(a)=\tanh\left(\frac{a-a_0}{w}\right) \tag{7}$$

where w is used to control the steepness of the transition from the lower asymptote to the higher asymptote and a_0 describes the offset of the inflection of function (see **Figure 2**). One application of the hyperbolic tangent function is the description for the expectation value of the magnetic moment [30]. This function can take on negative values before exponentially growing and approaching the horizontal asymptote. This can lead to modeling the universe where there is a transition from a negative to positive Newtonian gravitational parameter which has garnered some interest [31].

2.3. The Generalized Algebraic Sigmoid Function

The generalized algebraic sigmoid function [32] is of the form

$$f(a)=\frac{a}{\left(1+\left(a-a_0\right)^y\right)^{1/y}} \tag{8}$$

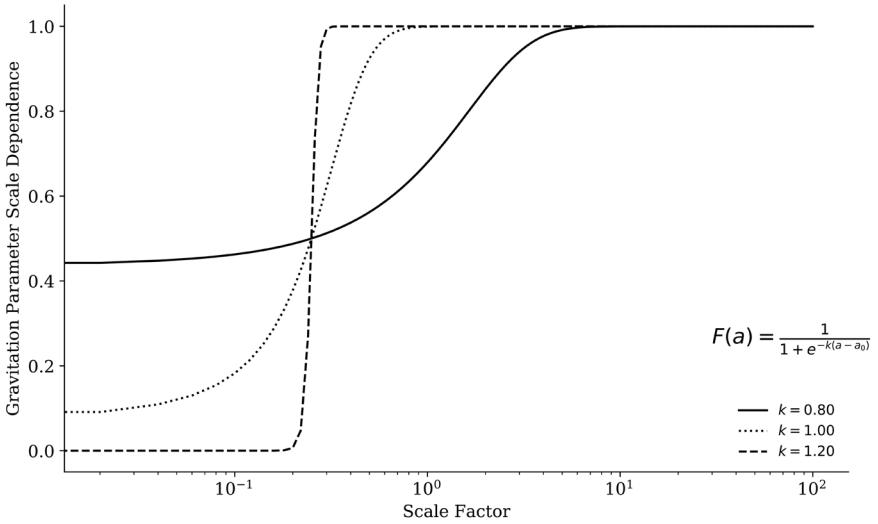

Figure 1. The figure above illustrates the logistic function described in the text.

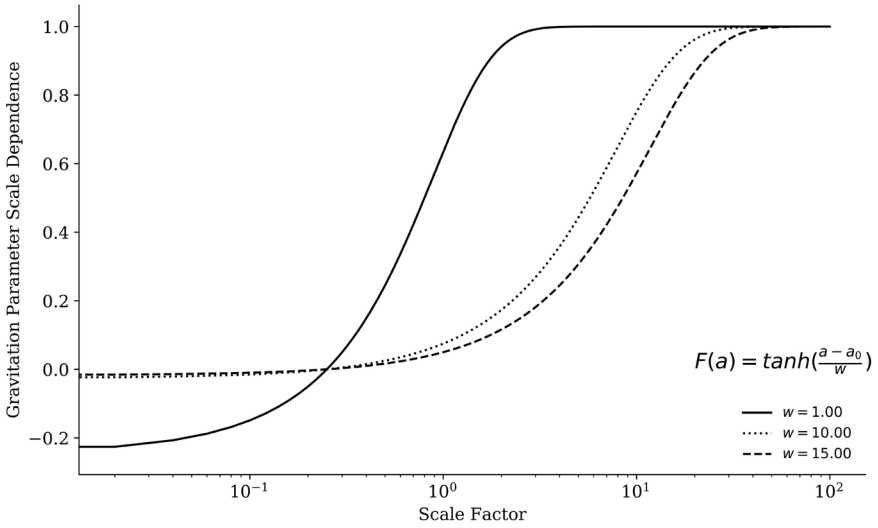

Figure 2. The figure above depicts the hyperbolic function described in the text.

where the y parameter describes the shape of how fast the curve approaches the asymptotes. Various values of the shape parameter correspond to a wide range of approximations of sigmoid functions such as $y = 1.5$ approximates the arctangent function, when $y = 2.9$ the function approximates the logistic function, and when $y = 3.4$ the function approximates the error function [33]. This can lead to modeling the universe where there is a transition from a non-negative value to the accepted value of the Newtonian gravitational constant today (**Figure 3**).

3. Newtonian Cosmology with a Varying Gravitational Parameter

Newtonian cosmology consists of a isotropic homogeneous sphere of radius R and mass M containing a gas of particles (galaxies). A test particle (galaxy) of

mass m is on the edge of the sphere. This sphere is expanding/contracting under the observed effects of Hubble's law (Figure 4),

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \dot{R} = H(t)R \tag{9}$$

where the dot indicates a time derivative and $H(t)$ is Hubble's parameter. The current value of Hubble's parameter, known as Hubble's constant, is denoted as $H_0 = H(t_0)$ where t_0 is the current epoch. The Lagrangian based on the work of Viera and Bezerra [34] for a particle (galaxy) in a Newtonian cosmology is

$$L(R,\dot{R}) = \frac{1}{2}m\dot{R}^2 + G(R)\frac{Mm}{R} + \frac{\Lambda}{6}mR^2 \tag{10}$$

where the first term is the kinetic energy, the second term is the gravitational potential, and the last term is associated with the dark energy, Λ.

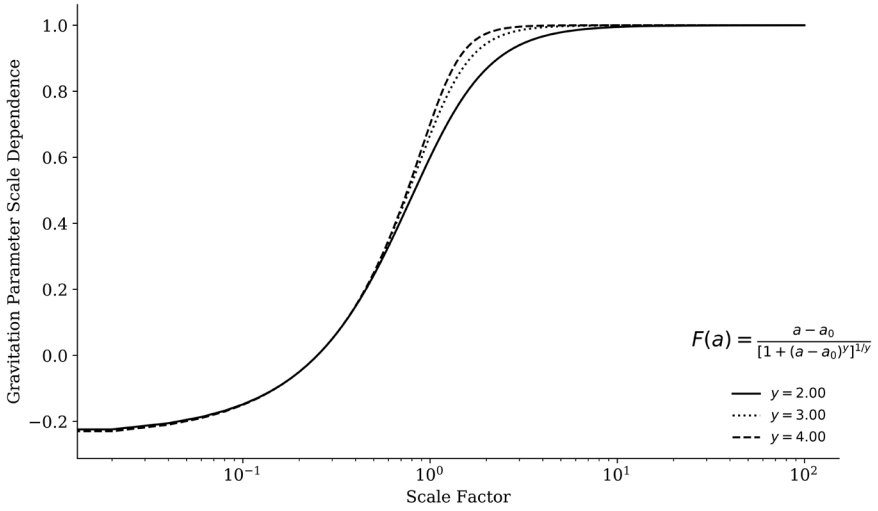

Figure 3. The figure above is a generalized algebraic sigmoid function described in the text.

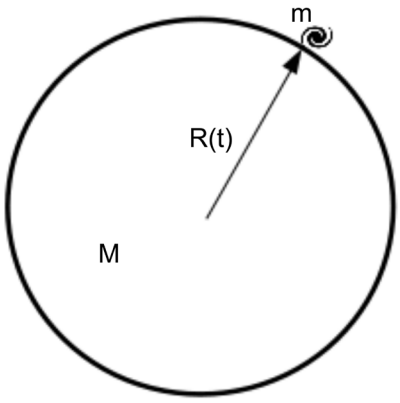

Figure 4. The center of the figure above represents some arbitrary point in the universe as the origin. A sphere defined by a radius $R(t)$ that depends on some time t contains within it a total mass M which remains constant as the sphere expands and/or contracts. A test particle (a galaxy) of mass m resides on the surface of the sphere. A Newtonian cosmology considers the interactions between the matter in the sphere and mass of the test particle.

The results of using the Euler-Lagrange equation with the Lagrangian given in Equation (10) leads to the first Friedmann equation

$$\ddot{R} = -G(R)\frac{M}{R^2} + \frac{1}{3}\Lambda R \tag{11}$$

where terms with derivatives of the Newtonian gravitational parameter have been neglected.

The energy, E, of the system can be determined by the Hamiltonian, $\mathcal{H}$ and using the conjugate momentum, $p = \frac{\partial L}{\partial \dot{R}} = m\dot{R}$, leading to

$$\mathcal{H} = E \tag{12}$$

$$= p\dot{R} - L \tag{13}$$

$$= \left(m\dot{R}\right)\dot{R} - \frac{1}{2}m\dot{R}^2 - G(R)\frac{Mm}{R} - \frac{\Lambda}{6}mR^2$$

$$= \frac{1}{2}m\dot{R}^2 - G(R)\frac{Mm}{R} - \frac{\Lambda}{6}mR^2. \tag{14}$$

Identifying the constant $k = -2\frac{E}{m}$ and using the fact that the mass within the sphere of interest can be written in terms of the density ρ as $M = \rho\left(\frac{4}{3}\pi R^3\right)$, allows the second Friedmann equation with a variable Newtonian gravitational parameter to be written as

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G(R)}{3}\rho - \frac{k}{R^2} - \frac{\Lambda}{3}. \tag{15}$$

Invoking the relationship, $R(t) = a(t)R_0$ where R_0 is the radius of the universe observed at some time t_0 and $a(t)$ is a dimensionless cosmic scale factor that represents the relative expansion of the universe such that $a(t_0) = 1$. Rewriting Equation (15) in terms of the scale factor and Equation (5) allows the separation of the modification of the scale dependency to Newton's gravitational parameter

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi F(a)G_0}{3}\rho - \frac{\kappa}{a^2} - \frac{\Lambda}{3} \tag{16}$$

where $\kappa = \frac{k}{R_0^2}$ represents the curvature constant of the universe. The curvature constant can take three values, $\kappa = 1, 0, -1$, which represent a positively, flat, or negatively curved universe.

Using the standard definitions for the critical density of the universe today as $\rho_{c,0} \equiv 3H_0^2/8\pi G_0$ and density of the dark energy $\rho_\Lambda \equiv \Lambda c^2/8\pi G_0$, the density parameters can be written as

$$\Omega_M(a) \equiv \rho(a)/\rho_{c,0}, \tag{17}$$

$$\Omega_\Lambda \equiv \rho_\Lambda/\rho_{c,0}, \tag{18}$$

$$\Omega_0 \equiv \Omega_M(1) + \Omega_\Lambda, \tag{19}$$

and it should be noted that the curvature can be identified as $-\kappa c^2 = H_0^2\left(1-\Omega_0\right)$.

This ultimately leads to a second Friedmann equation being written as

$$\frac{\dot{a}}{a} = H_0\left[F\left(a\right)\Omega_M\left(a\right)+\left(1-\Omega_0\right)a^{-2}+\Omega_\Lambda\right]^{\frac{1}{2}}. \tag{20}$$

4. Numerical Results of Newtonian Cosmology with a Varying Gravitational Parameter

The models considered in this study are based on a cosmology that consists only of matter (pressureless dust) with the possibility of spatial curvature that yielding the Friedmann equation of Equation (20) to be of the form

$$\dot{a} = H_0\left[F\left(a\right)\frac{\Omega_{M,0}}{a}+\Omega_{k,0}\right]^{\frac{1}{2}} \tag{21}$$

where $\Omega_{M,0}$ is the current matter density and $\Omega_{k,0}=1-\Omega_0$ is the current spatial curvature density. The variation in Newton's gravitational parameter is characterized by the function, $F(a)$, described by Equation (5) in Section 0. This first-order differential equation was numerically solved for the three modification functions, $F(a)$, represented by Equations (6), (7), and (8), using Adams' method [35]. This method was implement utilizing Open-source scientific SciPy modules [36] that are available though Python programming language [37]. The time evolution of three cosmological parameters were numerically calculated: the scale parameter, the Hubble parameter, and the deceleration parameter.

4.1. Scale Parameter

The temporal evolution of scale parameter describes how the size of the universe changes with time. All the universes under consideration begin from a singularity, a big bang event, at the cosmic time chosen to be zero. In the standard homogeneous isotropic matter-only Friedmann-Lemaitre-Robertson-Walker (FLRW) universe [38], the ultimate fate of the universe depends on the matter density. In a universe with less than the critical density, it will continue to expand for all time. In a universe with more than the critical density, the universe will grow to some maximum scale value, $a_{\max}$, and then begin to contract to a scale factor of zero. This event is known as the big crunch. Due to the numerical routine being employed, in scenarios where the universe undergoes a big crunch, the plots show this expansion until $a_{\max}$ is reached when $H\left(t\right)=0$, after which the contraction could not be calculated in the computer code. The contraction is symmetric in time and will result in a contraction that will mirror the expansion until $a_{\max}$. This fate of the universe is ruled out in the standard model of cosmology.

4.2. Hubble Parameter

The Hubble parameter, as described earlier, characterizes the inherent rate of expansion (or contraction) of the universe from the Friedmann equation. This cosmic expansion (or contraction) is measured by the Hubble parameter H that

is defined using the scale factor as

$$H = \frac{\dot{a}}{a}. \tag{22}$$

The Hubble constant, H_0, is the present value of the Hubble parameter in the current epoch, t_0. This factor describes how fast distant astronomical objects are moving away from an observer using Hubble's law described by Equation (9).

In this work, the Hubble parameter is taken to be $H_0 = \frac{1}{13.8\ \text{Gyr}} = 0.072\ \text{Gyr}^{-1}$. For a spatially flat, matter-only universe where power law solutions are assumed, this leads to the present age of the universe, t_0, being related to the Hubble parameter by

$$t_0 = \frac{2}{3} H_0^{-1} \tag{23}$$

which results in the present epoch to be 9.2 Gyr.

4.3. Deceleration Parameter

The deceleration parameter describes the rate of change of the rate of expansion of the universe. This dimensionless quantity is measured by the deceleration parameter, q, that is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}. \tag{24}$$

This quantity describes the acceleration of the expansion of the universe. For a universe that does not contain a cosmological constant term with negative pressure, the acceleration universe tends to decrease with increasing pressure and mass density.

In a spatially flat, matter-only universe, the deceleration parameter at the present epoch $q_0 = \frac{1}{2}$.

5. Numerical Results of Observables within a Newtonian Cosmology with a Varying Gravitational Parameter

Three cosmological parameters are utilized to describe cosmologies, the deceleration parameter q, the Hubble expansion parameter H, and the density parameter, Ω. These parameters connect the evolution of the universe and its ultimate fate with measured observables. The best model of the universe can be determined by analyzing observational data in conjunction with these parameters. It is one of the aims of this work to describe the possible effects a variable gravitational parameter would have on these important observables.

5.1. Logistic Dependent Gravitational Parameter Cosmology

Cosmologies with a logistic function-dependent gravitational parameter (LF) were investigated. An offset of $a_0 = 0.25$ had been chosen in order to enhance the observed effects of the gravitational parameter transitioning from a reduced

non-negative value to its current accepted value. The time scale of this transition period was controlled by the k parameter, which has been studied at values of 0.8, 1.0, and 1.2. In all of these models the gravitational parameter was attractive.

The LF cosmological models under consideration had a transition parameter k = 0.8 (1.0, 1.2) with a modification function $F(a)$ = 0.443 (0.091, 0.000) producing an initial gravitational parameter of 2.957 (0.607, 0.000) × 10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$. All three modification functions cross each other at the scale factor a_0 = 0.25 as required. The gravitational parameter for the transition parameter k = 0.8 (1.0, 1.2) reached the current accepted value at the scale factor of a = 0.3 (1.1, 10.0).

The numerical calculations of the scale factor for both the LF and standard FLRW matter-only cosmologies are shown in Figure 5 as a function of time. The LF cosmologies all follow the corresponding standard FLRW cosmologies very closely at a deviation of ~1% after ~25 Gyr. At the earliest times, around ~5 Gyr, the logistic-dependent gravitational parameter cosmologies have a Hubble parameter that is systematically lower by ~14%.

The numerical calculations of the observable Hubble parameter as a function of time for a matter-only universe are shown in Figure 6. The Hubble parameter for all the LF cosmologies tracks extremely close to that of the typical FLRW cosmologies at early times, before ~1 Gyr, and at later times, after ~10 Gyr. The greatest deviation between the two sets of cosmologies occurs ~5 Gyr where the LF cosmologies are systematically higher by ~15%.

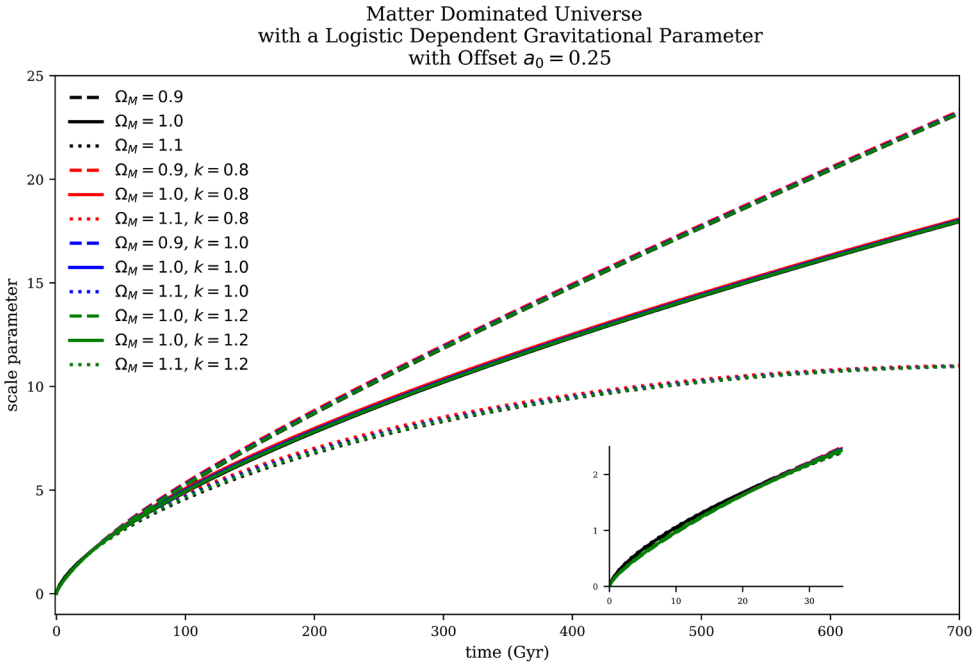

Figure 5. The cosmic scale factor (size) of a matter-only universe as a function of time with a logistic function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

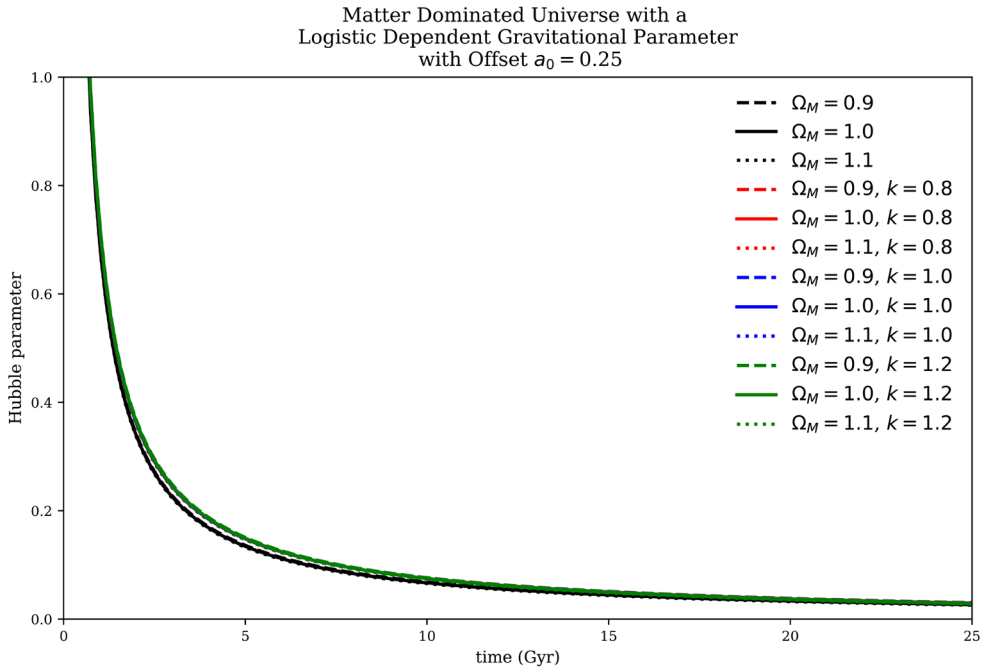

Figure 6. The Hubble parameter (expansion rate) of a matter-only universe as a function of time with a logistic function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

The numerical calculations of the deceleration parameter as a function of time for a matter-only universe are shown in **Figure 7**. In all of the models considered, FLRW and the LF cosmologies, the universes initially have a relatively steep deceleration in comparison to the standard FLRW cosmologies. At the current epoch, t_0 = 9.2 Gyr, the LF cosmologies have deceleration parameters in the neighborhood of 0.31 - 0.41.

5.2. Hyperbolic Tangent Gravitational Parameter Cosmology

Cosmologies with a hyperbolic tangent-dependent gravitational parameter (HT) were investigated. There was an offset of a_0 = 0.25 that had been employed in order to enhance the observed effects of the gravitational parameter transitioning from a negative to its current value. The time scale of this transition period is controlled by the w parameter, which had been studied at values of 1.0, 10.0, and 15.0. In this model, gravity is initially repulsive.

The cosmological model where the transition parameter w = 1.0 (10.0, 15.0), the modification function $F(a)$ = −0.226 (−0.023, −0.015) producing an initial repulsive gravitational parameter of −1.535 (−0.154, −0.100) × 10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$. All three modification functions cross each other at the scale factor a_0 = 0.25 as required. The gravitational parameter for the transition parameter w = 1.0 (10.0, 15.0) reaches its current value at the scale factor of a = 4.2 (49.1, 75.5).

The numerical calculations of the evolution of the scale of a matter-only universe are shown in **Figure 8** as a function of time. Initially, all the cosmologies

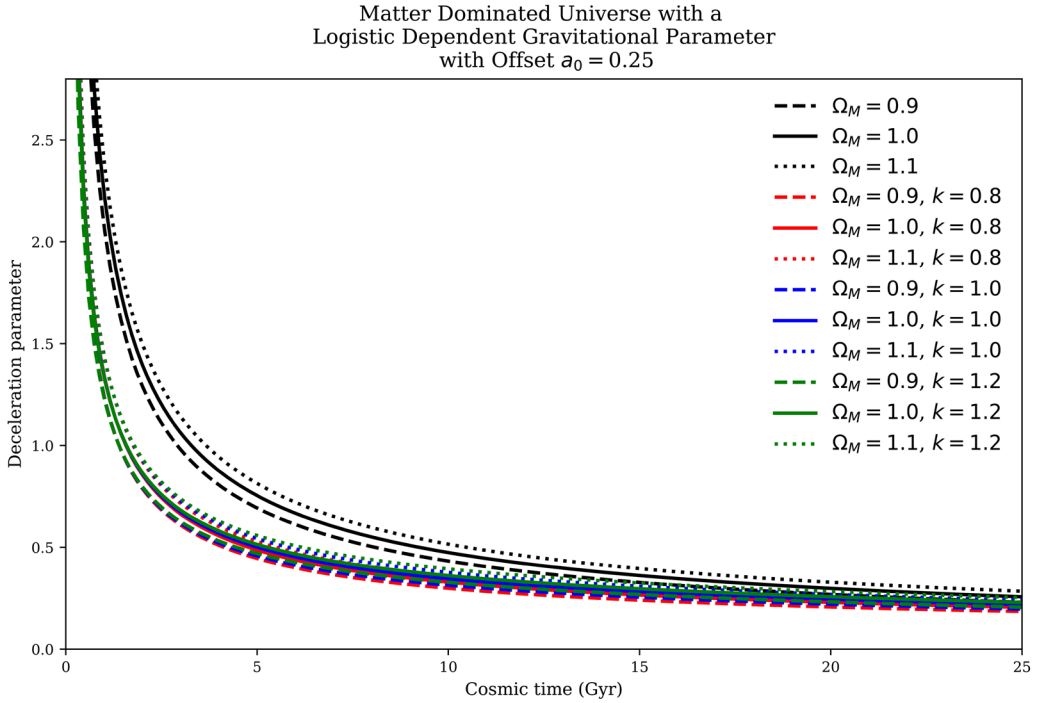

Figure 7. The deceleration parameter (acceleration rate) of a matter-only universe as a function of time with a logistic function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

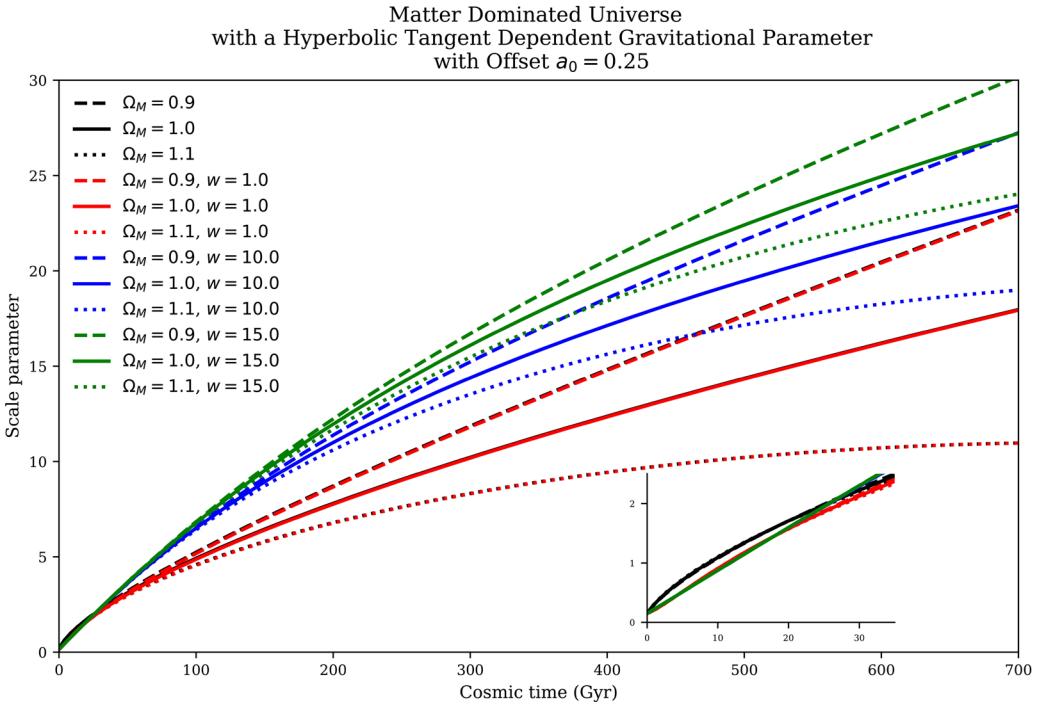

Figure 8. The cosmic scale factor (size) of a matter-only universe as a function of time with a hyperbolic tangent function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

with the HT gravitational parameter expand at a considerably slower rate than the FLRW cosmologies under consideration. The scale evolution of the HT cosmologies with transition parameter $w = 15.0$ track closely to their corresponding FLRW cosmologies after ~40 Gyr has been reached with the FLRW cosmologies. The HT scale parameters were consistently ~0.5% higher than the corresponding FLRW scale factors over the scale from ~40 - 700. The HT cosmologies with transition parameter $w = 10.0$ (1.0) continue to expand in a linear fashion until reaching the scale factor of ~40.0 (100.0). All four cosmologies under consideration with a matter density greater than the critical density exhibit the slowing down of the expansion rate to an eventual halt and then collapsing. The relative pattern between the open, flat, and closed cosmologies is consistent between all four models under consideration.

The numerical calculations of the time evolution of the Hubble parameter are shown in Figure 9. Initially, the Hubble parameters associated with the HT cosmologies with a transition parameter $w = 1$ show a Hubble parameter with a positive slope until an inflection point occurs ~0.3 Gyr. This owes to the initial repulsiveness of the gravitational parameter in this particular model. The HT cosmologies with transition parameter $w = 1.0$ and 10.0 are much milder, as expected. The time evolution of the HT cosmologies with transition parameters $w = 15.0$ and 10.0 track so closely for all matter densities examined that they appear as a single curve in Figure 9. After 25.0 Gyr, both the HT and FLRW models are approaching 0.035 Gyr^{-1}.

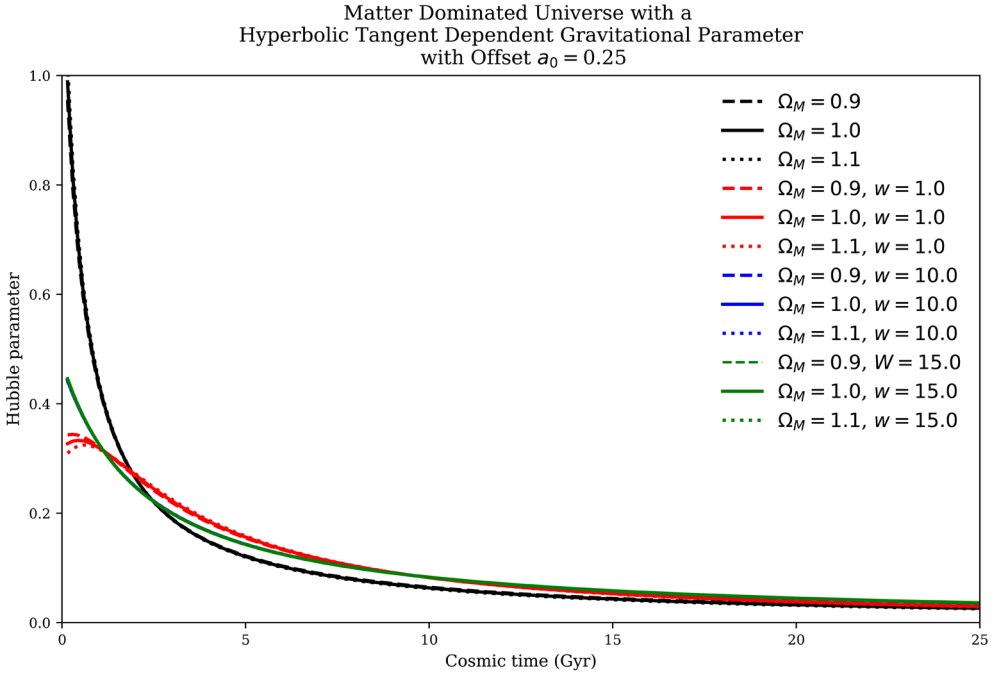

Figure 9. The Hubble parameter (expansion rate) of a matter-only universe as a function of time with a hyperbolic tangent function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

The numerical calculations of the time evolution of the deceleration parameter are shown in Figure 10. All HT cosmologies under consideration in this study have a gravitational parameter that is initially negative indicating the repulsive nature of the gravity under consideration in these models. The relative smallness associated with the acceleration of the HT cosmologies was related to the relative feebleness of the modification function associated with the gravitation parameter (note Figure 2 for w = 10.0 and w = 15.0) at the beginning of time with the chosen parameters in this model. The HT cosmologies with transition parameter w = 1.0 approaches the neighborhood values of the standard FLRW models ~q = 0.255 at 25.0 Gyr. HT cosmologies with transition parameters w = 10.0 and w = 15.0 are found to be in a fairly constant deceleration band of values between ~0.023 - 0.047 after passing ~12.0 Gyr.

5.3. Generalized Algebraic Sigmoid Gravitational Parameter

The cosmology with an algebraic generalized sigmoid function-dependent gravitational parameter (GAS) was investigated. There was an offset of a_0 = 0.25 that had been employed in order to enhance the observed effects of the gravitational parameter transitioning from a negative to its current value. The time scale of this transition period is controlled by the y parameter, which had been studied at values of 1.0, 2.0, and 3.0. The gravitational modification function at the beginning of time that was $F(a)$ ~ -0.23 leads to a gravitational parameter that initially is −1.535 (−0.154, −0.100)×10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$. In these models, the gravitational parameter becomes its current accepted value for the GAS modification function y = 2.0 (3.0, 4.0) at the scale factor ~22.5 (7.8, 3.7). In this model, gravity is initially repulsive.

The numerical calculations of the evolution of the scale of a matter-only universe are shown in Figure 11 as a function of time. The various GAS models followed the standard FLRW models quite closely within ~3% after ~50 Gyr had been reached. The analyzing power associated with these GAS models that transition in the early universe is associated with the behavior at early times. The scale parameter is systematically higher, ~30%, for the FLRW cosmologies compared to the GAS models at early times.

The numerical calculations of the evolution of the Hubble parameter of a matter-only universe are shown in Figure 12 as a function of time. All the GAS model Hubble parameters tracked closely with one another over the first 25 Gyr. The GAS models all had a systematically higher, 24.2%, Hubble parameter ~0.0894 Gyr^{-1} at the current epoch, t_0 = 9.2 Gyr as compared to the flat FLRW model value of 0.072 Gyr^{-1}. After 25 Gyr, the Hubble parameters of all models reach a value of 0.03 Gyr^{-1}.

The numerical calculations of the time evolution of the deceleration parameter are shown in Figure 13. The deceleration parameters associated with the GAS cosmologies all tracked closely together. Due to the initial repulsiveness of gravity in these GAS models the deceleration parameter was actually initially an

acceleration parameter. After 25 Gyr, both the FLRW and GAS models approached a deceleration value of ~0.24

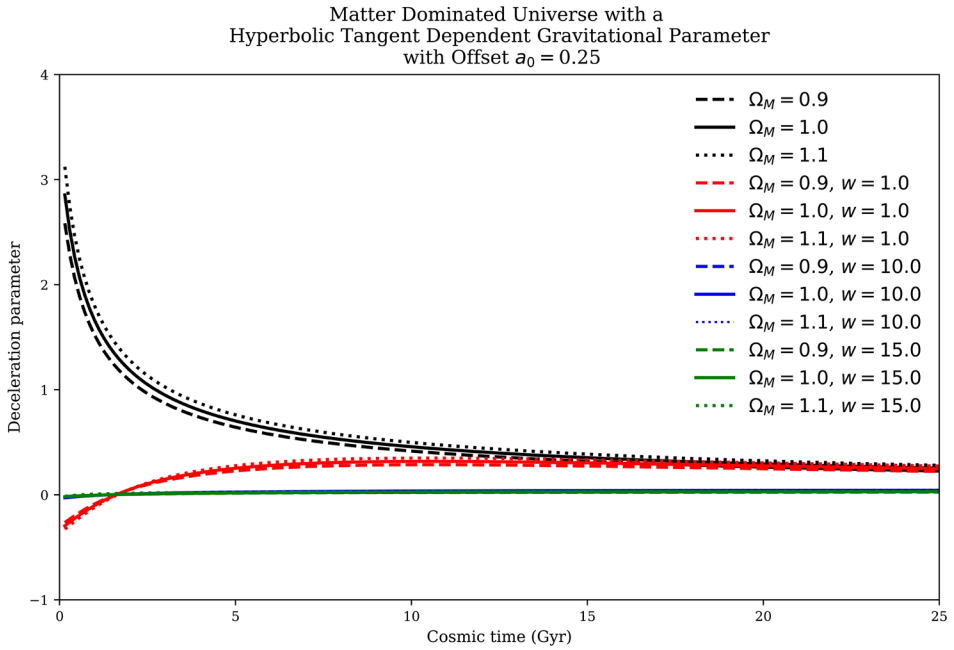

Figure 10. The deceleration parameter (acceleration rate) of a matter-only universe as a function of time with a hyperbolic tangent function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

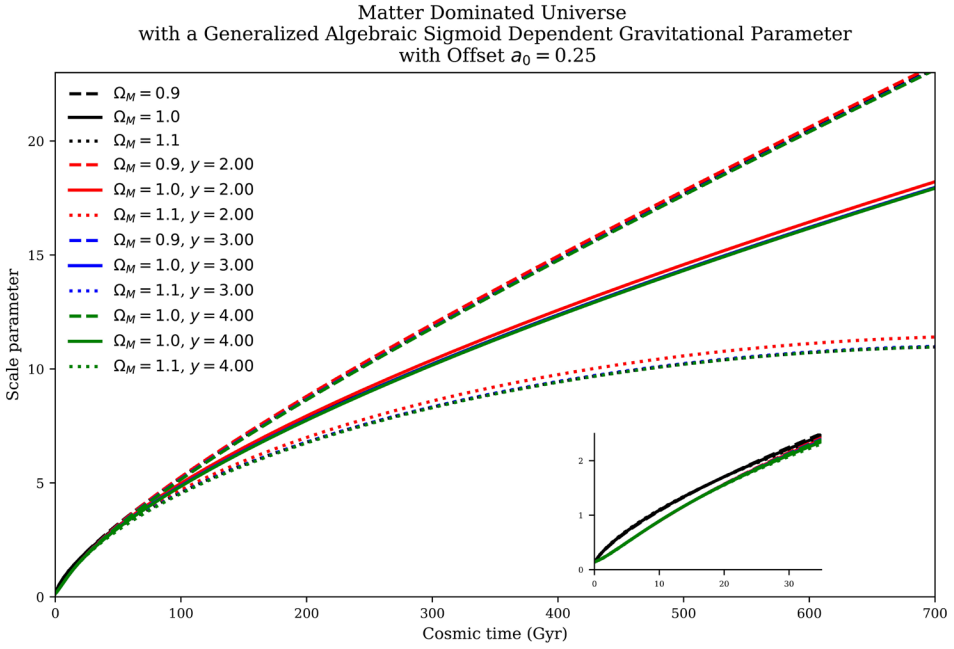

Figure 11. The cosmic scale factor (size) of a matter-only universe as a function of time with a generalized algebraic sigmoid function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

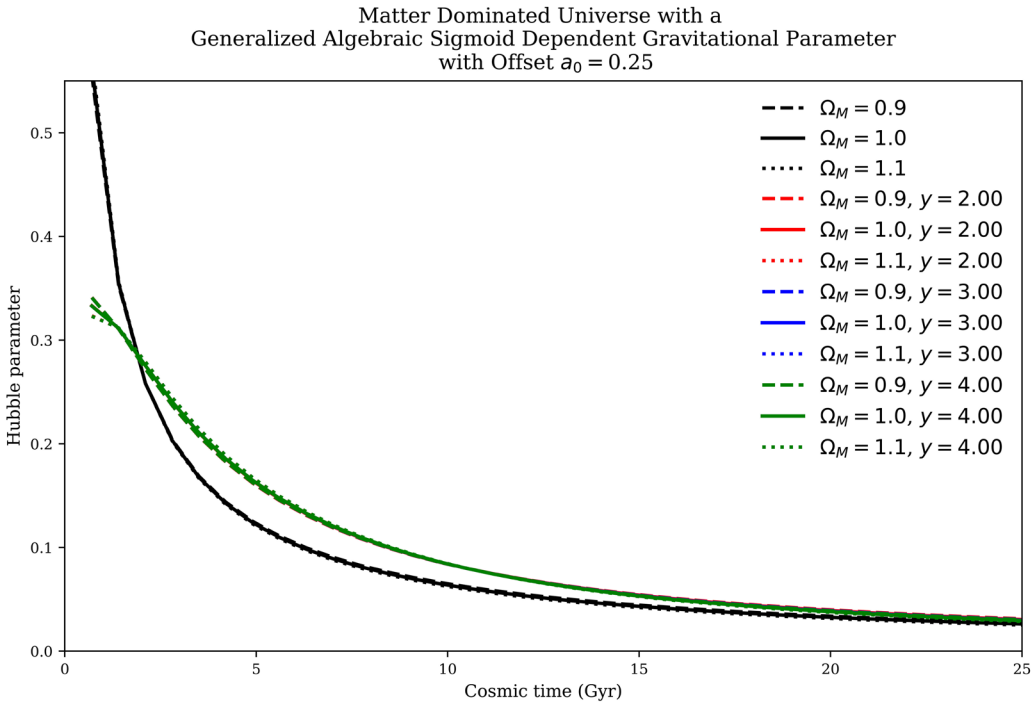

Figure 12. The Hubble parameter (expansion rate) of a matter-only universe as a function of time with a generalized algebraic sigmoid function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

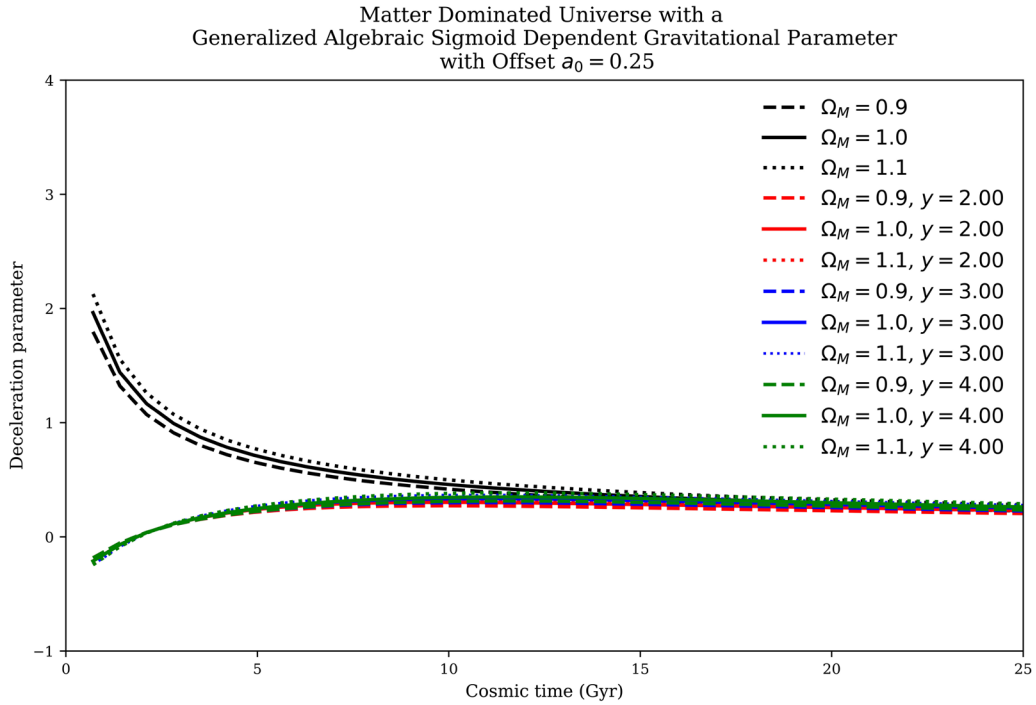

Figure 13. The deceleration parameter (acceleration rate) of a matter-only universe as a function of time with a generalized algebraic sigmoid function dependency for the gravitational parameter. The black data points represent the standard FLRW matter-only cosmology with less than, critical, and greater than average mass densities for comparison.

6. Conclusions and Suggestions

The idea of a phase transition in the early universe is not novel. An example of a phase transition in the early universe is the symmetry breaking of the electroweak interaction [39] [40] [41]. The early universe underwent a symmetry breaking phase transition in the neighborhood of the scale of about 100 GeV when the electroweak interaction was broken via the Higgs mechanism [42] [43] [44] [45] into the weak and electromagnetic interactions. Based upon the experimental success of the unification of the electroweak interaction, higher symmetry-based interactions have been hypothesized though never observed. Cosmologies with a variable gravitational parameter may be useful in describing early universe phenomena such as inflation or late universe phenomena such as the present transition from a matter-dominated to radiation-dominated universe.

Three matter-only cosmological toy models where the gravitational parameter undergoes a type of phase transition based upon the Hubble scale factor were considered. In the three models, the gravitational parameter was considered to begin at a negative, zero, or non-negative value and then approach the currently accepted value of the Newton constant. No speculation on behalf of the authors suggests the mechanism for the phase transition and all the models suffer from undesired free parameters. The Friedmann equation was solved numerically for a variety of parameters associated with each toy model. The cosmological kinematic Hubble and deceleration parameters were numerically calculated.

It is well known that a variation in Newton's gravitational parameter would have a multitude of consequences. Considering the impressive successes of classical cosmology, the authors turn their attention to future tests based on the early universe. Familiar applications to events from the early universe include cosmic recombination in the early universe after the photons that constitute the Cosmic Microwave Background today decoupled from the universe [10] [46] [47] [48] and the abundances of light elements from big bang nucleosynthesis [10] [49].

There are a number of future directions that can be explored based upon this work. A non-exhaustive list is:

- The inclusion of dark energy, radiation, and combinations of various multi-component universes that are more realistic representations of various different epochs of the observed universe.
- The inclusion of derivative terms associated with the gravitational parameter in the derivation of Equation (11) that were omitted in the approximation (see [50] and the references therein).
- The cavalier application of Equation (12) should be investigated more deeply.
- To examine the gravitational parameter in the epoch of the quantum universe.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Tiesinga, E., Mohr, P.J., Newell, D.B. and Taylor, B.N. (2021) CODATA Recommended Values of the Fundamental Physical Constants: 2018. *Reviews of Modern Physics*, **93**, Article ID: 025010. https://doi.org/10.1103/RevModPhys.93.025010 https://link.aps.org/doi/10.1103/RevModPhys.93.025010

[2] Weyl, H. (1917) Zur Gravitationstheorie. *Annalen der Physik*, **359**, 117-145. https://doi.org/10.1002/andp.19173591804

[3] Eddington, A.S. (1931) Preliminary Note on the Masses of the Electron, the Proton, and the Universe. *Mathematical Proceedings of the Cambridge Philosophical Society*, **27**, 15-19. https://doi.org/10.1017/S0305004100009269

[4] Dirac, P.A.M. (1937) The Cosmological Constants. *Nature*, **139**, 323. https://doi.org/10.1038/139323a0

[5] Dirac, P.A.M. (1938) A New Basis for Cosmology. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, **165**, 199-208. https://doi.org/10.1098/rspa.1938.0053

[6] Dirac, P.A.M. (1974) Cosmological Models and the Large Numbers Hypothesis. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, **338**, 439-446. https://doi.org/10.1098/rspa.1974.0095

[7] Hanimeli, E.T., Lamine, B., Blanchard, A. and Tutusaus, I. (2020) Time-Dependent G in Einstein's Equations as an Alternative to the Cosmological Constant. *Physical Review D*, **101**, Article ID: 063513. https://doi.org/10.1103/PhysRevD.101.063513 https://link.aps.org/doi/10.1103/PhysRevD.101.063513

[8] Dungan, R. and Prosper, H.B. (2011) Varying-G Cosmology with Type Ia Supernovae. *American Journal of Physics*, **79**, 57-62. https://doi.org/10.1119/1.3486585

[9] Sultana, J. (2015) Comment on "Varying-G Cosmology with Type Ia Supernovae" [Am. J. Phys. 79, 57-62 (2011)]. *American Journal of Physics*, **83**, 570-572. https://doi.org/10.1119/1.4907266

[10] Uzan, J.P. (2011) Varying Constants, Gravitation and Cosmology. *Living Reviews in Relativity*, **14**, Article No. 2. https://doi.org/10.12942/lrr-2011-2

[11] Goetz, A. and Secrest, J.A. (2021) Investigation of Arbitrary Large Spatial Dimensions in Anisotropic Bianchi Type-I Cosmologies with Variable Gravitational and Cosmological Parameters. *Reports on Mathematical Physics*, **88**, 73-88. https://doi.org/10.1016/S0034-4877(21)00057-4

[12] Barrow, J.D. (1999) Cosmologies with Varying Light Speed. *Physical Review D*, **59**, Article ID: 043515. https://doi.org/10.1103/PhysRevD.59.043515 https://link.aps.org/doi/10.1103/PhysRevD.59.043515

[13] Brans, C.H. and Dicke, R.H. (1961) Mach's Principle and a Relativistic Theory of Gravitation. *Physical Review*, **124**, 925-935. https://doi.org/10.1103/PhysRev.124.925 https://link.aps.org/doi/10.1103/PhysRev.124.925

[14] Garca-Bellido, J., Linde, A. and Linde, D. (1994) Fluctuations of the Gravitational Constant in the Inflationary Brans-Dicke Cosmology. *Physical Review D*, **50** 730-750. https://doi.org/10.1103/PhysRevD.50.730

https://link.aps.org/doi/10.1103/PhysRevD.50.730

[15] Occhionero, F. and Vagnetti, F. (1975) Variation of the Gravitational Constant in Cosmological Models. *Astronomy and Astrophysics*, **44**, 329-333.

[16] Torres, D.F. (1999) Stellar Footsteps of a Variable G. *Modern Physics Letters A*, **15**, 1007-1014. https://doi.org/10.1142/S0217732399001061

[17] Jamali, S. and Roshan, M. and Amendola, L. (2018) On the Cosmology of Scalar-Tensor-Vector Gravity Theory. *Journal of Cosmology and Astroparticle Physics*, **2018**, 48. https://doi.org/10.1088/1475-7516/2018/01/048

[18] Linde, A.D. (1980) Gauge Theories, Time-Dependence of the Gravitational Constant and Antigravity in the Early Universe. *Physics Letters B*, **93**, 394-396. https://doi.org/10.1016/0370-2693(80)90350-0 https://www.sciencedirect.com/science/article/pii/0370269380903500

[19] Raychaudhuri, A.K. and Bagchi, B. (1983) Temperature Dependent Gravitational Constant and Black Hole Physics. *Physics Letters B*, **124**, 168-170. https://doi.org/10.1016/0370-2693(83)91428-4 https://www.sciencedirect.com/science/article/pii/0370269383914284

[20] Gilles, G.T. (1997) The Newtonian Gravitational Constant: Recent Measurements and Related Studies. *Reports on Progress in Physics*, **60**, 151-225. https://doi.org/10.1088/0034-4885/60/2/001

[21] Shaw, P.E. and Davy, N. (1923) The Effect of Temperature on Gravitative Attraction. *Physical Review*, **21**, 680-691. https://doi.org/10.1103/PhysRev.21.680 https://link.aps.org/doi/10.1103/PhysRev.21.680

[22] Leszczyńska, K. (2017) Quantum Cosmology and Varying Physical Constants. *Universe*, **3**, Article No. 46. https://doi.org/10.3390/universe3020046

[23] Pilot, C. (2019) The Age of the Universe Predicted by a Time-Varying G Model? *Journal of High Energy Physics, Gravitation and Cosmology*, **5**, 928-934. https://doi.org/10.4236/jhepgc.2019.53048

[24] Pilot, C. (2019) Is Quintessence an Indication of a Time-Varying Gravitational Constant? *Journal of High Energy Physics, Gravitation and Cosmology*, **5**, 41-81. https://doi.org/10.4236/jhepgc.2019.51003

[25] Christodoulou, D.M. and Kazanas, D. (2019) Universal Expansion with Spatially Varying G. *Monthly Notices of the Royal Astronomical Society: Letters*, **487**, L53-L57. https://doi.org/10.1093/mnrasl/slz074

[26] Laake, J.L. and Lowry, M.S. and DeLong, R.L. and Melin, S.R. and Carretta, J.V. (2019) Population Growth and Status of California Sea Lions. *The Journal of Wildlife Management*, **82**, 583-595. https://doi.org/10.1002/jwmg.21405 https://wildlife.onlinelibrary.wiley.com/doi/abs/10.1002/jwmg.21405

[27] Tsoularis, A. and Wallace, J. (2002) Analysis of Logistic Growth Models. *Mathematical Biosciences*, **179**, 21-55. https://doi.org/10.1016/S0025-5564(02)00096-2 https://www.sciencedirect.com/science/article/pii/S0025556402000962

[28] Bose, T. and Trimper, S. (2009) Stochastic Model for Tumor Growth with Immunization. *Physical Review E*, **79**, Article ID: 051903. https://doi.org/10.1103/PhysRevE.79.051903 https://link.aps.org/doi/10.1103/PhysRevE.79.051903

[29] Dong, G., Lai, K.K. and Yen, J. (2010) Credit Scorecard Based on Logistic Regression with Random Coefficients. *Procedia Computer Science*, **1**, 2463-2468. https://doi.org/10.1016/j.procs.2010.04.278 https://www.sciencedirect.com/science/article/pii/S1877050910002796

[30] Guenault, A.M. (2007) Statistical Physics. 2nd Edition, Springer, Dordrecht.

[31] Ayuso, Ismael and Mimoso, Jos?? P. and Nunes, Nelson J. (2019) What If Newton's Gravitational Constant Was Negative? *Galaxies*, **7**, Article No. 38. https://doi.org/10.3390/galaxies7010038 https://www.mdpi.com/2075-4434/7/1/38

[32] Menon, A., Mehrotra, K., Mohan, C.K. and Ranka, S. (2010) Characterization of a Class of Sigmoid Functions with Applications to Neural Networks. *Neural Networks*, **5**, 819-835. https://doi.org/10.1016/0893-6080(95)00107-7 https://www.sciencedirect.com/science/article/pii/0893608095001077

[33] Dunning, A.J.., Kensler, J., Coudeville, L. and Bailleux, F. (2015) Some Extensions in Continuous Models for Immunological Correlates of Protection. *BMC Medical Research Methodology*, **15**, Article No. 107. https://doi.org/10.1186/s12874-015-0096-9

[34] Vieira, H.S. and Bezerra, V.B. (2014) Lagrangian Formulation of Newtonian Cosmology. *Revista Brasileira de Ensino de Física*, **36**, Article No. 3310. https://doi.org/10.1590/S1806-11172014000300010

[35] Jeffreys, H. and Jeffreys, B. (1999) Methods of Mathematical Physics. Cambridge University Press, Cambridge. https://doi.org/10.1017/CBO9781139168489

[36] Virtanen, P., Gommers, R., Oliphant, T.E., Haberland, M., Reddy, T., Cournapeau, D., Burovski, E., Peterson, P., Weckesser, W., Bright, J., van der Walt, S.J., Brett, M., Wilson, J., Millman, K.J., Mayorov, N., Nelson, A.R.J., Jones, E., Kern, R., Larson, E., Carey, C.J., Polat, İ. and Feng, Y., Moore, E.W., VanderPlas, J., Laxalde, D., Perktold, J., Cimrman, R., Henriksen, I., Quintero, E.A., Harris, C.R., Archibald, A.M., Ribeiro, A.H., Pedregosa, F. and van Mulbregt, P. and SciPy 1.0 Contributors (2020) SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. *Nature Methods*, **17**, 261-272. https://doi.org/10.1038/s41592-019-0686-2

[37] van Rossum, G. (1995) Technical Report CS-R9526. Centrum voor Wiskunde en Informatica (CWI), Amsterdam .

[38] Weinberg, S. (1972) Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. Wiley, New York.

[39] Glashow, G.L. (1961) Partial-Symmetries of Weak Interactions. *Nuclear Physics*, **22**, 579-588. https://doi.org/10.1016/0029-5582(61)90469-2 https://www.sciencedirect.com/science/article/pii/0029558261904692

[40] Weinberg, S. (1967) A Model of Leptons, *Physical Review Letters*, **19**, 1264-1266. https://doi.org/10.1103/PhysRevLett.19.1264 https://link.aps.org/doi/10.1103/PhysRevLett.19.1264

[41] Salam, A. (1980) Gauge Unification of Fundamental Forces. *Reviews of Modern Physics*, **52**, 525-538. https://doi.org/10.1103/RevModPhys.52.525

[42] Englert, F., and Brout, R. (1964) Broken Symmetry and the Mass of Gauge Vector Mesons. *Physical Review Letters*, **13**, 321-323. https://doi.org/10.1103/PhysRevLett.13.321

[43] Higgs, P. (1964) Broken Symmetries, Massless Particles and Gauge Fields. *Physics Letters*, **12**, 132-133. https://doi.org/10.1016/0031-9163(64)91136-9

[44] Higgs, P. (1964) Broken Symmetries and the Masses of Gauge Bosons. *Physical Review Letters*, **13**, 508-509. https://doi.org/10.1103/PhysRevLett.13.508

[45] Guralnik, G.S., Hagen, C.R. and Kibble, T.W.B. (1964) Global Conservation Laws and Massless Particles. *Physical Review Letters*, **13**, 585-587. https://doi.org/10.1103/PhysRevLett.13.585

[46] Peebles, P.J.E. (1968) Recombination of the Primeval Plasma. *The Astrophysical Journal*, **153**, 1-11. https://doi.org/10.1086/149628

[47] Datta, K.K., Kundu, A., Paul, A. and Bera, A. (2020) Cosmic Recombination History in Light of EDGES Measurements of the Cosmic Dawn 21-cm Signal. *Physical Review D*, **102**, Article ID: 083502. https://doi.org/10.1103/PhysRevD.102.083502 https://link.aps.org/doi/10.1103/PhysRevD.102.083502

[48] Moss, A.G. and Narimani, A. and Scott, D. (2010) Let's Talk about Varying *G*. *International Journal of Modern Physics D*, **19**, 2289-2294. https://doi.org/10.1142/S0218271810018396

[49] Alvey, J. and Sabti, N. and Escudero, M. and Fairbairn, M. (2020) Improved BBN Constraints on the Variation of the Gravitational Constant. *European Physical Journal C*, **80**, Article No. 148. https://doi.org/10.1140/epjc/s10052-020-7727-y

[50] Hanımeli, E.T. and Tutusaus, I. and Lamine, B. and Blanchard, A. (2020) Low-Redshift Tests of Newtonian Cosmologies with a Time-Varying Gravitational Constant. *Monthly Notices of the Royal Astronomical Society*, **497**, 4407-4415. https://doi.org/10.1093/mnras/staa2310