Scientific Research Publishing

Journal of High Energy Physics, Gravitation and Cosmology, 2022, 8, 265-268
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

Quantum Field Theory Deserves Extra Help

John R. Klauder

Department of Physics and Department of Mathematics, University of Florida, Gainesville, FL, USA
Email: klauder@ufl.edu

How to cite this paper: Klauder, J.R. (2022) Quantum Field Theory Deserves Extra Help. *Journal of High Energy Physics, Gravitation and Cosmology*, **8**, 265-268.
https://doi.org/10.4236/jhepgc.2022.82021

Received: August 25, 2021
Accepted: March 12, 2022
Published: March 15, 2022

Abstract

Today's quantum field theory (QFT) relies heavenly on canonical quantization (CQ), which fails for φ_4^4 leading only to a "free" result. Affine quantization (AQ), an alternative quantization procedure, leads to a "non-free" result for the same model. Perhaps adding AQ to CQ can improve the quantization of a wide class of problems in QFT.

Keywords

Quantum Field Theory, Canonical Quantization (CQ), Affine Quantization (AQ)

1. What is AQ?

The simplest way to understand AQ is to derive it from CQ. The classical variables, p & q, lead to self-adjoint quantum operators, P & Q, that cover the real line, *i.e.*, $-\infty < P \,\&\, Q < \infty$, and obey $[Q,P] \equiv QP - PQ = i\hbar\mathbb{1}$. Next we introduce several versions of $Q[Q,P] = i\hbar Q$, specifically

$$\begin{aligned}\{Q[Q,P]+[Q,P]Q\}/2 &= \{Q^2P - QPQ + QPQ - PQ^2\}/2 \\ &= \{Q(QP+PQ)-(QP+PQ)Q\}/2 = [Q, QP+PQ]/2.\end{aligned} \tag{1}$$

This equation serves to introduce the "dilation" operator $D \equiv (QP+PQ)/2$ [1] which leads to $[Q,D] = i\hbar Q$. While $P(=P^\dagger)\,\&\,Q(=Q^\dagger)$ are the foundation of CQ, $D(=D^\dagger)\,\&\,Q(=Q^\dagger)$ are the foundation of AQ. Another way to examine this story is to let $p,q \to P,Q$, while $d \equiv pq, q \to D,Q$.

Observe, for CQ, that while q & Q range over the whole real line, that is not possible for AQ. If $q \neq 0$ then d covers the real line, but if $q = 0$ then $d = 0$ and p is helpless. To eliminate this possibility we require $q \neq 0 \,\&\, Q \neq 0$. While

[1] Even if Q does not cover the whole real line, which means that $P^\dagger \neq P$, yet $P^\dagger Q = PQ$. This leads to $D = (QP + P^\dagger Q)/2 = D^\dagger$.

this may seem to be a problem, it can be very useful to limit such variables, like $0 < q \,\&\, Q < \infty$, or $-\infty < q \,\&\, Q < 0$, or even both.[2]

2. A Look at Quantum Field Theory

2.1. Selected Poor and Good Results

Classical field theory normally deals with a field $\varphi(x)$ and a momentum $\pi(x)$, where x denotes a spatial point in an underlying space.[3]

A common model for the Hamiltonian is given by

$$H(\pi,\varphi) = \int \left\{ \frac{1}{2}\left[\pi(x)^2 + \left(\vec{\nabla}(x)\right)^2 + m^2 \varphi(x)^2 \right] + g\varphi(x)^r \right\} \mathrm{d}^s x, \tag{2}$$

where $r \geq 2$ is the power of the interaction term, $s \geq 2$ is the dimension of the spatial field, and $n = s+1$, which adds the time dimension. Using CQ, such a model is nonrenormalizable when $r > 2n/(n-2)$, which leads to "free" model results [2]. Such results are similar for $r = 4$ and $n = 4$, which is a case where $r = 2n/(n-2)$ [3] [4] [5]. When using AQ, the same models lead to "non-free" results [2] [6].

Solubility of classical models involves only a single path, while quantization involves a vast number of paths, a fact well illustrated by path-integral quantization. The set of acceptable paths can shrink significantly when a nonrenormalizable term is introduced. Divergent paths of integration are like those for which $\varphi(x,t) = 1/z(x,t)$ when $z(x,t) = 0$. A procedure that forbids possibly divergent paths would eliminate nonrenormalizable behavior. As we note below, AQ provides such a procedure.

2.2. The Classical and Quantum Affine Story

Classical affine field variables are $\kappa(x) \equiv \pi(x)\varphi(x)$ and $\varphi(x) \neq 0$. The quantum versions are $\hat{\kappa}(x) \equiv \left[\hat{\varphi}(x)\hat{\pi}(x) + \hat{\pi}(x)\hat{\varphi}(x)\right]/2$ and $\hat{\varphi}(x) \neq 0$, with $\left[\hat{\varphi}(x), \hat{\kappa}(y)\right] = i\hbar\delta^s(x-y)\hat{\varphi}(x)$. The affine quantum version of (2) becomes

$$\mathcal{H}(\hat{\kappa},\hat{\varphi}) = \int \left\{ \frac{1}{2}\left[\hat{\kappa}(x)\hat{\varphi}(x)^{-2}\hat{\kappa}(x) + \left(\vec{\nabla}\hat{\varphi}(x)\right)^2 + m^2\hat{\varphi}(x)^2 \right] + g\hat{\varphi}(x)^r \right\} \mathrm{d}^s x. \tag{3}$$

The spacial differential term restricts $\hat{\varphi}(x)$ to continuous operator functions, maintaining $\hat{\varphi}(x) \neq 0$. In that case, it follows that $0 < \hat{\varphi}(x)^{-2} < \infty$ which implies that $0 < |\hat{\varphi}(x)|^r < \infty$ for all $r < \infty$, a most remarkable feature because it forbids nonrenormalizability![4]

Adopting a Schrödinger representation, where $\hat{\varphi}(x) \to \varphi(x)$, simplifies $\hat{\kappa}(x)\varphi(x)^{-1/2} = 0$, which also implies that $\hat{\kappa}(x)\Pi_y \varphi(y)^{-1/2} = 0$. This relation

[2]For example, affine quantization of gravity can restrict operator metrics to positivity, *i.e.*, $\hat{g}_{ab}(x)dx^a dx^b > 0$, straight away [1].

[3]In order to avoid problems with spacial infinity we restrict our space to the surface of a large, $(s+1)$-dimensional sphere.

[4]For Monte Carlo studies, concern for the term $\hat{\varphi}(x)^{-2} \neq 0$ has been resolved by successful usage of $\left[\hat{\varphi}(x)^2 + \varepsilon\right]^{-1}$, where $\varepsilon = 10^{-10}$ [2] [6].

suggests that a general wave function is like $\Psi(\varphi)=W(\varphi)\Pi_y\varphi(y)^{-1/2}$, as if $\Pi_y\varphi(y)^{-1/2}$ acts as the representation of a family of similar wave functions.

We now take a Fourier transformation of the absolute square of a regularized wave function that looks like[5]

$$F(f)=\Pi_{\mathbf{k}}\int\left\{e^{if_{\mathbf{k}}\varphi_{\mathbf{k}}}\left|w(\varphi_{\mathbf{k}})\right|^2\left(ba^s\right)\left|\varphi_{\mathbf{k}}\right|^{-\left(1-2ba^s\right)}\mathrm{d}\varphi_{\mathbf{k}}\right\}. \tag{4}$$

Normalization ensures that if all $f_{\mathbf{k}}=0$, then $F(0)=1$, which leads to

$$F(f)=\Pi_{\mathbf{k}}\int\left\{1-\int\left(1-e^{if_{\mathbf{k}}\varphi_{\mathbf{k}}}\right)\left|w(\varphi_{\mathbf{k}})\right|^2\left(ba^s\right)\mathrm{d}\varphi_{\mathbf{k}}\Big/\left|\varphi_{\mathbf{k}}\right|^{\left(1-2ba^s\right)}\right\}. \tag{5}$$

Finally, we let $a\to 0$ to secure a complete Fourier transformation[6]

$$F(f)=\exp\left\{-b\int \mathrm{d}^s x\left(1-e^{if(x)\varphi(x)}\right)\left|w(\varphi(x))\right|^2\mathrm{d}\varphi(x)/\left|\varphi(x)\right|\right\}. \tag{6}$$

This particular process side-steps any divergences that may normally arise in $\left|w(\varphi(x))\right|$ when using more traditional procedures.

3. The Absence of Nonrenormalizablity, and the Next Fourier Transformation

Observe the factor $\left|\varphi_{\mathbf{k}}\right|^{-\left(1-2ba^s\right)}$ in (4) which is prepared to insert a zero divergence for each and every $\varphi_{\mathbf{k}}$ when $a\to 0$. However, the factor ba^s in (4) turns that possibility into a very different story given in (6).

Another Fourier transformation can take us back to a suitable function of the field, $\varphi(x)$. That task involves pure mathematics, and it deserves a separate analysis of its own.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Klauder, J. (2020) Using Affine Quantization to Analyze Non-Renormalizable Scalar Fields and the Quantization of Einsteins Gravity. *Journal of High Energy Physics, Gravitation and Cosmology*, **6**, 802-816. https://doi.org/10.4236/jhepgc.2020.64053

[2] Fantoni, R. (2021) Monte Carlo Evaluation of the Continuum Limit of φ_3^{12}. *Journal of Statistical Mechanics*, **2021**, Article ID: 083102. https://doi.org/10.1088/1742-5468/ac0f69

[3] Freedman, B., Smolensky, P. and Weingarten, D. (1982) Monte Carlo Evaluation of the Continuum Limit of φ_4^4 and φ_3^4. *Physics Letters B*, **113**, 481. https://doi.org/10.1016/0370-2693(82)90790-0

[4] Aizenman, M. (1981) Proof of the Triviality of φ_d^4 Field Theory and Some Mean-Field Features of Ising Models for $d>4$. *Physical Review Letters*, **47**, 1-4.

[5]The remainder of this article updates and improves a recent article by the author [7].

[6]Any change of $w(\varphi)$ due to $a\to 0$ is left implicit.

https://doi.org/10.1103/PhysRevLett.47.1

[5] Fröhlich, J. (1982) On the Triviality of $\lambda\varphi_d^4$ Theories and the Approach to the Critical Point in $d \geq 4$ Dimensions. *Nuclear Physics B*, **200**, 281-296. https://doi.org/10.1016/0550-3213(82)90088-8

[6] Fantoni, R. and Klauder, J. (2021) Affine Quantization of φ_4^4 Succeeds While Canonical Quantization Fails. *Physical Review D*, **103**, Article ID: 076013. https://doi.org/10.1103/PhysRevD.103.076013

[7] Klauder, J. (2021) Evidence for Expanding Quantum Field Theory. *Journal of High Energy Physics, Gravitation and Cosmology*, **7**, 1157-1160. https://doi.org/10.4236/jhepgc.2021.73067