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The Principle of Equivalence: Periastron Precession, Light Deflection, Binary Star Decay, Graviton Temperature, Dark Matter, Dark Energy and Galaxy Rotation Curves

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Abstract

The nature of the principle of equivalence is explored. The path of gravitons is analyzed in an accelerating system equivalent to a gravitating system. The finite speed of the graviton results in a delay of the gravitational interaction with a particle mass. From the aberration found in the path of the graviton we derive the standard expression for the advancement of the periastron of the orbit of the mass around a star. In a similar way, by analysing the aberrations of the graviton and light paths in an accelerating reference frame, the expression for the deflection of light by a massive body is obtained identically to the standard result. We also examine the binary star system and calculate the decay in its orbital period. The decay is attributed to the redshift of the graviton frequency relative to the accelerating system. Here too, we obtain good agreement with experimental measurements. Also, hypothesizing that gravitons behave like photons, we determine the temperature of the gravitons in a binary star system and form the Bose-Einstein distribution. Finally, we show how the redshift of gravitons may be the source of dark matter, dark energy and flat line spiral galaxy rotation curves.

Keywords

Gravitons, Binary Stars, Galaxy Rotations, Dark Matter, Dark Energy

1. Introduction

This paper is based on the hypothesis that gravitons exist [1] [2], having physical properties revealed by the phenomena we will examine. We determine the advance of the periastron of a planet orbiting a star by calculating the aberration of gravitons in an equivalent accelerating system using the principle of equivalence. The finite speed of the graviton causes a delay in the travel time in the interaction of the graviton with the planet. This travel time delay leads to an aberration of the graviton path relative to the planet. From this aberration, the standard expression is obtained for the periastron advance of the planet's orbit.

Similarly, the same analysis is applied to the deflection of a beam of light grazing a star. Due to the finite speeds of the graviton and photon, there are delays in their travel times. We show that the delays cause aberrations of the graviton path and the light path. Combining these aberrations yields an expression for the deflection of light passing through the gravitational field of a star, identical to the standard value.

We also look at the binary star system and focus on the decay of its orbital period. By accounting for the Doppler redshift of the graviton frequency in free fall in a gravitaional field, which produces a decrease in graviton energy, we obtain an expression for the decay of the period based on Newtonian orbital mechanics, which gives close results when compared with the report on the binary pulsar B1913+16. A course fit to data from eight PSRs produces good agreement throughout the varied types of binary systems. A comparison is made with the General Relativity analysis of the binary star systems studied. Likewise, assuming that gravitons behave like photons, we determine the temperature of the gravitons in a binary star system and describe the Bose-Einstein statistics. We show how the decrease of graviton energy during the expansion of the universe may be the source of dark energy.

Finally, we briefly describe how graviton redshift in spiral galaxies may appear as dark matter causing flat lining of rotation curves.

Except for the concepts that the graviton is the agent particle of gravitation, that a beam of light is composed of photons, that both particles travel at a fixed speed c in vacuum in any reference frame, and that of the equivalence of mass and energy, only Newtonian and Galilean principles are invoked in this study.

This paper is based on an earlier publication by the same author [3]. We show that the earlier physical theories can go much further in the way of explaining certain phenomena which were thought to be only explainable by the General Relativity theory.

2. Planetary System Analyzed in an Accelerating Reference Frame

We seek to examine some essential properties of a gravitating system of a mass m orbiting a mass M, such as a planet orbiting a star. The equivalence of inertial and gravitational mass (Weak Equivalence Principle) has been proven experimentally to a high precision, $\delta(\text{Titanium, Plutonium}) = [-1 \pm 9(\text{stat}) \pm 9(\text{syst})] \times 10^{-15}$ [4]. This physical principle allows us to transform into a frame S accelerating at the same rate as the gravitational source we are studying. The acceleration a of the reference frame S is shown in **Figure 1**, and is upward along the negative x_i axis as viewed by an

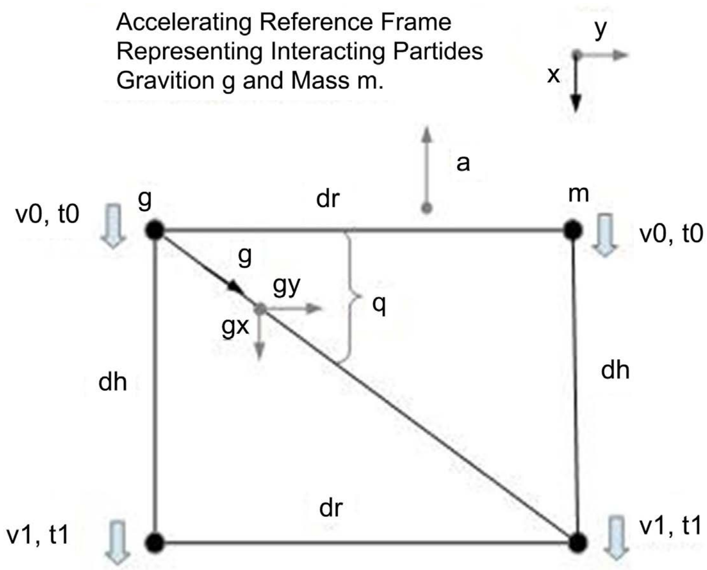

Figure 1. Interaction of graviton g and mass m in free fall relative to a reference frame accelerating at rate a in the direction of the negative x-axis.

inertial frame I. The infinitesimal travel time $\mathrm{d}t$, as viewed from an inertial frame I, is given by

$$\mathrm{d}t = \frac{\mathrm{d}r}{c}, \tag{1}$$

where $\mathrm{d}r$ is the separation distance along the y axis between the graviton due to mass M interacting with particle mass m, and c is the speed of gravity, where c is also the speed of light in vacuum. The graviton g and particle mass m are aligned along the x axis direction and are in free fall in the direction of the positive x axis of frame S. The acceleration a of frame S is small so that all free fall velocities are much less than the speed of light. Therefore, we can ignore the effects of Special Relativity and use Galilean and Newtonian physics.

In frame S at time t_0 the initial velocity of free fall is v_0. At time $t_1 = t_0 + \mathrm{d}t$ the free fall velocity is v_1 and the velocity change $\mathrm{d}v$ is

$$\mathrm{d}v = v_1 - v_0 = a(t_1 - t_0) = a\mathrm{d}t, \tag{2}$$

and the incremental distance $\mathrm{d}h$ of free fall of graviton g and mass m is

$$\mathrm{d}h = \frac{(v_1 - v_0)}{2}(t_1 - t_0) = \frac{a}{2}\mathrm{d}t^2, \tag{3}$$

where $a = \mathrm{d}v/\mathrm{d}t$. Referring to **Figure 1**, consider the path of graviton g traveling to particle m. The angle q that the graviton path makes with the y axis of frame S, without aberration, is given by

$$\tan(q) = \frac{\mathrm{d}h}{\mathrm{d}r}. \tag{4}$$

The speed of the graviton g is c and its square magnitude is given by

$$c^2 = gx^2 + gy^2, \tag{5}$$

where the component magnitudes in the x and y directions are given by, respectively,

$$gx = c\sin(q) \tag{6}$$

and

$$gy = c\cos(q). \tag{7}$$

Since the reference frame S is moving at a non-zero relative velocity $\mathrm{d}v = (v_1 - v_0) = a\mathrm{d}t$ when the observation is made at the time t_1, there will be an aberration of the graviton path viewed from S, analogous to the aberration of light [5]. The total angle of aberration $\Delta\varphi$ of the graviton g travel path for an observer in S, with respect to the mass m at the position (t_1, dh, dr) when the speed of free fall along the x axis is dv in frame S, is given by

$$\tan(\Delta\phi) = \frac{gx + \mathrm{d}v}{gy} = \frac{c\sin(q) + \mathrm{d}v}{c\cos(q)}. \tag{8}$$

Aberration of a Planet Orbiting a Star

Assume that the acceleration is equated to Newtonian gravitation, $a = GM/r^2$. Then, we have for the incremental velocity and distance traveled in the time interval $\mathrm{d}t = \mathrm{d}r/c$ between t_0 and t_1, where the graviton g travels to the particle m,

$$\mathrm{d}v = v_1 - v_0 = \left(\frac{GM}{r^2}\right)\mathrm{d}t = \left(\frac{GM}{cr^2}\right)\mathrm{d}r, \tag{9}$$

and

$$\mathrm{d}h = \left(\frac{GM}{2c^2r^2}\right)\mathrm{d}r^2, \tag{10}$$

and the angle that the photon makes is given by

$$\tan(q) = \frac{\mathrm{d}h}{\mathrm{d}r} = \left(\frac{GM}{2c^2r^2}\right)\mathrm{d}r. \tag{11}$$

Assuming that the angle q is very small in this analysis, as it would be for typical orbits around stars, we can make the approximations

$$\sin(q) \approx \tan(q) \approx \left(\frac{GM}{2c^2r^2}\right)\mathrm{d}r, \tag{12}$$

and

$$\cos(q) \approx 1. \tag{13}$$

Then, from (8), the aberration $\Delta\varphi$ of the graviton ray as seen at time t_1 in frame S at the particle m position is given by

$$\Delta\phi \approx \tan(\Delta\phi) = \frac{c\sin(q) + \mathrm{d}v}{c\cos(q)} = \left(\frac{GM}{2c^2r^2}\right)\mathrm{d}r + \left(\frac{GM}{c^2r^2}\right)\mathrm{d}r = \left(\frac{3}{2}\right)\left(\frac{GM}{c^2r^2}\right)\mathrm{d}r. \tag{14}$$

Assume that the mass m is a planet orbiting a star of mass M. The orbit will be an ellipse with periastron Rp and apastron Ra. The aberration measures the effect of the delay in response of the mass m to mass M, which appears as a phase shift in the orbit. The total aberration $\delta\Psi$ for a full orbit is obtained by multiplying by $\mathrm{d}\varphi$ and integrating over φ from 0 to 2π and over r between positive distances Rp and Ra, expressed by

$$\begin{aligned}\delta\Psi &= \int \Delta\phi \mathrm{d}\phi \\ &= \left(\frac{3GM}{2c^2}\right)\int_0^{2\pi}\left[\int_{Rp}^{\infty}\frac{\mathrm{d}r}{r^2}+\int_{Ra}^{\infty}\frac{\mathrm{d}r}{r^2}\right]\mathrm{d}\phi, \\ &= \left(\frac{3\pi GM}{c^2}\right)\left(\frac{1}{Rp}+\frac{1}{Ra}\right)\end{aligned} \quad (15)$$

where, since only the minimum and maximum radial distances are required, the integrals over r are integrated between from Rp to ∞ and from Ra to ∞. Equation (15) is also expressible in the form

$$\begin{aligned}\delta\Psi &= \int \Delta\phi \mathrm{d}\phi = \int_0^{2\pi}\int_r^{\infty}\left[\left(\frac{3GM}{2c^2u^2}\right)2\mathrm{d}u\right]\mathrm{d}\phi = \int_0^{2\pi}\left(\frac{3GM}{c^2 r}\right)\mathrm{d}\phi \\ &= \int_0^{2\pi}\left(\frac{3GM}{Lc^2}\right)\left[1+\varepsilon\cos(\phi)\right]\mathrm{d}\phi = \frac{6\pi GM}{Lc^2}\end{aligned} \quad (16)$$

where $1/L = 1/A\left(1-\varepsilon^2\right) = (1/2)(1/Rp + 1/Ra)$ is the semilatus rectum, and where A is the semi-major axis and ε is the eccentricity of the orbit. For the integration over temporary variable u, the full integral is twice the value of the integral with u going from to r to ∞, one for the minimum and one for the maximum radial distances, hence the differential expression ($2\mathrm{d}u$). This is similar to what was done in (15). Equations (15) and (16) both give the standard expression for the advancement of the periastron of the planet's orbit [6].

3. Photons Grazing a Star Analyzed in an Accelerating Reference Frame

Now we seek to examine some essential properties of a gravitating system of a photon p passing by a mass M, such as a star. Again, as in the previous sections, we invoke the equivalence principle of inertial and gravitational mass and transform into a frame S accelerating at the same rate as the gravitational property of the star we are studying. The acceleration a of the reference frame S is shown in **Figure 2**, and is upward along the negative x_i axis as viewed in an inertial frame I. As before, the infinitesimal travel time $\mathrm{d}t$, as viewed from the inertial frame I, is given by

$$\mathrm{d}t = \frac{\mathrm{d}r}{c}, \quad (17)$$

where $\mathrm{d}r$ is the separation distance along the y axis between the graviton due to mass M interacting with the photon of mass m, and c is the speed of gravity, where c is also the speed of light in vacuum. The photon mass $m = p/c$ where p is the photon momentum. The graviton g and photon p are aligned along the x axis direction and are in free fall in the direction of the positive x axis of frame S. The acceleration a of frame S is small so that all free fall velocities are much less

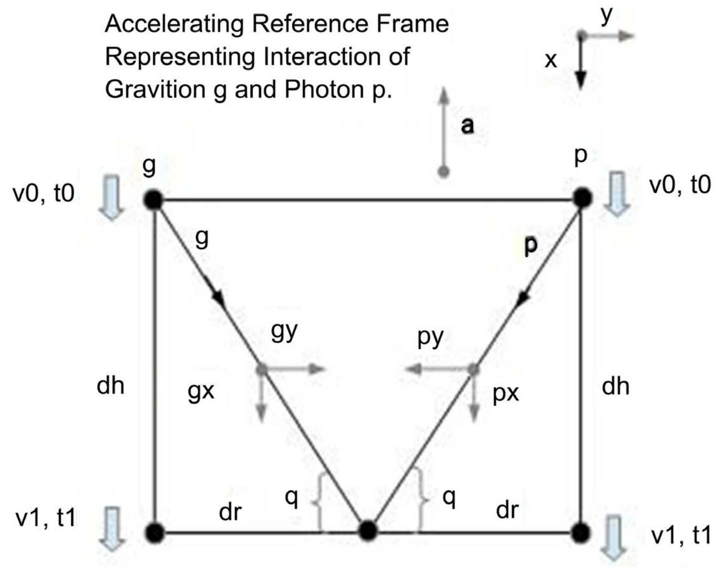

Figure 2. Interaction of graviton g and photon p in free fall relative to a reference frame accelerating at rate a in the direction of the negative x-axis.

than the speed of light. Therefore, we can ignore the effects of Special Relativity and use Galilean and Newtonian physics.

In frame S at time t_0 the initial velocity of free fall is v_0. At time $t_1 = t_0 + \mathrm{d}t$ the free fall velocity change $\mathrm{d}v$ in S is

$$\mathrm{d}v = v_1 - v_0 = a\left(t_1 - t_0\right) = a\mathrm{d}t\,, \tag{18}$$

and the incremental distance $\mathrm{d}h$ of free fall of graviton g and mass m is

$$\mathrm{d}h = \frac{\left(v_1 - v_0\right)}{2}\left(t_1 - t_0\right) = \frac{a}{2}\mathrm{d}t^2\,. \tag{19}$$

Referring to Figure 2., consider the path of graviton g traveling to photon p. The angle q that the graviton path makes with the y axis of frame S, without aberration, is given by

$$\tan\left(q\right) = \frac{\mathrm{d}h}{\mathrm{d}r}\,. \tag{20}$$

The speed of the graviton g is c and its square magnitude is given by

$$c^2 = gx^2 + gy^2\,, \tag{21}$$

where the component magnitudes in the x and y directions are given by, respectively,

$$gx = c\sin\left(q\right) \tag{22}$$

and

$$gy = c\cos\left(q\right). \tag{23}$$

Looking at Figure 2, consider the path of photon p traveling to graviton g. The angle q that the photon path makes with the y axis of frame S, without aberration, is given by

$$\tan(q) = \frac{dh}{dr}. \tag{24}$$

The speed of the photon p is c and its square magnitude is given by

$$c^2 = px^2 + py^2, \tag{25}$$

where the component magnitudes in the x and y directions, identical with the graviton, are given by, respectively,

$$px = c\sin(q) \tag{26}$$

and

$$py = c\cos(q). \tag{27}$$

Since the reference frame S is moving at a non-zero velocity $dv = (v_1 - v_0) = adt$ when the observation is made at the time t_1, there will be an aberration of the graviton and photon paths for an observer in S. The apparent angle $\Delta\varphi$ that the photon path makes with the y axis relative to the star of mass M at the interaction point (t_1, dh, dr), as observed in frame S, depends on the graviton angle, because the graviton gives the direction to the source. The tangent is given by the sum of the graviton and photon speeds in the x direction, plus the free fall speed relative to S, over the photon's speed in the y direction, expressed by,

$$\tan(\Delta\phi) = \frac{px + gx + dv}{py} = \frac{2c\sin(q) + dv}{c\cos(q)}. \tag{28}$$

Deflection of a Photon Beam Grazing a Star

Referring to **Figure 2**, assume that the acceleration is equated to Newtonian gravitation, $a = GM/r^2$. Then, we have for the velocity and distance traveled in the time interval $dt = dr/c$ between t_0 and t_1, where the graviton g travels to the photon p and the photon p travels to the graviton g,

$$dv = \left(\frac{GM}{r^2}\right)dt = \left(\frac{GM}{cr^2}\right)dr, \tag{29}$$

and

$$dh = \left(\frac{GM}{2c^2r^2}\right)dr^2. \tag{30}$$

The angle that the graviton and the photon paths each make with the *y-axis*, using (19), is given by

$$\tan(q) = \frac{dh}{dr} = \left(\frac{GM}{2c^2r^2}\right)dr. \tag{31}$$

Assuming that the angle q is very small in this analysis, as it would be for typical photons grazing a star, we can make the approximations

$$\sin(q) \approx \tan(q) = \left(\frac{GM}{2c^2r^2}\right)dr, \tag{32}$$

and

$$\cos(q) \approx 1. \tag{33}$$

Then, the total aberration $\Delta\varphi$ of the photon interacting with the graviton as seen at time t_1 in frame S at the interaction position (t_1, dh, dr), from (28), is given by

$$\Delta\phi \approx \tan(\Delta\phi) = \frac{2\sin(q)}{\cos(q)} + \frac{\mathrm{d}v}{c\cos(q)} = \left(\frac{GM}{c^2r^2}\right)\mathrm{d}r + \left(\frac{GM}{c^2r^2}\right)\mathrm{d}r = \left(2\frac{GM}{c^2r^2}\right)\mathrm{d}r \,. \tag{34}$$

Assume that the photon p is grazing a star of mass M and approaches the star from a great distance, making a closest approach of R from the center of the star and then continues away from the star to a great distance where observation of the total aberration angle (deflection angle) is determined. The total aberration $\delta\Psi$ for the photon's observed path is given by

$$\delta\Psi = \int \Delta\phi = \int_\infty^R \left(\frac{2GM}{c^2r^2}\right)(-\mathrm{d}r) + \int_R^\infty \left(\frac{2GM}{c^2r^2}\right)(+\mathrm{d}r) = \frac{4GM}{c^2R}, \tag{35}$$

where we specified $-\mathrm{d}r$ in the first integral because the integration is in the direction of decreasing r, whilst the second integral specified $+\mathrm{d}r$ for an integration with increasing r. Equation (35) is the standard expression for the deflection of a beam of light from a distant source grazing a star [6].

4. Graviton Physics

Relative to an accelerating reference frame, for a small increment in velocity δv due to the motion of the frame, the change observed in the angular frequency ω due to the Doppler effect upon a graviton in free fall in the frame is given by

$$\delta\omega = -\left(\frac{\delta v}{c}\right)\omega = -\left(\frac{a\delta t}{c}\right)\omega, \tag{36}$$

where the change in the frequency $\delta\omega$ is negative since it is red shifted because the velocity change δv is in the same direction as the motion of the graviton. The change in energy of the graviton is given by

$$\delta\xi = \hbar\delta\omega = -\left(\frac{a\delta t}{c}\right)\hbar\omega, \tag{37}$$

where $\hbar$ is Planck's constant over 2π. The change in momentum of the graviton is then given by

$$\delta p = \frac{\delta\xi}{c}. \tag{38}$$

The rate of momentum change is the force, which is given by

$$f = \frac{\delta p}{\delta t} = \frac{\delta\xi}{c\delta t} = -\left(\frac{a\delta t}{c^2}\right)\frac{\hbar\omega}{\delta t} = -\frac{a\hbar\omega}{c^2}. \tag{39}$$

Substituting $a = GM/r^2$, we get

$$f = -\left(\frac{GM}{c^2r^2}\right)\hbar\omega = \frac{-m_g MG}{r^2}, \tag{40}$$

where the mass m_g of the graviton is given by

$$m_g = \frac{\hbar\omega}{c^2}. \tag{41}$$

For the energy change,

$$\delta\xi = \hbar\delta\omega = -\left(\frac{a\delta t}{c}\right)\hbar\omega = \left(\frac{-GM}{cr^2}\right)\left(\frac{\delta r}{c}\right)\hbar\omega. \tag{42}$$

Assuming $\delta r = \lambda = 2\pi c/\omega$, where λ is the graviton wavelength, we obtain

$$\delta\xi = \left(\frac{-GM}{cr^2}\right)\left(\frac{2\pi}{\omega}\right)\hbar\omega = -\hbar\Omega(M,r), \tag{43}$$

where $\delta\xi$ is the energy decrease in the graviton due to a decrease in its frequency, where

$$\Omega(M,r) = \frac{2\pi GM}{cr^2} \tag{44}$$

is the frequency of that energy decrease in the graviton. The graviton's free fall velocity change δv in the gravity field GM/r^2 is always in the same direction as the graviton's motion, implying that the frequency change $\delta\omega < 0$ and the graviton's energy always decreases.

5. Period Decay of a Binary Star System

In the astronomical observation of a binary pulsar, as in a neutron star binary system, a key equation to obtain is the expression for the evolution of the orbital period T as a function of time t. The starting point is Kepler's third law given by

$$T^2(t) = \frac{4\pi^2 A^3(t)}{G(M_1 + M_2)}, \tag{45}$$

where G is Newton's gravitational constant, M_1 and M_2 are the masses which are assumed to remain constant, and A is the semi-major axis. Taking the derivative of (45) with respect to time, we get

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \left(\frac{3}{2T}\right)\left[\frac{4\pi^2 A^2}{G(M_1 + M_2)}\right]\frac{\mathrm{d}A}{\mathrm{d}t}. \tag{46}$$

Another equation we utilize is for the orbital energy E, given by

$$E = \frac{-GM_1M_2}{2A}. \tag{47}$$

Take the time derivative of (47), yielding

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \left(\frac{GM_1M_2}{2A^2}\right)\frac{\mathrm{d}A}{\mathrm{d}t}. \tag{48}$$

Substituting for $\mathrm{d}A/\mathrm{d}t$ from (48) into (46) and simplifying, we get

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \left(\frac{3}{T}\right)\left[\frac{4\pi^2 A^4}{G^2 M_1 M_2 (M_1 + M_2)}\right]\frac{\mathrm{d}E}{\mathrm{d}t}. \tag{49}$$

Equation (37) defines the energy change for a single graviton of frequency ω. We submit that all gravitons experience the same frequency decrease due to

Doppler redshift. Then, given that the total graviton energy Ξ is included in the total orbital energy E of (47), the change in the orbital energy due to the redshift of the graviton energy is expressed by

$$\frac{\mathrm{d}E}{\mathrm{d}t}=\frac{\delta\Xi}{\delta t}=-\Xi\left[\frac{GM_1M_2}{c(M_1+M_2)r^2}\right]=-\Xi\left[\frac{GM_1M_2\left(1+\varepsilon\cos(\phi')\right)^2}{c(M_1+M_2)\left(1-\varepsilon^2\right)^2A^2}\right], \quad (50)$$

where we applied the solution,

$$\frac{1}{r}=\frac{\left[1+\varepsilon\cos(\phi')\right]}{\left(1-\varepsilon^2\right)A}, \quad (51)$$

where ε is the orbit eccentricity and the phase

$$\phi'=\phi\left[1-3G(M_1+M_2)/c^2\left(1-\varepsilon^2\right)A\right], \quad (52)$$

where ϕ is the true anomaly, accounts for the periastron advancement. $\mathrm{d}E/\mathrm{d}t$ is the decrease in the graviton energy due to Doppler redshifting of the energy of the gravitons in free fall relative to the equivalent accelerating reference system.

We do not yet possess an expression for the total graviton energy, but we can use physical principles to try to approximate it. We begin with a constant of the motion of an elliptic orbit,

$$T\left(\frac{r^2}{2}\right)\frac{\mathrm{d}\phi}{\mathrm{d}t}=\pi AB=\frac{\pi L^2}{\left(1-\varepsilon^2\right)^{3/2}}, \quad (53)$$

where the semi-major axis $A=L/\left(1-\varepsilon^2\right)$, the semi-minor axis $B=L/\sqrt{1-\varepsilon^2}$ and L is the semi-latus rectum. Transform (53) in the following way and obtain

$$\left[\frac{M_1M_2}{(M_1+M_2)}\right]\left[\frac{G(M_1+M_2)}{c^2}\right]^2\left(\frac{AB}{L^2T^2}\right)=\left[\frac{M_1M_2}{(M_1+M_2)}\right]\left[\frac{G(M_1+M_2)}{\left(1-\varepsilon^2\right)^{3/4}c^2T}\right]^2. \quad (54)$$

Examination of (54) reveals that it is in the form of kinetic energy mv^2. Assuming the total energy of the gravitons is proportional to this, multiply the right hand side of (54) by $4k_0\pi^2/3\approx 64\pi^2/3$, where $k_0\approx 16$ is the fitted parameter, and define the total graviton energy Ξ by

$$\Xi=\left(\frac{64\pi^2}{3}\right)\left[\frac{M_1M_2}{(M_1+M_2)}\right]\left[\frac{G(M_1+M_2)}{\left(1-\varepsilon^2\right)^{3/4}c^2T}\right]^2, \quad (55)$$

where the first factor on the right hand side of (55) also accounts for the dual polarity of the gravitons.

Substituting (55) into (50) we get for $\mathrm{d}E/\mathrm{d}t$,

$$\frac{\mathrm{d}E}{\mathrm{d}t}=-\left(\frac{64\pi^2}{3}\right)\left[\frac{M_1M_2}{c^5(M_1+M_2)}\right]\left[\frac{\left(G(M_1+M_2)\right)^2\left(1+\varepsilon\cos(\phi')\right)^2}{\left(1-\varepsilon^2\right)^{7/2}}\right]\times\left[\frac{GM_1M_2}{(M_1+M_2)T^2A^2}\right]. \quad (56)$$

Inserting $\mathrm{d}E/\mathrm{d}t$ from (56) into (49) and substituting for $A=\left[G\left(M_1+M_2\right)T^2/4\pi^2\right]^{1/3}$, with some simplification, yields

$$\frac{\mathrm{d}T}{\mathrm{d}t}=-32\pi\left[\frac{\left(1+\varepsilon\cos\left(\phi'\right)\right)^2}{\left(1-\varepsilon^2\right)^{7/2}}\right]\left[\frac{\left(M_1+M_2\right)^2}{M_1M_2}\right]^{2/3}\left[\frac{2\pi GM_1M_2}{\left(M_1+M_2\right)c^3T}\right]^{5/3}. \quad (57)$$

Equation (57) is equivalent in form to the derivation given by [7], which was derived from General Relativity with gravitational wave emission for energy decay. For comparison, we put that equation into the form

$$\dot{P}_b=\frac{-192\pi}{5}\left[\frac{\left(1+\left(73/24\right)\varepsilon^2+\left(37/96\right)\varepsilon^4\right)}{\left(1-\varepsilon^2\right)^{7/2}}\right]\left[\frac{\left(m_p+m_c\right)^2}{m_pm_c}\right]^{2/3}\left[\frac{2\pi Gm_pm_c}{\left(m_p+m_c\right)c^3P_b}\right]^{5/3}, \quad (58)$$

where $P_b=T$, $\dot{P}_b=\mathrm{d}T/\mathrm{d}t$, $m_p=M_1$ and $m_c=M_2$. Except for the dependence on the eccentricity ε and proportionality factor, (57) and (58) are identical, due to the choices made for the form of the mass dependencies in (54).

Application to PSR B1913+16 and Other Binaries

We look at the report [8] on B1913+16, the Hulse-Taylor binary pulsar. This astronomical endeavor spanned 30 years of approximately yearly observations of the binary system.

The data for the system is as follows:

$$m_p=1.4408\pm0.0003M_{sol},$$
$$m_c=1.3873\pm0.0003M_{sol},$$
$$P_b=0.322997462727\pm5\times10^{-12}\,\text{day},$$
$$\dot{P}_b=\left(-2.4211\pm0.0014\right)\times10^{-12}\,\mathrm{s\cdot s^{-1}},$$
$$\varepsilon=0.6171338\pm0.0000004,$$

where m_p is the primary mass and m_c is the companion mass, $M_{sol}=1.988470\times10^{30}$ kg is the solar mass, P_b is the binary orbital period, $\dot{P}_b$ is the orbital period change and ε is the orbital eccentricity.

For this paper we refer to the masses as primary M_1 and companion M_2, the orbital period as T, the period change as $\mathrm{d}T/\mathrm{d}t$ and the eccentricity as ε. The initial orbital period is $T_0=P_b=0.322997462727$ day. For the gravitational constant we use the value $G=6.67430\times10^{-11}\,\mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}$ and $c=299792458\,\mathrm{m\cdot s^{-1}}$ for the speeds of gravity (graviton) and light (photon) in vacuum. Substituting these experimental values into (57) yields

$$\frac{\mathrm{d}T}{\mathrm{d}t}=-2.36415\times10^{-12}\,\mathrm{s\cdot s^{-1}}, \quad (59)$$

which is close to the GR theoretical value,

$$\dot{P}_{bGR}=\left(-2.40247\pm0.00002\right)\times10^{-12}\,\mathrm{s\cdot s^{-1}}. \quad (60)$$

and is also a fair match to the experimental value of Weisberg & Taylor (2005) of $\dot{P}_b=\left(-2.42011\pm0.0014\right)\times10^{-12}\,\mathrm{s\cdot s^{-1}}$.

In **Table 1**, we list eight PSR's, [8]-[16].

Table 1. Table of PSR binary systems. Column: description; 1) PSR name; 2) Pulsating star mass (solar mass) 3) Companion star mass (solar mass); 4) Orbit eccentricity; 5) Period (day) P_b or T; 6) $\mathrm{d}P_b/\mathrm{d}t$ (observed intrinsic value); 7) $\mathrm{d}P_b/\mathrm{d}t$ (General Relativity computation); 8) $\mathrm{d}T/\mathrm{d}t$ (due to graviton Doppler frequency shift).

PSR	M_1	M_2	Eccen	P_b (T)	$\mathrm{d}P_b/\mathrm{d}t$ intr	$\mathrm{d}P_b/\mathrm{d}t$ GR	$\mathrm{d}T/\mathrm{d}t$
	(Msol)	(Msol)		(day)	(10^{-12} s·s^{-1})	(10^{-12} s·s^{-1})	(10^{-12} s·s^{-1})
B1913+16	1.4408	1.3900	0.6171	0.3230	−2.4021	−2.4022	−2.3641
B1534+12	1.3332	1.3452	0.2737	0.4207	−0.1924	−0.1924	−0.2115
J1756-2251	1.341	1.230	0.1806	0.3196	−0.234	−0.2169	−0.2291
J0737-3039	1.3381	1.2489	0.0878	0.1023	−1.252	−1.2478	−1.2022
J1906+0746	1.291	1.322	0.0853	0.1660	−0.5650	−0.5645	−0.5421
J1141-6545	1.27	1.02	0.17	0.20	−0.403	−0.391	−0.410
J1012+5307	1.72	0.165	1.2×10^{-6}	0.6046	-1.5×10^{-2}	-1.158×10^{-2}	-0.965×10^{-2}
J0621-1002	1.70	0.97	0.0025	8.3187	<−5	-7.59×10^{-4}	-6.35×10^{-4}

For all the PSR's in Table 1, the mean error and unbiased standard deviation of the mean error between the observed intrinsic $\mathrm{d}P_b/\mathrm{d}t$ values and this paper's predicted $\mathrm{d}T/\mathrm{d}t$ values are $\text{Prediction Mean Error} = 0.1988 \pm 0.1178$. For a comparison with the standard GR gravitational wave emission theory, the mean error and unbiased standard deviation of the mean error are $\text{GR Prediction Mean Error} = 0.1669 \pm 0.1193$.

6. Quantized Graviton Energy Distribution

From (55) with (45) we obtain the total graviton energy Ξ in terms of the semi-major axis A,

$$\Xi = \frac{16}{3\left(1-\varepsilon^2\right)^{3/2}}\left[\frac{M_1 M_2 c^2}{\left(M_1+M_2\right)}\right]\left[\frac{G\left(M_1+M_2\right)}{c^2 A}\right]^3. \tag{61}$$

Assume that the gravitons in a binary star system are confined to a conical frustum volume contained by the space between the two stars in the orbit. The volume V depends on the distance between the stars, r, and the diameters d_1 and d_2 of the masses M_1 and M_2, respectively. The volume is given by,

$$V = \frac{\pi r}{12}\left(d_1^2 + d_1 d_2 + d_2^2\right). \tag{62}$$

Then, with the graviton energy density $\rho = \Xi/V$, the graviton field temperature T is obtained from the Stephan-Boltzmann law,

$$T = \left(\frac{\rho}{a_B}\right)^{1/4}, \tag{63}$$

where $a_B = \pi^2 k_B^4 / 15 c^3 \hbar^3$ and k_B is Boltzmann's constant. Since gravitons are bosons of spin 2, they can be described by the Bose-Einstein distribution, identical to the photons. Then, the energy of gravitons in the frequency interval

$[\omega, \omega + \mathrm{d}\omega]$ per unit volume is given by,

$$u(\omega)\mathrm{d}\omega = \left(\frac{2\hbar\omega^3}{\pi c^3}\right)\left[\frac{\mathrm{d}\omega}{\left(\exp\left(\hbar\omega/k_B T\right)-1\right)}\right]. \tag{64}$$

By Wein's Displacement law, the frequency ω_p where $u(\omega)$ peaks is given by

$$\omega_p \approx \frac{2.82144 k_B T}{\hbar}. \tag{65}$$

For B1913+16, assuming the mass diameters are $D \approx d \approx 22$ km, by (62)-(65), the temperature of the graviton field is $T = 1.778 \times 10^7$ K and the peak density is at frequency

$$\omega_p = 6.568 \times 10^{18}\ \mathrm{s}^{-1}$$

where $u(\omega_p) = 44.686 \times 10^{-6}\ \mathrm{J \cdot s \cdot m^{-3}} = 278.910 \times 10^6\ \mathrm{eV \cdot s \cdot cm^{-3}}$.

In **Table 2** we list the characteristics of the graviton Bose-Einstein statistics for the eight binary pulsars. Most binary NS diameters are assumed to be 22 km. PSR J0737-3039 is an eclipsing binary which enabled the determination of the companion diameter (18.1 km) [11]. Also, PSR J1012+5307 has a white dwarf companion whose diameter was also determined $(0.094 R_{sol})$ [15].

7. Gravitons as the Source of Dark Matter and Dark Energy

Consider the universe as a sphere of interior mass M with a thin spherical shell of mass m. The masses M and m are constants. The thin shell has a radius $r(t)$ at time t. Only the mass interior to the shell has an effect on the shell.

Define the graviton energy $\Xi(t)$ within the shell at time t by

$$\Xi(t) = \frac{GMm}{r(t)}. \tag{66}$$

Table 2. Characteristics of the graviton Bose-Einstein distributions. Column: description; 1) PSR name; 2) Primary mass diameter; 3) Companion mass diameter; 4) Temperature of graviton field; 5) Frequency at peak density; 6) Peak density (Joules); 7) Peak density (electron volts). PSR J0737-3039 is an eclipsing binary which enabled the companion diameter to be determined (Kaspi *et al.* 2004). PSR J1012+5307 has a white dwarf companion whose diameter was also determined (Mata Sánchez *et al.* 2020).

PSR	Prim. diam	Comp. diam	Temp.	Peak freq	u (peak freq)	u (peak freq)
	km	km	K (10^7)	s^{-1} (10^{18})	$\mathrm{J \cdot s \cdot m^{-3}}$ (10^{-6})	$\mathrm{eV \cdot s \cdot cm^{-3}}$ (10^6)
B1913+16	22	22	1.78	6.57	44.69	278.91
B1534+12	22	22	1.06	3.90	9.36	58.41
J1756-2251	22	22	1.17	4.33	12.81	79.95
J0737-3039	22	18.1	2.54	9.39	130.65	815.46
J1906+0746	22	22	1.78	6.56	44.51	277.81
J1141-6545	22	22	1.49	5.49	26.09	162.87
J1012+5307	22	6.54×10^4	1.01×10^{-2}	3.75×10^{-2}	8.31×10^{-6}	5.18×10^{-5}
J0621-1002	22	22	1.24×10^{-1}	4.59×10^{-1}	1.52×10^{-2}	9.51×10^{-2}

7.1. Dark Matter

Now we account for the gravitational redshift of gravitons in the galaxies which are the points of masses in the universe we are imagining, the possible source of dark matter [17]. The energy loss $\Delta\Xi_{dm}(t)$ due to the redshift of graviton energy in galaxies is modeled by the expression

$$\Delta\Xi_{dm} = \Xi(t)(-\gamma) = \frac{GMm}{r(t)}(-\gamma), \tag{67}$$

where γ is a constant which can be determined as follows: for a galaxy of mass M_k, the loss in gravitational energy due to the gravitational redshift of a graviton of energy ξ_k between the distances $r_{0,k}$ and $r_{1,k}$, where $r_{0,k} \le r_{1,k}$, is given by,

$$\Delta\xi_k = \left(\frac{\xi_k}{c^2}\right)\left(\frac{GM_k}{r_{1,k}} - \frac{GM_k}{r_{0,k}}\right) = \left(\frac{-\xi_k GM_k}{c^2}\right)\left[\frac{r_{1,k} - r_{0,k}}{r_{1,k} r_{0,k}}\right] = \left[\frac{-\xi_k GM_k}{c^2 r_{1,k}}\right](\gamma_k), \tag{68}$$

where

$$\gamma_k = \frac{(r_{1,k} - r_{0,k})}{r_{0,k}}. \tag{69}$$

The distance $r_{0,k}$ for any galaxy is identified by observing the leveling of the velocity rotation curve [18]. We show in a subsequent section how r_0 can be determined for any galaxy for which the Tully-Fisher relation is known [19]. This feature may not be observable for every galaxy but energy loss due to graviton redshift is assumed to affect every galaxy.

Then, for N galaxies, γ in (67) can be approximated by

$$\gamma = \frac{1}{N}\sum_{k=1}^{N}\gamma_k. \tag{70}$$

7.2. Dark Energy

We define the *cosmological graviton energy loss* of $\Delta\Xi_{de}(t)$ at time t by the expression

$$\Delta\Xi(t)_{de} = \Xi(t)\left(\frac{-\beta c^2 r^3(t)}{6GM}\right) = -\frac{1}{6}\left(mc^2\beta r^2(t)\right), \tag{71}$$

where β is a constant with dimensionality $[\text{length}]^{-2}$ and the second factor of the middle equality,

$$\frac{-\beta c^2 r^3(t)}{6GM}, \tag{72}$$

is the *cosmological graviton energy loss rate*, which has the dimensionless form $-[\text{velocity}]/c$.

7.3. Equation of the Expanding Universe

The total energy of the shell of mass m, having kinetic energy, gravitational potential energy, galaxies graviton redshift energy loss (67) and the cosmological graviton energy loss (71), is expressed by

$$\frac{1}{2}mv^2 - \frac{GMm}{r} + \Delta\Xi_{dm} + \Delta\Xi_{de} \\ = \frac{1}{2}mv^2 - \frac{GMm}{r} - \frac{\gamma GMm}{r} - \frac{1}{6}mc^2\beta r^2 = -\frac{1}{2}mc^2 k\bar{a}^2 \tag{73}$$

where k is a constant (curvature) with dimensionality [length]$^{-2}$ and $\bar{a}$ is the constant comoving distance. Define the interior mass M (baryon, non-relativistic mass) by

$$M = \left(\frac{4\pi r^3(t)}{3}\right)\rho(t), \tag{74}$$

where $\rho(t)$ is the mass density. Since the mass M is constant, this implies that $\rho(t) \propto r^{-3}(t)$. Substituting (74) for M and multiplying (73) by $2/mr^2$ and simplifying, we get the expression for the expansion of the shell,

$$\frac{v^2}{r^2} = \frac{8\pi G}{3}(1+\gamma)\rho + \frac{\beta c^2}{3} - \frac{kc^2\bar{a}^2}{r^2}. \tag{75}$$

Define the distance r by

$$r = \bar{a}a, \tag{76}$$

where $\bar{a}$ is the constant comoving distance and the time varying scale factor a is dimensionless. Using (76), the velocity v takes the form

$$v = \frac{\mathrm{d}r}{\mathrm{d}t} = \bar{a}\frac{\mathrm{d}a}{\mathrm{d}t}. \tag{77}$$

Substituting (77) into (75) our expansion equation takes the form

$$\left(\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 = \frac{8\pi G}{3}(1+\gamma)\rho + \frac{\beta c^2}{3} - \frac{kc^2}{a^2}. \tag{78}$$

Except for the dark matter component (γ), (78) is identical to the Friedmann equation [20] if we take

$$\beta = \Lambda, \tag{79}$$

where Λ is the cosmological constant of the General Relativity theory.

Physically, since $\left|\Delta\Xi(t)_{de}\right| \le \Xi(t)$ then $0 \le \left|\frac{\Delta\Xi(t)_{de}}{\Xi(t)}\right| \le 1$. From (71), applying (74) for baryon mass M, this implies that, at the present epoch,

$$0 \le \beta \le \frac{6GM}{c^2 r^3} = \frac{8\pi G\rho_B}{c^2} \approx \frac{0.15H_0^2}{c^2}, \tag{80}$$

where we used the approximation for the baryon density $\rho_B \approx 0.05\rho_c \approx \left(0.15H_0^2/8\pi G\right)$, where $\rho_c = 3H_0^2/8\pi G$ is the critical mass density.

To obtain the acceleration of the expansion, assuming $\rho = \rho_B/a^3$, take the time derivative of (78) and simplify the result to obtain

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} = -\left(\frac{4\pi G}{3}\right)(1+\gamma)\frac{\rho_B}{a^2} + \left(\frac{\beta c^2}{3}\right)a. \tag{81}$$

As scale factor a increases, the mass density term in (81) decreases in magnitude and the acceleration rate increases positively by

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} \approx \left(\frac{\beta c^2}{3}\right) a, \tag{82}$$

which expresses the accelerated expansion of the universe.

8. Spiral Galaxy Rotation Curves

The graviton energy gravitational redshift in a spiral galaxy can be shown to cause the flattening of the galaxy rotation velocity curve, the phenomenon which initiated the need for dark matter [18]. The gravitational redshift of the energy of gravitons between two radial distances r_0 and r, where $r_0 \le r$ is given by

$$\Delta\Xi_{dm} = \frac{GMm}{r} - \frac{GMm}{r_0} = \frac{-GMm}{r}\left[\frac{(r-r_0)}{r_0}\right], \tag{83}$$

where M is the galaxy mass within radial distance r and m is the mass of a particle in orbit with radius r. In the interaction of interior galaxy mass M with object mass m, we require the total mass of gravitons m_g in the interaction equal m, *i.e.*; $m_g = m$. The equation for the energy of an object orbiting a galaxy, including (83), is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{r} + \Delta\Xi_{dm} = \frac{1}{2}mv^2 - \frac{GMm}{r} - \frac{GMm}{r}\left(\frac{r-r_0}{r_0}\right) = -E. \tag{84}$$

Multiplying (84) by $2/m$ and putting the velocity on the left hand side, and simplifying, we have, for $r_0 \le r$,

$$v^2 = \frac{2GM}{r} - \frac{2GM}{r} + \frac{2GM}{r_0} - \frac{2E}{m} = \frac{2GM}{r}(1+\gamma) - \frac{2E}{m} = \frac{2GM}{r_0} - \frac{2E}{m}, \tag{85}$$

where $\gamma = (r-r_0)/r_0$, showing that the velocity becomes fixed at the value for radial distance r_0.

The baryon Tully-Fisher relation (BTFR) [21] is given by,

$$M_o = AV_o^x, \tag{86}$$

where M_o is the galaxy baryon mass in solar masses and V_o is the flat velocity in $\mathrm{km \cdot s^{-1}}$. The rotation curve parameters A and x are determined by extensive observations of spiral galaxies.

From [21], the value of parameters $x = 3.90 \pm 0.43 \approx 4$ and $\log(A) = 1.92 \pm 0.75$. Drop the ($2E/m$) term from (85) and square it to solve for mass M, which gives

$$M = \left(\frac{r_0^2}{4G^2 M}\right) V^4 = \left(\frac{1}{a_0 G}\right) V^4, \tag{87}$$

where

$$a_0 = \frac{4GM}{r_0^2} \tag{88}$$

is an acceleration. From (86) and (87) we get

$$a_0 = \frac{1}{AG}(\text{conversion factor}) = \left(\frac{1}{10^{1.92} G}\right)\left(\frac{1000^4}{M_{sol}}\right) \approx 1.222 \times 10^{-10}\ \text{m} \cdot \text{s}^{-2}, \quad (89)$$

where the *conversion factor* converts from $(\text{m/km})^4$/(Msolkg/Solar mass) since in (86), M is in kilograms and V is in [m·s^{-1}]. The definition of a_0 and its value are consistent with the MOND theory [22].

Though the effort to determine γ of (70) is beyond the scope of this present work, we may apply a bit of hypothetical experimentation by use of (69), $\gamma = (r_1 - r_0)/r_0$ for a range $r_1 = wr_0, 2 \le w \le 10$, uniformly distributed. This gives values for γ uniformly distributed in the range $1 \le \gamma \le 9$, which has an average value of $\gamma = 5$. The true value of γ is given by the ratio $\Omega_{dm}/\Omega_b \approx 0.2589/0.0486 = 5.327$ [23].

9. Conclusions

Applying the principle of equivalence has given us the shift in the orbital periastron and the stellar deflection of light. Both of these phenomena are well known, although we have derived these results in a non-traditional manner. The novel ingredient is in revealing the role that the graviton plays in all of this. The new result is in showing that, if they exist, the gravitons in a binary star system are continuously Doppler shifted, *i.e.*, gravitationally redshifted, to lower energies that lead to a decay in the period of the orbit, having comparable magnitudes as observed for the binary pulsar systems that we cited. This challenges the hypothesis that the orbital decays in binary pulsar systems are due to the emission of gravitational radiation.

We showed how the graviton field behaves as massless bosons in a volume contained by the two masses in orbit and that the peak frequencies of the gravitons are on the order of $\omega \approx 10^{18}\,\text{s}^{-1}$ for the binary systems studied and the temperatures are on the order of $T \approx 10^7\,\text{K}$.

We described how gravitational redshifts of gravitons can apply in the expansion of the universe, being the possible source for dark matter and dark energy. Gravitons fulfill the requirement of dark matter in that they interact with light and baryonic matter but, since they have yet to be observed, possibly do not emit light in the interaction process. And, graviton decay fulfills the behavior of dark energy, in which the gravitational energy loss results in an accelerated rate of expansion.

In the case of individual galaxies, where dark matter was invented to explain flat rotation curves, we show how the redshift of gravitons interacting with matter in the outer regions of a galaxy leads to a leveling of the orbital rotation velocity, in agreement with the baryonic Tully-Fisher relation.

The *Extended Theories of Gravity* [24], such as massless Scalar-Tensor Gravity and $f(R)$ theories, endeavors to quantify the phenomena of dark matter, dark energy and the flattening of galaxy rotation curves as well as predicting additional polarizations of gravitational radiation beyond the quadrupole predicted by General Relativity. Perhaps a theory for the graviton that includes the ideas discussed in this paper can emerge from these extended theories.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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