

A Solution of the Cosmological Constant and DE Using Breakup of Primordial Black Holes, via a Criteria Brought up by Dr. Freeze of Austin, Texas, Which Initiates DE as Linked to Inflation

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Abstract

We reduplicate the Book “Dark Energy” by M. Li, X-D. Li, and Y. Wang, given zero-point energy calculation with an unexpected “length” added to the “width” of a graviton wave just prior to specifying the creation of “gravitons”, while using Karen Freeze’s criteria as to the breakup of primordial black holes to give radiation era contributions to GW generation. The GW generation will be when there is sufficient early universe density so as to break apart Relic Black holes of the order of Planck mass (10^{15} grams) which is about when the mass of relic black holes is created, 10^{-27} or so seconds after expansion starts. Needles to state a key result will be in the initial potential V calculated, in terms of other input variables.

Keywords

Minimum Scale Factor, Cosmological Constant, DE, Arrow of Time

1. Introduction

What we are doing is to try to confirm if we can apply the techniques of the following reference to the problem of DE and the arrow of time, and heavy gravity. After work I did in [1] was allegedly not credible, due to people having doubts as to the existence of a multiverse and equating two first integrals as I did, via early pre Planckian space-time, the following reference was accessed [2], and then applied to [3] and the work on heavy gravity in [4]. In doing so we will keep in mind the ‘t Hooft memorandum as to the arrow of time, which is in [5] as a basic organizational principle for our discussion, *i.e.* formation of our program is assuming initial conditions for using [4] in the expansion of the universe say af-

ter 10^{-42} seconds.

$$m_g = \frac{\hbar\sqrt{\Lambda}}{c} \tag{1}$$

This release of conditions for massive gravity should be in line with what is in Freeze *et al.*

$$\rho_{BH} = \frac{m_p^4}{32\pi} \cdot \left(\frac{m_p}{m}\right)^4 \cdot \frac{1}{|1+3w_Q|} \tag{2}$$

If the conditions of an early universe, are greater than this value, for Equation (2) [6] then according to Freeze, *et al.* primordial black holes would break apart. We state that this break up of primordial black holes would be enough to create an initial “sea” of gravitons, due to Equation (1) which would then add up to be in effect a value for a sufficient number of early universe gravitons, which would be added up per unit volume, to in fact sum up to an energy density equivalent to Equation (1) so we have massive gravitons and DE. Hence we will be adding up the number of gravitons which may be released due to Equation (2) and [7] which states the number of gravitons which may be emitted due to a black hole as given in its page 47 is 0.1 percent of emitted energy from a nonrotating black hole. Keep in mind that this is for black holes, as given in [7] with mass:

$$M_{\text{black hole primordial}} \sim 10^{15} \times \left(\frac{t}{10^{-23}\text{s}}\right) \cdot \text{g (grams)} \tag{3}$$

For a 10^{-5} gram black hole, t would have to be about 10^{-33} seconds, and according to inflation expands space by a factor of 10^{26} over a time of the order of 10^{-33} to 10^{-32} seconds. Meaning we had 10^{-5} gram black holes at the end of inflation, and at the time the density of space would be greater than Equation (2) we would have a breakup of black holes if we had space-time density greater than or equal to about Equation (2) then we have .1% of the mass of the broken BH contributing to gravitons, which after we review it may be relevant to Equation (1) above. One Planck mass is about 10^{-5} grams.

And it is worth noting in our development when we go past inflation, that we have Black holes growing to the value of about 10^{11} grams, after 10^{-27} seconds which is for a radii of approximately 10^6 kilometers, whereas we can and will define Black holes of 10^{-5} grams which would be for less than a centimeter radii just after the end of inflation.

The discussion below will reflect these values and put them in the context of answering how the breakup of Black holes, due to the phenomenon brought up in [7] allows for the production of gravitons which we conflate with DE. The radius of the inflationary universe at 10^{-33} seconds would be about 10^{-3} centimeters to at most 10^{-1} centimeters. If it was the latter, we have:

$$a(t) = a_{\text{initial}} t^\gamma \tag{4}$$

$$\rho \approx \frac{\dot{\phi}^2}{2} + V(\phi) \equiv \frac{\gamma}{8\pi G} \cdot t^2 + V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \tag{5}$$

$$P_G = \frac{\gamma \cdot t^2}{8\pi G} - V_0 \cdot \left\{ \frac{\sqrt{8\pi G V_0}}{\gamma \cdot (3\gamma - 1)} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \tag{6}$$

$$\rho_{BH} = \frac{m_p^4}{32\pi} \cdot \left(\frac{m_p}{m} \right)^4 \cdot \frac{1}{\left| 1 + 3 \frac{P_G}{\rho_G} \right|} \tag{7}$$

What we will be looking at would be having a glimpse of:

$$\frac{P_G}{\rho_G} \approx \frac{\frac{\gamma \cdot t^2}{8\pi G} - V_0 \cdot \left\{ \frac{\sqrt{8\pi G V_0}}{\gamma \cdot (3\gamma - 1)} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}}}{\frac{\gamma \cdot t^2}{8\pi G} + V_0 \cdot \left\{ \frac{\sqrt{8\pi G V_0}}{\gamma \cdot (3\gamma - 1)} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}}} \tag{8}$$

Doing this would be at a minimum of having, a distance greater than or equal to:

$$\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}} \tag{9}$$

Assume then we have, for the sake of argument:

$$\frac{P_G}{\rho_G} \approx \frac{1 - V_0 \cdot \left\{ \frac{8\pi G \sqrt{8\pi G V_0}}{\gamma^2 \cdot (3\gamma - 1)} \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \cdot t^{-2 + \sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}}}{1 + V_0 \cdot \left\{ \frac{8\pi G \sqrt{8\pi G V_0}}{\gamma^2 \cdot (3\gamma - 1)} \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \cdot t^{-2 + \sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}}} \approx -10^{\epsilon} \tag{10}$$

The particulars of the coefficient of the right hand side of Equation (10), if $\hbar = G = \ell_p = t_p = 1$, then if we set,

$$\frac{\gamma}{4\pi} - 2\sqrt{\frac{\gamma}{4\pi}} - 1 = 0 \tag{11}$$

So then we have if we wish to neutralize sensitivity to time itself at first approximation,

$$\gamma = 4\pi \cdot (1 \pm \sqrt{2})^2 \tag{12}$$

We can then look at the following:

$$\frac{1 - V_0 \cdot \left\{ \frac{8\pi \sqrt{8\pi V_0}}{16\pi^2 \cdot (1 \pm \sqrt{2})^4 \cdot (12\pi \cdot (1 \pm \sqrt{2})^2 - 1)} \right\}^{(1 \pm \sqrt{2}) - \frac{\sqrt{2}}{(1 \pm \sqrt{2})}}}{1 + V_0 \cdot \left\{ \frac{8\pi \sqrt{8\pi V_0}}{16\pi^2 \cdot (1 \pm \sqrt{2})^4 \cdot (12\pi \cdot (1 \pm \sqrt{2})^2 - 1)} \right\}^{(1 \pm \sqrt{2}) - \frac{\sqrt{2}}{(1 \pm \sqrt{2})}}} \approx -10^{\epsilon} \tag{13}$$

The above equation can be used, to locate appropriate values for V_0 in units where $\hbar = G = \ell_p = t_p = m_p = c = 1$.

Given an approximate value for V_0 we will then proceed to come up with examining,

$$\frac{P_G}{\rho_G} \approx \frac{1 - V_0 \cdot \left\{ \frac{8\pi G \sqrt{8\pi G V_0}}{\gamma^2 \cdot (3\gamma - 1)} \right\} \sqrt{\frac{\gamma}{4\pi G}} \sqrt{\frac{8\pi G}{\gamma}} t^{-2+\sqrt{\frac{\gamma}{4\pi G}} \sqrt{\frac{8\pi G}{\gamma}}}}{1 + V_0 \cdot \left\{ \frac{8\pi G \sqrt{8\pi G V_0}}{\gamma^2 \cdot (3\gamma - 1)} \right\} \sqrt{\frac{\gamma}{4\pi G}} \sqrt{\frac{8\pi G}{\gamma}} t^{-2+\sqrt{\frac{\gamma}{4\pi G}} \sqrt{\frac{8\pi G}{\gamma}}}} \quad (14a)$$

$$\rho_{BH} = \frac{m_p^4}{32\pi} \cdot \left(\frac{m_p}{m}\right)^4 \cdot \frac{1}{|1 - 3 \cdot 10^{-\zeta}|} \quad (14b)$$

$$M_{\text{black hole primordial}} \sim 10^{15} \times \left(\frac{t}{10^{-23} \text{ s}}\right) \cdot g \text{ (grams)}$$

Whereas we ask for initial conditions for the arrow of time, and Λ and DE formation once we understand what happens if Equation (14) is exceeded by the initial density of the early universe.

2. Reviewing Conditions for the Early Universe, Assuming a Volume of Inflation at Its End and the Beginning of Radiation Domination to Employ Equation (15) Depending upon What Is Put into m at $t \approx 10^{-33}$ s

The easiest solution to Equation (14) is to look at when $\zeta = 0$ when we have Dark Energy, and then look at when an object of mass m , and radius R is pulled apart. From [7], page 154:

$$m \approx \frac{8\pi R^3 (\text{radius}) \cdot \rho}{3} \quad (15)$$

With this generalized to being for black holes being pulled apart when,

$$\begin{aligned} m &\approx -\left(\frac{4\pi\rho}{3}\right) \cdot \left(1 + 3 \cdot \frac{P_G}{\rho_G}\right) \cdot \frac{8m^3}{m_p^6} \\ \Rightarrow \text{if } P_G &\approx -\rho_G \\ m(\text{black holes}) &\geq 10^5 m_p \text{ break apart} \\ \text{after } t &\approx 10^{-27} \text{ sec} \end{aligned} \quad (16)$$

We then have, say that there are a large number of black holes of about 10^5 Planck Mass at a time just after the end of the inflationary era, which are pulled apart, and if we then look at the formula of 1/1000 of the mass of a black hole we are talking about a contribution of about $10^2 m_p$ in black hole generation of a flux of gravitons after 10^{-27} seconds, *i.e.* for the regime from 10^{-33} seconds to 10^{-27} seconds, we get:

$$10^2 m_p (\text{gravitons}) \Rightarrow 10^{67} \text{ gravitons per } 10^5 m_p (\text{black hole}) \quad (17)$$

3. Making DE Equivalent to a Sea of Initial Gravitons, in Regime 10^{-33} to 10^{-27} Seconds

Roughly put, one hydrogen atom is about 1.66 times 10^{-24} grams. The weight of

a massive graviton is about 10^{-65} grams [8] [9], hence we are talking about 10^{-22} grams, or about 10^{44} gravitons, with each graviton about $6 \times 10^{-32} \text{ eV}/c^2$. After 10^{-27} seconds, the following in the set of equations given below are Equivalent, and that these together will lead to a cosmological Constant, Λ of the sort which we will be able to refer to later,

$$1 \text{ graviton} \approx 10^{-65} \text{ g} \tag{17a}$$

$$M_{\text{PBH}} \sim 10^{15} \times \left(\frac{t = 10^{-27} \text{ s}}{10^{-23} \text{ s}} \right) \cdot \text{g (grams)} \approx 10^{11} \text{ g} \approx 10^{16} m_p \tag{17b}$$

Assuming that gravitons contribute to the Dark Energy value will lead to us using the Karen Freeze model, with gravitons being released in the early universe by the breakup of early universe black holes which have a maximum value of about 10^{11} g, as opposed to the value of the Sun which has about 10^{33} grams, *i.e.* by making use of the following:

$$\text{DE (from broken black hole)} \sim 7 \times 10^{-30} \text{ g/cm}^3 \approx 7 \times 10^{12} \text{ g} / (10^9 \text{ km})^3 \tag{17c}$$

$$\text{Universe (black hole)} \sim 7 \times 10^{15} \text{ g} / (10^9 \text{ km})^3 \approx 7 \times 10^4 M_{\text{PBH}} / (10^9 \text{ km})^3 \tag{17d}$$

We claim that the above Equation (17c), will be able to yield a DE value $\Leftrightarrow 10^{80}$ gravitons in a region of space for which we have a “sphere” of radius 10^9 km at 10^{-27} seconds as a way to support the existence of the mass of DE. *i.e.* about 104 number of black holes given in Equation (17d) initially.

And now we can look at the to the calculation given by the following as by [3],

$$\frac{1}{2} \cdot \sum_i \omega_i \equiv V(\text{volume}) \cdot \int_0^{\tilde{\lambda}} \sqrt{k^2 + m^2} \frac{k^2 dk}{4\pi^2} \approx \frac{\tilde{\lambda}^4}{16\pi^2} \tag{18}$$

$$\xrightarrow{\tilde{\lambda} = M_{\text{Planck}}} \rho_{\text{boson}} \approx 2 \times 10^{71} \text{ GeV}^4 \approx 10^{119} \cdot \left(\rho_{\text{DE}} = \frac{\Lambda}{8\pi G} \right)$$

In stating this we have to consider that by [10] the cosmological “constant” should be small and positive:

$$\rho_{\text{DE}} = \frac{\Lambda}{8\pi G} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{\text{DE}}^4} \tag{18a}$$

so then that we can consider we have after 10^{-27} seconds,

$$\lambda_{\text{DE}} \approx 10^{30} \ell_{\text{Planck}} \tag{19}$$

We then have to consider how to reach the experimental conditions for when,

$$\rho_{\text{DE}} = \frac{\Lambda}{8\pi G} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{\text{DE}}^4} \tag{20}$$

After we fill in more details of this procedure, we will be considering the implications of the paper by Alves [11] which conflates DE with gravitons which is the approach we are using and the idea is that one uses gravitons in say a sphere of about 1 million kilometers to give the same energy “density” as gravitons in a way to recover the DE density. From here, we should realize that Equation (19)

is commensurate with a “length” within a few orders of magnitude similar to a centimeter, making the above “analysis” similar to DE being formed at the start near the end of inflation.

4. What about Representative Wavefunctions within the Radii of 10^{-3} Centimeters to 10^6 Kilometers?

The cleanest representation the author found is given in [12] where on page 47 the Hartle-Hawking wave function for minisuperspace cosmology, with a no boundary condition would lead to, if,

$a^2(t) \cdot V(\phi(t)) < 1$, a value of, if $V(\phi(t))$ is defined by:

$$V(\phi(t)) = V_0 \cdot \left(\sqrt{\frac{8\pi G V_0}{v \cdot (3v - 1)}} \cdot t \right)^{\frac{1}{2} \sqrt{\frac{v}{\pi G}} - 4 \sqrt{\frac{\pi G}{v}}} \tag{21}$$

Then, use from [12] a saddle point solution to the Wheeler De Witt equation of the form:

$$\Psi_{HH}(a, \phi(t)) \approx \exp\left[\frac{1}{3V(\phi(t))}\right] \times \exp\left[\frac{-(1 - a^2(t) \cdot V(\phi(t)))}{3V(\phi(t))}\right] \tag{22}$$

At inflation, $z(\text{red shift}) \sim 10^{-25}$, implying a tiny scale factor so Equation (22) would work. Having said that, the term V_0 could be set by Equation (13) especially, and the main work for future work would be comparing Equation (22) with the so called wave function of a black hole as given by [13], by:

$$\begin{aligned} \Psi_{BH} &\approx P \cdot \exp(-4\pi M) \exp(-I_2) \\ M &\approx \text{mass of black hole plus background} \\ I_2 &= \text{dynamics of local deg of freedom} \\ &\approx \frac{1}{2} \cdot \sum_{\chi} \omega_{\chi} \cdot \left\{ \tanh(\beta \omega_{\chi} / 4) f_{\chi,+}^2 + \frac{f_{\chi,-}^2}{\tanh(\beta \omega_{\chi} / 4)} \right\} \end{aligned} \tag{23}$$

Here, we have that we can also look at [14] where we can look at:

$$\begin{aligned} S_{BH} &\approx k_B \tilde{A} / 4 \ell_P = k_B M^2 G^2 / c^4 \\ R_{\text{Schwartzschild}} &= 2MG / c^2 \end{aligned} \tag{24}$$

If M is $\sim 10^{-18}$ solar mass, about 10^{15} grams, it means that the dynamics of local degrees of freedom of the black hole [15] that the approximation used is that exists:

$$A_n = 4 \cdot (\ln 3) \cdot n_{\text{quantum}} \tag{24a}$$

$$\mathbb{N} = (\ln 3) \cdot A_n \tag{24b}$$

$$M = \mathbb{N} \cdot (\ln 3) \cdot \frac{T}{2} \tag{24c}$$

$$T = (h/2\pi) c^3 / (8\pi G M k) \tag{24d}$$

Then by Equation (24) we can write out the following:

$$\begin{aligned} T_{BH} &= (h/2\pi) c^3 / (8\pi G M k) \approx 10^{12} \text{ kelvin} \\ \text{if } M_{BH} &\approx 10^{15} \text{ g or } 10^{-18} M_{\text{Sun}} \end{aligned} \tag{25}$$

IF this is true, then we have the situation, where the Black holes being broken apart by the reasons given in [7] would lead to, in first order an effective black hole wave function approximately set and looking like, when $M_{BH} \approx 10^{15} \text{ g}$ or $10^{-18} M_{\text{Sun}}$ that an individual black hole wave function is,

$$\Psi_{BH} \approx P \cdot \exp(-4\pi M) \tag{26}$$

With $M_{BH} \approx 10^{15} \text{ g}$ or $10^{-18} M_{\text{Sun}}$ assumed to be equivalent to Equation (24c), and then M has data from Degrees of freedom already included.

This value of Equation (26) needs to be compared to, if we have say 10^4 or so black holes being broken up in the first 10^{-27} seconds in order to get DE according to Equation (20) and Equation (17c), and Equation (26) would be for each of the up to 10^4 black holes releasing gravitons, with perhaps a maximum value of up to 10^{80} gravitons released as to obtain a mass-energy value for DE which should be equivalent to Equation (18a) and Equation (19).

Meanwhile, the up to 10^4 black holes represented are such that conceivably a relationship between Equation (13) and V_0 exists, so then that one can identify, using Planckian units of the following type $\hbar = G = \ell_p = t_p = m_p = c = 1$ a “magnitude” of V_0 exists, so we can examine if there a Relationship linkage as follows:

- 1) Specify V_0 as given above, for a given time $t \sim 10^{27}$ seconds.
- 2) Use that to give a value, then try to satisfy this criteria this inhomogeneity requirement of [16],

$$\frac{H^2}{\dot{\phi}} \approx \sqrt{\frac{4\pi G}{v}} \cdot V_0 \cdot \left(\sqrt{\frac{8\pi G V_0}{v \cdot (3v-1)}} \right)^{\frac{1}{2} \sqrt{\frac{v}{\pi G}} - 4 \sqrt{\frac{\pi G}{v}}} t^{\frac{1}{2} \sqrt{\frac{v}{\pi G}} - 4 \sqrt{\frac{\pi G}{v}} + 1} \approx 10^{-5} \tag{27}$$

One can calculate and set a value of the coefficient of expansion using this, for γ as used in Equation (4).

- 3) After we have this value satisfied, set:

$$1 - V_0 \cdot \left\{ \frac{8\pi G \sqrt{8\pi G V_0}}{\gamma^2 \cdot (3\gamma - 1)} \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} t^{-2 + \sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \approx -10^5 \cdot \left(1 + V_0 \cdot \left\{ \frac{8\pi G \sqrt{8\pi G V_0}}{\gamma^2 \cdot (3\gamma - 1)} \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} t^{-2 + \sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \right) \tag{28}$$

- 4) Use this , if ζ set small to isolate out values of frequency for which we can have at $t \sim 10^{-27}$ s.

$$V = V_0 \cdot \left\{ \frac{8\pi G V_0}{\gamma(3\gamma - 1)} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \xrightarrow{t_{\text{Planck}} = \hbar = G = 1} V_0 \cdot \left\{ \frac{8\pi V_0}{\gamma(3\gamma - 1)} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi}} - \sqrt{\frac{8\pi}{\gamma}}} \approx \omega \propto \left(\frac{2}{3a_{\text{min}}} \right)^{-1/\gamma} \tag{29}$$

This frequency value, say of radiation will be to look at the frequency range of DE, with Equation (29) allowing for inputs of $V(\phi(t))$ of Equation (29) into Equation (22), with $V(\phi(t)) \approx \omega_{DE}$.

5) Afterwards, with this value of frequency, examine,

$$\rho \approx \frac{\dot{\phi}^2}{2} + V(\phi) \equiv \frac{\gamma}{8\pi G} \cdot t^2 + V_0 \cdot \left\{ \frac{\sqrt{8\pi G V_0}}{\sqrt{\gamma(3\gamma-1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \quad (30)$$

Whereas the frequency is for, roughly looking at what we will be thinking of if we put in Equation (12) when $G = 1 = \hbar$ is: for a time between t set as 10^{-33} seconds to 10^{-27} seconds.

$$\begin{aligned} \rho &\approx \frac{\gamma t^2}{8\pi G} + V_0 \cdot \left\{ \frac{\sqrt{8\pi G V_0}}{\gamma^2 \cdot (3\gamma-1)} \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \cdot t^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \\ &\approx \frac{t^2}{\sqrt{2}} + V_0 \text{ if } \gamma = \frac{8\pi}{\sqrt{2}} \end{aligned} \quad (31)$$

At $t \sim 10^{17}$ times Planck time, the first term is overshadowed by V_0 with V_0 proportional to the frequency ω where this can be for gravitons emitted from black holes of mass 10^{15} g, or initially having wavelength $\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}$, which corresponds to frequency about 10^{13} Hz, which if we look at the following relation, namely,

$$(1 + z_{\text{Radii} \sim 1 \text{ billion km}}) \omega_{\text{Earth orbit}} \approx z_{\text{Radii} \sim 1 \text{ billion km}} \omega_{\text{Earth orbit}} \approx \omega_{\text{Radii} \sim 1 \text{ billion km}} \propto 10^4 \text{ GHz} \quad (32)$$

and,

$$z_{\text{Radii} \sim 1 \text{ billion km}} \approx 10^4 \Rightarrow \omega_{\text{Earth orbit}} \approx 1 \text{ GHz} \quad (33)$$

The figure of red shift about $z \sim 10^4$ for a radii of about one billion kilometers at $t \sim 10^{-27}$ s isto be seen and compared to $z \sim 1100$ about 100,000 years after the start of the inflationary era. Point in fact is that at the end of inflation, that acceleration ceased, for a time with the zero value of acceleration reached at 10^{-32} - 10^{33} seconds, so then the one billion km at $t \sim 10^{-27}$ s represents a slowing down of the rate of expansion of the universe, whereas $z \sim 10^{25}$ at the end of inflation whereas z drops 21 orders of magnitude to $z \sim 10^4$ at about $t \sim 10^{-27}$ s whereas z futher “shrinks” to about 1100, or roughly 10^3 at about $t \sim 100,000$ years.

5. What Can Be Inferred as to Subsequent Growth of Entropy from Early Times to Today?

At about $z \sim 10^4$, at about time $\sim 10^{-27}$ seconds we have about 10^{80} gravitons, and from here [17],

$$S(k) = \ln(\Gamma/\Gamma_0) = r_k + \ln 2 + 1/2 \ln n(k) \propto 10^{80} \quad (34)$$

The value of the “final” phase state, Γ would be the “final” phase state, about 10^{-27} s after inflation. The value of the “initial” phase state, Γ_0 would be the phase state, about 10^{-32} after inflation The ratio would be due to $\Gamma = 17621117.2689$ “bigger” than Γ_0 , whereas we would scale:

$$\Gamma/\Gamma_0 \approx 1.762 \times 10^8 \tag{35}$$

This is the ratio of expansion of the “graviton” based entropy, forming a value of DE which may be commensurate with the number of gravitons released in a volume of space, from radius of 1 centimeter to a radius of 1 times 10⁹ kilometers.

As from $z \sim 0$, at about 13.8 billion years $\sim 43,5485937613.56$ times 10¹⁶ seconds to 10⁻³² seconds. The value of Γ_{Today} would be the Today’s phase state at 43,5485937613.56 times 10¹⁶ seconds.

The value of the “initial” phase state, Γ_0 would be the phase state, about 10⁻³² after inflation,

$$S_{\text{Today}}(k) = \ln(\Gamma_{\text{Today}}/\Gamma_0) \propto 10^{105} \tag{36}$$

For what it is worth, the radius of the universe, TODAY is about 8.8×10^{23} kilometers from about 10⁹ kilometers at 10⁻²⁷ seconds, and about 0.0001 kilometers at 10⁻³² seconds *i.e.* roughly 10²⁸ times the radial distance larger than the value of the radius at end of inflation. And, if have this baseline, we can compare it against purely thermal treatments of cosmology.

6. Examining What Was Done by Rosen, in 1991, Which Had Some Fidelity with the Same Issues with the Idea of Using a Breakup of Black Holes, for DE

The key point of this mini chapter will be to summarize the derivation of the temperature [2]:

$$T = (\rho_p/\sigma)^{1/4} \cdot \frac{\tilde{a}r^7}{(\tilde{a}^4 + r^4)^2} \tag{36}$$

Whereas $(\rho_p/\sigma)^{1/4} = 1.574 \times 10^{32}$ K (kelvin), and $\tilde{a} = 10^{-3}$ cm, whereas

$$\begin{aligned} r_{\text{initial}} &= (3/8 \pi \rho_p)^{1/2} = 5.58 \times 10^{-34} \text{ cm} \\ \Rightarrow T_{\text{initial}} &= 2.65 \times 10^{-180} \text{ K (kelvin)} \end{aligned} \tag{36a}$$

$$\begin{aligned} r_{\text{DE formation}} &= \tilde{a} = 10^{-3} \text{ cm} \\ \Rightarrow T_{\text{DE formation}} &= 7.41 \times 10^{31} \text{ K (kelvin)} \end{aligned} \tag{36b}$$

Is Equation (36b) going to be feasible as to conditions as to the breakup of black holes, which may produce DE We will be deriving Equation (36) as a summary of what to expect in this treatment of nonsingular space-time To do so we start off with [2] in pre matter and radiation periods with entropy S , $\rho = \rho(T), P = P(T)$.

$$dS(V, T) = \frac{1}{T} \cdot [d(\rho V) + PdV] \tag{37}$$

$$V = V(\text{volume}) = 2\pi^2 r^3 \tag{37a}$$

And an integrability condition on Equation (36) leading to:

$$\frac{dP}{dT} = \frac{1}{T} \cdot (\rho + P) \tag{37b}$$

Then the integral of Equation (37) is given as:

$$S = \frac{V}{T} \cdot (\rho + P) \quad (37c)$$

Also, we look at a given value of pressure as given in [2] for which,

$$P = \frac{\rho}{3} \cdot \left(1 - \frac{4\rho}{\rho_p} \right) \quad (38)$$

Put Equation (37d) into Equation (37b) and then one will get after integrating Equation (37b),

$$\rho \cdot \left(1 - \frac{\rho}{\rho_p} \right)^7 = \sigma T^4 \quad (38a)$$

Here, [2] treated σ as the Stephan-Boltzman constant, and so then if we add in the energy equation,

$$\dot{\rho} + 3 \cdot (\dot{r}/r) \cdot (\rho + P) = 0 \quad (38b)$$

Then we put Equation (8) into Equation Equation (8b) we obtain:

$$\rho = \tilde{a}^4 \rho_p / (\tilde{a}^4 + r^4) \quad (38c)$$

We claim that Equation (38c) put into Equation (38a) we will then obtain Equation (36) So what is the problem with the Rosen derivation? And using Temperature? See this one:

$$E \approx \frac{d(\text{dim}) k_B T}{2} \quad (39)$$

Taking this as the defining value at Planck temperature, at say the end of inflation, may lead to a value in some semblance of 10^{32} Kelvin, and from that high temperature it would be hard to ascertain when the separate DE would form. Not impossible, but the procedure outlined as to a breakup of black holes, as done up to a radii of 9 million kilometers at least is not solely dependent upon temperature itself but upon DENSITY. We also assume that at the end of inflation, there is a drop in temperature and an end to acceleration of the speed of the rate of expansion of the universe of expansion of the universe. But, reference [6] has it that there is a critical density we need to consider, and the author is assuming breakup of black holes, initially, may lead to DE.

Does Rosen with his near Planck Temperature at the end of inflation, as he modeled that it has overlap with breaking up of primordial black holes? It is unclear if this is the case, or even possible.

7. What Rosen Model Would Give Us and Not Give Us as to Entropy, The arrow of Time and DE

1) We will be able to come up with an initial temperature of 10^{-180} Kelvin, at a radius of about Planck length, in value, almost absolute zero.

2) The temperature of space-time will be of the order of Planck Temperature after expansion of about 10^{30} times from the initial nonsingular configuration.

3) For making effective use of [2] we will be looking at what is measured after measured after 10^{-42} seconds, which is roughly Planck Time, in this model. *I.e.* the convention is that we will be using is that we are starting off, in [2] in what is called the pre-matter radiation transition point, in the history of the universe. We should also keep in mind that A. and B. and C will allow an arrow of time forming due to the reasons brought up in [5] whereas we have the following Entropy value of [18] [19],

$$S \sim 3 \cdot [1.66\sqrt{g_*}]^2 T^3 \quad (40)$$

However, having fidelity with respect to Equation (18), Equation (18a) , Equation (19) and Equation (39) will be difficult.

8. Conclusion, Our Approach to DE and If It Is Commensurate with Rosen, and Ng's Infinite Quantum Statistics?

The Rosen approach, as of section 5, and section 6 satisfies the Arrow of time criteria as given in [5]. We do have though that the way to combine DE with entropy would be to utilize Ng infinite quantum statistics [20] as far as counting,

$$S \sim n \quad (41)$$

as n is a graviton count, which presumably would be increasing with the expansion of the universe. However, this would entail in comparing with and making use of the Hawking statement of the arrow of time as commensurate with the expansion of the universe [21] and would need that as a Necessary & sufficient condition to tie in the Rosen approach with DE. And that may be very difficult.

The conclusion the author draws is that any full reconciliation between sort of cosmology presented by Rosen in [2] and the procedure as to how the author is trying to construct DE would be in reconciling the counting procedure of Ng, as used for graviton count with Fay Dowker's causal structure as given in [22]. Once that is done, it may be feasible to state DE as given by the author, and Rosen's thermal cosmology are commensurate sides to the same animal. In doing so, the author hopes to explore if the causal structure idea of [21] may be employed to make a linkage between DE, the arrow of time, and Entropy as has been suggested by the author in other publications. In addition, ideas in [23] [24], and [25] should be seriously investigated finally and not least the author is well aware of Lagrangian representations of GR, as in [26] [27], and [28]. The author hopes that refinement of the issues given in this text will in the spirit of [29], pp 120-125, allow creating full Lagrangian GR development for a more refined Hamiltonian treatment of GR which moves toward a new synthesis of conserved and non conserved Quantities which allows for making a linkage between GR and QM from the primordial regime of space-time. Possibly, to compliment [30], and later improve upon some of the ideas in Loop quantum gravity [31], the idea also should be in investigating if this research is commensurate with the ideas of Eternal inflation as given in [32].

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Conflicts of Interest

The author declares that there is no conceivable conflict of interest in this document.

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