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Probable Decay Modes of Superheavy Nuclei and Cluster Radioactivity

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Abstract

Cluster radioactivity is a very rare phenomenon. Among the various parameters that may determine the cluster radioactivity or cluster decay, the most important parameters are the binding energy $B(A,Z)$ of the parent nucleus and the binding energies of the nuclei that constitute the decay products. These parameters are crucial in determining the nuclear reaction energies (Q-values). Calculations for Q-values for even-even super heavy nuclei whose atomic numbers (Z) range from $Z=100$ to $Z=124$ were computed and the sign of the quantity of Q-values was used to determine whether the super heavy nuclei could undergo cluster radioactivity. The modified Bethe-Weizsäcker formula was used to calculate the binding energy of the nuclei involved in the cluster radioactivity, and the Coulomb energy was calculated using the modified Coulomb energy formula. It was found that nuclei whose Q-values are positive but relatively small fall under the category of super heavy nuclei which undergo cluster radioactivity while those nuclei whose Q-values are very high are significantly unstable, thus, they release energy that is sufficient to cause spontaneous fission. It is also noted that all the parent nuclei whose atomic numbers lie between $Z=100$ to $Z=124$ satisfy the conditions for spontaneous fission, however, for $Z>124$ the possibility for spontaneous fission diminishes. Based on our results, we conclude that nuclei with very high Q-values are significantly unstable, and the energy released is sufficient to cause spontaneous fission. Additionally, the boundary between cluster radioactivity and spontaneous fission remains elusive and an open question.

Keywords

Cluster Radioactivity, Binding Energy, Modified Coulomb Interaction,

Nuclear Reaction Energy, Super Heavy Nuclei

1. Introduction

In the old nuclear physics, the disintegration of heavy nucleons by spontaneous emission of alpha, beta and gamma rays is governed by the Universal Decay Law (UDL). According to this law $N = N_0 e^{-\lambda t}$, where N_0 is the number of nuclei at time $t = 0$, N is the number of nuclei at any time t and λ is the decay constant. In the case of superheavy nuclei, these nuclei can undergo deformation such that in the process of disintegration they emit particles whose mass is more than the mass of alpha-particles. This kind of disintegration is called cluster radioactivity and the emitted nuclei include ${}^{12}_{6}\mathrm{C}$, ${}^{16}_{8}\mathrm{O}$, ${}^{20}_{10}\mathrm{Ne}$, ${}^{24}_{12}\mathrm{Mg}$, ${}^{28}_{12}\mathrm{Mg}$, ${}^{28,32,34}_{14}\mathrm{Si}$, ${}^{46}_{18}\mathrm{Ar}$ and so on [1] [2] depending upon the mass number (A) of the superheavy nucleus, and the magnitude of the atomic number Z. Cluster radioactivity can also be defined as that in which heavy or superheavy atomic nuclei emit small clusters of neutrons and protons with a structure more than that of alpha-particle, but less than a typical binary fission fragment. Thus, cluster radioactivity is a form of disintegration intermediate between alpha decay and spontaneous fission (S.F). The condition for spontaneous fission depends on the magnitude of A and Z such that Z^2/A lies between 17.6 and 50 [3].

Research on Cluster radioactivity has recently gained a lot of attention by nuclear Physicists both experimentally and theoretically [4] [5]. Some of the recent experimental observations have detected the emission of even deuterons and ${}^{6}_{3}\mathrm{Li}$ [6]. These experiments have contributed to the knowledge on nuclear structure physics and cluster structures of the disintegrating nuclei.

Experimentally, cluster radioactivity was discovered in 1984 by Rose and Jones [7] who discovered a new type of nuclear decay. In the alpha-decay of parent nucleus ${}^{227}_{90}\mathrm{Th}$, and mother nucleus or daughter nucleus ${}^{223}_{88}\mathrm{Ra}$, a few events were observed in which ${}^{14}_{6}\mathrm{C}$ was also a decay product. This phenomenon was called cluster radioactivity since the light mass fragments that were emitted had masses in the intermediate range of atomic nuclei. Later, another 20 cluster emitters were discovered [8] in which the decay nuclei ranged from ${}^{12}_{6}\mathrm{C}$ to ${}^{34}_{14}\mathrm{Si}$, and these were emitted by somewhat less heavy nuclei in the range ${}^{221}_{87}\mathrm{Fr}$ to ${}^{242}_{96}\mathrm{Cm}$. It is not easy to observe cluster radioactivity being a rare phenomenon, and it is roughly 9 orders of magnitude weaker than the competing alpha particle decay [5].

In the last three decades, new elements in the periodic table have been synthesized [9]-[13]. As a result, the subject of superheavy nuclei gained enormous interest in nuclear Physics both experimentally and theoretically, with focus on the island of stability. Despite the fact that the discovery of superheavy elements pushed the boundary of nuclear science, the increased number of protons leads to instability of the nuclei decaying within seconds or milliseconds due to the strong electrostatic repulsion within the nucleus and complex interplay of nuclear forces.

The primary objective of this research is to study the theoretical possibility of a strong cluster decay compared to alpha-particle decay for some superheavy nuclei with $Z \geq 100$. Additionally, this study aims at investigating the condition for the superheavy nucleus to decay by the emission of nuclei with mass more than that of alpha-particles. Since superheavy nuclei may decay through various modes such as alpha decay, cluster radioactivity and spontaneous fission (SF), it is important to understand whether there is any preformation of emitted clusters just before disintegration. If the parent nucleus was symmetric, does it become asymmetric after the preformation of clusters? Simultaneously, how does the preformation of clusters affect the Coulomb interaction in the nucleus? [14].

To explore these questions, a few superheavy nuclei were picked and their cluster decay process was studied by calculating the reaction energies (Q-values) of the disintegrating nucleus [15]. The Bethe-Weizsäcker formula was used to calculate the binding energy of the nuclei involved in the cluster radioactivity, and the Coulomb energy was calculated using the modified Coulomb energy formula [16]. Consequently, the sign of the quantity of Q-values was used to determine whether the superheavy nuclei can undergo cluster radioactivity.

2. Theoretical Derivations

2.1. Binding Energies

The binding energy $B(A,Z)$ of an atomic nucleus is composed of many different forms of energy [17] such that $B(A,Z)$ is given by;

$$B(A,Z) = a_1 A - a_2 A^{\frac{2}{3}} - a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_4 \frac{(A-2Z)^2}{A} \pm \delta(A) \tag{1}$$

In Equation (1), the first term is the volume energy, the second term is the surface energy, the third term is the Coulomb energy term, the fourth term is asymmetry term and the fifth term is the pairing energy term. Due to the strong Coulomb repulsion in the nucleus of superheavy nuclei, the third term in Equation (1) is modified. Coulomb energy (E_C) which is as a long-range interaction is written as;

$$E_C = \frac{3}{5} \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r} \tag{2}$$

In Equation (2) e is the electron charge, Z_1 and Z_2 are the proton nuclear charges, r is the distance between the charges and ε_0 is the permittivity of free space. The modified Coulomb potential (E_{CM}) is given by [18];

$$E_{CM} = \frac{3}{5} \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o R_0} e^{\frac{R_0^n}{nR^n}} \tag{3}$$

where R is the radius of the nucleus and it is given by $R = r_0 A^{\frac{1}{3}}$ $(r_0 \simeq 1.5 \times 10^{-13}\ \text{cm})$, $R_0 = r_0 (2Z)^{\frac{1}{3}}$ is the cut off range of the Coulomb potential inside the nucleus and n varies between $n = 0$ and $n = 21$. The value of R_0

may be core radius (the core has $2Z$ particles, and $N-Z$ in the surface region) or charge radius.

For $n>21$, Equation (3) becomes,

$$E_{CM}=a_C\frac{z(z-1)}{(2Z)^{\frac{1}{3}}}\quad\text{where}\quad a_C=0.66\ \text{MeV} \tag{4}$$

Substituting Equation (4) in Equation (1) we obtain,

$$B(A,Z)=a_1A-a_2A^{\frac{2}{3}}-a_3\frac{Z(Z-1)}{(2Z)^{\frac{1}{3}}}-a_4\frac{(A-2Z)^2}{A}\pm\delta(A) \tag{5}$$

where $a_1=15.85\ \text{MeV}$, $a_2=18.34\ \text{MeV}$, $a_3=a_C=0.66\ \text{MeV}$, $a_4=23.21\ \text{MeV}$ and $\delta(A)=a_5A^{-\frac{1}{2}}$ where $a_5=34\ \text{MeV}$ [17] and $\delta(A)$ is zero for odd A nuclei, positive for even-even nuclei and negative for odd-odd nuclei.

2.2. Nuclear Reaction Energies

The nuclear reaction energy (Q-value) is the net energy change in a nuclear reaction. This form of energy is crucial in determining the amount of energy of the emitted particles in a nuclear reaction and provides insights into the stability of the superheavy elements and their decay behaviour. The Q-value of a decay process can also be defined as the increase or decrease in the binding fraction of the decay products which translates to a release or absorption of stored potential energy.

The energy released manifests as kinetic energy of the decay particles or gamma radiation while the energy absorbed manifests as increased kinetic energy of the products or excitation energy in the resulting products which may lead to emission of gamma radiation as the particle returns to a stable state.

In this study, the nuclear reaction energy is calculated for different cluster radioactive decays such as ${}^{8}_{4}\text{Be}$, ${}^{12,14}_{6}\text{C}$, ${}^{28}_{12}\text{Mg}$, ${}^{30,32}_{14}\text{si}$, ${}^{48}_{20}\text{Ca}$, ${}^{66,68}_{28}\text{Ni}$, and so on [15]. As a hypothetical example, we consider the decay of the nucleus ${}^{252}_{100}\text{Fm}$ such that,

$${}^{252}_{100}\text{Fm}\rightarrow{}^{204}_{80}\text{Hg}+{}^{48}_{20}\text{Ca}+Q \tag{6}$$

The decay reaction in Equation (6) can be expressed in terms of binding energies such that,

$$Q=B(A,Z)(Hg)+B(A,Z)(\text{Ca})-B(A,Z)(\text{Fm}) \tag{7}$$

The binding energies $\{B(A,Z)\}$ of ${}^{252}_{100}\text{Fm}$, ${}^{204}_{80}\text{Hg}$ and ${}^{48}_{20}\text{Ca}$ are calculated using Equation (5) and substituted in Equation (7) to obtain the Q-value. Therefore, the general reaction energy equation can be written as;

$$Q=B(Dn)+B(Ec)-B(Pn) \tag{8}$$

where $B=B(A,Z)$ is the binding energy, Dn stands for Daughter nucleus, Ec stands for Emitted cluster and Pn stands for Parent nucleus.

The sign of Q-value is either positive or negative. If the rest mass of the reactants exceeds the rest masses of the products, the Q-value of the reaction is positive implying that energy is released while if Q-value of the reaction is negative it indicates that energy is absorbed. Therefore, the Q-values of radioactive decays whose atomic numbers lie between $Z = 100$ to $Z = 124$ were calculated to determine which nuclei may be stable and become part of the island of stability.

3. Results and Discussions

In this study, the binding energies and reaction energies were calculated using Equation (5) and Equation (8) respectively. The Q-values for selected isotopes of known heavy cluster emitters [19] are shown in **Table 1**.

Table 1. Calculated values of reaction energies for heavy cluster emitters.

Parent Nuclei	Daughter Nuclei	Emitted Cluster	Q-Values (MeV)
$^{222}_{88}\text{Ra}$	$^{208}_{82}\text{Pb}$	$^{14}_{6}\text{C}$	21.909
$^{224}_{88}\text{Ra}$	$^{210}_{82}\text{Pb}$	$^{14}_{6}\text{C}$	21.487
$^{226}_{88}\text{Ra}$	$^{212}_{82}\text{Pb}$	$^{14}_{6}\text{C}$	21.039
$^{228}_{90}\text{Th}$	$^{208}_{82}\text{Pb}$	$^{20}_{8}\text{O}$	33.930
$^{230}_{90}\text{Th}$	$^{206}_{80}\text{Hg}$	$^{24}_{10}\text{Ne}$	47.723
$^{232}_{92}\text{U}$	$^{210}_{82}\text{Pb}$	$^{22}_{10}\text{Ne}$	49.005
$^{234}_{92}\text{U}$	$^{210}_{82}\text{Pb}$	$^{24}_{10}\text{Ne}$	50.298
$^{234}_{92}\text{U}$	$^{208}_{82}\text{Pb}$	$^{26}_{10}\text{Ne}$	46.966
$^{234}_{92}\text{U}$	$^{206}_{80}\text{Hg}$	$^{28}_{12}\text{Mg}$	63.421
$^{236}_{94}\text{Pu}$	$^{208}_{82}\text{Pb}$	$^{28}_{12}\text{Mg}$	67.409
$^{238}_{94}\text{Pu}$	$^{206}_{80}\text{Hg}$	$^{32}_{14}\text{Si}$	79.018

From the reaction energies obtained in **Table 1**, it is noted that all the Q-values are positive and they increase in magnitude with increase in nuclear sizes. The positive Q-values signify that energy is released in the cluster decay process. By definition, the Q-value is the difference between the sums of the masses of initial products from the final products. Alternatively, the Q-value can be defined as the difference between the kinetic energy of the initial products from the kinetic energy of the final products. In our model, the Q-values of the decay process correspond to the increase in binding energies of the decay products, which translates to the positive values of reaction energies shown in **Figure 1**.

Figure 1 clearly illustrates a relationship between atomic numbers of emitted clusters and the magnitude of the Q-values. It is noted that larger clusters such as $^{28}_{12}\text{Mg}$ and $^{32}_{14}\text{Si}$ correspond to higher Q-values due to the mass difference between the parent nucleus and the sum of the daughter nucleus and the emitted cluster, resulting in greater energy release. Similarly, higher atomic numbers among the emitted clusters lead to the stronger repulsive Coulomb force which increases the magnitude of the Q-values since additional energy is required to overcome the Coulomb repulsion.

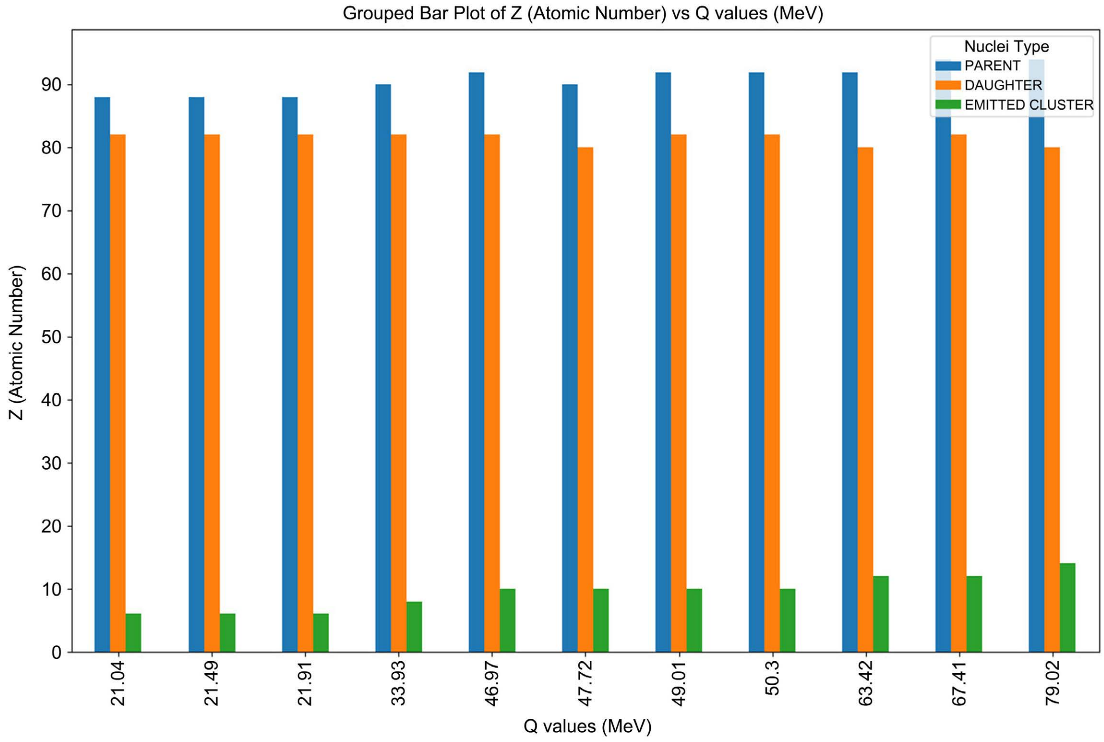

Figure 1. Graph of some known cluster emitters.

Additionally, calculations of Q-values for even-even superheavy nuclei with atomic numbers (Z) ranging from $Z = 100$ to $Z = 124$ were calculated to identify some probable superheavy cluster emitters or/and nuclei that are likely to reside in the island of nuclear stability. The results for these calculations are shown in Table 2.

The calculations of reaction energies for some probable Superheavy cluster emitters shown in Table 2 illustrate that all the Q-values are positive and increase in magnitude with increasing nuclear sizes. The increase in magnitude of the Q-values is associated with increase in decay probabilities and the reduced likelihood of cluster radioactivity occurring among all the superheavy elements. Undoubtedly, cluster radioactivity is a rare phenomenon among the superheavy elements and it lies between alpha decay and fission fragmentation [5] [20]. However, our calculations have shown that very small values of Q-values are also obtained among the superheavy nuclei such as ${}^{16}_{6}\mathrm{C}$ emitted by ${}^{284}_{114}\mathrm{Fl}$, ${}^{12}_{4}\mathrm{Be}$ emitted by ${}^{292}_{116}\mathrm{Lv}$ and ${}^{16}_{6}\mathrm{C}$ emitted by ${}^{294}_{118}\mathrm{Og}$. This shows that superheavy nuclei may emit smaller clusters as they decay thus confirming the dominance of cluster radioactivity among some of the superheavy nuclei [21]. Overall, higher values of reaction energies are observed among the superheavy nuclei due to the strong repulsive Coulomb force. Therefore, it is necessary to define the possible limits of cluster radioactivity in order to distinguish it from other decay modes.

The limits of spontaneous fission are well known and its condition depends on the magnitude of A and Z such that $\frac{Z^2}{A}$ lie between 17.6 to 50 [3]. In this study, the determination for spontaneous fission (S.F) among all the parent nuclei in Table 2 was carried out and the results are shown in Figure 2. It is noted that all the parent nuclei with atomic numbers between $Z = 100$ to $Z = 124$ satisfy the conditions for S.F, however, for $Z > 124$ the possibility for S.F diminishes.

Table 2. Reaction energies for some probable Superheavy cluster emitters.

Parent Nuclei	Daughter Nuclei	Emitted Cluster	Q-Values (MeV)
$^{252}_{100}Fm$	$^{222}_{86}Rn$	$^{30}_{14}Si$	83.354
	$^{220}_{86}Rn$	$^{32}_{14}Si$	89.218
	$^{180}_{72}Hf$	$^{72}_{28}Ni$	176.610
	$^{248}_{98}Cf$	$^{4}_{2}He$	10.935
	$^{198}_{78}Pt$	$^{54}_{22}Ti$	145.411
	$^{202}_{80}Cf$	$^{48}_{20}Ca$	122.572
	$^{202}_{82}Pb$	$^{50}_{18}Ar$	103.865
	$^{188}_{74}W$	$^{64}_{26}Fe$	168.602
$^{252}_{102}No$	$^{204}_{82}Pb$	$^{48}_{20}Ca$	139.097
	$^{202}_{82}Pb$	$^{50}_{20}Ca$	137.136
	$^{178}_{72}Hf$	$^{74}_{30}Zn$	197.108
	$^{170}_{70}Yb$	$^{82}_{32}Ge$	201.982
	$^{168}_{68}Er$	$^{84}_{34}Se$	214.606
	$^{166}_{66}Dy$	$^{86}_{36}Kr$	221.298
	$^{158}_{64}Gd$	$^{94}_{38}Sr$	228.514
	$^{188}_{76}Os$	$^{64}_{26}Fe$	176.123
$^{254}_{104}Rf$	$^{204}_{82}Pb$	$^{50}_{22}Ti$	156.757
	$^{190}_{78}Pt$	$^{50}_{20}Ca$	182.440
	$^{194}_{76}Os$	$^{60}_{28}Ni$	178.596
	$^{190}_{74}W$	$^{64}_{30}Zn$	185.180
	$^{180}_{72}Hf$	$^{74}_{32}Ge$	212.724
	$^{158}_{66}Dy$	$^{96}_{38}Sr$	234.504
	$^{138}_{58}Ce$	$^{116}_{46}Pd$	253.410
	$^{130}_{54}Xe$	$^{124}_{50}Sn$	260.116
$^{268}_{106}Sg$	$^{216}_{82}Pb$	$^{52}_{24}Cr$	156.290
	$^{210}_{80}Hg$	$^{58}_{26}Fe$	173.880
	$^{198}_{78}Pt$	$^{70}_{28}Ni$	197.222
	$^{180}_{72}Hf$	$^{88}_{34}Se$	224.724
	$^{172}_{64}Gd$	$^{96}_{42}Mo$	234.543
	$^{166}_{62}Sm$	$^{102}_{44}Ru$	242.956
	$^{132}_{54}Xe$	$^{136}_{52}Te$	264.331
	$^{126}_{50}Sn$	$^{142}_{56}Ba$	267.859

Continued

${}^{272}_{108}Hs$	${}^{262}_{104}Rf$	${}^{10}_{4}Be$	18.607
	${}^{256}_{102}No$	${}^{16}_{6}C$	29.334
	${}^{240}_{94}Pu$	${}^{32}_{14}Si$	101.852
	${}^{208}_{82}Pb$	${}^{64}_{26}Fe$	191.888
	${}^{186}_{72}Hf$	${}^{86}_{36}Kr$	241.243
	${}^{154}_{62}Sm$	${}^{118}_{46}Pd$	272.539
	${}^{138}_{54}Xe$	${}^{134}_{54}Xe$	280.677
	${}^{130}_{52}Te$	${}^{142}_{56}Ba$	280.910
${}^{280}_{110}Ds$	${}^{216}_{84}Nd$	${}^{64}_{26}Fe$	196.401
	${}^{208}_{82}Pb$	${}^{72}_{28}Ni$	207.459
	${}^{202}_{82}Pb$	${}^{78}_{28}Ni$	190.998
	${}^{216}_{80}Hg$	${}^{64}_{30}Zn$	187.276
	${}^{188}_{70}Yb$	${}^{92}_{40}Zr$	248.928
	${}^{170}_{66}Dy$	${}^{110}_{44}Ru$	278.074
	${}^{168}_{62}Sm$	${}^{112}_{48}Cd$	267.232
	${}^{156}_{60}Nd$	${}^{124}_{50}Sn$	287.743
${}^{284}_{112}Cn$	${}^{280}_{110}Ds$	${}^{4}_{2}He$	13.408
	${}^{212}_{84}Po$	${}^{72}_{28}Ni$	213.674
	${}^{158}_{62}Sm$	${}^{126}_{50}Sn$	301.148
	${}^{160}_{62}Sm$	${}^{124}_{50}Sn$	300.056
	${}^{168}_{66}Dy$	${}^{116}_{46}Pd$	293.786
	${}^{166}_{66}Dy$	${}^{118}_{46}Pd$	292.917
	${}^{150}_{60}Nd$	${}^{134}_{52}Te$	302.313
	${}^{142}_{56}Ba$	${}^{142}_{56}Ba$	305.334
${}^{284}_{114}Fl$	${}^{280}_{112}Cn$	${}^{4}_{2}He$	14.434
	${}^{268}_{108}Hs$	${}^{16}_{6}C$	32.960
	${}^{198}_{80}Hg$	${}^{86}_{34}se$	257.631
	${}^{196}_{80}Hg$	${}^{88}_{34}Se$	254.026
	${}^{146}_{60}Nd$	${}^{138}_{54}Xe$	316.653
	${}^{144}_{58}Ce$	${}^{140}_{56}Ba$	320.294
	${}^{142}_{58}Ce$	${}^{142}_{56}Ba$	319.014
	${}^{206}_{82}Pb$	${}^{78}_{32}Ge$	248.472
${}^{292}_{116}Lv$	${}^{280}_{112}Cn$	${}^{12}_{4}Be$	9.108
	${}^{282}_{112}Cn$	${}^{10}_{4}Be$	22.047
	${}^{206}_{82}Pb$	${}^{86}_{34}se$	264.753
	${}^{154}_{62}Sm$	${}^{138}_{54}Xe$	328.960
	${}^{200}_{80}Hg$	${}^{92}_{36}Kr$	274.374
	${}^{184}_{74}W$	${}^{108}_{42}Mo$	299.980
	${}^{166}_{66}Dy$	${}^{126}_{50}Sn$	324.394
	${}^{152}_{60}Nd$	${}^{140}_{56}Ba$	331.294

Continued

$^{294}_{118}\mathrm{Og}$	$^{278}_{112}\mathrm{Cn}$	$^{16}_{6}\mathrm{C}$	35.504
	$^{210}_{86}\mathrm{Rn}$	$^{84}_{32}\mathrm{Ge}$	253.976
	$^{186}_{76}\mathrm{Os}$	$^{108}_{42}\mathrm{Mo}$	308.753
	$^{174}_{70}\mathrm{Yb}$	$^{120}_{48}\mathrm{Cd}$	332.140
	$^{166}_{68}\mathrm{Er}$	$^{128}_{50}\mathrm{Sn}$	333.767
	$^{160}_{66}\mathrm{Dy}$	$^{134}_{52}\mathrm{Te}$	335.922
	$^{152}_{62}\mathrm{Sm}$	$^{142}_{56}\mathrm{Ba}$	343.951
	$^{172}_{66}\mathrm{Dy}$	$^{130}_{52}\mathrm{Te}$	378.596
$^{296}_{120}\mathrm{X}$	$^{202}_{82}\mathrm{Pb}$	$^{94}_{38}\mathrm{Sr}$	304.571
	$^{204}_{80}\mathrm{Hg}$	$^{92}_{40}\mathrm{Zr}$	307.808
	$^{200}_{78}\mathrm{Pt}$	$^{96}_{42}\mathrm{Mo}$	313.871
	$^{216}_{84}\mathrm{Po}$	$^{80}_{36}\mathrm{Kr}$	280.480
	$^{186}_{76}\mathrm{Os}$	$^{110}_{44}\mathrm{Ru}$	330.145
	$^{190}_{74}\mathrm{W}$	$^{106}_{46}\mathrm{Pd}$	328.878
	$^{178}_{70}\mathrm{Yb}$	$^{118}_{50}\mathrm{Sn}$	344.893
	$^{188}_{70}\mathrm{Yb}$	$^{108}_{50}\mathrm{Sn}$	302.540
$^{298}_{122}\mathrm{X}$	$^{238}_{94}\mathrm{Pu}$	$^{60}_{28}\mathrm{Ni}$	277.855
	$^{214}_{84}\mathrm{Po}$	$^{84}_{38}\mathrm{Sr}$	342.187
	$^{250}_{102}\mathrm{No}$	$^{48}_{20}\mathrm{Ca}$	225.522
	$^{242}_{96}\mathrm{Cm}$	$^{56}_{26}\mathrm{Fe}$	265.625
	$^{228}_{92}\mathrm{U}$	$^{70}_{30}\mathrm{Zn}$	306.213
	$^{214}_{90}\mathrm{Th}$	$^{84}_{32}\mathrm{Ge}$	306.487
	$^{224}_{88}\mathrm{Ra}$	$^{74}_{34}\mathrm{Se}$	317.678
	$^{214}_{86}\mathrm{Rn}$	$^{84}_{36}\mathrm{Kr}$	343.841
$^{306}_{124}\mathrm{X}$	$^{202}_{80}\mathrm{Hg}$	$^{104}_{44}\mathrm{Ru}$	332.524
	$^{210}_{82}\mathrm{Pb}$	$^{96}_{42}\mathrm{Mo}$	348.082
	$^{272}_{108}\mathrm{Hs}$	$^{34}_{16}\mathrm{S}$	142.351
	$^{208}_{82}\mathrm{Pb}$	$^{98}_{42}\mathrm{Mo}$	337.53
	$^{220}_{88}\mathrm{Ra}$	$^{86}_{38}\mathrm{Sr}$	316.537
	$^{276}_{110}\mathrm{Ds}$	$^{30}_{14}\mathrm{Si}$	124.744
	$^{280}_{112}\mathrm{Cn}$	$^{26}_{12}\mathrm{Mg}$	106.27
	$^{212}_{84}\mathrm{Po}$	$^{94}_{40}\mathrm{Zr}$	329.628

From this perspective, it can be inferred that nuclear decay (cluster radioactivity and spontaneous fission) has limiting conditions that depend on its nuclear composition and binding energies. For nuclei with positive but relatively small Q-values (close to zero but not exceeding certain threshold such as 100 MeV) as shown in **Figure 3** and **Table 2**, such parent nuclei are marginally unstable with respect to cluster emission hence they fall under the category of superheavy nuclei which undergo cluster radioactivity rather than fission.

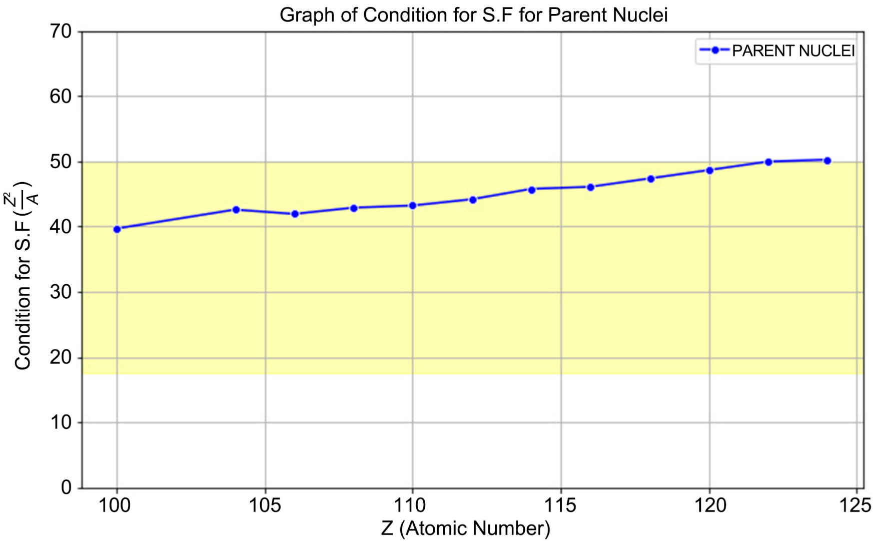

Figure 2. Graph showing the condition for spontaneous fission.

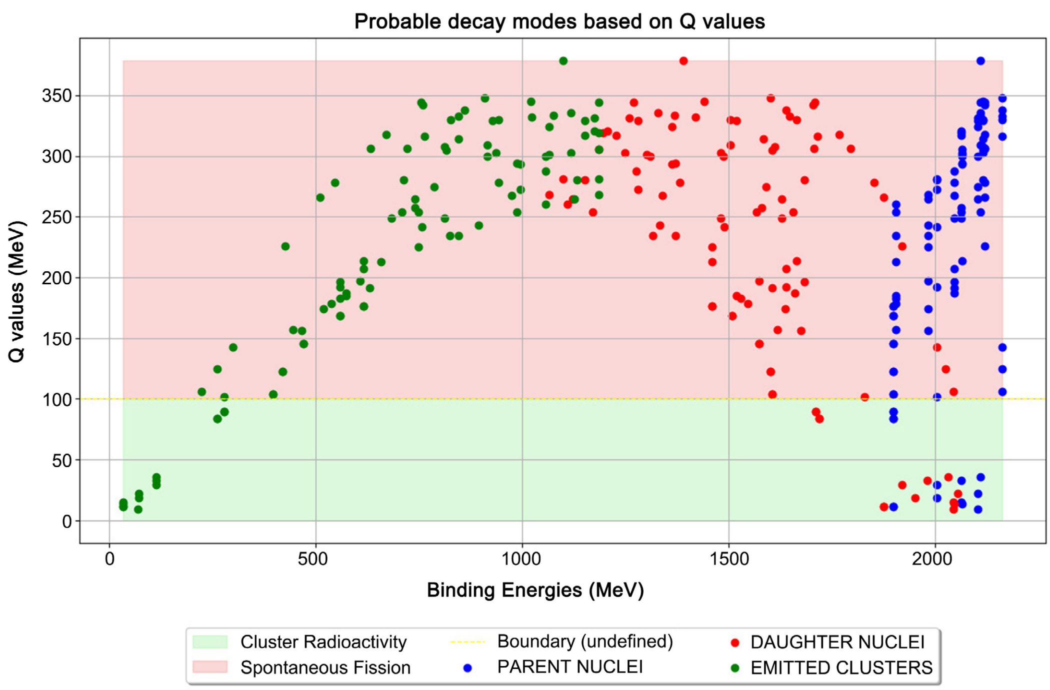

Figure 3. Graph of Q-values against Binding energies showing the probable decay modes.

On the other hand, when the Q-values are very high (greater than 100 MeV) such parent nuclei are significantly unstable. In such cases, the energy released is sufficient to cause spontaneous fission, where the nucleus splits into two nearly equal parts rather than emitting a cluster.

Therefore, we propose that the binding energies of the parent nuclei and the resultant Q-values are the key parameters that determine the condition for cluster radioactivity and spontaneous fission.

4. Conclusions

Generally, a theory explains already experimentally observed phenomena. But cluster decay is one of the rare examples of a phenomena that was theoretically predicted in 1980, prior to its experimental discovery in 1984 by Rose and Jones [7] [19]. Several models have been proposed to explain the decay of heavy and superheavy nuclei into masses intermediate between fission fragments and alpha particles [22]-[25]. While the general mechanism of cluster radioactivity is well understood, accurately predicting decay rates and half-lives for various cluster emissions remains an unsolved challenge.

Our calculations show that both smaller and larger clusters are emitted by superheavy nuclei, with Q-values varying depending on the mass difference between the parent nucleus and the sum of the daughter nucleus and emitted cluster. Generally, nuclei with positive but relatively small Q-values fall under the category of superheavy nuclei that undergo cluster radioactivity rather than nuclear fission. Conversely, nuclei with very high Q-values are significantly unstable, releasing sufficient energy to trigger spontaneous fission.

On the other hand, nuclei with very high Q-values are significantly unstable, and the energy released is sufficient to cause spontaneous fission. It is noted that all the parent nuclei with atomic numbers between $Z = 100$ to $Z = 124$ satisfy the conditions for spontaneous fission. However, for $Z > 124$ the likelihood of spontaneous fission diminishes. In this regard, the boundary between cluster radioactivity and spontaneous fission remains elusive and an open question.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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