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Looking at Quantization Conditions, for a Wormhole Wavefunction, While Considering Differences between Magnetic Black Holes, Versus Standard Black Holes as Generating Signals from a Wormhole Mouth

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Abstract

We utilize how Weber in 1961 initiated the process of quantization of early universe fields to the problem of what may be emitted at the mouth of a wormhole. While the wormhole models are well developed, there is as of yet no consensus as to how, say GW or other signals from a wormhole mouth could be quantized or made to be in adherence to a procedure Weber cribbed from Feynman, in 1961. In addition, we utilize an approximation for the Hubble parameter parameterized from Temperature using Sarkar's H ~ Temperature relations, as given in the text. We review what could be a game changer, *i.e.* magnetic black holes as brought up by Maldacena, in early 2021, at the mouth of the wormhole, and compare this with more standard black holes, at the mouth of a wormhole, while considering also the Bierman battery effect of an accreditation disk moving charges around a black hole as yet another way to have signals generated. The Maldacena article has good order of estimate approximations as to the strength of a magnetic monopole which we can use, and we also will go back to the signal processing effects which may be engendered by the Weber quantization of a wormhole to complete our model.

Keywords

Minimum Scale Factor, Cosmological Constant, Space-Time Bubble, Bouncing Cosmologies

1. Introduction

The template of what we will be looking at will be a wormhole, using a wave-function quantization procedure, as given in [1] which may also be enhanced by using the suggestion by [2] as to a magnetic wormhole to generate fields for our perusal and signal generation edification. Keep in mind, that [2] concludes as to the following "If Q is the integer magnetic charge, the fermions lead to order Q massless two-dimensional fermions moving along the magnetic field lines. These greatly enhance Hawking radiation effects". Greatly enhanced Hawking radiation combined with Weber quantization of a wave functional may after certain tweaks allow for observable macroscopically detected quantum gravity effects. In doing so we also will be considering what if a wormhole also has black holes generating a magnetic field according to the Biermann battery effect, where moving charges in an accreditation disk outside the black hole generate a given magnetic field [3]. This also can be compared with what will happen if we have higher dimensional black holes, not necessarily magnetic which can be affected by two different generalized uncertainty principles, [4] [5] whereas the higher dimensions of black hole [6] in the mouth of the wormhole may also give verifiable quantum effects without the need of a magnetic field generating black hole in the mouth of a wormhole [7] and also considering [8] and [9] issues, as in [9] we have a way to make a temperature dependent estimation of effects, and we will be also examining conditions in which a BEC (Bose Einstein condensate) approximation of black holes is as condensate of gravitons in order to estimate in part optimal GW and graviton production from black holes in the mouth of the wormholes. Since Gravitons are quantum mechanical in origin, this will tie into Quantum Gravity in a very natural way [10]. And as a bonus in the conclusion, as far as a black hole in the mouth of a wormhole picture, we will make use of the idea of comparing what we get as a signal from a wormhole mouth, with at least one black hole present to the issue brought up in [11] of what happens if thermal quanta are mined from the so called "atmosphere" of a black hole as seen in page 340, Equation (8.119).

2. That Business of the Weber Technique, Summarized

We bring up this study first a result given by Weber, in 1961 [1] as to getting an initial wavefunction given in [12], which may be able to model behavior of what happens in the mouth of a wormhole if we assume as given in [13] that H (Hubble's parameter) is proportional to Temperature, and then go to Energy ~ Temperature. The last part will be enough to isolate, up to first principles a net frequency value.

The behavior of frequency, versus certain conditions at the mouth of a wormhole may give us clues to be investigated later as to polarization states relevant to the wormhole [14] as well as examining what may be relevant to measurement of signals from a wormhole [15].

In doing all of this, the idea is that we are evolving from the Einstein-Rosen bridge [16] to a more complete picture of GR which may entail a new representation of the Visser "Chronology protection" paper as in [17].

What we are seeing is a version of convolution, which may allow for quantization.

3. Looking at the Weber Book as to Reformulate Quantization Imposed Alteration of the Wave Function

Using [1] a statement as to quantization for a would-be GR term comes straight from

$$\Psi_{\text{Later}} = \int \sum_{H} \mathrm{e}^{(iI_H/\hbar)(t,t^0)} \Psi_{\text{Earlier}}\left(t^0\right) \mathrm{d}t^0 \quad (1)$$

The approximation we are making is to pick one index, to have

$$\Psi_{\text{Later}} = \int \sum_{H} \mathrm{e}^{(iI_H/\hbar)(t,t^0)} \Psi_{\text{Earlier}}\left(t^0\right) \mathrm{d}t^0 \xrightarrow[H \to 1]{} \int \mathrm{e}^{(iI_{H_{\text{FIXED}}}/\hbar)(t,t^0)} \Psi_{\text{Earlier}}\left(t^0\right) \mathrm{d}t^0 \quad (2)$$

This corresponds to say being primarily concerned as to GW generation, which is what we will be examining in our ideas, via using

$$\mathrm{e}^{(iI_{H_{\text{FIXED}}}/\hbar)(t,t^0)} = \exp\left[\frac{i}{\hbar} \cdot \frac{c^4}{16\pi G} \cdot \int_M \mathrm{d}t \cdot \mathrm{d}^3 r \sqrt{-g} \cdot (\Re - 2\Lambda)\right] \quad (3)$$

We will use the following, namely, if Λ is a constant, do the following for the Ricci scalar [18] [19]

$$\Re = \frac{2}{r^2} \quad (4)$$

If so then we can write the following, namely: Equation (3) becomes, if we have an invariant Cosmological constant, so we write $\Lambda \xrightarrow[\text{all time}]{} \Lambda_0$ everywhere, then

$$\mathrm{e}^{(iI_{H_{\text{FIXED}}}/\hbar)(t,t^0)} = \exp\left[\frac{i}{\hbar} \cdot \frac{c^4 \cdot \pi \cdot t^0}{16 G} \cdot \left(r - r^3 \Lambda_0\right)\right] \quad (5)$$

Then, we have that Equation (1) is re written to be

$$\begin{aligned} \Psi_{\text{Later}} &= \int \sum_{H} \mathrm{e}^{(iI_H/\hbar)(t,t^0)} \Psi_{\text{Earlier}}\left(t^0\right) \mathrm{d}t^0 \\ &\xrightarrow[\text{at wormhole}]{} \int \exp\left[\frac{i}{\hbar} \cdot \frac{c^4 \cdot \pi \cdot t^0}{16 G} \cdot \left(r - r^3 \Lambda_0\right)\right] \Psi_{\text{Earlier}}\left(t^0\right) \mathrm{d}t^0 \end{aligned} \quad (6)$$

4. Examining the Behavior of the Earlier Wavefunction in Equation (6)

[18] states a Hartle-Hawking wavefunction which we will adapt for the earlier wavefunction as stated in Equation (6) to read as follows

$$\Psi_{\text{Earlier}}\left(t^0\right) \approx \Psi_{HH} \propto \exp\left(\frac{-\pi}{2GH^2} \cdot \left(1 - \sinh(Ht)\right)^{3/2}\right) \quad (7)$$

Here, making use of Sarkar [13], we set, if say g_* is the degree of freedom allowed [19]

$$H = 1.66\sqrt{g_*}T_{\text{temp}}^2 \big/ M_{\text{Planck}} \tag{8}$$

We assume initially a relatively uniformly given temperature, that H is constant.

So then we will be attempting to write out an expansion as to what the Equation (6) gives us while we use Equation (7) and Equation (8), with H approximately constant. If so then.

5. Method Used in Calculating Equation (6), with Interpretation of the Results

We will be considering how, to express Equation (6). And in doing this we will be looking at having a constant value for Equation (8). If so then using numerical integration, [20] [21] [22] on page 751 of this [22] citation

$$\begin{aligned} &\Psi_{\text{Later}} \xrightarrow[t_M \to \varepsilon^+]{} \int_0^{t_M} \mathrm{e}^{i\cdot(\tilde{\alpha}1)\cdot t - (\tilde{\alpha}2)\cdot(1-\sinh(Ht))^{3/2}} \mathrm{d}t \\ &\approx \frac{t_M}{2}\cdot\left(\mathrm{e}^{i\cdot(\tilde{\alpha}1)\cdot t_M - (\tilde{\alpha}2)\cdot(1-\sinh(H\cdot t_M))^{3/2}} - 1\right) \\ &\tilde{\alpha}1 = \left[\frac{c^4\cdot\pi}{16G\hbar}\cdot\left(r - r^3\Lambda_0\right)\right],\ \tilde{\alpha}2 = \frac{\pi}{2GH^2} \end{aligned} \tag{9}$$

6. Using This Wavefunction in the Face of Choices for What Sort of Black Hole May Be in the Wormhole Mouth. Case 1, the Magnetic Monopole Based Black Hole as Given by Maldacena

First, we should consider what to do if there is a Magnetic Black hole at the mouth of the wormhole. Then what if there is a Bierman battery generated B field. Then the case of when there is a non-B field generating black hole, which may (or may not) have higher dimensions. The first case to consider is what to do if there is a magnetic "monopole" based black hole generating magnetic field, using [2].

In [2] the supposition is that the following will be used for a magnetic charge, as given by

$$e\cdot|B| = \frac{e^2}{2\pi l_P^2 Q} \tag{10}$$

Here, we have that the charge, Q as so stated by [2] will lead to an energy, E

$$E(\text{black hole}) \approx m_H Q^{3/2} = m_H\cdot\left(\frac{e^2}{2\pi l_P^2\cdot e\cdot|B|}\right)^{3/2} \tag{11}$$

The implication, rides as to the m_H value picked which will be as the mass of the Higgs **boson** to be 125.35 GeV [23], whereas we can make use of a simple uncertainty principle to obtain a first order time contribution to the wave function Equation (9) above [24].

$$\Delta t \approx \frac{\hbar}{m_H} \cdot \left(2\pi l_P^2 \cdot e^{-1} \cdot |B|\right)^{3/2} \tag{12}$$

This value for time will be placed in Equation (9) above, whereas the time for initial formation of the uncertainty principle for the GUP as in [4] and [5] for refinement will be given in the concluding statements of this document, but it is interesting to note that the strength of a magnetic field, will determine the initial times step as in Equation (9). This magnetic field strength will also be commensurate with the issue of what may be expected in Graviton production due to a "flux" of black holes through/about the wormhole mouth. Note that the *B* field is not specified here, explicitly but is assumed to be a measurable conundrum to be faced by data set analysis. And to first order, according to [3] the *B* field would

$$B = \frac{\aleph \cdot \sqrt{M}}{r^2} \approx \frac{\mu^0 q_m \cdot \sqrt{\dfrac{n_{\text{monopoles}}}{\rho_{\text{PBH}}}} \cdot \sqrt{M}}{4\pi \cdot r^2} \tag{13}$$

Here, $n_{\text{monopoles}}$ is the number of magnetic monopoles associated with a magnetic black hole, while, q_m is a unit of magnetic monopole charge, [25] [26], and *M* is the mass of the black hole, and ρ_{PBH} is the relevant density of black holes in a wormhole throat area. In addition, *M* is likely in this configuration to be of the order of 1 to a few Planck masses.

7. What If We Have a Biermann Battery *B* Field Generation and We Are Looking at the Time Interval as Compared to Equation (12) for Equation (9) Wavefunction?

In the Biermann battery, the mere act of electric charges in an accreditation disk about a black hole will create magnetic fields. In the case of a magnetic monopole, the *B* field as for Equation (12) above at least for a short period of time, before decay of the magnetic black hole would be "approximately" constant. This in line with a charge, *Q*, not decaying rapidly as to what is seen in [27] whereas one has the following situation, *i.e.*

Quote

Here we show that magnetic fields can be generated in initially unmagnetized accretion disks around PBHs through the Biermann battery mechanism, and therefore provide the small-scale seeds of magnetic field in the universe. The radial temperature and vertical density profiles of these disks provide the necessary conditions for the battery to operate naturally. The generated seed fields have a toroidal structure with opposite sign in the upper and lower half of the disk. In the case of a thin accretion disk around a rotating PBH, the field generation rate increases with increasing PBH spin. At a fixed *r*/risco, where *r* is the radial distance from the PBH and risco is the radius of the innermost stable circular orbit, the battery scales as *M*–9/4, where *M* is the PBH's mass.

End of quote

The idea here would be in moving electric charges in a dynamically rotating disc. If we ascertain what is relevant here, the Bierman battery would necessitate a movement beyond the innermost regime of the throat of the wormhole, and would necessitate interfacing with the shape function of the wormhole, as seen in [28].

What [27] and [28] imply is that if one is in the restricted wormhole throat region, that the necessary accreditation disc for the Biermann battery would not form, but if the black hole were say a distance, after Δt traveling time past the wormhole throat then perhaps the geometry of a wormhole shape function [28] would permit forming an accreditation disk, and have movement of electric charges, in a manner about a wormhole which would allow for a B field to form. In doing so, the B field for the accreditation disc would likely linearly grow, as of the form given by [29] and what we have is that there would be a linear growth in the magnitude of the magnetic field, as given by

$$B(t) \approx \frac{m_e c}{e} \cdot \frac{v_{the}^2}{L_t L_n} \cdot t \tag{14}$$

With length of gradients (of material in the Biermann disc defined by)

$$\begin{aligned} L_t &= T_e / \nabla T \\ L_n &= n / \nabla n \end{aligned} \tag{15}$$

I.e. the B field would grow linearly, as the black hole exited the Wormhole throat regime, whereas we could have an overall magnitude of the B field as established by

$$\frac{B}{\sqrt{8\pi P_e}} \approx \frac{c}{\sqrt{2} \cdot L_n \cdot \omega_{pe}} \tag{16}$$

where we would set c/ω_{pe} as so-called electron inertial length, and ω_{pe} as an electron plasma frequency

Making use of Equation (14) we could have a net magnetic field strength as looking like

$$E_{p,m} = -\mathbf{m} \cdot \mathbf{B} \tag{17}$$

where the term, $\mathbf{m}$, in Equation (17) is a dipole moment, but we can get what we want via the old standby [30]

$$\rho_{\text{magnetic}} \equiv \frac{1}{2 \cdot \mu} \cdot B^2 \tag{18}$$

whereas we can, up to a point calculate the generated minimum uncertainty of energy and time via

$$\left(\frac{V_{\text{volume}}}{2 \cdot \mu} \cdot B^2 \right) \cdot \Delta t \approx \frac{V_{\text{volume}}}{2 \cdot \mu} \cdot \left(\frac{m_e c}{e} \cdot \frac{v_{the}^2}{L_t L_n} \cdot \Delta t \right)^2 \cdot \Delta t \approx \hbar \tag{19}$$

Then in this situation, unlike what is in Equation (12) and Equation (13) the Biermann battery approximation would yield an initial delta t value for Equation (9) which is not crazy. This would likely necessitate numerical simulation work. And

$$\Delta t \approx \left(\frac{e \cdot L_t L_n}{v_{the}^2 \cdot m_e c} \right)^{2/3} \cdot \left(\frac{2 \cdot \mu}{V_{\text{volume}}} \right)^{1/3} \tag{20}$$

Obviously, the shape of the wormhole function would have to be employed to ascertain a value for $(V_{\text{volume}})^{1/3}$. And we then would compare Equation (20) to Equation (13).

Needless to say, for this situation, the Biermann battery approximation for the magnetic field, for a black hole in a wormhole throat would have a linear link to time and would be growing and would NOT be constant which is tandem to using magnetic dipole approximations, whereas if we have a magnetic monopole, likely up to an initial approximation for the first iteration of Equation (9) the B field would be presentable as a constant. Then spatially it would be decreasing as given in Equation (13). *i.e.*, in the Biermann approximation it is likely that the B field would grow linearly in time, t, whereas it would decrease

8. Examining What to Expect in the Case of a Nonmagnetic Black Hole in a Wormhole Configuration of the Weight of about a Planck Mass

So far what we have done is to configure energy values associated with a black hole in the absence of, say a strong magnetic field.

A black hole weighing 606,000 metric tons (6.06×10^8 kg) would have a Schwarzschild radius of (0.9×10^{-18} m), a power output of 160 petawatts (160×10^{15} W, or 1.6×10^{17} W), and a 3.5-year lifespan. This is without looking at say a magnetic field, Building on this, if we look at a Planck mass sized black hole, At this stage, a black hole would have a Hawking temperature of (5.6×10^{32} K), which means an emitted Hawking particle would have an energy comparable to the mass of the black hole. If so then the time

$$\Delta t \approx \text{Planck time} = t_P = \sqrt{\frac{\hbar G}{c^5}} \approx 5.391247(60) \times 10^{-44}\,\text{s} \tag{21}$$

This would be the unit of time placed into Equation (9) above, *i.e.* assuming we are not looking at magnetic fields, and black holes in the mouth of a wormhole.

Having specified the input of a brief time interval as to black holes through worm holes, let us guess what they should entail in terms of the number of black holes going through the wormhole mouth, First the case of nonmagnetic field black holes and their rate of production and flow through the wormhole mouth, and then the magnetic field Black hole case which is far harder. We begin with the easy case first.

9. A First Order Guess as to the Rate of Production of Planck Sized Black Holes through a Wormhole, without Referring to Magnetic Fields

In order to do this, we will be estimating that the temperature would be of the order of Planck temperature, *i.e.* using ideas from [31] [32] [33] [34]

$$\frac{\omega_p}{T_p} \equiv \frac{\sqrt{G k_B^2}}{\hbar} \xrightarrow[\hbar = G = k_B = 1]{} 1 \tag{22}$$

If so, then there would be to first order the following rate of production

$$\Gamma_{\text{rate of production}} \approx e \approx 2\text{ - }3 \tag{23}$$

Some of the considerations given in this could be related to [32] as an afterthought whereas the author in [33] estimated for an LHC that there would be about 3000 gravitons produced per second. Assuming a figure from [34] as to the percentage of black hole mass decaying into gravitons, *i.e.* [34], and [35] *i.e.*, 1/1000 of the mass of a Planck sized black hole would delve into gravitons, so if one had 3000 gravitons produced per second, as measured on Earth, one would likely have 2 - 3 black holes of mass of about 10^−5 grams per black hole, producing say 10^57 gravitons, produced per black hole of mass about 10^−62 grams per black hole [36] [37]

$$\Gamma \approx \exp\left(\omega_{\text{signal}} / T_{\text{temperature}}\right) \tag{24}$$

whereas we have from [38] a probability for "scalar" particle production from the wormhole given by

$$\Gamma \approx \exp\left(-E / T_{\text{temperature}}\right) \tag{25}$$

We next then examine what we can expect if we have black holes producing magnetic fields, and how that would change Equation (23) from considering Equation (24) as a template. Before doing so, let us review what can be stated as far as signal frequencies, as far as Equation (24) and a counterpart, for magnetic field generating black holes.

10. Examining Signal Frequencies in the Case of a Magnetic Monopole Constituent Black Hole and Its Relevance to Black Hole Flux through a Wormhole "Mouth"

As stated in [2], page 10, the evaporation timescale of a Schwarzschild black hole of a given radii, of a given radius is Q times larger than the evaporation timescale of a charged (magnetically speaking) black hole. See [39] whereas we also can look at the frequency via the following rule, [40] which has on page 20, via Formula (7.4) and quoting [41]

$$|\omega| \approx \frac{n_{\text{quantum}} + \frac{1}{2}}{4GM} \tag{26}$$

whereas if we look at what M is, in the case of magnetic black holes, there is in [2], page 10, Formula (4.1) a mass expression as to collapse to mass extremality for a black hole which we can write as

$$\tilde{M} = M - M_e \propto \exp\left(-t/\tau\right) \tag{27}$$

$$\tau \approx \frac{8\pi^{5/2} Q^2 l_P}{3 \cdot \left(g'^3\right)} \tag{28}$$

If we make the substitution of $M \to \tilde{M}$ in Equation (26) we arrive at

$$|\omega| \approx \left(\frac{n_{\text{quantum}} + \frac{1}{2}}{4G} \right) \cdot \exp\left(t \Big/ \left\{ \tau = \left[\frac{8\pi^{5/2} Q^2 l_P}{3 \cdot \left(g'^3 \right)} \right] \right\} \right) \tag{29}$$

If n_{quantum} is set equal to zero, we have then that

$$|\omega| \approx \frac{\exp\left(t \Big/ \left\{ \tau = \left[\frac{8\pi^{5/2} Q^2 l_P}{3 \cdot \left(g'^3 \right)} \right] \right\} \right)}{8G} \tag{30}$$

The larger t gets, despite the value of Q, the larger the frequency, and we can then compare this Equation (30) with

$$\omega \approx \frac{m_H Q^{3/2}}{\hbar} = \frac{m_H}{\hbar} \cdot \left(\frac{e^2}{2\pi l_P^2 \cdot e \cdot |B|} \right)^{3/2} \tag{31}$$

Then

$$\Gamma \approx \exp\left(\frac{m_H}{\hbar T} \cdot \left(\frac{e^2}{2\pi l_P^2 \cdot e \cdot |B|} \right)^{3/2} \right) \tag{32}$$

This will lead to the production rate of Equation (32) being at least Q of Equation (23) per second.

At the same time, we have that [39] also states that the time of decay decreased by $1/Q$, for the black holes, with the time of decay for a non-Magnetic black hole given by, in extra dimensions, of value D

$$E \sim \frac{D(\dim)^2}{4\pi R_H} \equiv \frac{D(\dim)^2 T_H}{D(\dim) - 3} \approx \frac{\hbar}{\Delta t} \tag{33}$$

Then, roughly, the decay in the case of a magnetic field is to first approximation

$$\Delta t \approx \frac{D(\dim) - 3}{D(\dim)^2 T_H \cdot \hbar} \xrightarrow{\text{Magnetic Black hole}} Q^{-1} \cdot \frac{D(\dim) - 3}{D(\dim)^2 T_H \cdot \hbar} \tag{34}$$

If a Planck sized black hole disappears after delta t seconds, this means that the same Plank sized black hole will disappear in roughly delta t/Q seconds.

We will though when having this more rapid decay, have a situation for which there will be Q times the rate of black hole appearance in the throat of the wormhole as given in Equation (32). This is at least Q times the value of the rate of black hole appearance as given in Equation (23), hence the amount of transfer of the black hole stuff through the wormhole remains roughly invariant.

11. Formal Bounding of the Cosmological Constant, in Terms of Two Wavefunctions Plus Analysis of Initial Wormhole Frequency Values

Doing so, leads to setting

$$r \equiv \hat{B} \cdot r_P \xrightarrow[r_P \to 1]{} \hat{B} \tag{35}$$

If so, then we have the following bounding as far as the value of the cosmological "constant", namely

$$\begin{aligned}
&\Psi_{\text{Later}} \xrightarrow[t_M \to \varepsilon^+]{} \int_0^{t_M} \mathrm{e}^{i\cdot(\tilde{\alpha}1)\cdot t-(\tilde{\alpha}2)\cdot(1-\sinh(Ht))^{3/2}} \mathrm{d}t \\
&\approx \frac{t_M}{2}\cdot\left(\mathrm{e}^{i\cdot(\tilde{\alpha}1)\cdot t_M-(\tilde{\alpha}2)\cdot(1-\sinh(H\cdot t_M))^{3/2}}-1\right) \\
&\Psi_{1,\kappa=n=0} \approx \sqrt{\frac{\omega}{\pi}}\cdot\left[\frac{1}{\omega+i\cdot(t+r)}-\frac{1}{\omega+i\cdot(t-r)}\right] \\
&\tilde{\alpha}1=\left[\frac{c^4\cdot\pi}{16G\hbar}\cdot\left(r-r^3\Lambda_0\right)\right],\ \tilde{\alpha}2=\frac{\pi}{2GH^2}
\end{aligned} \tag{36}$$

We will be looking at comparing the real values of Equation (36) to obtain a bound on the cosmological constant, to get a bound on the Cosmological constant as given by

$$\begin{aligned}
\Lambda_0 &\approx \tilde{B}^{-2}-\frac{16}{\pi}\cdot\tilde{B}^{-2}\cdot\left(\tilde{\alpha}2\cdot\left(1-\sinh\left(H\cdot\tilde{B}\right)\right)^{3/2}\right) \\
&-\frac{16}{\pi}\cdot\tilde{B}^{-2}\cdot\cos^{-1}\left[\frac{2\cdot 8^{3/4}\cdot\pi^{1/4}}{8\pi+\left(1+\tilde{B}\right)^2}-\frac{2\cdot 8^{3/4}\cdot\pi^{1/4}}{8\pi+\left(1-\tilde{B}\right)^2}\right]
\end{aligned} \tag{37}$$

In doing this, considering the Planck units and their normalization, we also need to keep in consideration the frequency, which we will denote here as

$$\omega_{\text{signal}} \approx \frac{k_B\cdot\sqrt{M_{\text{Planck}}H}}{\hbar\sqrt{1.66\sqrt{g_*}}} \xrightarrow[\hbar=\ell_P=G=t_P=k_B=1]{} \frac{\sqrt{H}}{\sqrt{1.66\sqrt{g_*}}} \approx \frac{T_{\text{temperature}}}{2} \tag{38}$$

Whereas what we will be doing, after we obtain a frequency of a signal near the mouth of a wormhole is to use the following scaling of frequency, near Earth Orbit from this wormhole. First if the wormhole is right at the start of the Universe [8], we use

$$\begin{aligned}
&\left(1+z_{\text{initial era}}\right) \equiv \frac{a_{\text{today}}}{a_{\text{initial era}}} \approx \left(\frac{\omega_{\text{Earth orbit}}}{\omega_{\text{initial era}}}\right)^{-1} \\
&\Rightarrow \left(1+z_{\text{initial era}}\right)\omega_{\text{Earth orbit}} \approx 10^{25}\,\omega_{\text{Earth orbit}} \approx \omega_{\text{initial era}}
\end{aligned} \tag{39}$$

If we are say far closer to the Earth, or the Solar system, then we would likely see [8]

$$10\cdot\omega_{\text{Earth orbit signal}} \approx \omega_{\text{wormhole mouth signal}} \tag{40}$$

Our derivation so far is to obtain the initial signal frequency for Equation (39) and Equation (40). Our next task is to obtain some considerations as to the Polarization, of say GW to observe and look for, in conclusion of this document.

12. The Big Picture, Polarization of Signals from a Wormhole Mouth May Affect GW Astronomy Investigations

We have a rate of production from the worm hole mouth we can quantify as

$$\Gamma \approx \exp\left(\omega_{\text{signal}} / T_{\text{temperature}}\right) \tag{41}$$

whereas we have from [17] a probability for "scalar" particle production from the wormhole given by

$$\Gamma \approx \exp\left(-E / T_{\text{temperature}}\right) \tag{42}$$

whereas if we assume that there is a "negative" temperature in Equation (41) and say rewrite Equation (42) as obeying having

$$\left(\omega_{\text{signal}} / T_{\text{temperature}}\right) \approx \left(-E / T_{\text{temperature}}\right) \tag{43}$$

This is specifying a rate of particle production from the wormhole. And so then:

Whereas what we are discussing in Equation (41) and Equation (42) is having a rate of, from a wormhole mouth, presumably from graviton production. If as an example, we are examining the mouth of a wormhole as being equivalent of a linkage between two black holes, or a black hole—white hole pair, we are presuming a release from the mouth of the wormhole commensurate with an eye to "white holes" for a black hole model as of probability for "scalar" particle production given as, if M is the mass of the black(white) hole, m is the mass of an emitted "particle", ω is frequency of emitted particles,

$$\Gamma \propto \exp\left(-8\pi M \cdot \omega \cdot \left[1 + \frac{\beta}{4} \cdot \left(m^2 + 4\omega^2\right)\right]\right) \tag{44}$$

whereas we define the parameter β via a modified energy expression, as in [18] given by $\tilde{E}$ as a modified energy expression in [18] [19]

$$\tilde{E} = E \cdot \left(1 - \beta \cdot \left(p^2 + m^2\right)\right) \tag{45}$$

Our Equations (28), (41) and (42), which are for wormholes, as well as Equation (43) should encompass the same information of Equation (44) which would be consistent with a white hole [20] [21] at the mouth of a worm hole, as would be expected from Equation (44), whereas reviewing a linkage between black holes and white holes as may be for forming a wormhole may give more credence to the information loss criteria as given in [22].

Our next step is to ask if this permits speaking of say GW polarization in the mouth of a worm hole. To do this, first of all, note that in [23] that the simplest version of a worm hole is one of two universes connected by a "throat" of the form of a "ball" given by πb^2, whereas the term *b*, is in a diagram, consigned to be the radius, or shape of the initial "ball" joining two "universes".

In the case of extending b to become the "shape" of the mouth of a wormhole, we would likely be using [24] for what is called by Visser the "shape" function of the wormhole [25], whereas what we are referring to in Equation (46) below comes straight from [23].

$$b(r) = \left[r_0^{\frac{\gamma-1}{\gamma}} + \gamma \cdot \frac{(8\pi G)^{\frac{\gamma-1}{\gamma}}}{\tilde{\tilde{\omega}}^{1/\gamma}} \cdot \left(r^3 - r_0^3\right) \right]^{\frac{\gamma}{\gamma-1}} \xrightarrow[r \to r_0]{} r_0 \tag{46}$$

whereas we need to keep in mind the equation of state for pressure and density of [24]

$$p = \tilde{\tilde{\omega}}(r) \cdot \rho \tag{47}$$

The long and short of it is as follows. Following [24] we have that

$$\rho_{\tilde{\alpha}} = \frac{M}{(4\pi\tilde{\alpha})^{3/2}} \cdot \exp\left(-r^2/4\tilde{\alpha}\right) \tag{48}$$

whereas the b coefficient in the case of noncommutative geometry is chosen [26]

$$\begin{aligned} b(r) &= \frac{2r_s}{\sqrt{\pi}} \cdot \hat{\gamma}\left(\frac{3}{2}, \frac{r^2}{4\tilde{\alpha}}\right) \\ &\equiv \frac{2r_s}{\sqrt{\pi}} \cdot \left(\frac{r^2}{4\tilde{\alpha}}\right)^{3/2} \cdot \tilde{\Gamma}(3/2) \cdot e^{-3/2} \cdot \sum_{k=0}^{\infty} \left(\frac{\left(\frac{r^2}{4\tilde{\alpha}}\right)^k}{\tilde{\Gamma}((3/2)+k+1)} \right) \end{aligned} \tag{49}$$

This is called the incomplete lower gamma function, with $\tilde{\Gamma}$ being a gamma function [27].

From here, using that Equation (49) is to be included in the following metric, as given by.

The coefficient $[\tilde{\alpha}] = [r^2]$ in terms of dimensional analysis is chosen so that the dimensions of $[\tilde{\alpha}] = [r^2]$ are chosen to contain M as mass in a wormhole. *i.e.*, the denominator of Equation (48) $(4\pi\tilde{\alpha})^{3/2}$ is chosen so that M is within the volume of space so subscribed. And this is for line element [26]. With Equation (48) and Equation (49) fully described in [26] and [28].

$$dS^2 = -\exp(-2\Phi(r))dt^2 + \frac{dr^2}{1-b(r)/r} + r^2 \cdot \left(d\theta^2 + \left(\sin^2\theta\right)d\varphi^2\right) \tag{50}$$

If we refer to black holes, with extra dimension, n, of Planck sized mass, we have a lifetime of the value of about

$$\tau \sim \frac{1}{M_*}\left(\frac{M_{\text{BH}}}{M_*}\right)^{\frac{n+3}{n+1}} \xrightarrow[M_{\text{BH}} \approx M_{\text{Planck}}]{} 10^{-26}\,\text{seconds}$$
M_* is the low energy scale, (51)
which could be as low as a few TeV,

The idea would be that there would be n additional dimensions, as given in Equation (51) which would then lay the door open to investigating [29] and [30] in terms of applications, with [30] of additional polarization states to be investigated, as to signals from the mouth of the wormhole. We will next then go into some predictions into first, the strength of the signals, the frequency range, and several characteristics as to the production rate of Planck sized black holes.

13. Conclusion: A First Order Guess as to the Rate of Production of Planck Sized black Holes through a Wormhole

To do this, we will be estimating that the temperature would be of the order of

Planck temperature, *i.e.*, using ideas from [30] and [31]

$$\frac{\omega_p}{T_p} \equiv \frac{\sqrt{Gk_B^2}}{\hbar} \xrightarrow[\hbar=G=k_B=1]{} 1 \tag{52}$$

If so, then there would be to first order the following rate of production

$$\Gamma_{\text{rate of production}} \approx e \approx 2 - 3 \tag{53}$$

Some of the considerations given in this could be related to [32] as an afterthought whereas the author in [33] estimated for an LHC that there would be about 3000 gravitons produced per second. Assuming a figure from [34] as to the percentage of black hole mass decaying into gravitons, *i.e.* [34], *i.e.*, 1/1000 of the mass of a Planck sized black hole would delve into gravitons, so if one had 3000 gravitons produced per second, as measured on Earth, one would likely have 2 - 3 black holes of mass of about 10^−5 grams per black hole, producing say 10^57 gravitons, produced per black hole of mass about 10^−62 grams per black hole [35]

Having said, that what about frequencies? Here, if we have a wormhole throat of about 2 - 3 Planck lengths in diameter, with a frequency of emitted gravitons of about 10^{19} GHz initially, it is realistic, using the following, to expect in many cases a redshift downscaling of frequencies of about 10^−18, if the worm holes are close to the initial near singularity, so then that we could be looking at approximately 10 to 12 GHz, on Earth, for frequencies, of initially about 10^19 GHZ. So then note at inflation we have

$$\begin{aligned} &\left(1+z_{\text{initial era}}\right) \equiv \frac{a_{\text{today}}}{a_{\text{initial era}}} \approx \left(\frac{\omega_{\text{Earth orbit}}}{\omega_{\text{initial era}}}\right)^{-1} \\ &\Rightarrow \left(1+z_{\text{initial era}}\right)\omega_{\text{Earth orbit}} \approx 10^{25}\,\omega_{\text{Earth orbit}} \approx \omega_{\text{initial era}} \end{aligned} \tag{54}$$

In our situation, the figure would likely be instead of 10^25 times Earth orbit detected frequency, something closer to 10^18 to 10^19 times Earth orbit GW frequencies detected as given by [36]. The relative GW strength of the signal, if one uses [36] while assuming approximately 10 to 12 GHz, for initially about 10^19 GHz GW signals would be about h ~ 10^−26 and this could change an order of magnitude given instrument sensitivity. In any case it would be well worth our while to look closely at [37] [38] [39] [40] for additional clues and insights to consider while commencing this investigation, as well as details given in [41]. Finally, the references [42]-[72] are referencing situations which are natural extensions of this present document and which will be used in future publications. We include them for the readers to review as to consider on their own what may be following up to our first order approximations given for Equation (52), Equation (53) and Equation (54).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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