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The Theoretical Spectrum of Mass of the Light Mesons without Strangeness via the Quarks' Geometric Model

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Abstract

Using the "Aureum Geometric Model" (AGM) of quarks, we formulate the structure equations describing mesons and, by a mathematic procedure, we calculate the theoretical spectrum of mass values of light mesons without strangeness.

Keywords

Quark, Structure Equation, Geometric Structure, Golden Number, Massive Coupling, Interpenetration, IQuO, Pion, Meson, Photon

1. Introduction

In previous papers [1] [2] [3], we have shown quarks to be geometric structures of coupling quantum oscillators. By these quarks, we have built the mesons (π) and (η) [3] [4]. We will prove in this article that all light mesons not strange, following the pion, are elaborated structures of pion compositions and a $\{d, \underline{d}\}$-lattice of d-quarks. In essence, we refer to this as to the hypothesis of hadronic lattice. This hypothesis induces us to admit an appropriate structure equation [4], which describes all hadrons and thus also mesons. This new way of seeing the particles allows to solve the problem of hadron mass determined by quarks with small values of mass [5] [6]. In fact, by structure equations and an appropriate mathematics procedure [3] [4], we can calculate the masses of light mesons and also nucleons. Moreover, the structure equation allows us to identify the possible decays of a particle and its spin value. Nevertheless, one can achieve the idea of a geometric structure of quarks if we assume a quark made by a *non-separable* set of coupled quantum oscillators with "*Aurea*" (golden) geometric structure [1] [2] [3]. We referred to this as the Aureum Geometric Model (AGM) of quarks [4]. Thanks to the quarks geometric hypothesis, it is possible to explain fundamental issues such as the origin of the mass of the hadrons, the role of gluons in constructing bonded quarks and the existence of *molecules of pions*. We can achieve all of this without turning to the QCD and its calculations by colour potentials [7] [8]. Nevertheless, this new path in particles' theory needs to revise the mass conception in physics and to introduce a new idea of mass calculation ($\underline{\otimes}$-operation). This idea of calculation takes into account both interactions between quarks and a possible *interpenetration* [3] [4] of the quarks. Then, the global mass calculation must take into account all the possible configurations of quark components. So, the hypothesis of the quark geometrics structure introduces a "*new paradigm*" [4] in the phenomenology of hadronic interactions. The new paradigm allows to describe the hadrons in a more structured way and straight of the QCD. The first success of this idea is the possibility of calculating the mass spectrum of light mesons with values very close (if not even equal) to the experimental ones. In this paper, exactly, we will calculate, by theoretical physics aspects and mathematical procedure, the mass of mesons (η', ρ, ω, ϕ) which make, with the pions and η-meson, the light component but strangeness of the octet of fundamental mesons.

2. The Quarks' Interpenetration

2.1. The Elastic Tensions of Binding Gluons

The aims of this paper are to calculate the mass values of light mesons using a way different to one of QCD potentials. Instead of incorporating the binding energy of the quarks in suitable potentials [9], we incorporated [4] it into the quarks' mass which compose the pion. Nevertheless, we have not considered the bare masses of quarks [9], but appropriate values of them masses derived from a "geometric structure" of pions [2] [4] and of same quarks. These "dressed" mass values are very greater than that of free quarks. This way, the bonded quarks inside the hadron would have enough mass to reach the hadronic mass, provided that we correctly sum them up. Since the pion, the lightest meson is made of two bonded states of quarks, its mass could be placed as base mass to determine the mass of the most massive mesons and hadrons. Although the procedure is different from that of QCD, we can see that the two models (QCD and AGM) have some points of connection. In fact, in QCD [9] the quarks (u, d) would show an interaction potential [$V(r) \approx kr$], expressed by an interaction strong force with modality [$F \approx -k$], while, at a short distance, could be [$V(r) \approx (-4\alpha/3r + \ldots)$] with force [$F \approx 4\alpha/3r^2$]. This aspect would be similar to the interaction force in AGM, where the binding gluons in the geometric structure of two quarks (u, d) are equivalent to simple oscillators [10] with elastic force [$F \approx -k$]. If it is [$F \approx -kr$] then it would be better. Speaking of elastic tensions in bound quarks q_i, we can so admit that in the elastic parameters ($\underline{k}_i$) are contained the mass defects; in this way the parameter k replaces the potential $V(r)$).

2.2. The $\underline{\otimes}$-Operation

In the interpenetration of quarks and their dynamics interactions, we used, see ref. [3] [4], a new $\underline{\otimes}$-*operation* of combination (or coupling) of quarks and particles: ($a_i \underline{\otimes} b_j$). The new operation ($\underline{\otimes}$) indicates a composition of two operations ($\otimes$, $\oplus$) or [$\underline{\otimes} \equiv (\otimes \cup \oplus)$], where $\otimes$-operation describes the proper interpenetration of the quarks and follows, in algebraic calculations, the properties of multiplication. Instead, $\oplus$-operation describes dynamics interactions and follows, in algebraic calculations, the properties of the sum. In a table, see **Table 1**, we list some algebraic properties of the $\underline{\otimes}$-operation which will be used in the calculations of mesons' masses.

2.3. The F_m Function of Partial Mass and $F_{\Delta m}$ Mass Defect

In general, for every X-system (composed by more particles), we used [3] [4] a partial mass Function (F_m) applied to *structure equation* of X-system, with

$$X\left[\left(A_1, A_2, \cdots, A_n\right)_{\otimes}; \left(B_1, B_2, \cdots, B_n\right)_{\oplus}\right],$$

where $\left[\left(A_i\right)_{\otimes}, \left(B_j\right)_{\oplus}\right]$ are the "base components" of the structure. The (F_m) is an application on the structure components (A, B), which gives us the mass values

Table 1. Algebraic properties of two operations ($\otimes$, $\oplus$).

$\otimes$-operation $(a_i, a_j) \in \{a\}$	$\oplus$-operation $(a_i, a_j) \in \{a\}$
1) $(a_i \otimes a_i) = (a)$	1) $(a_i \oplus a_i) = 0$
2) $(a_i, a_j) \in \{a\}$ $(a_i \otimes a_j) = (a_j \otimes a_i)$	2) $(a_i \oplus a_j) = (a_j \oplus a_i)$ If $(a_i \oplus a_j)_{\oplus} = \left[(a_i \oplus a_j) \oplus a_k\right]$ $\rightarrow (a_i \oplus a_j)_{\oplus} \neq (a_j \oplus a_i)_{\oplus}$
3) $(n, m) \in N$ $(na_i \otimes a_j) = n(a_i \otimes a_j)$ $(na_i \otimes ma_j) = (n \times m)(a_i \otimes a_j)$	3) $\left[2(a_i) \oplus (a_j)\right] \equiv \left[(a_i) + (a_i)\right] \oplus a_j$ $= \left[(a_i \oplus a_j) + (a_i \oplus a_j)\right]$ $= 2(a_i \oplus a_j)$
4) transitive property if $\left[(a \otimes b), (b \otimes c)\right] \rightarrow \left[(a \otimes c)\right]$	4) transitive property if $\left[(a \otimes b), (b \otimes c), (a \otimes c)\right]$ if $\left[(a \oplus b), (b \oplus c)\right] \rightarrow \left[(a \oplus c)\right]$
5) $\left[(a_i \otimes a_j) \otimes a_k\right] = \left[(a_i \otimes a_k) \otimes a_j\right]$ $= \left[(a_k \otimes a_j) \otimes a_i\right]$	5) $\left[(a_i \oplus a_j) \oplus a_k\right] = \left[(a_i \oplus a_k) \oplus (a_j \oplus a_k)\right]$
$\underline{\otimes} \equiv [\otimes, \oplus] \leftrightarrow$ composed operation	
6) If $\left\{\left[A = (a \oplus b)\right], \left[B = (c \oplus d)\right]\right\} \rightarrow (A \underline{\otimes} B) = (a \oplus b) \otimes (c \oplus d)$	
7) $(a \oplus b) \otimes (c \oplus d) = (a \otimes c) \oplus (a \otimes d) \oplus (b \otimes c) \oplus (b \otimes d)$	
8) $\left[(a_i \otimes a_j) \oplus a_k\right] \neq \left[(a_j \otimes a_i) \oplus a_k\right] \leftrightarrow (a_i \otimes a_j)_{\oplus} \neq (a_j \otimes a_i)_{\oplus}$	

(m_i) of these components of base. The (F_m) operates on X, in the following way:

$$F_m(X)=\left\{\sum_{(i,j)=1}^{n}F_m\left[(A_i)_{(\otimes)},(B_j)_{(\oplus)}\right]\right\}=\left[\sum_{i=1}^{n}m(A_i)_{(\otimes)}\right]_A+\left[\sum_{j=1}^{m}m(B_j)_{(\oplus)}\right]_B$$
$$=\left[m(a\otimes b)_{A_1}+\cdots+m(w\otimes z)_{A_n}\right]_A+\left[m(a\oplus b)_{B_1}+\cdots+m(w\oplus z)_{B_m}\right]_B \tag{1}$$

We will have the following applications:

$$\left\{\begin{array}{c} F_m\left(A_{(\otimes)}\right)=F_m\left[(a)\otimes(b)\right]_A=\langle m(a,b)\rangle=\langle m(a),m(b)\rangle \\ F_m\left(B_{(\oplus)}\right)=F_m\left[(a)\oplus(b)\right]_B=m\left((a)\oplus(b)\right)=m(a)+m(b) \end{array}\right\} \tag{2}$$

Note that the mass of two interpenetrating particles $[a\otimes b]$ will be obtained by the average value of individual masses $[\langle m(a,b)\rangle]$, while the mass of two interacting particles $[a\oplus b]$ will be obtained by the sum of the masses of each particle. To obtain the total mass of a structure, it needs to add eventual (m_{kin}) relativistic kinetic mass and mass defects (Δm). To the exception of some cases (which we will specify) $m_{kin}\ll m_0$, therefore we will have: $\left[m_{tot}=m_{part}\pm\Delta m_i\right]$.

The mass defect will be:

$$\Delta m=\Delta m_g+\Delta m_{em}$$

where Δm_g is the mass defect due to binding by gluons. Nevertheless, the Δm_g has been englobed in masses of the charged pion (see ref. [3] [4]) or in quark (u_π, d_π)); therefore, we consider only electromagnetic mass defect: $\Delta m = \Delta m_{em}$.

To obtain the mass defects ($\Delta m > 0$, $\Delta m < 0$) we will use a Function ($F_{\Delta m}$) of mass defect applied to structure equation so defined:

$$\begin{aligned} &F_{\Delta m}(A_1,A_2,\cdots,A_n)\\ &=\left\{\sum_{(i,j)=1}^{n}F_{\Delta m}\left[(A_i)_{(\otimes)},(B_j)_{(\oplus)}\right]\right\}\\ &=\left[\sum_{i=1}\Delta m(A_i)_{(\otimes)}+\sum_{j=1}\Delta m(B_i)_{(\oplus)}\right]+\left[\sum_{(i=1,j=1)}^{n}\Delta m\left(A_i\oplus B_j\right)\right]_{(\text{I}^\circ\text{-degree})}\\ &+\left[\sum_{\left(\substack{i=1,j=1,k=1\\ i\neq j\neq k}\right)}^{n}\Delta m\left(A_i\oplus\left(A_j\oplus A_k\right)\right)\right]_{(\text{II}^\circ\text{-deg.})}\\ &+\left[\sum_{\left(\substack{i=1,j=1,k=1\\ i\neq j\neq k}\right)}^{n}\Delta m\left(B_i\oplus\left(B_j\oplus B_k\right)\right)\right]_{(\text{II}^\circ\text{-deg.})} \end{aligned} \tag{3}$$

where "i-degree" points out the degree of coupling of two and more couplings. Here the terms of degree superior to II° are omitted. It needs to consider that:

$$\begin{aligned} &\Delta m\left[(a)\otimes(b)\right]_{A_i}=\left\{\begin{array}{c}0\\ \Delta m(a,b)_{(a\cap b)}\neq 0\end{array}\right\}\\ &\Delta m\left[(a)\oplus(b)\right]_{B_i}=\Delta m(a,b)_{\text{interaction}} \end{aligned} \tag{4}$$

where the (a, b) point out "base particles" of the (A_i, B_j)-component, as *i.e.* the pions π or the quarks q. Note mass defect is zero if there is the only interpenetration between the two particles (a, b) without interacting parts. Instead, the mass defect cannot be zero if there are some parts of (a, b) dynamically interacting ($a \cap b$), see the neutral pion in diagonal (AB), see **Figure 1**, where the d-quark and u-quark, exchange energy quanta along the diagonal. Now, we can project the properties of **Table 1** in the space of the functions (F_m, $F_{\Delta m}$) to obtain the properties of the operations ($\otimes$, $\oplus$) for calculating the masses and mass defects (see **Table 2**).

2.4. The Bare Masses and Dressed of Quarks (u, d)

In article [3], we have calculated the bare masses of quark, by the annihilation energy of quark (u, d) belonging to neutral pion:

$$\left[m(u)_{\text{bare}}=(3.51)\,\text{MeV}, m(d)_{\text{bare}}=(5.67)\,\text{MeV}\right] \tag{5}$$

These values are inside the range anticipated by literature [11] [12]. Besides, see the ref. [3], we calculated also the values of dressed masses of quarks inside to pion:

$$\left[m(u)_{\text{dressed}}=(53.31)\,\text{MeV}, m(d)_{\text{dressed}}=(86.26)\,\text{MeV}\right] \tag{6}$$

It is so evident that the values of masses of quarks, both bare and dressed inside pion, could be used to obtaining the mass spectrum of light mesons. If the values in Equation (6) are that of bound quarks in pion by gluons, then we can think of incorporate the potential $V(r)$ of gluons in QCD into mass values of the

Table 2. The fundamental operations of functions (F_m, $F_{\Delta m}$).

($\otimes$-operation) in (F_m, $F_{\Delta m}$) representations	$\oplus$-operation in (F_m, $F_{\Delta m}$) representations
4) $F_m(a_i \otimes a_j)=\langle m(a_i,a_j)\rangle =\langle m(a_i),m(a_j)\rangle$ $F_m(a_i \otimes a_i)=F_m(a_i)=m(a_i)$	1) $F_m(a_i \oplus a_j)=F_m(a_i)+F_m(a_j) =m(a_i)+m(a_j)$ $F_m(a_i \oplus a_i)=F_m(a_i)+F_m(a_i) =m(a_i)+m(a_i)=2m(a_i)$
2) $F_{\Delta m}(a_i \otimes a_j)=0$ $F_{\Delta m}(a_i \otimes a_j)_{\oplus}\neq 0$ with $(a_i \otimes a_j)_{\oplus}=(a_i \otimes a_j)\oplus a_k$	2) $F_{\Delta m}(a\oplus b\oplus c) =F_{\Delta m}(a)+F_{\Delta m}(b)+F_{\Delta m}(c)+F_{\Delta m}(a \underline{\oplus} b)_{ab} +F_{\Delta m}(a \underline{\oplus} c)_{ac}+F_{\Delta m}(b \underline{\oplus} c)_{bc}$
3) $F_{\Delta m}(a_i \otimes a_j)_{\oplus}=F_{\Delta m}(a_i)$ $F_{\Delta m}((a_i \otimes a_j)\oplus a_k)=F_{\Delta m}((a_i)\oplus a_k)$	3) $F_{\Delta m}\left[(a_i \oplus a_j)\oplus a_k\right] =F_{\Delta m}\left[(a_i \oplus a_k)\oplus(a_j \oplus a_k)\right]$
4) $F_m\left[(a_{i1}\otimes a_{j2})\oplus(a_{i2}\otimes a_{j1})\right] =F_m\left[2(a_i \otimes a_j)_{12}\right]$	4) $F_{\Delta m}\left[(a_i \otimes a_j)\oplus(a_k \otimes a_r)\right] =F_{\Delta m}\left[(a_i \otimes a_j)\oplus(a_k)\right] +F_{\Delta m}\left[(a_i \otimes a_j)\oplus(a_r)\right]$
5) $F_{\Delta m}(2a_i \otimes a_j)=F_{\Delta m}(2(a_i \otimes a_j))$ $F_{\Delta m}(n(a_i \otimes a_j)\oplus m(a_k \otimes a_r))$ $=(n\times m)F_{\Delta m}((a_i \otimes a_j)\oplus(a_k \otimes a_r))$	6) $F_{\Delta m}\left[a_i \oplus a_j\right]+F_{\Delta m}\left[a_i \oplus a_k\right] =F_{\Delta m}\left[a_i \oplus(a_j + a_k)\right]$

Equation (6). So, the bound masses of quarks playing a fundamental rule inside pions. By pion ($\pi \equiv (u, \underline{d})$) we can build the mesons' structures. To make this, it needs so to admit the presence of a lattice of bound "virtual pions" $\{\pi^+, \pi^-\}$, with an elementary cell having the form of the $\left\{\left(\pi_r^0\right) = \left(\pi^+ \oplus \pi^-\right)\right\}$, which represents a "*molecule*" of pions, you see **Figure 1**.

However, this lattice, in turn, could have a background lattice of *d*-quark pairs $\{d, \underline{d}\}$, see, in fact, the structure of neutral pion. In this way, both the $\left\{\pi_r^0\right\}$-lattice and $\{d, \underline{d}\}$ participating in building the mass spectrum of light mesons.

Here we report **Table 3** which summarizes all properties of the operations (F_m, $F_{\Delta m}$), (see **Table 1** and **Table 2**) with pions.

2.5. The Matrix of Dynamics Couplings

In equivalent two ways, see the ref. [4], we can treat the binding energy between quarks inside pions. The function $F_{\Delta m}$ of mass defects expresses these two possibilities. The structure equation contains the considerations on the various dynamics couplings and interpenetrations. The first way is considering without

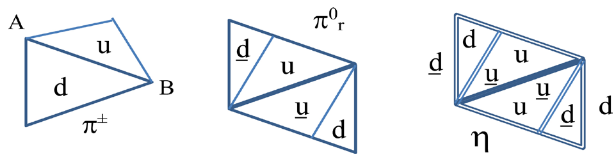

Figure 1. Pion, Pion Molecule, η-meson.

Table 3. Applications of the function (F_m, $F_{\Delta m}$) to the pions.

Properties of pions in (⊗) operations	Properties of pions in (⊕) operations
1) $\left[\pi^\pm = \left(u \underline{\otimes} d\right)\right]$	4) $m\left(\pi^+ \oplus \pi^-\right) = m\left(\pi^+\right) + m\left(\pi^-\right)$
2) $(\pi)^0 = \left[\left(\pi^+\right)\underline{\otimes}\left(\pi^-\right)\right] = \left[\left(u \oplus \underline{d}\right)\underline{\otimes}\left(\underline{u} \oplus \underline{d}\right)\right]$ $= \left[\left(\pi^+\right)\otimes\left(\pi^-\right)\right] - \left[\left(u \oplus \underline{u}\right)\otimes\left(d \oplus \underline{d}\right)\right]$	5) $\left\{\left(\pi_r^0\right) = \left(\pi^+ \oplus \pi^-\right)\right\}; m\left(\pi_r^0\right) = 2m\left(\pi^\pm\right)$ 6) $F_m\left[\left(\pi^+ \otimes \pi^-\right) \oplus \left(\pi^+ \otimes \pi^-\right)\right] = F_m\left[2\left(\pi^\pm\right)^0\right]$
3) $\left(\pi^\pm\right)^0 = \left[\left(\pi^+\right)\otimes\left(\pi^-\right)\right]$ virtual neutral pion	8) $\Delta m\left[\left(\pi_1^\pm \oplus \pi_2^\mp\right) \otimes \pi^0\right] = \Delta m\left[\left(\pi_1^\pm \oplus \pi_2^\mp\right)_{\pi^0}\right]$
4) $m\left[\left(\pi^+\right)\otimes\left(\pi^-\right)\right] = \left(\pi^\pm\right)^0 = m\left(\pi^\pm\right)$ Equation (11)	$\left[\Delta m(\pi)/4 = (1.15)\,\mathrm{MeV}\right]$
5) $m\left(\pi^+ \otimes \pi^+\right) = m\left(\pi^+\right)$	
6) $m\left(\pi^+ \otimes \pi^-\right) = \left\langle m\left(\pi^+\right), m\left(\pi^-\right)\right\rangle$	
7) $\left\{\Delta m(\pi) = \left[m\left(\pi^\pm\right) - m\left(\pi^0\right)\right]\right\} = (4.59)\,\mathrm{MeV}$	
8) $\left[\left(\pi_1^\pm \otimes \pi_2^\pm\right) \oplus \pi_3^\pm\right]$	
$\rightarrow \Delta m\left(\pi_i^\pm \otimes \pi_j^\pm\right)_\oplus = \Delta m\left(\pi_j^\pm\right)$	
9) $2\pi^\pm \equiv 2\left(\pi_1^\pm \underline{\otimes} \pi_2^\pm\right)$	
10) $\left(2\pi_1^\pm \underline{\otimes} 2\pi_2^\pm\right) = (2 \times 2)\left(\pi_1^\pm \underline{\otimes} \pi_2^\pm\right)$	
11) $\Delta m\left(\pi^+ \otimes \pi^-\right) = 0$	12) $\left[\left(\pi_a^\pm \otimes \pi_b^\pm\right) \otimes \pi_c^\pm\right] \oplus \left[\left(\pi_a^\pm \otimes \pi_b^\mp\right) \otimes \pi_c^\mp\right]$
12) $\Delta m\left(\pi_i^\pm \otimes \pi_j^\pm\right)_\oplus = \Delta m\left(\pi_j^\pm\right)$	$= \left[\left(\pi_a^\pm \otimes \pi_b^\pm\right) \oplus \left(\pi_a^\mp \otimes \pi_b^\mp\right)\right] \oplus \left[\pi_c^\pm \oplus \pi_c^\mp\right]$

dynamical couplings all the interpenetrations between two pions, but not the ones between two pairs of pions. Vice versa, the second way is considering without dynamical couplings all the interpenetrations between two pairs of pions, but not the ones between two pions. This last aspect implies that all interactions of the different pairs are attributable to the individual couplings between two pions. In this second case, we represent all possible dynamics couplings by a matrix: $\|A_{ij}\| = \left[\left(\pi_i^{\pm} \otimes \pi_j^{\pm}\right)_{\oplus}\right]$.

We will show the two possible ways in the calculations of meson masses.

3. The Light Mesons

3.1. The Pseudo-Scalar Mesons (η')

η'-meson **((957.78) MeV)**

In ref. [4], we have calculated the mass of *η-meson*. The pions molecules place this meson to first level of energy. One could build a state of 2-level η' with combination $((\pi^0)_r, \eta^*)$ where the pion molecule $(\pi^0)_r$ overlapping to η^*-combination, with $\left[\eta^* = \left(\pi^0\right)_r \otimes \left(\pi^0\right)_r\right]$; we well have: $\left[\eta' = \left(\pi^0\right)_r \otimes \left(\eta^*\right)\right]$. We need to reduce the structure equation in sum of elementary components (A_i, B_i), see Equation (1).

It follows, by 7-property in **Table 1**:

$$\begin{aligned}
\eta' &= \left\{\left[\left(\pi_1^0\right)_r\right] \otimes \left[\eta^*\right]\right\} = \left[\left(\pi_1^0\right)_r\right] \otimes \left[\left(\pi_2^0\right)_r \otimes \left(\pi_3^0\right)_r\right] \\
&= \left(\pi_1^0\right)_r \otimes \left[\left(\pi^+ \oplus \pi^-\right)_2 \otimes \left(\pi^+ \oplus \pi^-\right)_3\right]_{\eta^*} \\
&= \left\{\left[\left(\pi^+ \oplus \pi^-\right)_1\right] \otimes \left[\left(\pi^+ \oplus \pi^-\right)_2 \otimes \left(\pi^+ \oplus \pi^-\right)_3\right]_{\eta^*}\right\}_{(1)} \\
&= \left\{\left[\left(\pi^+ \oplus \pi^-\right)_1\right] \otimes \left[\left(\pi^+ \otimes \pi^+\right)_{23} \oplus \left(\pi^+ \otimes \pi^-\right)_{23}\right.\right. \\
&\qquad \left.\left. \oplus \left(\pi^- \otimes \pi^+\right)_{23} \oplus \left(\pi^- \otimes \pi^-\right)_{23}\right]_{\eta^*}\right\}_{(1)}
\end{aligned} \tag{7}$$

We apply the F_m function:

$$\begin{aligned}
m(\eta') &= F_m\Big(\Big\{\left[\left(\pi^+ \oplus \pi^-\right)_1\right]_B \otimes \left[\left(\pi^+ \otimes \pi^+\right)_{23} \oplus \left(\pi^+ \otimes \pi^-\right)_{23}\right. \\
&\qquad \left. \oplus \left(\pi^- \otimes \pi^+\right)_{23} \oplus \left(\pi^- \otimes \pi^-\right)_{23}\right]_{\eta^*}\Big\}\Big) \\
&= F_m\left(B \otimes \eta^*\right)
\end{aligned}$$

Then in the F_m-representation, we can have

$$\begin{aligned}
m\left(\eta^*\right) &= F_m\left(\left[\left(\pi^+ \otimes \pi^+\right)_{23} \oplus \left(\pi^+ \otimes \pi^-\right)_{23} \oplus \left(\pi^- \otimes \pi^+\right)_{23} \oplus \left(\pi^- \otimes \pi^-\right)_{23}\right]_{\eta^*}\right) \\
&= F_m\left(\left[\left(\pi^+\right) + 2\left(\pi^+ \otimes \pi^-\right) + \left(\pi^-\right)\right]\right)
\end{aligned}$$

where we have used the $1_{(\otimes)}$-properties of F_m in **Table 2**. Well, we can apply the same property to the following equation:

$$m(\eta') = F_m\left(\left\{\left[\left(\pi^+ \oplus \pi^-\right)\right]_B \otimes \left[\left(\pi^+\right) \oplus 2\left(\pi^+ \otimes \pi^-\right) \oplus \left(\pi^-\right)\right]_{\eta^*}\right\}\right)$$
$$= F_m\left(\left[\left(\pi^+ \otimes \pi^+\right) \oplus \left(\pi^+ \otimes 2\left(\pi^\pm\right)^0\right) \oplus \left(\pi^+ \otimes \pi^-\right)\right]_{B_1}\right.$$
$$\left.\oplus\left[\left(\pi^- \otimes \pi^+\right) \oplus \left(\pi^- \otimes 2\left(\pi^\pm\right)^0\right) \oplus \left(\pi^- \otimes \pi^-\right)\right]_{B_2}\right)$$
$$= F_m\left(\left(\pi^+\right) \oplus 2\left(\pi^\pm \otimes 2\left(\pi^\pm\right)^0\right) \oplus 2\left(\pi^\pm\right)^0 \oplus \left(\pi^-\right)\right)$$

Moreover, we have $\left[\left(\pi^\pm\right)^0 = \left(\pi^+ \otimes \pi^-\right)\right]$ and $\left[2\left(\pi^\pm\right)^0 = \left(\pi^+ \otimes \pi^-\right) \oplus \left(\pi^+ \otimes \pi^-\right)\right]_{F_m}$ and $F_m\left(\pi^+ \otimes \pi^+\right) = F_m\left(\pi^+\right)$, see ref. [4] and **Table 3**; the 0-superscriptum point out a neutral pion having mass equal to that charged pion ($\pi^\pm$): $\left[m\left(\pi^\pm\right)^0 = m\left(\pi^+ \otimes \pi^-\right) = m\left(\pi^\pm\right)\right]$, see **Table 3**. Note, the number two (2) in the components becomes a multiplicative factor in F_m-function, see the $1_{(\oplus)}$-property in **Table 2**. Then, the partial mass is:

$$\begin{aligned} m(\eta')^* &= F_m\left\{2\left(\pi^\pm \otimes 2\left(\pi^\pm\right)^0\right) \oplus \left(2\left(\pi^\pm\right)^0\right) \oplus \left(\pi^+\right) \oplus \left(\pi^-\right)\right\}_{(5)} \\ &= \left(2\left\langle m\left(\pi^\pm, 2\left(\pi^\pm\right)^0\right)\right\rangle\right) + \left(2\left\langle m\left(\pi^+, \pi^-\right)\right\rangle\right) + m\left(\pi^+\right) + m\left(\pi^-\right) \end{aligned} \tag{8}$$

where it is:

$$\left\langle m\left(\pi^\pm, 2\left(\pi^\pm\right)^0\right)\right\rangle = \left[\frac{m\left(\pi^\pm\right) + m\left(2\left(\pi^\pm\right)^0\right)}{2}\right] = (209.36)\,\mathrm{MeV} \tag{9}$$

Thus the partial mass value is:

$$m(\eta')^* = 2(209.36)\,\mathrm{MeV} + 4(139.57)\,\mathrm{MeV} = (976.99)\,\mathrm{MeV} \tag{10}$$

Mass defect of (η') can be (see Equation (3)):

$$\Delta m(\eta') = \Delta m\left(\left(\pi^0\right)_r \otimes \eta^*\right) = \Delta m\left(\left(\pi^0\right)_r \otimes \left(\pi^0\right)_r \otimes \left(\pi^0\right)_r\right)$$

Note that $\Delta m\left(\eta^*\right) = \Delta m(\eta)$, *see in ref.* [4] *the Equations* (14) *and* (25). Then, this equation tells us that the mass defect $\Delta m(\eta)$ of η-meson must be inserted inside $\Delta m(\eta')$, because in η' there is the product of interpenetration (⊗), see Equation (14), ref. [4]:

$$\left\{\left(\pi^0\right)_{r_1} \otimes \left(\pi^0\right)_{r_2} \otimes \left(\pi^0\right)_{r_3}\right\}$$
$$= \left\{\left(\pi^0\right)_{r_1} \otimes \left[\left(\pi^0\right)_{r_2} \otimes \left(\pi^0\right)_{r_3}\right]_{\eta_{23}}\right\}$$
$$= \left\{\left[\left(\pi^0\right)_{r_1} \otimes \left(\pi^0\right)_{r_2}\right]_{\eta_{12}} \otimes \left(\pi^0\right)_{r_3}\right\}$$

Thus, in $\Delta m(\eta')$ we need to insert $2\Delta m(\eta)$; it follows, with $\Delta m(\eta^*) = \Delta m(\eta)$:

$$\begin{aligned}\Delta m(\eta') &= \Delta m\left(\left(\pi^0\right)_r \otimes \eta^*\right) = \Delta m\left(\left(\pi^+ \oplus \pi^-\right) \otimes \eta\right) \\ &= \Delta m\left(\left(\pi^+ \otimes \eta\right)_{A_1} \oplus \left(\pi^- \otimes \eta\right)_{A_2}\right) \\ &= \Delta m\left(\left(\pi^+ \otimes \eta\right)_{A_1}\right) + \Delta m\left(\left(\pi^- \otimes \eta\right)_{A_2}\right) \\ &\quad + \Delta m\left(\left[\left(\pi^+ \otimes \eta\right) \underline{\oplus} \left(\pi^- \otimes \eta\right)\right]_{B_{12}}\right) \\ &= 2\Delta m\left(\eta_\pm\right)_{(A_1+A_2)} + \Delta m\left(\left[\left(\pi^+ \otimes \eta\right) \underline{\oplus} \left(\pi^- \otimes \eta\right)\right]_{B_{12}}\right)\end{aligned} \tag{11}$$

Here, we apply the 2-property of **Table 2**, where ($\underline{\oplus}$) indicates the dynamic coupling at components no-separable: we cannot apply the 2-property again.

Besides, $\left[\eta_\pm = \left(\pi^\pm\right) \otimes (\eta)\right]$ and

$\Delta m\left(\left[\pi^\pm \otimes \eta\right]\right) = \Delta m\left((\eta)_{\pi^\pm}\right) \approx \Delta m(\eta) = (10.35)\,\text{MeV}$

This because η is a neutral particle and, thus the interaction between ($\pi^\pm$) and (η) during them interpenetration is not comparable with $\Delta m(\eta)$. The value of B_{12}-component in Equation (11) we will be:

$$\begin{aligned}\Delta m\left(B_{12}\right) &= \Delta m\left(\left(\left[\pi^+ \otimes \eta\right] \underline{\oplus} \left[\pi^- \otimes \eta\right]\right)_{B_{12}}\right) \\ &= \Delta m\left(\left(\eta \underline{\otimes} \left[\pi^+ \oplus \pi^-\right]\right)_{B_{12}}\right) \\ &= \Delta m\left(\left[\left(\pi^0\right)_r \otimes \left(\pi^0\right)_r\right]_\eta \underline{\otimes} \left[\pi^+ \oplus \pi^-\right]\right) \\ &= \Delta m\left(\left[\left(\pi^+ \oplus \pi^-\right)_a \otimes \left(\pi^+ \oplus \pi^-\right)_b\right]_\eta \underline{\otimes} \left(\pi^+ \oplus \pi^-\right)_c\right)\end{aligned} \tag{12}$$

Solving individual products:

$$\begin{aligned}&\Delta m\left(\left[\left(\pi^+ \oplus \pi^-\right)_a \otimes \left(\pi^+ \oplus \pi^-\right)_b\right]_\eta \underline{\otimes} \left(\pi^+ \oplus \pi^-\right)_c\right) \\ &= \Delta m\left(\left[\left(\pi_a^+ \otimes \pi_b^+\right) \oplus \left(\pi_a^+ \otimes \pi_b^-\right) \oplus \left(\pi_a^- \otimes \pi_b^+\right) \oplus \left(\pi_a^- \otimes \pi_b^-\right)\right]_\eta \underline{\otimes} \left(\pi_c^+ \oplus \pi_c^-\right)\right) \\ &= \Delta m\left(\left[\left(\pi_a^+ \otimes \pi_b^+\right) \underline{\otimes}\, \pi_c^+ \oplus \left(\pi_a^+ \otimes \pi_b^-\right) \underline{\otimes}\, \pi_c^+ \oplus \left(\pi_a^- \otimes \pi_b^+\right) \underline{\otimes}\, \pi_c^+ \oplus \left(\pi_a^- \otimes \pi_b^-\right) \underline{\otimes}\, \pi_c^+\right]_{\eta,c}\right) \\ &\quad + \Delta m\left(\left[\left(\pi_a^+ \otimes \pi_b^+\right) \underline{\otimes}\, \pi_c^- \oplus \left(\pi_a^+ \otimes \pi_b^-\right) \underline{\otimes}\, \pi_c^- \oplus \left(\pi_a^- \otimes \pi_b^+\right) \underline{\otimes}\, \pi_c^- \oplus \left(\pi_a^- \otimes \pi_b^-\right) \underline{\otimes}\, \pi_c^-\right]_{\eta,c}\right)\end{aligned} \tag{13}$$

Where we can note two terms at repulsive interaction

$$\begin{aligned}&\left(\pi^+ \otimes \pi^+\right) \underline{\otimes}\, \pi^+ \Leftrightarrow \text{component at repulsive action} \\ &\left(\pi^+ \otimes \pi^-\right) \underline{\otimes}\, \pi^+ = \left(\pi^\pm\right)^0 \underline{\otimes}\, \pi^+; \quad \left(\pi^- \otimes \pi^+\right) \underline{\otimes}\, \pi^+ = \left(\pi^\pm\right)^0 \underline{\otimes}\, \pi^+ \\ &\left(\pi^- \otimes \pi^-\right) \underline{\otimes}\, \pi^+ = \pi^- \underline{\otimes} \left(\pi^\pm\right)^0; \quad \left(\pi^+ \otimes \pi^+\right) \underline{\otimes}\, \pi^- = \pi^+ \underline{\otimes} \left(\pi^\pm\right)^0 \\ &\left(\pi^+ \otimes \pi^-\right) \underline{\otimes}\, \pi^- = \left(\pi^\pm\right)^0 \underline{\otimes}\, \pi^-; \quad \left(\pi^- \otimes \pi^+\right) \underline{\otimes}\, \pi^- = \left(\pi^\pm\right)^0 \underline{\otimes}\, \pi^- \\ &\left(\pi^- \otimes \pi^-\right) \underline{\otimes}\, \pi^- \Leftrightarrow \text{component at repulsive action}\end{aligned}$$

The components expressing an interpenetration between neutral pion $(\pi^\pm)^0$ and charged pion ($\pi^\pm$) have a zero value of mass defect. Only the two charged components have a negative mass defect. Therefore, the B_{12}-component has a final negative value of mass defect: $\Delta m(B_{12}) < 0$. We need calculating the numerical value of $\Delta m(B_{12})$, which, now, is:

$$\Delta m\left(B_{12}\right)=\Delta m\left(\left[\left(\pi_a^+\otimes\pi_b^+\right)\underline{\otimes}\,\pi_c^+\right]_{rep}\oplus\left[\left(\pi_a^-\otimes\pi_b^-\right)\underline{\otimes}\,\pi_c^-\right]_{rep}\right) \tag{14}$$

In B_{12}-component there is an interaction ($\oplus$) between the three interpenetrated pions (π^+) and the three interpenetrated pions π^-; therefore, there is also a partial interaction between $\left(\pi_c^+\right)$ and $\left(\pi_c^-\right)$: $\left(\pi_c^+\right)\otimes\left(\pi_c^-\right)$.

The same happens between the two pairs $\left[\left(\pi_a^+\otimes\pi_b^+\right)\leftrightarrow\left(\pi_a^-\otimes\pi_b^-\right)\right]$ of Equation (14); it follows: $\left[\left(\pi_a^+\otimes\pi_b^+\right)\oplus\left(\pi_a^-\otimes\pi_b^-\right)\right]$.

Therefore, we could exchange the terms of equation (see **Table 3**):

$$\begin{aligned}\Delta m\left(B_{12}\right)&=\Delta m\left(\left[\left(\pi_a^+\otimes\pi_b^+\right)\underline{\otimes}\,\pi_c^+\right]_{rep}\oplus\left[\left(\pi_a^-\otimes\pi_b^-\right)\underline{\otimes}\,\pi_c^-\right]_{rep}\right)_{(1)}\\&=\Delta m_{rep}\left(\left[\left(\pi_a^+\otimes\pi_b^+\right)\oplus\left(\pi_a^-\otimes\pi_b^-\right)\right]_{B_1}\underline{\otimes}\left[\left(\pi_c^+\oplus\pi_c^-\right)\right]_{B_2}\right)_{(2)}\\&=\Delta m_{rep}\left(\left[\left(\pi_a^+\otimes\pi_b^+\right)\oplus\left(\pi_a^-\otimes\pi_b^-\right)\right]_{B_1}\underline{\otimes}\left(\pi_{r_c}^0\right)_{B_2}\right)_{(3)}\end{aligned}$$

Besides, note that the B_2-component $\left[\left(\pi_c^+\oplus\pi_c^-\right)=\pi_r^0\right]$ is neutral and partially could shield the attractive reciprocal interaction between the two couples (π_{ab}^+) and (π_{ab}^-), as also the reciprocal repulsive interaction between the internal pions of each couple: the 3-component expresses just this aspect. To express the shielding action of π_r^0 on the B_1-component, we will write the 3-component in the following way:

$$\begin{aligned}\Delta m\left(B_{12}\right)&=\Delta m\left(\left[\left(\pi_a^+\otimes\pi_b^+\right)\oplus\left(\pi_a^-\otimes\pi_b^-\right)\right]_{B_1}\underline{\otimes}\left(\pi_{r_c}^0\right)_{B_2}\right)_{(3)}\\&=\Delta m\left(\left[\left(\pi_a^+\otimes\pi_b^+\right)\oplus\left(\pi_a^-\otimes\pi_b^-\right)\right]_{\left(\pi_r^0\right)}\right)_{(4)}\end{aligned} \tag{15}$$

By Equation (22-6) *in ref.* [4], we can have:

$$\Delta m\left(\left[\left(\pi_1^+\otimes\pi_2^+\right)\oplus\left(\pi_1^-\otimes\pi_2^-\right)\right]_{\left(\pi_r^0\right)}\right)=\Delta m\left(\left[\left(\pi_{12}^+\right)\oplus\left(\pi_{12}^-\right)\right]_{\left(\pi_r^0\right)}\right) \tag{16}$$

The shielding action of (π_r^0) lowers the value of mass defect, see *in ref.* [4] *the Equation* (22-6); we conjecture a value of mass defect:

$$\Delta m\left(\left[\left(\pi_{12}^+\right)\oplus\left(\pi_{12}^-\right)\right]_{\left(\pi_r^0\right)}\right)=-\left(\frac{\Delta m(\pi)}{n^m}\right)=-\left(\frac{(4.60)}{2^2}\right)\text{MeV} \tag{17}$$

This, ($n = 4$), because the neutral molecule (π_r^0) is a double particle which acting on two pions: the submultiple of $\Delta m(\pi)$ is 2^2.

Thus, the total (η')-mass by Equations [(11), (17)] and *in ref.* [4] *the* Equations [(22-6), (27)] and omitting (O_{Ci}) is:

$$\begin{aligned}m(\eta')\approx m(\eta')^*-\Delta m&=\left\{(976.99)-\left[2(10.35)-(1.15)\right]\right\}\text{MeV}\\&=(957.44)\text{MeV}\end{aligned} \tag{18}$$

Here too we obtain un value very close to the experimental one.

Also here we can build (see **Table 4**) the matrix $\|A_{ij}\|$ of all couplings between pions which compose the structure equation in η'-meson:

The mass defect is, see the ref. [4] Equation (26):

$$\begin{aligned}\Delta m(\eta') = & \left[\Delta m\left(\left(\pi_1^- \otimes \pi_1^+\right)_\oplus\right) + \Delta m\left(\left(\pi_2^- \otimes \pi_2^+\right)_\oplus\right) + \Delta m\left(\left(\pi_3^- \otimes \pi_3^+\right)_\oplus\right)\right]_G \\ & + \left[\Delta m\left(\left(\pi_2^- \otimes \pi_1^+\right)_\oplus\right) + \Delta m\left(\left(\pi_1^- \otimes \pi_2^+\right)_\oplus\right)\right]_Y \\ & + \left[\Delta m\left(\left(\pi_3^- \otimes \pi_1^+\right)_\oplus\right) + \Delta m\left(\left(\pi_1^- \otimes \pi_3^+\right)_\oplus\right)\right]_Y \\ & + \left[\Delta m\left(\left(\pi_3^- \otimes \pi_2^+\right)_\oplus\right) + \Delta m\left(\left(\pi_2^- \otimes \pi_3^+\right)_\oplus\right)\right]_Y \\ & - \left[\Delta m\left(\left(\pi_2^+ \otimes \pi_1^+\right)_\oplus\right) + \Delta m\left(\left(\pi_2^- \otimes \pi_1^-\right)_\oplus\right)\right]_B \\ & - \left[\Delta m\left(\left(\pi_3^+ \otimes \pi_1^+\right)_\oplus\right) + \Delta m\left(\left(\pi_3^- \otimes \pi_1^-\right)_\oplus\right)\right]_B \\ & - \left[\Delta m\left(\left(\pi_3^+ \otimes \pi_2^+\right)_\oplus\right) + \Delta m\left(\left(\pi_3^- \otimes \pi_2^-\right)_\oplus\right)\right]_B \\ & - \left[\Delta m\left(\left(\pi_i^\pm \otimes \pi_i^\pm\right)_\oplus\right)\right]_R\end{aligned}$$

$$\begin{aligned}\Delta m(\eta') & = \left\{\left[3(4.6)_{(i,i)G} + 6(2.3)_{(i,j)Y}\right]_{(\pm,\mp)} - \left[6(1.15)_{(i,j)B} + (1.15)_{(i,i)R}^{(\pm,\pm)}\right]\right\}\text{MeV} \\ & = (19.55)\,\text{MeV}\end{aligned} \tag{19}$$

Obtaining the same value of mass defect of that Equation (18). As already find, also here we will have:

$m(\eta') \approx m(\eta')^* - \Delta m = \{(976.99) - (19.55)\}\,\text{MeV} = (957.44)\,\text{MeV}$

The most probable decays [11] are (see also structure Equation (7)):

$$\left[\eta' \to \eta + \left(\pi^+ + \pi^-\right)\right]_{(43\%)},\ \left[\eta' \to \eta + \left(\pi^0 + \pi^0\right)\right]_{(23\%)},$$

$$\left[\eta' \to \rho + \gamma\right]_{(30\%)},\ \left[\eta' \to \omega + \gamma\right]_{(4\%)}$$

Moreover, other decays are possible (see more forward) for available energy.

Note the spins of (η, η') are at zero values: $\left[s(\eta) = 0, s(\eta') = 0\right]$. This result

Table 4. The matrix of the couplings between pions in η-meson.

	π_1^+	π_2^+	π_3^+	π_1^-	π_2^-	π_3^-
π_1^+	$(\pi_1^+,\pi_1^+)^R$	(π_1^+,π_2^+)	(π_1^+,π_3^+)	(π_1^+,π_1^-)	(π_1^+,π_2^-)	(π_1^+,π_3^-)
π_2^+	$(\pi_2^+,\pi_1^+)^B$	$(\pi_2^+,\pi_2^+)^R$	(π_2^+,π_3^+)	(π_2^+,π_1^-)	(π_2^+,π_2^-)	(π_2^+,π_3^-)
π_3^+	$(\pi_3^+,\pi_1^+)^B$	$(\pi_3^+,\pi_2^+)^B$	$(\pi_3^+,\pi_3^+)^R$	(π_3^+,π_1^-)	(π_3^+,π_2^-)	(π_3^+,π_3^-)
π_1^-	$(\pi_1^-,\pi_1^+)^G$	$(\pi_1^-,\pi_2^+)^Y$	$(\pi_1^-,\pi_3^+)^Y$	$(\pi_1^-,\pi_1^-)^R$	(π_1^-,π_2^-)	(π_1^-,π_3^-)
π_2^-	$(\pi_2^-,\pi_1^+)^Y$	$(\pi_2^-,\pi_2^+)^G$	$(\pi_2^-,\pi_3^+)^Y$	$(\pi_2^-,\pi_1^-)^B$	$(\pi_2^-,\pi_2^-)^R$	(π_2^-,π_3^-)
π_3^-	$(\pi_3^-,\pi_1^+)^Y$	$(\pi_3^-,\pi_2^+)^Y$	$(\pi_3^-,\pi_3^+)^G$	$(\pi_3^-,\pi_1^-)^B$	$(\pi_3^-,\pi_2^-)^B$	$(\pi_3^-,\pi_3^-)^R$

derives from the zero value of the spin [3] [4] of a molecule of pions: $\left[s\left(\pi^{+}\right)+s\left(\pi^{-}\right)=0\right]$

Recall the interpenetration of two triangular structure (see the pion) has spin zero because the relative rotations in interpenetration are always opposites [3].

If we consider the superposition of two η-meson, as η'-meson, we have:

$$\eta''=\left\{\left[\eta^{*}\right]\otimes\left[\eta^{*}\right]\right\}=\left[\left(\pi^{0}\right)_{r}\otimes\left(\pi^{0}\right)_{r}\right]\otimes\left[\left(\pi^{0}\right)_{r}\otimes\left(\pi^{0}\right)_{r}\right]$$

Moreover, if we calculate the partial mass, then one finds the following value m = (1954.00) MeV, which is next to f_2-meson.

3.2. The Vectorial Light Mesons (ρ, ω, ϕ) Scalar

ρ-meson **(775.26 MeV)**

Note, generally, the existence of light mesons implies the presence of a background lattice of pion molecules $\{(\pi^0)_r\}$. Nevertheless, this π-lattice, in turn, is built on another "primary" lattice $\{d, \underline{d}\}$, as now we will show. This lattice can be important for binding more molecules of pions. Not only that, but we could think the presence of these lattices $\{d, \underline{d}\}$ and of molecules $\{(\pi^0)_r\}$ is connected to the "hadronization process" in strong interactions. The external energy flows into a hadron and "immediately" transformed, through these lattices, into further hadrons. In the same way, these lattices can also participate in building the more massive hadrons. Then we think that resonance states involve a "primary" background lattice $\{d, \underline{d}\}$. In fact, into structure the ρ-meson we add the pair $(d, \underline{d})_\pi$ and so having:

$$\rho=\left[\left(d,\underline{d}\right)_{\pi}\right]\otimes\left\{\left[\left(2\pi^{+}\oplus\pi_{r}^{0}\right)_{B_1}\otimes\left(2\pi^{-}\oplus\pi_{r}^{0}\right)_{B_2}\right]_{A}\right\} \tag{20}$$

where $(\pi^0)_r$ is the pion molecule and $\left[\left(d,\underline{d}\right)_{\pi}=\left(d\otimes\underline{d}\right)_{\pi}\right]$ is the pair of d_π-quark. Note that, see Equation (2) and Table 2:

$$m\left(d\otimes\underline{d}\right)_{\pi}=\left\langle m\left(d,\underline{d}\right)_{\pi}\right\rangle=\left[\frac{m\left(d_{\pi}\right)+m\left(\underline{d}_{\pi}\right)}{2}\right]=m\left(d_{\pi}\right)$$

Besides the multiplicative number (2) in B_i-component becomes a multiplicative factor in F_m-function, see η'-meson and the $9_{(\oplus)}$-property in Table 3:

$$\left[2\pi^{\pm}\equiv2\left(\pi_{1}^{\pm}\otimes\pi_{2}^{\pm}\right)\right]$$

We develop the structure equation:

$$\begin{aligned}\rho&=\left[\left(d,\underline{d}\right)_{\pi}\right]\otimes\left\{\left[\left(2\pi^{+}\oplus\pi_{r}^{0}\right)_{B_1}\otimes\left(2\pi^{-}\oplus\pi_{r}^{0}\right)_{B_2}\right]\right\}\\&=\left[\left(d,\underline{d}\right)_{\pi}\right]\otimes\left\{\left[4\left(\pi_{1}^{+}\otimes\pi_{2}^{-}\right)_{A_1}\oplus\left(2\pi_{1}^{+}\otimes\pi_{r_2}^{0}\right)_{A_2}\right.\right.\\&\qquad\left.\left.\oplus\left(\pi_{r_1}^{0}\otimes2\pi_{2}^{-}\right)_{A_3}\oplus\left(\pi_{r_1}^{0}\otimes\pi_{r_2}^{0}\right)_{A_4}\right]_{B}\right\}\end{aligned} \tag{21}$$

where $\left[2\pi^{+}\otimes2\pi^{-}=4\left(\pi^{+}\otimes\pi^{-}\right)\right]$, see the $10_{\otimes}$-property in Table 3. We apply the F_m function:

$$
\begin{aligned}
m(\rho) &= F_m\left(\left[(d,\underline{d})_\pi\right]\otimes\left\{\left[4\left(\pi_1^+\otimes\pi_2^-\right)\oplus\left(2\pi_1^+\otimes\pi_{r_2}^0\right)\right.\right.\right.\\
&\qquad\left.\left.\left.\oplus\left(\pi_{r_1}^0\otimes 2\pi_2^-\right)\oplus\left(\pi_{r_1}^0\otimes\pi_{r_2}^0\right)\right]_B\right\}\right)\\
&= F_m\left(\left[(d,\underline{d})_\pi\right]\otimes\left\{\left[4\left(\pi^\pm\right)_{12}^0\oplus 2\left(2\pi^\pm\otimes\pi_r^0\right)_{12}\oplus\left(\pi_{r_1}^0\otimes\pi_{r_2}^0\right)\right]_B\right\}\right)\\
&= F_m\left(\left\{\left[(d,\underline{d})_\pi\otimes 4\left(\pi^\pm\right)_{12}^0\right]_{A_1}\oplus\left[(d,\underline{d})_\pi\otimes 2\left(2\pi^\pm\otimes\pi_r^0\right)_{12}\right]_{A_2}\right.\right.\\
&\qquad\left.\left.\oplus\left[(d,\underline{d})_\pi\otimes\left(\pi_{r_1}^0\otimes\pi_{r_2}^0\right)\right]_{A_3}\right\}\right)
\end{aligned}
\tag{22}
$$

where $\left[\left(\pi^+\otimes\pi^-\right)\equiv\left(\pi^\pm\right)^0\right]$ and (see 4-property in **Table 3**); the partial mass is:

$$
\begin{aligned}
m(\rho) &= m\left\{\left[(d,\underline{d})_\pi\otimes 4\left(\pi^\pm\right)_{12}^0\right]_{A_1}\oplus\left[(d,\underline{d})_\pi\otimes 2\left(2\pi^\pm\otimes\pi_r^0\right)_{12}\right]_{A_2}\right.\\
&\qquad\left.\oplus\left[(d,\underline{d})_\pi\otimes\left(\pi_r^0\otimes\pi_r^0\right)\right]_{A_3}\right\}\\
&= \left\{\left\langle m\left((d,\underline{d}),4\left(\pi^\pm\right)^0\right)\right\rangle+\left\langle m\left((d,\underline{d}),2\left(2\pi^\pm\otimes\pi_r^0\right)\right)\right\rangle\right.\\
&\qquad\left.+\left\langle m\left((d,\underline{d}),\left(\pi_r^0\otimes\pi_r^0\right)\right)\right\rangle\right\}\\
&= \left[\frac{m(d)+4m\left(\left\langle\left(\pi^\pm\right)^0\right\rangle\right)}{2}+\frac{m(d)+2m\left(\left\langle 2\pi^\pm,\pi_r^0\right\rangle\right)}{2}+\frac{m(d)+m\left(\left\langle\pi_r^0,\pi_r^0\right\rangle\right)}{2}\right]\\
&= \frac{3}{2}m(d)+2m\left(\pi^\pm\right)+\left[m\left(\pi^\pm\right)+\frac{1}{2}m\left(\pi_r^0\right)\right]+\frac{1}{2}m\left(\pi_r^0\right)\\
&= \frac{3}{2}m(d)+3m\left(\pi^\pm\right)+m\left(\pi_r^0\right)
\end{aligned}
\tag{23}
$$

Its partial mass value is:

$$
\begin{aligned}
m(\rho)^* &= \frac{3}{2}m(d)+3m\left(\pi^\pm\right)+m\left(\pi_r^0\right)\\
&= \left[\frac{3}{2}(86.26)+3(139.57)+2(139.57)\right]\text{MeV}\\
&= (827.24)\text{MeV}
\end{aligned}
\tag{24}
$$

About mass defect by Equation (20) we have:

$$
\begin{aligned}
\Delta m(\rho)_{(d,\underline{d})} &= F_{\Delta m}\left\{(d,\underline{d})\otimes\left[\left(2\pi^+\oplus\pi_r^0\right)_1\otimes\left(2\pi^-\oplus\pi_r^0\right)_2\right]_A\right\}\\
&= F_{\Delta m}\left\{\left[\left(2\pi^+\oplus\pi_r^0\right)_1\otimes\left(2\pi^-\oplus\pi_r^0\right)_2\right]_{(d,\underline{d})}\right\}\\
&= F_{\Delta m}\left\{\left[\left(2\pi_1^+\otimes 2\pi_2^-\right)_{A_1}\oplus\left(2\pi_1^+\otimes\pi_{r2}^0\right)_{A_2}\right.\right.\\
&\qquad\left.\left.\oplus\left(\pi_{r1}^0\otimes 2\pi_2^-\right)_{A_3}\oplus\left(\pi_{r1}^0\otimes\pi_{r2}^0\right)_{A_4}\right]_B\right\}_{(d,\underline{d})}
\end{aligned}
\tag{25}
$$

where $\left[(d,\underline{d})\otimes A = A_{(d,\underline{d})}\right]$, see Equation (16). Processing the mass defect one obtains:

$$\begin{aligned}\Delta m(\rho)_{(d,\underline{d})} &= F_{\Delta m}\Big\{\Big[\left(2\pi_1^+\otimes 2\pi_2^-\right)_{A_1}\oplus\left(2\pi_1^+\otimes\pi_{r2}^0\right)_{A_2}\\&\quad\oplus\left(\pi_{r1}^0\otimes 2\pi_2^-\right)_{A_3}\oplus\left(\pi_{r1}^0\otimes\pi_{r2}^0\right)_{A_4}\Big]_B\Big\}_{(d,\underline{d})}\\&=\{\Delta m(A_1)+\Delta m(A_2)+\Delta m(A_3)+\Delta m(A_4)+\Delta m(A_{12})\\&\quad+\Delta m(A_{13})+\Delta m(A_{14})+\Delta m(A_{23})+\Delta m(A_{24})+\Delta m(A_{34})\}_{(d,\underline{d})}\end{aligned}\tag{26}$$

In a specific way, it is: $\left[\Delta m(A_1)=0,\Delta m(A_4)=0\right]$.

Here we cannot pose $\left[\left(\pi_r^0\otimes\pi_r^0\right)=\eta\right]$ and to have ($\Delta m(\eta)\neq 0$), because η-meson do not belong to structure equation. Besides, by 5-property in **Table 2**.

$$\begin{aligned}&\Delta m(A_2)+\Delta m(A_3)\\&=\Delta m\left(\left(2\pi_1^+\otimes\pi_{r2}^0\right)_{A_2}\right)+\Delta m\left(\left(\pi_{r1}^0\otimes 2\pi_2^-\right)_{A_3}\right)\\&=\Delta m\left(\left(2\pi_1^+\otimes\left(\pi_2^+\oplus\pi_2^-\right)\right)_{A_2}\right)+\Delta m\left(\left(2\pi_2^-\otimes\left(\pi_1^+\oplus\pi_1^-\right)\right)_{A_3}\right)\\&=\Delta m\left(\left[\left(2\pi_1^+\otimes\pi_2^+\right)\oplus\left(2\pi_1^+\otimes\pi_2^-\right)\right]_{A_2}\right)+\Delta m\left(\left[\left(2\pi_2^-\otimes\pi_1^+\right)\oplus\left(2\pi_2^-\otimes\pi_1^-\right)\right]_{A_3}\right)_{A_3}\\&=4\Delta m\left(\left[\left(\pi_1^+\otimes\pi_2^+\right)\oplus\left(\pi_1^+\otimes\pi_2^-\right)\right]_{A_2}\right)+4\Delta m\left(\left[\left(\pi_2^-\otimes\pi_1^+\right)\oplus\left(\pi_2^-\otimes\pi_1^-\right)\right]_{A_3}\right)\\&=\left[4(1.15)+4(1.15)\right]\text{MeV}\end{aligned}$$

By 5-property in **Table 2** we will have:

$$\begin{aligned}&\Delta m(A_{12})+\Delta m(A_{13})\\&=\Delta m\left(\left[\left(2\pi_1^+\otimes 2\pi_2^-\right)_{A_1}\oplus\left(2\pi_1^+\otimes\pi_{r2}^0\right)_{A_2}\right]\right)_{A_{12}}\\&\quad+\Delta m\left(\left[\left(2\pi_1^+\otimes 2\pi_2^-\right)_{A_1}\oplus\left(\pi_{r1}^0\otimes 2\pi_2^-\right)_{A_3}\right]\right)_{A_{13}}\\&=\Delta m\left(\left[\left(2\pi_1^+\otimes 2\pi_2^-\right)_{A_1}\oplus\left[\left(2\pi_1^+\otimes\pi_2^+\right)_{A_1'}\oplus\left(2\pi_1^+\otimes\pi_2^-\right)_{A_2'}\right]_{A_2}\right]\right)_{A_{12}}\\&\quad+\Delta m\left(\left[\left(2\pi_1^+\otimes 2\pi_2^-\right)_{A_1}\oplus\left[\left[\left(2\pi_2^-\otimes\pi_1^+\right)_{A_1''}\oplus\left(2\pi_2^-\otimes\pi_1^-\right)_{A_2''}\right]\right]_{A_3}\right]\right)_{A_{13}}\\&\qquad=\Delta m\left(\left[4\left(\pi_1^+\otimes\pi_2^-\right)_{A_1}\oplus\left[2\left(\pi_1^+\otimes\pi_2^+\right)_{A_1'}\right]_{A_2}\right]\right)_{A_{12}}\\&\qquad\quad+\Delta m\left(\left[4\left(\pi_1^+\otimes\pi_2^-\right)_{A_1}\oplus\left[2\left(\pi_1^+\otimes\pi_2^-\right)_{A_2'}\right]_{A_2}\right]\right)_{A_{12}}\\&\qquad\quad+\Delta m\left(\left[4\left(\pi_1^+\otimes\pi_2^-\right)_{A_1}\oplus\left[2\left(\pi_2^-\otimes\pi_1^+\right)_{A_1''}\right]_{A_3}\right]\right)_{A_{13}}\\&\qquad\quad+\Delta m\left(\left[4\left(\pi_1^+\otimes\pi_2^-\right)_{A_1}\oplus\left[2\left(\pi_2^-\otimes\pi_1^-\right)_{A_2''}\right]_{A_3}\right]\right)_{A_{13}}\end{aligned}$$

$$= 8\Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^+ \otimes \pi_2^+\right)_{A_2}\right]\right)_{A_{12}}$$
$$+ 8\Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_1^+ \otimes \pi_2^-\right)_{A_2}\right]\right)_{A_{12}}$$
$$+ 8\Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_2^- \otimes \pi_1^+\right)_{A_3}\right]\right)_{A_{13}}$$
$$+ 8\Delta m\left(\left[\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left(\pi_2^- \otimes \pi_1^-\right)_{A_3}\right]\right)_{A_{13}}$$
$$= 32(1.15)\,\text{MeV}$$

Note $\left[\Delta m(A_{12}) = \Delta m(A_{13})\right]$; once more by 6-property in Table 2, it follows:

$$\Delta m\left(\left[4\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left[2\left(\pi_1^+ \otimes \pi_2^+\right)_{A_1'}\right]_{A_2}\right]\right)_{A_{12}}$$
$$+ \Delta m\left(\left[4\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left[2\left(\pi_1^+ \otimes \pi_2^-\right)_{A_2'}\right]_{A_2}\right]\right)_{A_{12}}$$
$$+ \Delta m\left(\left[4\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left[2\left(\pi_2^- \otimes \pi_1^+\right)_{A_1''}\right]_{A_3}\right]\right)_{A_{13}}$$
$$+ \Delta m\left(\left[4\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left[2\left(\pi_2^- \otimes \pi_1^-\right)_{A_2''}\right]_{A_3}\right]\right)_{A_{13}}$$

$$= \Delta m\left(4\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left\{2\left(\pi_1^+ \otimes \pi_2^+\right) + 2\left(\pi_1^+ \otimes \pi_2^-\right)\right.\right.$$
$$\left.\left. + 2\left(\pi_2^- \otimes \pi_1^+\right) + 2\left(\pi_2^- \otimes \pi_1^-\right)\right\}_{A_{23}}\right)$$
$$= \Delta m\left(4\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left\{\left[2\left(\pi_1^+ \otimes \pi_2^+\right) + 2\left(\pi_2^- \otimes \pi_1^-\right)\right]\right.\right.$$
$$\left.\left. + \left[2\left(\pi_1^+ \otimes \pi_2^-\right) + 2\left(\pi_2^- \otimes \pi_1^+\right)\right]\right\}_{A_{23}}\right)$$
$$= \Delta m\left(4\left(\pi_1^+ \otimes \pi_2^-\right)_{A_1} \oplus \left\{\left[4\left(\pi_{12}^\pm \otimes \pi_{12}^\pm\right)\right] + \left[4\left(\pi_1^\pm \otimes \pi_2^\mp\right)\right]\right\}_{A_2'}\right)$$

Thus the only component A_2' represents two components (A_2, A_3):

$$A_2' = (A_2 + A_3) = \left(\left[4\left(\pi_{12}^\pm \otimes \pi_{12}^\pm\right)\right] + \left[4\left(\pi_1^\pm \otimes \pi_2^\mp\right)\right]_{A_2'}\right)$$

Then, by 3-property in Table 2, it follows:

$$\Delta m(A_{24}) + \Delta m(A_{34}) = \Delta m(A_2' \oplus A_4) = \Delta m(A_{24}')$$
$$= F_{\Delta m}\left\{\left[\left\{\left[4\left(\pi_{12}^\pm \otimes \pi_{12}^\pm\right)\right] + \left[4\left(\pi_1^\pm \otimes \pi_2^\mp\right)\right]\right\}_{A_2'} \oplus \left(\pi_{r1}^0 \otimes \pi_{r2}^0\right)_{A_4}\right]_B\right\}$$
$$= F_{\Delta m}\left\{\left[4\left(\pi_{12}^\pm \otimes \pi_{12}^\pm\right) \oplus \left(\pi_{r1}^0 \otimes \pi_{r2}^0\right)\right]_B\right\}$$
$$+ F_{\Delta m}\left\{\left[\left[4\left(\pi_1^\pm \otimes \pi_2^\mp\right)\right]_{A_2'} \oplus \left(\pi_{r1}^0 \otimes \pi_{r2}^0\right)_{A_4}\right]_B\right\}$$
$$= F_{\Delta m}\left\{\left[4\left(\pi_{12}^\pm \otimes \pi_{12}^\pm\right) \oplus \left(\pi_{r1}^0\right)\right]_B\right\} + F_{\Delta m}\left\{\left[4\left(\pi_{12}^\pm \otimes \pi_{12}^\pm\right) \oplus \left(\pi_{r2}^0\right)\right]_B\right\}$$

$$
\begin{aligned}
&+F_{\Delta m}\left\{\left[\left[4\left(\pi_1^{\pm}\otimes\pi_2^{\mp}\right)\right]_{A_2'}\oplus\left(\pi_{r1}^{0}\right)\right]_B\right\}+F_{\Delta m}\left\{\left[\left[4\left(\pi_1^{\pm}\otimes\pi_2^{\mp}\right)\right]_{A_2'}\oplus\left(\pi_{r2}^{0}\right)_{A_4}\right]_B\right\}\\
&=F_{\Delta m}\left\{\left[4\left(\pi_{12}^{\pm}\otimes\pi_{12}^{\pm}\right)\oplus\left(\pi_1^{+}\oplus\pi_1^{-}\right)\right]_B\right\}+F_{\Delta m}\left\{\left[4\left(\pi_{12}^{\pm}\otimes\pi_{12}^{\pm}\right)\oplus\left(\pi_2^{+}\oplus\pi_2^{-}\right)\right]_B\right\}\\
&+F_{\Delta m}\left\{\left[\left[4\left(\pi_1^{\pm}\otimes\pi_2^{\mp}\right)\right]_{A_2'}\oplus\left(\pi_1^{+}\oplus\pi_1^{-}\right)\right]_B\right\}\\
&+F_{\Delta m}\left\{\left[\left[4\left(\pi_1^{\pm}\otimes\pi_2^{\mp}\right)\right]_{A_2'}\oplus\left(\pi_2^{+}\oplus\pi_2^{-}\right)\right]_B\right\}\\
&=16\left(2.30\right)\text{MeV}^{q}
\end{aligned}
$$

The A_{14}-component is:

$$
\begin{aligned}
\Delta m\left(A_{14}\right)&=\Delta m\left(\left\{4\left(\pi_1^{+}\otimes\pi_2^{-}\right)_{A_1}\underline{\oplus}\left(\pi_{r1}^{0}\otimes\pi_{r2}^{0}\right)_{A_4}\right\}_{A_{14}}\right)\\
&=\Delta m\left(\left\{4\left(\pi_1^{+}\otimes\pi_2^{-}\right)\oplus\left(\pi_{r1}^{0}\right)\right\}_{A_{14}}\right)+\Delta m\left(\left\{4\left(\pi_1^{+}\otimes\pi_2^{-}\right)\oplus\left(\pi_{r2}^{0}\right)\right\}_{A_{13}}\right)\\
&=\Delta m\left(\left\{4\left(\pi_1^{+}\otimes\pi_2^{-}\right)\oplus\left(\pi_1^{+}\oplus\pi_1^{-}\right)\right\}_{A_{13}}\right)\\
&\quad+\Delta m\left(\left\{4\left(\pi_1^{+}\otimes\pi_2^{-}\right)\oplus\left(\pi_2^{+}\oplus\pi_2^{-}\right)\right\}_{A_{13}}\right)\\
&=\left[4\left(2.30\right)+4\left(2.30\right)\right]\text{MeV}
\end{aligned}
$$

where we have used once more by 4-property in **Table 2**. Besides, always using the Equations (15) and (17), see also **Table 3**:

$$
\begin{aligned}
\Delta m\left(A_{23}\right)&=\Delta m\left(\left[\left(2\pi_1^{+}\otimes\left(\pi_r^{0}\right)_2\right)\underline{\oplus}\left(\left(\pi_r^{0}\right)_1\otimes2\pi_2^{-}\right)\right]_{A_{23}}\right)\\
&=\Delta m\left(\left[\left(\left(\pi_r^{0}\right)_2\otimes2\pi_1^{+}\right)\underline{\oplus}\left(\left(\pi_r^{0}\right)_1\otimes2\pi_2^{-}\right)\right]_{A_{23}}\right)\\
&=\Delta m\left(\left[\left(\left(\pi_r^{0}\right)_1\otimes2\pi_1^{+}\right)\underline{\oplus}\left(\left(\pi_r^{0}\right)_2\otimes2\pi_2^{-}\right)\right]_{A_{23}}\right)\\
&=\Delta m\left(\left(\pi_r^{0}\right)_{12}\otimes\left[\left(2\pi_1^{+}\right)\underline{\oplus}\left(2\pi_2^{-}\right)\right]_{A_{23}}\right)\\
&=4\Delta m\left(\left[\left(\pi_1^{+}\oplus\pi_2^{-}\right)_{\pi_r^{0}}\right]_{A_{23}}\right)=4\left(4.6/4\right)\text{MeV}=\left(4.6\right)\text{MeV}
\end{aligned}
$$

where we have exchanged $\left[\left(\pi_r^{0}\right)_1\leftrightarrow\left(\pi_r^{0}\right)_2\right]$ because of the dynamic interactions in couplings not change; besides, we need admitting that the neutral π_r^{0} -component shields the dynamics coupling between two charged pions. The shielding lowers the value of mass defect of the A_{23}-component of (1/4). It so follows:

$$
\begin{aligned}
\Delta m\left(\rho\right)&=\Delta_m\left(A_1\right)+\left[\Delta_m\left(A_2\right)+\Delta_m\left(A_3\right)+\Delta_m\left(A_{23}\right)\right]\\
&\quad+\Delta_m\left(A_4\right)+\Delta_m\left(A_{12}\right)+\Delta_m\left(A_{13}\right)+\Delta_m\left(A_{14}\right)+\Delta_m\left(A_{234}\right)\\
&=\left[\left(0\right)_{A_1}+\left[8\left(1.15\right)\right]_{\left(A_2+A_3\right)}+\left(0\right)_{A_4}+32\left(1.15\right)_{\left(A_{12}+A_{13}\right)}\right.\\
&\quad\left.+\left(4.6\right)_{A_{23}}+16\left(2.30\right)_{\left(A_{24}+A_{34}\right)}+8\left(2.30\right)_{A_{14}}\right]\text{MeV}\\
&\approx\left(105.8\right)\text{MeV}
\end{aligned}
$$

In A_i-component the ($d\otimes\underline{d}$) could shield the electromagnetic interaction between two charged pions; see the Equations (15) and (17), then we can have

$$\Delta m(\rho)_{(d,\underline{d})} \approx (105.8/2)\,\mathrm{MeV} = (52.9)\,\mathrm{MeV}$$

The presence of shielding neutral pair $(d\otimes \underline{d})_\pi$ instead of charged pions $(\pi^\pm)$ leads us to conjecture, see Equations (32) and (46):

$$\left\{\begin{array}{l} \Delta m\Big((d,\underline{d})\otimes\big[(\pi^+\otimes\pi^-)\oplus(\pi^-\otimes\pi^+)\big]\Big) \\ = \Delta m\Big(\big[(\pi^+\otimes\pi^-)_{(d,\underline{d})}\oplus(\pi^-\otimes\pi^+)_{(d,\underline{d})}\big]\Big) = \left[\dfrac{(1.15)}{2}\right]\mathrm{MeV} \\ \Delta m\Big((d,\underline{d})\otimes\big[(\pi^+\otimes\pi^-)\oplus(\pi^-\oplus\pi^+)\big]\Big) \\ = \Delta m\Big(\big[(\pi^+\otimes\pi^-)_{(d,\underline{d})}\oplus(\pi^-\oplus\pi^+)_{(d,\underline{d})}\big]\Big) = \left[\dfrac{(2.30)}{2}\right]\mathrm{MeV} \\ \Delta m\Big((d,\underline{d})\otimes(\pi^+\oplus\pi^-)\Big) = \Delta m\Big((\pi^+\oplus\pi^-)_{(d,\underline{d})}\Big) = \left[\dfrac{(4.59)}{2}\right]\mathrm{MeV} \end{array}\right\} \tag{27}$$

Note that the interpenetration between the pair $(d\otimes \underline{d})_\pi$ and pions $(\pi^\pm)$ lows the values (*see in ref.* [4] *the* Equation (22)), we suppose that the values are halve because the neutral pair $(d, \underline{d})$ partially shields the electrodynamics interactions between pions. Recall the neutral pion is composed by two pairs $[(u, \underline{u}), (d, \underline{d})]$: this determines that the neutral pion shields more than the pair $(d, \underline{d})$.

Then, we will have in ρ-meson: $\Delta m(\rho)_{(d,\underline{d})} \approx (105.8/2)\,\mathrm{MeV} = (52.9)\,\mathrm{MeV}$

Finally:

$$m(\rho) \approx \big[(827.24)\,\mathrm{MeV} - (52.9)\,\mathrm{MeV}\big] = (774.34)\,\mathrm{MeV} \tag{28}$$

Besides, see the ref. [4] in **Table 4**, if we consider the missing mass defects in Equation (3) (the terms of superior degree at II°), then we will have: $\left[(O_{\Delta m})_{(>\mathrm{II}^\circ)} = (1.15)\,\mathrm{MeV}\right]$. Then, we obtain so:

$$m(\rho^0) = m(\rho^0)^* - \Delta m(\rho^0) - O_{\Delta m} = (775.49)\,\mathrm{MeV} \tag{29}$$

Very next to the experimental value [11]. We elaborate on the structural equation of ρ-meson, see Equation (21), in the following way:

$$\begin{aligned} &\left\{\left[(2\pi^+\oplus\pi_r^0)_{B_1}\otimes(2\pi^-\oplus\pi_r^0)_{B_2}\right]\right\} \\ &= \left\{\left[(2\pi_1^+\otimes 2\pi_1^-)_{A_1}\oplus(2\pi_1^+\otimes\pi_{r_2}^0)_{A_2}\oplus(\pi_{r_3}^0\otimes 2\pi_1^-)_{A_3}\oplus(\pi_{r_2}^0\otimes\pi_{r_3}^0)_{A_4}\right]\right\} \\ &= \left\{\left[\big(2\pi_1^+\otimes(\pi_2^+\oplus\pi_2^-)\big)_{A_2}\oplus\big(2\pi_1^-\otimes(\pi_3^+\oplus\pi_3^-)\big)_{A_3}\right]\right\} \\ &\quad\oplus\left\{\left[(2\pi_1^+\otimes 2\pi_1^-)_{A_1}\oplus\big((\pi_2^+\oplus\pi_2^-)\otimes(\pi_3^+\oplus\pi_3^-)\big)_{A_4}\right]\right\} \\ &= \left\{\left[(2\pi_1^+\otimes\pi_2^+)\oplus(2\pi_1^+\otimes\pi_2^-)\right]_{A_2}\oplus\left[(2\pi_1^-\otimes\pi_3^+)\oplus(2\pi_1^-\otimes\pi_3^-)\right]_{A_3}\right\} \\ &\quad\oplus\left\{\left[4(\pi_1^+\otimes\pi_1^-)_{A_1}\oplus\big((\pi_2^+\oplus\pi_2^-)\otimes(\pi_3^+\oplus\pi_3^-)\big)_{A_4}\right]\right\} \end{aligned}$$

By this structure, we build, see **Table 5**, the matrix $\|A_{ij}\|$ of all couplings between pions which compose the structure equation in ρ-meson:

However, we must not forget the presence of the pair (d, $\underline{d}$) in structural equation. The presence of this pair has a double effect:

1) as we have already seen, the pair act as a shield between the interactions of pions

2) it makes the interactions no-commutative between two pions

$$\left[\left(\left(\pi_i \otimes \pi_j\right)_{\oplus}\right) \neq \left(\left(\pi_j \otimes \pi_i\right)_{\oplus}\right)\right].$$

This last aspect pushes us to consider all elements of matrix $\left\Vert A_{ij}\right\Vert$: in the calculations, it is necessary to double the numerical values. Then we can use the following matrix, see Table 6:

The mass defect will be:

$$\Delta m\left(\rho^*\right) = \sum_{\left((i,j)\in\left\Vert A_{ij}\right\Vert\right)} \Delta m\left(\pi_i \otimes \pi_j\right)_{\oplus}$$

Then we will have:

$$\begin{aligned}\Delta m(\rho) &= \left[18(4.6)_{(i,i)G} + 16(2.3)_{(1,2)Y} + 2(2.3)_{(2,3)Y} - 18(1.15)_{(\pm,\pm)B} - (1.15)_{R}\right]\text{MeV} \\ &= (102.35)\,\text{MeV}\end{aligned}$$

$$\Delta m(\rho)_{(d,\underline{d})} = (51.18)\,\text{MeV}$$

$$m(\rho) = (827.24)\,\text{MeV} - (51.18)\,\text{MeV} = (776.06)\,\text{MeV}$$

Table 5. The matrix of couplings between pions in ρ-meson.

	$2\pi_1^+$	π_2^+	π_3^+	$2\pi_1^-$	π_2^-	π_3^-
$2\pi_1^+$	$4(\pi_1^+,\pi_1^+)$	$2(\pi_1^+,\pi_2^+)$	$2(\pi_1^+,\pi_3^+)$	$4(\pi_1^+,\pi_1^-)$	$2(\pi_1^+,\pi_2^-)$	$2(\pi_1^+,\pi_3^-)$
π_2^+	$2(\pi_2^+,\pi_1^+)$	(π_2^+,π_2^+)	(π_2^+,π_3^+)	$2(\pi_2^+,\pi_1^-)$	(π_2^+,π_2^-)	(π_2^+,π_3^-)
π_3^+	$2(\pi_3^+,\pi_1^+)$	(π_3^+,π_2^+)	(π_3^+,π_3^+)	$2(\pi_3^+,\pi_1^-)$	(π_3^+,π_2^-)	(π_3^+,π_3^-)
$2\pi_1^-$	$4(\pi_1^-,\pi_1^+)$	$2(\pi_1^-,\pi_2^+)$	$2(\pi_1^-,\pi_3^+)$	$4(\pi_1^-,\pi_1^-)$	$4(\pi_1^-,\pi_2^-)$	$4(\pi_1^-,\pi_3^-)$
π_2^-	$2(\pi_2^-,\pi_1^+)$	(π_2^-,π_2^+)	(π_2^-,π_3^+)	$2(\pi_2^-,\pi_1^-)$	(π_2^-,π_2^-)	(π_2^-,π_3^-)
π_3^-	$2(\pi_3^-,\pi_1^+)$	(π_3^-,π_2^+)	(π_3^-,π_3^+)	$4(\pi_3^-,\pi_1^-)$	(π_3^-,π_2^-)	(π_3^-,π_3^-)

Table 6. The elements of $\Vert A_{ij}\Vert$ used for calculate the mass defects ρ-meson.

	$4\pi_1^+$	π_2^+	π_3^+	$4\pi_1^-$	π_2^-	π_3^-
$4\pi_1^+$	$16(\pi_1^+,\pi_1^+)$	$\Vert$	$\Vert$	$\Vert$	$\Vert$	$\Vert$
π_2^+	$4(\pi_2^+,\pi_1^+)$	(π_2^+,π_2^+)	$\Vert$	$\Vert$	$\Vert$	$\Vert$
π_3^+	$4(\pi_3^+,\pi_1^+)$	(π_3^+,π_2^+)	(π_3^+,π_3^+)	$\Vert$	$\Vert$	$\Vert$
$4\pi_1^-$	$16(\pi_1^-,\pi_1^+)$	$4(\pi_1^-,\pi_2^+)$	$4(\pi_1^-,\pi_3^+)$	$16(\pi_1^-,\pi_1^-)$	$\Vert$	$\Vert$
π_2^-	$4(\pi_2^-,\pi_1^+)$	(π_2^-,π_2^+)	(π_2^-,π_3^+)	$4(\pi_2^-,\pi_1^-)$	(π_2^-,π_2^-)	$\Vert$
π_3^-	$4(\pi_3^-,\pi_1^+)$	(π_3^-,π_2^+)	(π_3^-,π_3^+)	$4(\pi_3^-,\pi_1^-)$	(π_3^-,π_2^-)	(π_3^-,π_3^-)

Here too it is necessary consider the $\left[(O_{\Delta m})_{(d,\underline{d})}=(1.15)\,\text{MeV}\right]$, we obtain so:

$$m(\rho^0)=m(\rho^0)^*-\Delta m(\rho^0)-O_{\Delta m}=(775.76)\,\text{MeV} \tag{30}$$

There is another possibility in combinations between basic pions of lattice $\{\pi_r^0\}$: a charged pion ($\pi^\pm$) can interpenetrate a molecule π_r^0 : [$\pi^\pm \to \pi_r^0$].

We have two possibilities: $\rho^+=\left[(\pi^+\otimes\pi^-)\oplus\pi^+\right]$, $\rho^-=\left[(\pi^-\otimes\pi^+)\oplus\pi^-\right]$.

Then one can have: $\pi_r^\pm=\left[\pi^+\oplus(\pi^-\otimes\pi^\pm)\right]$. So, it is possible having:

$$\begin{aligned}
\rho^\pm &= \left[(d,\underline{d})_\pi\right]\otimes\left\{\left[(2\pi^+\oplus\pi_r^0)_{B_1}\otimes(2\pi^-\oplus\pi_r^\pm)_{B_2}\right]_A\right\}\\
&=\left[(d,\underline{d})_\pi\right]\otimes\left\{\left[4(\pi^+\otimes\pi^-)\oplus(2\pi^+\otimes\pi_r^\pm)\oplus(\pi_r^0\otimes 2\pi^-)\oplus(\pi_r^0\otimes\pi_r^\pm)\right]_B\right\}\\
&=\left[(d,\underline{d})_\pi\right]\otimes\left\{\left[4(\pi^\pm)^0\oplus(2\pi^+\otimes\pi_r^\pm)\oplus(\pi_r^0\otimes 2\pi^-)\oplus(\pi_r^0\otimes\pi_r^\pm)\right]_B\right\}\\
&=\left[(d,\underline{d})_\pi\right]\otimes\left\{\left[4(\pi^\pm)^0\oplus(2\pi^+\otimes\pi_r^\pm)\oplus(2\pi^-\otimes\pi_r^0)\oplus(\pi_r^0\otimes\pi_r^\pm)\right]_B\right\}
\end{aligned} \tag{31}$$

Note that:

$$\begin{aligned}
m(\pi_r^\pm)&=m\left(\left[\pi^+\oplus(\pi^-\otimes\pi^\pm)\right]\right)=m(\pi^+)+m\left(\langle\pi^+,\pi^\pm\rangle\right)\\
&=m(\pi^+)+\left(\frac{m(\pi^+)+m(\pi^\pm)}{2}\right)=2m(\pi^+)=m(\pi_r^0)
\end{aligned}$$

Appling the F_m with this relation, it is:

$$\begin{aligned}
F_m(\rho^\pm)&=F_m\left(\left[(d,\underline{d})_\pi\right]\otimes\left[4(\pi^\pm)^0\oplus(2\pi^+\otimes\pi_r^\pm)\oplus(2\pi^-\otimes\pi_r^0)\oplus(\pi_r^0\otimes\pi_r^\pm)\right]_B\right)\\
&=F_m\left(\left[(d,\underline{d})_\pi\right]\otimes\left[4(\pi^\pm)^0\oplus 2(2\pi^\pm\otimes\underline{\pi}_r^\pm)\oplus(\pi_r^0\otimes\pi_r^\pm)\right]_B\right)\\
&\equiv F_m\left(\left\{\left[(d,\underline{d})_\pi\otimes 4(\pi^\pm)_{12}^0\right]_{A_1}\oplus\left[(d,\underline{d})_\pi\otimes 2(2\pi^\pm\otimes\underline{\pi}_r^\pm)_{12}\right]_{A_2}\right.\right.\\
&\qquad\left.\left.\oplus\left[(d,\underline{d})_\pi\otimes(\pi_{r_1}^0\otimes\pi_r^\pm)\right]_{A_3}\right\}\right)
\end{aligned} \tag{32}$$

where $F_m(\underline{\pi}_r^\pm)\equiv F_m(\pi_r^\pm)\equiv F_m(\pi_r^0)$. This equation is equal to Equation (22), because $m(\pi_r^\pm)=m(\pi_r^0)$. It follows partial masses are equal: $m^*(\rho^\pm)=m^*(\rho^0)$. Thus we have: $m(\rho^\pm)^*=(827.24)\,\text{MeV}$.

Note that the structure equations are different only in the interpenetration of $\pi^\pm$ with a charged pion π^- of the structure; therefore, an eventual mass defect could be only in this interpenetration:

$$\begin{aligned}
&\left\{\rho^+\leftrightarrow\left[(\pi^+\otimes\pi^-)\oplus\pi^+\right]\text{ or }\rho^-\leftrightarrow\left[(\pi^-\otimes\pi^+)\oplus\pi^-\right]\right\}\\
&\leftrightarrow\left\{\rho^\pm\leftrightarrow\left[(\pi^\pm\otimes\pi^\mp)\oplus\pi^\pm\right]\right\}
\end{aligned}$$

In Equation (26) we admitted $\Delta m(A_4)=0$ with $\left[A_4=(\pi_r^0)\otimes(\pi_r^0)\right]$, now we have $\left[(A_4)^\pm=(\pi_r^0\otimes\pi_r^\pm)\right]$; therefore, we will have:

$$\begin{aligned}
\Delta m(A_4)_{(d,\underline{d})} &= F_{\Delta m}\left(\left[\left(\pi_r^0 \otimes \pi_r^\pm\right)\right]_{A_4}\right)_{(d,\underline{d})} \\
&= F_{\Delta m}\left(\left\{\left(\pi^+ \oplus \pi^-\right) \otimes \left[\left(\pi^\pm \otimes \pi^\mp\right) \oplus \pi^\pm\right]\right\}_{A_4}\right)_{(d,\underline{d})} \\
&= F_{\Delta m}\left(\left(\pi^+ \oplus \pi^-\right)\right)_{(d,\underline{d})} + F_{\Delta m}\left(\left[\left(\pi^\pm \otimes \pi^\mp\right) \oplus \pi^\pm\right]\right)_{(d,\underline{d})} + O'_{\Delta m} \\
&= \left(\frac{1}{2}\right) F_{\Delta m}\left(\left(\pi^+ \oplus \pi^-\right)\right) + \left(\frac{1}{2}\right) F_{\Delta m}\left(\left[\left(\pi^\pm \oplus \pi^\mp\right) \otimes \pi^\pm\right]\right) + O'_{\Delta m} \\
&= \left(\frac{1}{2}\right) F_{\Delta m}\left(\left(\pi^+ \oplus \pi^-\right)\right) + \left(\frac{1}{2}\right) F_{\Delta m}\left(\left[\left(\pi^\pm \oplus \pi^\mp\right)_{\pi^\pm}\right]\right) + O'_{\Delta m} \\
&= \left\{\left(\frac{1}{2}\right)(4.60)\,\text{MeV} + \left(\frac{1}{2}\right)\left[-(4.60/2)\right]\text{MeV}\right\} + O'_{\Delta m} \qquad (33) \\
&= (1.15)\,\text{MeV} + O'_{\Delta m}
\end{aligned}$$

The value (−4, 6/2) is due to anti-shield of subscript $\pi^\pm$. We calculate $O'_{\Delta m}$

$$\begin{aligned}
O'_{\Delta m} &= F_{\Delta m}\left(\left\{\left(\pi^+ \oplus \pi^-\right) \underline{\otimes} \left[\left(\pi^\pm \otimes \pi^\mp\right) \oplus \pi^\pm\right]\right\}_{A_4}\right)_{(d,\underline{d})} \\
&= F_{\Delta m}\left(\left\{\left(\pi^+ \oplus \pi^-\right) \underline{\otimes} \left[\left(\pi^\pm \oplus \pi^\mp\right)_{\pi^\pm}\right]\right\}\right)_{(d,\underline{d})} \\
&= F_{\Delta m}\left(\left[\left(\pi^+ \oplus \pi^-\right) \underline{\otimes} \left(\pi^\pm \oplus \pi^\mp\right)_{\pi^\pm}\right]\right)_{(d,\underline{d})} \\
&= F_{\Delta m}\left(\left[\left(\pi^\pm \oplus \pi^\mp\right)_{\pi^\pm}\right]\right)_{(d,\underline{d})} = \left(\frac{1}{2}\right)(4.60)_{\pi^\pm}\,\text{MeV} \\
&= +\left(\frac{1}{2}\right)\left[-(4.60/2)\right]\text{MeV} = -(1.15)\,\text{MeV}
\end{aligned}$$

It follows: $m(\rho^\pm) = m(\rho^\pm)^* - \Delta m(\rho^\pm) - O_{\Delta m} = (775.76)\,\text{MeV}$.

So we obtain (for $\rho^\pm$) the same result (the mass defect) of ρ^0-meson. In this way, it could be here a perfect symmetry between $\rho^\pm$ and ρ^0.

In this quarks' model, one could so obtain the mass symmetry in the triplet ($\rho^\pm$, ρ^0), inside the experimental approximation. Besides, these procedures show correctly the existence of d-quark lattice or $\{d, \underline{d}\}$ where mesons are immersed, see the neutral molecule of pions. By structure equation, we have the following decay:

$$\rho = \left[(d,\underline{d})_\pi\right] \otimes \left\{\left[\left(2\pi^+ \oplus \pi_r^0\right)_{B_1} \otimes \left(2\pi^- \oplus \pi_r^0\right)_{B_2}\right]_A\right\} \quad \text{with} \quad (\rho^0) \to (\pi^+ + \pi^-)$$

$$\rho^\pm = \left[(d,\underline{d})_\pi\right] \otimes \left\{\left[\left(2\pi^+ \oplus \pi_r^0\right)_{B_1} \otimes \left(2\pi^- \oplus \pi_r^\pm\right)_{B_2}\right]_A\right\} \quad \text{with} \quad (\rho^\pm) \to (\pi^\pm + \pi^0)$$

This because $\left[(d,\underline{d})_\pi \otimes \pi_r^0\right] \leftrightarrow (\pi^0)$ and $\left[\pi_r^0 = (\pi^+ \oplus \pi^-)\right]$

A second channel, with η-meson $(\pi_r^0 \otimes \pi_r^0)$, could be $[\rho \to \eta + \gamma]$, where the (γ) is originated by (d, $\underline{d}$), and another $[\rho \to \eta + \pi]$, but they have low probability ≈ $(10)^{-4}$. With high probability, there is only the first channel: in fact, the ρ-meson decays in two pions, see ref. [11]. Spin of ($\rho^\pm$, ρ^0), see structure equation, is given by couple (d, $\underline{d}$) because the system [(η) ⊕ ($\pi^\pm$)] has zero spin while the pair (d, $\underline{d}$) being potentially a (matter-antimatter) pair shows a spin (s = 1) in correspondence to the virtual γ-photon:

$$S\left(\rho^{\pm}\right)=S\left(d,\underline{d}\right)_{\gamma}+\left[S\left(\eta\right)+S\left(\pi^{\pm}\right)\right]=S\left(\gamma_{v}\right)=1.$$

where (γ_v) is virtual photon because of binding energy. Therefore ρ-mesons show a spin: [s = +1]. Note the ρ-mesons are vectorial mesons. Also in $\rho^{\pm}$ meson, we treat an individual interaction between each pion (π_i) with other pions (π_j), $\left[\left(\pi_{i}^{\pm}\otimes\pi_{j}^{\pm}\right)_{\oplus}\right]$, where we can consider that

$$\Delta m\left(A_{12}\right)=\Delta m\left(A_{13}\right)=\Delta m\left(A_{14}\right)=\Delta m\left(A_{23}\right)=\Delta m\left(A_{24}\right)=\Delta m\left(A_{34}\right)=0$$

We elaborate the structure equation of ρ-meson, see Equation (21):

$$\begin{aligned}\rho^{\pm}=&\left\{\left[\left(2\pi_{1}^{+}\otimes\pi_{2}^{+}\right)\oplus\left(2\pi_{1}^{+}\otimes\pi_{2}^{-}\right)\right]\oplus\left[\left(2\pi_{1}^{-}\otimes\pi_{3}^{+}\right)\oplus\left(2\pi_{1}^{-}\otimes\pi_{3}^{-}\right)\right]\right\}\\&\oplus\left\{\left[4\left(\pi_{1}^{+}\otimes\pi_{1}^{-}\right)\oplus\left(\left(\pi_{2}^{+}\oplus\pi_{2}^{-}\right)\otimes\left(\pi_{3}^{\pm}\oplus\left(\pi_{3}^{\mp}\otimes\pi_{4}^{\pm}\right)\right)\right)\right]\right\}\end{aligned}\tag{34}$$

Note $\left\{\rho^{+}\leftrightarrow\left[\left(\pi^{+}\otimes\pi^{-}\right)\oplus\pi^{+}\right]\text{ or }\rho^{-}\leftrightarrow\left[\left(\pi^{-}\otimes\pi^{+}\right)\oplus\pi^{-}\right]\right\}\leftrightarrow$ $\left\{\rho^{\pm}\leftrightarrow\left[\left(\pi^{\pm}\otimes\pi^{\mp}\right)\oplus\pi^{\pm}\right]\right\}$

with

$$\left(\pi_{3}^{\pm}\oplus\left(\pi_{3}^{\mp}\otimes\pi_{4}^{\pm}\right)\right)=\left(\left(\pi_{3}^{\pm}\oplus\pi_{3}^{\mp}\right)_{\pi_{4}^{\pm}}\right)$$

The subscript $\pi_4^{\pm}$ lows the mass defect between the two pion $\left(\pi^{\pm}\oplus\pi^{\mp}\right)$:

$$\Delta m\left[\left(\pi^{\pm}\oplus\pi^{\mp}\right)_{\pi_{4}^{\pm}}=\left(2.3\right)\text{MeV}\right]$$

Then we need to have a symmetric matrix; it follows, processing Table 7, that:

The mass defect is:

$$\begin{aligned}\Delta m\left(\rho^{\pm}\right)=&\left[17\left(4.6\right)_{\left(i,i\right)_{G}}+\left(4.6/4\right)_{\left(\pi_{4}^{0}\right)_{G}}+8\left(2.3\right)_{\left(1,2,3\right)_{Y}}+10\left(2.3/2\right)_{\left(1,2,3\right)_{Y}}\right.\\&\left.-8\left(1.15\right)_{\left(\pm,\pm\right)_{B}}+10\left(1.15/2\right)_{\left(\pm,\pm\right)_{B}}-2\left(1.15\right)_{33R}\right]\text{MeV}\\=&\left(105.25\right)\text{MeV}\end{aligned}$$

$$\Delta m\left(\rho^{\pm}\right)_{\left(d,\underline{d}\right)}=\left(52.63\right)\text{MeV}$$

$$m\left(\rho^{\pm}\right)=\left(827.24\right)\text{MeV}-\left(53.20\right)\text{MeV}=\left(774.61\right)\text{MeV}$$

Besides, if we consider, as it is in the neutral ρ-meson, the term of the mass defects $O'_{\Delta m}$, with $\left[\left(O_{\Delta m}\right)_{d,\underline{d}}=-\left(1.15\right)\text{MeV}\right]$, this because $\rho^{\pm}$-meson has an electric charge (see its structure equation). So we will have:

$$m\left(\rho^{\pm}\right)-\left(1.15\right)\text{MeV}=\left[\left(774.61\right)+\left(1.15\right)\right]\text{MeV}=\left(775.76\right)\text{MeV}$$

Here too, we obtain the symmetry between ($\rho^{\pm}$, ρ^{0}).

ω-meson (782.7 MeV)

Further proof of existence of lattices {d, $\underline{d}$} and {π^0} is in the neutral ω-meson which could be defined by the connection:

Table 7. The elements of $||A_{ij}||$ used for calculate the mass defects in $\rho^{\pm}$ meson.

$\rho^{\pm}$	$4\pi_1^{+}$	π_2^{+}	$\pi_3^{+}\otimes\pi_4^{+}$	$4\pi_1^{-}$	π_2^{-}	$\pi_3^{-}\otimes\pi_4^{-}$
$4\pi_1^{+}$	$16(\pi_1^{+},\pi_1^{+})$	\\	\\	\\	\\	\\
π_2^{+}	$4(\pi_2^{+},\pi_1^{+})$	(π_2^{+},π_2^{+})	\\	\\	\\	\\
$\pi_3^{+}\otimes\pi_4^{+}$	$4(\pi_3^{+},\pi_1^{+})_{\pi_4^{+}}$	$(\pi_3^{+},\pi_2^{+})_{\pi_4^{+}}$	$(\pi_3^{+},\pi_3^{+})_{2\pi_4^{+}}$	\\	\\	\\
$4\pi_1^{-}$	$16(\pi_1^{-},\pi_1^{+})$	$4(\pi_1^{-},\pi_2^{+})$	$4(\pi_1^{-},\pi_3^{+})_{\pi_4^{+}}$	$16(\pi_1^{-},\pi_1^{-})$	\\	\\
π_2^{-}	$4(\pi_2^{-},\pi_1^{+})$	(π_2^{-},π_2^{+})	$(\pi_2^{-},\pi_3^{+})_{\pi_4^{+}}$	$4(\pi_2^{-},\pi_1^{-})$	(π_2^{-},π_2^{-})	\\
$\pi_3^{-}\otimes\pi_4^{-}$	$4(\pi_3^{-},\pi_1^{+})_{\pi_4^{-}}$	$(\pi_3^{-},\pi_2^{+})_{\pi_4^{-}}$	$(\pi_3^{-},\pi_3^{+})_{\pi_4^{0}}$	$4(\pi_3^{-},\pi_1^{-})_{\pi_4^{-}}$	$(\pi_3^{-},\pi_2^{-})_{\pi_4^{-}}$	$(\pi_3^{-},\pi_3^{-})_{2\pi_4^{-}}$

$$\omega = \left[(d,\underline{d})_\pi\right]\otimes\left\{\left[\left(2\pi^{+}\oplus 2\pi^{0}\right)\otimes\left(2\pi^{-}\oplus 2\pi^{0}\right)\right]\right\}.$$

We have:

$$\begin{aligned}
\omega &= \left[(d,\underline{d})_\pi\right]\otimes\left\{\left[\left(2\pi_1^{+}\oplus 2\pi_1^{0}\right)_{B_1}\otimes\left(2\pi_2^{-}\oplus 2\pi_2^{0}\right)_{B_2}\right]_A\right\} \\
&= \left[(d,\underline{d})_\pi\right]\otimes\left\{\left[4\left(\pi_1^{+}\otimes\pi_2^{-}\right)\oplus\left(2\pi_1^{+}\otimes 2\pi_2^{0}\right)\right.\right. \\
&\qquad \left.\left.\oplus\left(2\pi_1^{0}\otimes 2\pi_2^{-}\right)\oplus\left(\left(2\pi_1^{0}\right)\otimes\left(2\pi_2^{0}\right)\right)\right]_B\right\} \\
&= \left[(d,\underline{d})_\pi\right]\otimes\left\{\left[4\left(\pi^{\pm}\right)_{12}^{0}\oplus 2\left(2\pi^{\pm}\otimes 2\pi^{0}\right)_{12}\oplus\left(2\pi_{12}^{0}\right)\right]_B\right\} \\
&= \left\{\left[(d,\underline{d})_\pi\otimes 4\left(\pi^{\pm}\right)^{0}\right]_{A_1}\oplus\left[(d,\underline{d})_\pi\otimes 2\left(2\pi^{\pm}\otimes 2\pi^{0}\right)\right]_{A_2}\right. \\
&\qquad \left.\oplus\left[(d,\underline{d})_\pi\otimes\left(2\pi_{12}^{0}\right)\right]_{A_3}\right\}
\end{aligned} \tag{35}$$

where we use the $3_\otimes$-property and 4, in **Table 2**, in prospective to apply the F_m. In fact, note that

$$m\left(\left(2\pi_1^{0}\right)\otimes\left(2\pi_2^{0}\right)\right) = m\left(\left\langle\left(2\pi_1^{0}\right),\left(2\pi_2^{0}\right)\right\rangle\right) = \frac{m\left(2\pi_1^{0}\right)+m\left(2\pi_1^{0}\right)}{2} = m\left(2\pi_1^{0}\right)$$

The partial mass is:

$$\begin{aligned}
m(\omega)^{*} &= m\left\{\left[(d,\underline{d})_\pi\otimes 4\left(\pi^{\pm}\right)^{0}\right]_{A_1}\oplus\left[(d,\underline{d})_\pi\otimes 2\left(2\pi^{\pm}\otimes 2\pi^{0}\right)\right]_{A_2}\right. \\
&\qquad \left.\oplus\left[(d,\underline{d})_\pi\otimes\left(2\pi^{0}\right)\right]_{A_3}\right\} \\
&= \left\{\left\langle m\left((d,\underline{d}),4\left(\pi^{\pm}\right)^{0}\right)\right\rangle+\left\langle m\left((d,\underline{d}),2\left(2\pi^{\pm}\otimes 2\pi^{0}\right)\right)\right\rangle+\left\langle m\left((d,\underline{d}),\left(2\pi^{0}\right)\right)\right\rangle\right\} \\
&= \left[\frac{m(d)+m\left(4\pi^{\pm}\right)^{0}}{2}+\frac{m(d)+2m\left(\left\langle 2\pi^{\pm},2\pi^{0}\right\rangle\right)}{2}+\frac{m(d)+m\left(2\pi^{0}\right)}{2}\right]
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3}{2}m(d) + 2m(\pi^{\pm}) + m(\pi^{\pm}) + m(\pi^{0}) + m(\pi^{0}) \\
&= \frac{3}{2}m(d) + 3m(\pi^{\pm}) + 2m(\pi^{0}) \\
&= \frac{3}{2}(86.26) + 3(139.57) + 2(134.97)\,\text{MeV} \\
&= (818.04)\,\text{MeV}
\end{aligned} \tag{36}
$$

The mass defect can calculate with more facility by matrix $\|A_{ij}\|$.

Then we can write the structure equation in the following way:

$$
\begin{aligned}
\omega &= \left[(d,\underline{d})_{\pi}\right] \otimes \left\{\left[\left(2\pi_1^{+} \oplus 2\pi_2^{0}\right) \otimes \left(2\pi_3^{-} \oplus 2\pi_4^{0}\right)\right]\right\} \\
&= \left\{\left[\left(2\pi_1^{+} \otimes 2\pi_3^{-}\right) \oplus \left(2\pi_1^{+} \otimes 2\pi_4^{0}\right) \oplus \left(2\pi_2^{0} \otimes 2\pi_3^{-}\right) \oplus \left(\left(2\pi_2^{0}\right) \otimes \left(2\pi_4^{0}\right)\right)\right]\right\}_{(d,\underline{d})} \\
&= \left(\omega_B\right)_{(d,\underline{d})}
\end{aligned}
$$

where

$$
\begin{aligned}
\omega_B &= \left\{\left[\left(2\pi_1^{+} \otimes 2\pi_3^{-}\right) \oplus \left(2\pi_1^{+} \otimes 2\pi_4^{0}\right) \oplus \left(2\pi_2^{0} \otimes 2\pi_3^{-}\right) \oplus \left(\left(2\pi_2^{0}\right) \otimes \left(2\pi_4^{0}\right)\right)\right]\right\} \\
&= \left\{\left[\left(2\pi_1^{+} \otimes 2\pi_3^{-}\right) \oplus 4\left(\pi_4^{0}\right)_{\pi_1^{+}} \oplus 4\left(\pi_2^{0}\right)_{\pi_3^{-}} \oplus \left(\left(2\pi_2^{0}\right) \otimes \left(2\pi_4^{0}\right)\right)\right]\right\} \\
&= \left\{\left[\left(2\pi_1^{+} \otimes 2\pi_3^{-}\right) \oplus 4\left(\pi_4^{0^{+}}\right) \oplus 4\left(\pi_2^{0^{-}}\right) \oplus \left(\left(2\pi_2^{0}\right) \otimes \left(2\pi_4^{0}\right)\right)\right]\right\}
\end{aligned}
$$

Recall that in the calculations it is necessary to double the numerical values, see matrix $\|A_{ij}\|$, see Table 8.

We need to consider all elements of the matrix because in interaction with shield particles could be $\left[\left(\left(\pi_i \otimes \pi_j\right)_{\oplus}\right) \neq \left(\left(\pi_j \otimes \pi_i\right)_{\oplus}\right)\right]$; The mass defect is:

$$
\begin{aligned}
\Delta m(\omega_B) &= \left[8(4.6)_{(-,+)} + 16(2.3)_{(\pm,0^{\pm})} + 8(1.15)_{(0^{\pm},0^{\pm})}\right]_{(G)}\,\text{MeV} \\
&\quad + \left[32(1.15)_{(0,\pm)} + 24(0.58)_{(0,0^{\pm})}\right]_{Y}\,\text{MeV} \\
&\quad + \left[-16(2.3)_{(\pm,0^{\pm})} - 8(1.15)_{(0^{-},0^{-})}\right]_{(B)}\,\text{MeV} + \left[\approx 0\right]_{R}\,\text{MeV} \\
&= (87.52)\,\text{MeV}
\end{aligned}
$$

Table 8. The elements of $\|A_{ij}\|$ used for calculate the mass defects in ω-meson.

	$2\pi_1^{+}$	$2\pi_4^{0+}$	$2\pi_3^{-}$	$2\pi_2^{0-}$	$2\pi_2^{0}$	$2\pi_4^{0}$
$2\pi_1^{+}$	$8(\pi_1^{+},\pi_1^{+})$	\\	\\	\\	\\	\\
$2\pi_4^{0+}$	$8(\pi_4^{0+},\pi_1^{+})$	$8(\pi_4^{0+},\pi_4^{0+})$	\\	\\	\\	\\
$2\pi_3^{-}$	$8(\pi_3^{-},\pi_1^{+})$	$8(\pi_3^{-},\pi_4^{0+})$	$8(\pi_3^{-},\pi_3^{-})$	\\	\\	\\
$2\pi_2^{0-}$	$8(\pi_2^{0-},\pi_1^{+})$	$8(\pi_2^{0-},\pi_4^{0+})$	$8(\pi_2^{0-},\pi_3^{-})$	$8(\pi_2^{0-},\pi_2^{0-})$	\\	\\
$2\pi_2^{0}$	$8(\pi_2^{0},\pi_1^{+})$	$8(\pi_2^{0},\pi_4^{0+})$	$8(\pi_2^{0},\pi_3^{-})$	$8(\pi_2^{0},\pi_2^{0-})$	$8(\pi_2^{0},\pi_2^{0})$	\\
$2\pi_4^{0}$	$8(\pi_4^{0},\pi_1^{+})$	$8(\pi_4^{0},\pi_4^{0+})$	$8(\pi_4^{0},\pi_3^{-})$	$8(\pi_4^{0},\pi_2^{0-})$	$8(\pi_4^{0},\pi_2^{0})$	$8(\pi_4^{0},\pi_4^{0})$

$$\Delta m(\omega_B)_{(d,\underline{d})} = (43.76)\,\text{MeV}$$
$$m(\omega) = (827.24)\,\text{MeV} - (43.76)\,\text{MeV} = (783.48)\,\text{MeV}$$

The mass defect in the elements in red is almost zero for the presence of neutral pions which shield the repulsive actions. If we consider $O_{\Delta m}$ will be:

$$m(\omega) = m^*(\omega) - O_{\Delta m} = (783.48)\,\text{MeV} - (1.15)\,\text{MeV} = (782.33)\,\text{MeV} \quad (37)$$

In perfect accord with experimental value [11]. The most probable decay (see structure equation) can be:

$$\omega \to \left[\pi^+ + \pi^- + \pi^0\right]_{90\%};\ \ \omega \to \left[\pi^0 + \gamma\right]_{8\%};\ \ \omega \to \left[\pi^+ + \pi^-\right]_{2\%}$$

Here, the photon (γ) is originated by lattice $\{d, \underline{d}\}$. Spin of (ω), see structure equation, is given by couple (d, $\underline{d}$) which shows a spin(s = 1), because potentially it is in correspondence to the virtual γ-photon: $S(\omega) = S(d,\underline{d})_\gamma = S(\gamma_v) = 1$

Therefore ω-mesons is vectorial mesons ($J^P = 1^-$). To think to particles made of quarks in interpenetration with lattices $\{\pi^0\}$, the pair$\{d, \underline{d}\}$ can induce us to see the mesons with a structure more articulated or "molecules" of quarks.

ϕ -meson (1019.41 MeV)

Another meson with by lattice $\{\pi^0\}$ is the ϕ -meson:

$$\phi^0 = \left\{\left[(d,\underline{d}) \otimes (2\pi^0 \oplus \eta)\right] \oplus (\eta)\right\}$$

Then

$$\begin{aligned}\phi^0 &= \left\{\left[(d,\underline{d}) \otimes (2\pi^0 \oplus \eta)\right] \oplus \eta\right\} \\ &= \left\{\left[\left((d,\underline{d}) \otimes 2\pi^0\right) \oplus \left((d,\underline{d}) \otimes \left(\pi_r^0 \otimes \pi_r^0\right)_\eta\right)\right] \oplus \left(\pi_r^0 \otimes \pi_r^0\right)_\eta\right\}\end{aligned} \quad (38)$$

The partial mass is

$$\begin{aligned} m(\phi)^* &= m\left(\left\langle\left((d,\underline{d}), 2\pi^0\right)\right\rangle\right) + m\left(\left\langle\left((d,\underline{d}), (\eta)\right)\right\rangle\right) + m(\eta) \\ &= (1/2)\left[2m(d) + m(2\pi^0) + 3m(\eta)\right] \\ &= \left[(86.26) + (134.98) + (821.91)\right]\text{MeV} \\ &= (1043.15)\,\text{MeV}\end{aligned} \quad (39)$$

There could be a mass defect $\Delta m(\phi)$. We adapt the equation

$$\begin{aligned}\phi^0 &= \left\{\left[(d,\underline{d}) \otimes (2\pi^0 \oplus \eta)\right] \oplus \eta\right\} \\ &= \left\{\left[(2\pi^0 \oplus \eta)_{(d,\underline{d})}\right] \oplus \eta\right\} = \left\{\left[(2\pi_a^0 \oplus \eta_c)_{(d,\underline{d})}\right] \oplus \eta_c\right\} \\ &= \left\{\left[\left(2\pi_a^0 \oplus \left(\pi_{r_d}^0 \otimes \pi_{r_e}^0\right)_b\right)_{(d,\underline{d})}\right] \oplus \left(\pi_{r_f}^0 \otimes \pi_{r_g}^0\right)_c\right\} \\ &= \left\{\left[\left(2\pi_a^0 \oplus \left(\pi_{r_e}^0\right)_{\pi_{r_d}^0}\right)_{(d,\underline{d})}\right] \oplus \left(\pi_{r_f}^0\right)_{\pi_{r_g}^0}\right\} \\ &= \left\{\left[\left(2\left(\pi_1^+ \underline{\otimes} \pi_1^-\right)_b \oplus \left(\pi_2^+ \oplus \pi_2^-\right)_{\pi^0}\right)_{(d,\underline{d})}\right] \oplus \left(\pi_3^+ \oplus \pi_3^-\right)_{\pi^0}\right\}\end{aligned} \quad (40)$$

The matrix is, see **Table 9**.

Table 9. The elements of $||A_{ij}||$ used for calculate the mass defects in ϕ^0-meson.

ϕ	$2\pi_1^+$	$(1/4)\pi_2^+$	$(1/2)\pi_3^+$	$2\pi_1^-$	$(1/4)\pi_{2(d,\underline{d})}^-$	$(1/2)\pi_3^-$
$2\pi_1^+$	$4(\pi_1^+,\pi_1^+)$	\\	\\		\\	\\
$(1/4)\pi_{2(d,\underline{d})}^+$	$(1/2)(\pi_2^+,\pi_1^+)_{(d,\underline{d})}$	$(1/16)(\pi_2^+,\pi_2^+)_{(d,\underline{d})}$	\\		\\	\\
$(1/2)\pi_3^+$	(π_3^+,π_1^+)	$(1/8)(\pi_3^+,\pi_2^+)$	$(1/4)(\pi_3^+,\pi_3^+)$		\\	\\
$2\pi_1^-$	$4(\pi_1^-,\pi_1^+)$	$(1/2)(\pi_1^-,\pi_2^+)$	(π_1^-,π_3^+)	$4(\pi_1^-,\pi_1^-)$		
$(1/4)\pi_2^-$	$2(1/4)(\pi_2^-,\pi_1^+)_{(d,\underline{d})}$	$(1/16)(\pi_2^-,\pi_2^+)_{(d,\underline{d})}$	$(1/8)(\pi_2^-,\pi_3^+)_{(d,\underline{d})}$	$(1/2)(\pi_2^-,\pi_1^-)_{(d,\underline{d})}$	$(1/16)(\pi_2^-,\pi_2^-)_{(d,\underline{d})}$	\\
$(1/2)\pi_3^-$	(π_3^-,π_1^+)	$(1/8)(\pi_3^-,\pi_2^+)$	$(1/4)(\pi_3^-,\pi_3^+)$	(π_3^-,π_1^-)	$(1/8)(\pi_3^-,\pi_2^-)$	$(1/4)(\pi_3^-,\pi_3^-)$

The mass defect will be:

$$\begin{aligned}\Delta m(\phi) &= \left[\frac{69}{16}\right]_G (4.6)\,\text{MeV} + \left[\frac{26}{8}\right]_Y (2.3)\,\text{MeV} \\ &\quad - \left[\frac{26}{8}\right]_B (1.15)\,\text{MeV} - \left[(1.15)\right]_R \text{MeV} \\ &= \left[\frac{69+26}{16}\right]_G (4.6)\,\text{MeV} - \left[\frac{26}{8}\right]_B (1.15)\,\text{MeV} - \left[(1.15)\right]_R \text{MeV} \\ &= \left[(27.31)-(3.74)-(1.15/4)\right]\text{MeV} = (23.28)\,\text{MeV}\end{aligned}$$

Then, the mass is:

$$m(\phi) = m(\phi)^* - \Delta m(\phi) = \left\{(1043.15)-\left[(23.28)\right]\right\}\text{MeV}/c^2 = (1019.87)\,\text{MeV}/c^2 \quad (41)$$

Spin of (ϕ), see structure equation, is given by couple (d, $\underline{d}$) because the system $\left\{\left[\left(2\pi^0 \oplus \eta\right)\right] \oplus (\eta)\right\}$ has zero spin while the pair (d, $\underline{d}$) shows a spin ($s = 1$) in correspondence to the virtual γ-photon:

$S(\rho^\pm) = S(d,\underline{d})_\gamma + \left[S(\eta) + S(\pi^\pm)\right] = S(\gamma_v) = 1$.

Therefore ϕ -mesons show a spin: [$s = +1$]. Note the ϕ -mesons are vectorial mesons.

3.3. Mesons' Spectrum

If we look with attention the structure equations of mesons, one can deduce a representative spectrum built by pion "quanta" which overlap each other, see the ref. [4]. Looking at structure equation, we can see a meson as constituted by of an integer number of elementary pions. The increasing of mass in mesons is so incorrespondence with an increase in the number of constituent pions. The law of increasing of mass (or of pions) has an intuitive graphics representation, **Figure 2**.

Where (π^0) is the neutral pion, while $(\pi^0)_r$ is [$\pi^+ \oplus \pi^-$]. We could think to 6-level others mesons as *i.e.* $x = \left\{3(d,\underline{d}) \otimes \cdots \left[\left(\pi^0\right)_r \otimes \cdots (\eta \oplus \eta \oplus \eta)\right] \cdots\right\}$.

By this model, it is possible to calculate all others mesons with higher mass. It needs to think the structure equation, calculating the global mass defect and thus

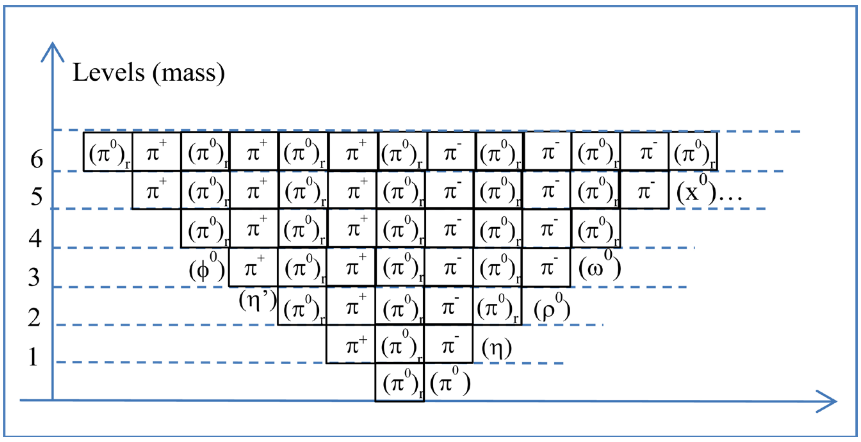

Figure 2. Configuration of meson spectrum.

Table 10. Table of synthesis of the Mass values.

Meson	Level	Structure Equation	Theory Mass (MeV)	Exper. Mass (MeV)
π^0	1	$\left[\pi^+\otimes\pi^-\right]\oplus\left[(u\oplus\underline{u})\otimes(d\oplus\underline{d})\right]$	(134.97)	(134.97)
η^0	2	$(\pi^0)_r\otimes(\pi^0)_r=(\pi^+\oplus\pi^-)\otimes(\pi^+\oplus\pi^-)$	(547.95)	(547.86)
η'	3	$\{(\pi^0)_r\otimes(\eta)\}=(\pi^0)_r\otimes(\pi^0)_r\otimes(\pi^0)_r$	(957.44)	(957.78)
ρ	3	$\left[(d,\underline{d})_\pi\right]\otimes\left\{\left[(2\pi^+\oplus\pi_r^0)\otimes(2\pi^-\oplus\pi_r^0)\right]\right\}$	(775.76)	(775.26)
$\rho^\pm$	3	$\left[(d,\underline{d})_\pi\right]\otimes\left\{\left[(2\pi^+\oplus\pi_r^0)\otimes(2\pi^-\oplus\pi_r^\pm)\right]\right\}$	(775.76)	(775.26)
ω	4	$\left[(d,\underline{d})_\pi\right]\otimes\left\{\left[(2\pi^+\oplus2\pi^0)\otimes((2\pi^-\oplus2\pi^0)\right]\right\}$	(782.33)	(782.65)
ϕ^0	4	$\phi^0=\left\{\left[(d,\underline{d})\otimes(2\pi^0\oplus\eta)\right]\oplus(\eta)\right\}$	(1019.87)	(1019.41)

finding the global mass value. Note the following Table 10 of confronting:

Furthermore, this quarks' theory allows of calculating the masses of nucleons, as it will be shown in the next study. In essence, a meson is a lattice of quark pairs (see $\{d, \underline{d}\}$, $\{(\pi^0)_r\}$) and internal gluons $\{g\}$ of coupling [4].

4. Conclusions

In the next study, we will show the equation structure of the nucleons and barions without strangeness. The structure equations will allow us to calculate their masses.

The hypothesis of structure allows us of describe the various transformations of particles, as the hadron production accompanied by the production of the W, Z bosons, in the beta decay or the production of hadronic jets:

$$\left(e^-+e^+\right)\to\left(g+g\right)\to q+q\to hadrons$$

How is it possible for photons to create quarks? A comprehensive answer could then be to assign a geometric structure (of quantum oscillators) to all elementary particles (that is not composed of sub-particles) and that it is possible to transform a structure into another. In this way, we introduce a new paradigm in the phenomenology of any interaction, which opens up new perspectives for resolving the various problems of this. On this basis, we can discuss the "internal structure" of quarks with the physical awareness that this structure is realizable only through "particular" quantum oscillators [4] [13]. These oscillators are the IQuO [13] [14]: quantum oscillators with a "sub-structure" or two sub-units of oscillation or "sub-oscillators". Not only, but the energy of the "quanta" should be distributed between these oscillating components. The presence of more components in an oscillator causes the splitting of its quanta of energy into two, and more, sub-oscillators: this introduces the idea of half-quanta.

To treat the particles as IQuO structures can explain the origin of some fundamental physics greatness as the electric charge [13] [14], spin [4] [15], isospin [2] and colour charge [16].

So, the IQuO idea constitutes a new paradigm in physics that allows us to describe with depth the physical phenomenon of particles and to open new descriptive scenarios of interactions between particles.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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