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A Dark Matter Theory by Quantum Gravitation for Galaxies and Clusters

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Abstract

This paper develops an original theory of dark matter in the current ΛCDM framework, whose main hypothesis is that DM is generated by the own gravitational field, according to an unknown quantum gravitational phenomenon. This work is the best version of the theory, which I have been developing and publishing since 2014. The hypothesis of DM by quantum gravitation, DMbQG hereafter, has two main consequences: the first one is that the law of DM generation has to be the same, in the halo region, for all the galaxies and the second one is that the haloes are unbounded, so the total DM goes up without limit as the gravitational field is unbounded as well. The first one consequence is backed by the fact that M31 and MW has a fitted function with the same power exponent for the rotation curve at the halo region and both giant galaxies are the only ones whose rotation curves at the halo region may be studied with accuracy. This paper is firstly developed all the theory with M31 rotation curve data up to Chapter 9. The most important formula of the theory is the called Direct mass, which calculates the total mass at a specific radius into the halo region. Chapter 10 is dedicated to apply the theory to Milky Way, it is calculated its total mass at different radius into the halo and such results have been validated successfully using the data of masses at different radius published by two researcher teams. In Chapter 11, it is calculated the direct mass for the Local Group, and it is shown how the DMbQG theory is able to calculate the total mass at 770 kpc, that the dynamical methods estimate to be $5\times10^{12} M_\odot$. In Chapter 12, it is shown a method to estimate the Direct mass formula for a cluster of galaxies, using only its virial mass and virial radius. By this method, it is estimated the parameter a^2 of the Local Group, which match with the one calculated in previous chapter by a different method. Also are calculated the parameters a^2 associated to Virgo and Coma clusters. In Chapter 13, it is demonstrated how the DE is able to counterbalance the DM at cluster scale, as the Direct mass grows up with the square root of radius whereas the

DE grows up with the cubic power. The chapter is an introduction to the DMbQG theory for cluster of galaxies, which has been developed fully by the author in other works. This theory aims to be a powerful method to study DM in the halo region of galaxies and cluster of galaxies and conversely the measures in galaxies and clusters offer the possibility to validate the theory.

Keywords

Dark Matter, Dark Energy, Galaxies, Local Group of Galaxies, Clusters

1. Introduction

Since 2014 up to 2024, I have published several papers studying DM in galactic halos, especially in M31 and Milky Way although also I have published some papers studying other galaxies and cluster, see bibliography.

As reader knows, M31 is the twin galaxy of Milky Way in the Local Group of galaxies. According to [1] Sofue, Y. 2015, its baryonic masses are $M_{M31} = 1.61\times10^{11} M_{\Theta}$ and $M_{MILKY\ WAY} = 1.4\times10^{11} M_{\Theta}$ where M_{Θ} is the Solar mass equal to 1.99E30 kg.

The DM by Quantum Gravitation, DMbQG hereafter, theory was introduced in [2] Abarca, M.2014. *Dark matter model by quantum vacuum.* It considers that DM is generated by the own gravitational field according to an unknown quantum gravitational phenomenon.

In order to study purely the phenomenon, it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible. *i.e.* radius bigger than 30 kpc for MW and 40 kpc for M31, as it will be shown in Chapter 5.

This hypothesis has two main consequences: the first one is that the law of dark matter generation, in the halo region, has to be the same for all the galaxies and the second one is that the haloes are unlimited so the total dark matter goes up without limit. Chapter 13 will be solved this apparent divergence of the total mass, thanks to the Dark energy.

The first consequence before mentioned, dark matter generated by a Universal law, it has been studied through all my papers. In fact, I could develop rigorously the theory because the rotation curve of M31 at halo region decreased with a power regression fitted curve whose exponent is −1/4. However, with the data published for Milky Way at the same paper, [1] Sofue, Y.2015, it was not possible to fit rigorously the rotation curve with such exponent.

Fortunately, in a new paper, [3] Sofue, Y. 2020, the author gives a new rotation curve data for Milky Way at halo region whose fitted curve has an exponent −1/4. Such result was good news for the DMbQG theory, because the theory states a universal law of DM generation in the halo region of galaxies or clusters.

In this paper, it is firstly developed all the theory with M31 rotation curve data up to the chapter 9 and the chapter 10 is dedicated to apply the theory to Milky Way and also its Direct mass formula is validated successfully using the data of masses published at different radius by two researcher teams.

Chapter 11 is estimated the total mass of the local Group, using the Direct mass and considering MW, M31 and its main galaxy satellites: the Large Magellan Cloud and M33 respectively. The total mass calculated for MW, LMC, M31 and M33 is $4.97\times10^{12} M_\Theta$. The mass at 770 kpc for MW and M31 is accepted to be $5\times10^{12} M_\Theta$. See [4] Azadeh Fattahi, Julio F. Navarro.2020. So it is possible to state that both results match perfectly. The importance of these findings is high because, as far I know, there is not any other theory able to explain such amount of mass offering a physic nature of dark matter.

In Chapter 12, it is shown a method to estimate the Direct mass formula for a cluster of galaxies, using only its virial mass and virial radius. By this method it is estimated the parameter a^2 of the Local Group, which match with the one calculated in previous chapter by a different method. The parameter a^2 determines the Direct mass associates to a galactic halo, see formula (8.5). Also are calculated the parameters a^2 associated to Virgo and Coma clusters.

Chapter 13 is crucial to demonstrate how the Dark energy is able to counterbalance the DM at cluster scale, because the Direct mass grows up with the square root of radius whereas the DE grows up with the cubic power and it is equivalent to a negative mass.

This chapter is an introduction to extend the theory to cluster of galaxies. In the paper [5] Abarca, M. 2024, it is developed fully the DMbQG theory in clusters with remarkable theoretical findings validated with results published by well known researcher teams.

As I have mentioned before, this theory has been developed assuming the hypothesis that DM is a quantum gravitational effect. However, it is possible to remain into the Newtonian framework to develop the theory. In my opinion, there are two factors to manage the DM conundrum with a quite simple theory.

The first one, that it is developed into the halo region, where baryonic matter is negligible. The second one, that the mechanics movements of celestial bodies are very slow regarding velocity of light, which is supposed to be the speed of gravitational bosons.

It is known that community of physics is researching a quantum gravitation theory since many years ago, but it does not exist yet, however I think that my works in this area support strongly that DM is a quantum gravitation phenomenon.

Use a more simple theory instead the general theory is a typical procedure in physics.

For example, the Kirchhoff ‘s laws are the consequence of Maxwell theory for direct current and remain valid for alternating current, introducing complex impedances, on condition that signals must have low frequency. However, these laws do not work for electromagnetic microwaves because of its high frequency.

Thanks the possibility to study the gravitational effect of DM pure, in the halo regions of M31 and MW, it have been possible to develop a theory mathematically simple.

The coincidence of the same exponent to the fitted function for the rotation curves at the halo region for both galaxies is crucial in order to state that dark matter is generated according to an Universal law because MW and M31 are the only giant galaxies whose rotation curve at the halo region are known with accuracy.

2. Observational Data for M31 Galaxy from Sofue 2015 Data

Figure 1 comes from [1] Sofue, Y. 2015. The axis for radius has logarithmic scale. Although the Sofue rotation curve ranges from 0.1 kpc up to 352 kpc, the range of dominion considered for this work is only the halo region where the ratio baryonic matter versus dark matter is negligible. In the chapter 5, it will be shown that this happens for radius bigger than 40 kpc. From Figure 1, it is got the Table 1 data set.

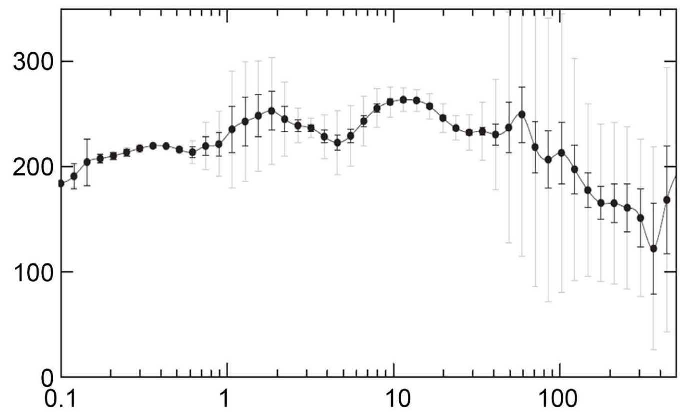

Figure 1. Velocity (km/s) vs radius (kpc).

Table 1. M31 Rotation curve.

kpc	km/s
40.5	229.9
49.1	237.4
58.4	250.5
70.1	219.2
84.2	206.9
101.1	213.5
121.4	197.8
145.7	178.8
175	165.6
210.1	165.6
252.3	160.7
302.9	150.8

The measure at 352 kpc has been rejected because has a velocity too high, so does not match with the other measures. This measure may belong to a celestial object captivated by the gravitational field of M31 afterwards to M31 formation and it is right to think that it is not in dynamical equilibrium with M31. So it is a good criterion to consider only a data set with high statistical correlation.

Power Regression to the Rotation Curve

In Table 2, the first and second column represent the data set of Table 1, the third and fourth columns are the same data into the I.S. of units. The fifth column shows the velocity by the fitted function and the last column shown the relative difference between velocity data and velocity fitted by the power regression function.

Table 2. Relative difference Fitted velocity curve vs velocity measures.

Radius kpc	Vel. km/s	Radius m	Vel. m/s	Vel. fitted	Relative Diff.
40.5	229.9	1.250E+21	2.299E+05	2.510E+05	8.397E−02
49.1	237.4	1.515E+21	2.374E+05	2.393E+05	7.777E−03
58.4	250.5	1.802E+21	2.505E+05	2.292E+05	−9.304E−02
70.1	219.2	2.163E+21	2.192E+05	2.190E+05	−8.154E−04
84.2	206.9	2.598E+21	2.069E+05	2.093E+05	1.138E−02
101.1	213.5	3.120E+21	2.135E+05	2.000E+05	−6.755E−02
121.4	197.8	3.746E+21	1.978E+05	1.911E+05	−3.500E−02
145.7	178.8	4.496E+21	1.788E+05	1.826E+05	2.107E−02
175	165.6	5.400E+21	1.656E+05	1.745E+05	5.115E−02
210.1	165.6	6.483E+21	1.656E+05	1.668E+05	7.100E−03
252.3	160.7	7.785E+21	1.607E+05	1.594E+05	−8.307E−03
302.9	150.8	9.347E+21	1.508E+05	1.523E+05	9.891E−03

In Table 3 are shown the parameters of the fitted function velocity versus radius, third and fourth columns, using a power regression function.

Table 3. Power regression for M31 rot. Curve.

$V = a \cdot r^b$	
a	4.32928×10^{10}
b	−0.24822645
Correlation coeff.	0.96

In Figure 2 are plotted the data set of Table 2 in grey colour, also it is represented the fitted function by a power regression whose parameters are shown in Table 3.

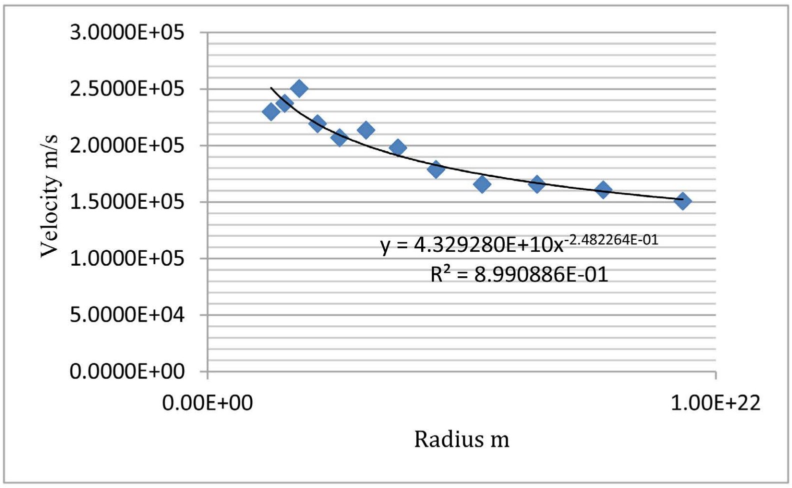

Figure 2. Fitted function to rotation curve.

The correlation coefficient is 0.96 which is a superb result especially when dominion measures are up to 303 kpc.

There is not any other galaxy with a rotation curve data set so wide.

According to the DMbQG theory, the galaxy halo is unbounded, but as the distance MW and M31 is 770 kpc, it is right to consider that inside the data set radius of M31, *i.e.* from 40 kpc up to 300 kpc the gravitational influence of Milky Way may be neglected as a first approximation, using a model of spherical symmetry for the gravitational field.

3. Direct Formula for DM Density on M31 Halo Got From Rotation Curve

3.1. Getting the Direct DM Density from Newtonian Dynamics

In the halo region, the rotation curve it is fitted by power regression with a high correlation coefficient according to formula $v = a \cdot r^b$. As in the Newtonian framework, $M_{DYNAMIC}(<r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius *r*, by substitution of velocity results $M = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$. Hereafter this formula will be called Direct Mass

$$M_{DIRECT}(<r) = \frac{a^2 \cdot r^{2b+1}}{G} \tag{3.1}$$

because it has been got rightly from rotation curve.

If it is considered the halo region where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In the next chapter will be show that for $r > 40$ kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{\mathrm{d}m}{\mathrm{d}V}$ where $\mathrm{d}m = \frac{a^2 \cdot (2b+1) \cdot r^{2b} \mathrm{d}r}{G}$ and $\mathrm{d}V = 4\pi r^2 \mathrm{d}r$ then results $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$. Writing $L = \frac{a^2 \cdot (2b+1)}{4\pi G}$

results

$$D_{DM}(r) = L \cdot r^{2b-2} \tag{3.2}$$

This formula will be called Direct DM density. If $b = -1/2$ then the L parameter is zero and the DM density will be zero, which is the Keplerian rotation.

3.2. Direct DM Density for M31 Halo

Parameters a & b from power regression of M31 rotation curve, see Table 3, allow calculate easily the direct DM density.

Direct DM density for M31 halo 40 < r < 300 kpc (3.3)

$D_{DM}(r) = L \cdot r^{2b-2}$ kg/m³ being L = 1.1255E+30 and $2b - 2 = -2.4964529$.

It is important to highlight that this formula is only a statistical approximation of DM density able to explain the rotation curve, into the dominion 40 kpc up to 300 kpc.

4. Dark M. Density as Power of Gravitational Field *E*

As the independent variable for this function is E, the gravitational field, previously will be studied the formula for E in the following paragraph.

4.1. Gravitational Field *E*

As it is known the total gravitational field, considering spherical symmetry, is the centripetal acceleration, $E = v^2/R$ whose unit is m/s² (I.S.). Hereafter, the gravitational field will be represented by E.

The key to use the centripetal acceleration to calculate E is the dynamical equilibrium. It is supposed that celestial bodies are quite close to dynamical equilibrium, because the most of celestial bodies belong to a specific galactic system from its formation times.

By substitution of $v = a \cdot r^b$ into the formula $E = \frac{v^2}{r}$ it is got $E = \frac{a^2 \cdot r^{2b}}{r}$, briefly

$$E = a^2 \cdot r^{2b-1} \tag{4.1}$$

4.2. Dark Matter Density as Power of Gravitational Field

According to the hypothesis of dark matter by quantum vacuum

$$D_{DM} = A \cdot E^B \tag{4.2}$$

i.e. the DM density at a point depend on the gravitational field according to a power function. The parameters A & B have to be calculated. This hypothesis has been widely studied by the author in previous papers: [2] Abarca, M.2014. [6] Abarca, M. 2016. [7] Abarca, M.2019 and [8] Abarca, M. 2023. This hypothesis fulfils the physic meaning of D.M. Density formula in the halo region because it is supposed that such D.M. density is generated as a consequence of gravitational field propagation in the framework of a quantum gravitation theory.

The formula (3.2) is $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$, the direct DM density depend on a & b parameters which come from power regression formula for velocity. In the previous epigraph has been got the formula for $E = a^2 \cdot r^{2b-1}$ (4.1) which depend on a & b as well.

As D_{DM} depend on parameters a, b similarly to gravitational field E, then it is possible to get the formulas for the variable change of D_{DM}. *i.e.* D_{DM} depending on radius, formula (3.2) is changed into D_{DM} depending on E, formula $D_{DM} = A \cdot E^B$ (4.2).

Through a simple mathematical treatment it is possible to get the coefficients A & B depending on a & b, specifically these formulas are:

$$A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G} \quad \& \quad B = \frac{2b-2}{2b-1}, \tag{4.3}$$

According to parameters, a & b shown in Table 3, in Table 4 are shown A & B coefficients for M31 galaxy in the halo region.

Table 4. M31 galaxy.

	$D_{DM} = A \cdot E^B$
A	3.6559956×10^{-6}
B	1.6682469

Conversely

$$b = \frac{B-2}{2B-2} \quad \text{and} \quad a = \left[\frac{4\pi GA(B-1)}{2B-3}\right]^{\frac{2b-1}{2}} \quad \text{being} \quad B \neq 1 \quad \text{and} \quad B \neq 3/2, \tag{4.4}$$

As conclusion, in this chapter has been demonstrated that a power law for the rotation velocity $v = a \cdot r^b$ *is mathematically equivalent to a power law for DM density depending on E.* $D_{DM} = A \cdot E^B$ *on condition that D.M. is generated by the gravitation field.*

4.3. The Importance of $D_{DM} = A \cdot E^B$

This formula is the cornerstone for DMbQG theory because it is supposed that DM is generated locally according to an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ and $E = a^2 \cdot r^{2b-1}$ have been got rightly from rotation curve. Therefore it can be considered more specific for each galaxy. However the formula $D_{DM} = A \cdot E^B$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be similar for different galaxies on condition that the galaxies are similar. In further chapters will be got that power B is exactly the same for M31 and Milky Way although parameter A will be a bit different.

However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters. As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example, inside the Solar system it is clear that Newton and General Relativity Theory are able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore, it is right to conclude that DM is non negligible when gravitational interaction takes a longer time to link the matter. Namely, at galactic scale.

5. Ratio Baryonic Mass versus Dark Matter Mass Depending on Radius for M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy. According to [1] Sofue, Y.2015, the data for M31 disk are (Table 5):

Table 5. M31 disk parameters.

Baryonic Mass at disk	a_d	Σ_0
$M_d = 2\pi \cdot \Sigma_0 \cdot a_d^2$		
$M_d = 1.26 \times 10^{11} M_\Theta$	5.28 kpc	1.5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents the superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi \cdot r\Sigma(r) \cdot \mathrm{d}r = 2\pi \cdot \Sigma_0 \cdot a_d^2$

To convert superficial baryonic density to volume density it is right using the formula:

$$D_{BARYONIC}^{VOLUME} = \frac{\Sigma(r)}{2r} \quad \text{So} \quad D_{BARYONIC}^{VOLUME}(40\text{ kpc}) = 3.1 \times 10^{-25}\text{ kg/m}^3, \tag{5.1}$$

The definitive formula of Direct Dark matter density is got in epigraph 8.2. $D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ Being L = 1.33E+30 formula (8.3).

For example

$$D_{DM}(40\text{ kpc}) = 2.5\text{E} - 23\text{ kg/m}^3, \tag{5.2}$$

So the ratio baryonic (5.1) versus DM (5.2) densities at 40 kpc is 0.0124.

In conclusion it is right to consider negligible the baryonic density at 40 kpc, therefore it is possible to estate that halo dominion begins at 40 kpc in M31.

6. A Bernoulli Differential Equation for the Gravitational Field E

6.1. Introduction

The formula (4.2) $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \frac{G \cdot M(<r)}{r^2} = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ has been got through $v = a \cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, the D_{DM} formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves $M(<r)$ which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get $E(r)$ involves $M(<r)$ which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A \cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

6.2. A Bernoulli Differential Equation for the Gravitational Field in the Halo

As it is known in this formula $\vec{E} = -G\frac{M(r)}{r^2}\hat{r}$, $M(r)$ represents mass enclosed by a sphere with radius r. If it is considered a region where does not exist any baryonic matter, such as any galactic halo, then the derivative of $M(r)$ depend on dark matter density essentially and therefore

$$M'(r) = 4\pi r^2 D_{DM}(r), \tag{6.1}$$

If $E = G\frac{M(r)}{r^2}$, vector modulus, is differentiated then it is got

$$E'(r) = G\frac{M'(r) \cdot r^2 - 2rM(r)}{r^4}$$

If $M'(r) = 4\pi r^2 D_{DM}(r)$ is replaced above then it is got

$E'(r) = 4\pi G D_{DM}(r) - 2G\frac{M(r)}{r^3}$ As $D_{DM}(r) = A \cdot E^B(r)$ it is right to get

$$E'(r) = 4\pi \cdot G \cdot A \cdot E^B(r) - 2\frac{E(r)}{r} \tag{6.2}$$

which is a Bernoulli differential equation.

$E'(r) = K \cdot E^B(r) - 2\frac{E(r)}{r}$ Being

$$K = 4\pi \cdot G \cdot A \tag{6.3}$$

Calling y to E, the differential equation is written in this simple way

$$y` = K \cdot y^B - \frac{2 \cdot y}{r} \tag{6.4}$$

The Bernoulli differential Equation (6.4) may be converted into a differential linear equation with this variable change $u = y^{1-B}$ and this is the linear equation:

$$\frac{u`}{1-B} + \frac{2u}{r} = K \tag{6.5}$$

The homogenous equation is

$$\frac{u`}{1-B}+\frac{2u}{r}=0 \tag{6.6}$$

and its general solution is

$$u=C\cdot r^{2B-2} \tag{6.7}$$

being C the integration constant.

If it is searched a particular solution for the complete differential Equation (6.5) with a simple linear function $u=z\cdot r$ then it is got that

$$z=\frac{K\cdot(1-B)}{3-2B}. \tag{6.8}$$

Therefore the general solution for (6.5) is $u=C\cdot r^{2B-2}+z\cdot r$

When it is inverted the variable change it is got the general solution for (6.3)

And it is got

$$E(r)=\left(Cr^{2B-2}+\frac{K(1-B)\cdot r}{3-2B}\right)^{\frac{1}{1-B}} \text{ with } B\neq 1 \text{ and } B\neq 3/2 \tag{6.9}$$

where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha=2B-2$, $\beta=\frac{1}{1-B}$ and $D=\frac{K(1-B)}{3-2B}$ then

$$E(r)=\left(Cr^{\alpha}+Dr\right)^{\beta} \tag{6.10}$$

Calculus of parameter C through initial conditions R_0 and E_0

Suppose R_0 and E_0 are the specific initial conditions for radius and E, then

$$C=\frac{E_0^{1/\beta}-D\cdot R_0}{R_0^{\alpha}} \tag{6.11}$$

Final comment

It is clear that the Bernoulli solution contains implicitly the fact that it is supposed there is not any baryonic matter inside the radius dominion and the only DM matter is added by $D_{DM}(r)=A\cdot E^{B}(r)$. Therefore this solution for field works only in the halo region and R_0 and E_0 could be the border radius of galactic disk where it is supposed begins the halo region and the baryonic density is negligible, although it is possible to select any other point belonging to the halo.

7. Dimensional Analysis for D.M. Density as Power of *E* Formula

7.1. Power of *E* Using Only One Pi Monomial

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D, density of DM whose units are kg/m^3, E gravitational field whose units are m/s^2, G and finally h.

Table 6 developed dimensional expression for these four elements D, E, G and h.

Table 6. Dimensional expression for D, E, G, h.

	G	h	E	D
M	−1	1	0	1
L	3	2	1	−3
T	−2	−1	−2	0

According to Buckingham theorem it is got the following formula for Density $D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ Being K a dimensionless number which may be understood as a coupling constant between field E and DM density.

As it is shown in Table 4, for M31 galaxy, the parameter B = 1.6682469

So the relative difference between B = 1.6682469 and 10/7 ≈ 1.428 is 16.7% which is not excessive, however in the following epigraph will be demonstrated the impossibility of this formula in the framework of the theory.

7.2. Impossibility of D.M. Density Formula with Only One PI Monomial Theorem

The formulas (4.3) are $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \frac{2b-2}{2b-1}$. Being a, b the parameters got to fit rotation curve of velocities $v = a \cdot r^b$

As A has to be a positive quantity, see formula (4.2), then $2b+1>0$. As $2b+1 = \frac{2B-3}{B-1} > 0$ therefore mathematically $B \in (-\infty, 1) \cup (3/2, \infty)$.

If B = 3/2 then $2b$ + 1 = 0 and A = 0 so dark matter density is cero which is the Keplerian rotation case.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore the experimental data got in several galaxies match perfectly with mathematical findings made in this paragraph, namely for $B \in (3/2, \infty)$.

The main consequence of this mathematical analysis is that formula

$$D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$$

got with only one pi monomial has to be rejected because B = 10/7 < 3/2.

7.3. Power E Formula for DM Density with Two PI Monomials

As consequence of the previous theorem it is compulsory to explore a new formula with two pi monomials.

As this formula come from quantum gravitation theory it is right to include c, the velocity of light as the additional Universal constant required in this analysis.

So the elements to make dimensional analysis are D, E, G, h and c (Table 7).

Table 7. Dimensional expression for D, E, G, h, c.

	G	h	E	D	c
M	−1	1	0	1	0
L	3	2	1	−3	1
T	−2	−1	−2	0	−1

According to Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous epigraph and the second one involves G, h, E and c. These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So according to the Buckingham theorem the formula for DM density through two pi monomials will be $D_{DM} = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown function, which cannot be calculated by the dimensional analysis theory.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of DM density. As the hypothesis of the theory estates that DM is generated through a quantum gravitation mechanism it is right to consider not only G and h but also c as it is supposed that gravitons are virtual particles whose velocity is c.

7.4. Looking for a D.M. Density Function Coherent with Dimensional Analysis

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E. In other words, to select the function $f(\pi_2)$ it is used the Ockham's razor principle.

Taking in consideration the A & B parameters fitted to M31 halo galaxy, see Table 4, it is right to consider 5/3 as the best simple number closer to the empirical value $B = 1.6682$

To achieve this goal the power for π_2 must be −5/6. This way, power of E in formula $D_{DM} = A \cdot E^B$ will be 5/3 and the formula of Density with two pi monomial $D_{DM} = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes

$$D_{DM} = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h}} E^{\frac{5}{3}} \tag{7.1}$$

being M a dimensionless number.

8. Recalculating Formulas in M31 Halo with $B = 5/3$

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent. Therefore it is needed to rewrite all the formulas considering $B = 5/3$. Furthermore, with $B = 5/3$, a lot of parameters of the theory become simple fraction numbers. In other words, the theory gains simplicity.

In the chapter 4 was shown that the relation between a & b parameters and A & B parameters are:

$$A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G} \quad \& \quad B = \frac{2b-2}{2b-1}. \text{ Formulas (4.3)}$$

Considering $B = 5/3$ It is right to get

$$b = \frac{B-2}{2B-2} = -\frac{1}{4} \quad \text{and} \quad A = \frac{a^{\frac{-4}{3}}}{8\pi G} \tag{8.1}$$

Therefore, the central formula of theory (4.2) becomes

$$D_{DM} = A \cdot E^{\frac{5}{3}} = \frac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}} \tag{8.2}$$

In the chapter 10 will be studied the rotation curve of Milky Way according to the data published by [3] Sofue. 2020, and it will be shown that in the halo region the rotation curve decreases with the same exponent $b = -1/4$. This fact is crucial for DMbQG theory because both giant galaxies are the only ones with accuracy measures in the halo region.

8.1. Recalculating the Parameter *a* in M31 Halo

Table 3 comes from chapter 2 and represents regression curve of velocity depending on radius.

Due to the Buckingham theorem it is needed that $b = -1/4$. Therefore, it is needed to recalculate the parameter a in order to find a new couple of values a &b that fit perfectly to the experimental measures in M31 halo.

Recalculating A with the Minumun Square Method

When it is searched the parameter a, a method widely used is called the minimum squared method. So it is searched a new parameter a for the formula $v = a \cdot r^{0.25}$ on condition that $\sum_e (v - v_e)^2$ has a minimum value. Where v represents the value fitted for velocity formula, r_e represents each radius measure and v_e represents its velocity associated. So using the data set in **Table 2**, third and fourth columns, it is right to get the value for parameter a.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 4.727513 \times 10^{10} \text{ m}^{5/4}/\text{s}$$

From now on it is possible to consider the dimension precise for parameter a, as this parameter is the coefficient for the formula of velocity $v = a \cdot r^{-1/4}$ so its dimension will be $\text{m}^{5/4}/\text{s}$ and for the parameter a^2 its dimension will be $\text{m}^{5/2}/\text{s}^2$.

This parameter a^2 will be used continuously in the following chapters.

8.2. Formulas of Direct D.M. for Density for Mass and Field E

With these new parameters recalculated, see Table 8, it is going to get the new direct formulas for DM density, field E and total mass.

Table 8. New parameters a & b and A & B for M31 galaxy.

B	5/3
$b=\frac{B-2}{2B-2}$	$b=-1/4$
a new	$4.727513 \times 10^{10}\ \mathrm{m}^{5/4}/\mathrm{s}$
A new	3.488152×10^{-6}

The formula (3.3) for DM Density depending on radius now becomes:

$$D_{DM}(r)=L\cdot r^{2b-2}=L\cdot r^{\frac{-5}{2}} \tag{8.3}$$

Being $L=\frac{a^2\cdot(2b+1)}{4\pi G}=\frac{a^2}{8\cdot\pi\cdot G}=1.33\times10^{30}\ \mathrm{kg}/\mathrm{m}^{1/2}$

The formula (4.1) for field E now becomes:

$$E=a^2\cdot r^{2b-1}=a^2\cdot r^{\frac{-3}{2}} \tag{8.4}$$

Being $a^2=2.235\times10^{21}\ \mathrm{m}^{5/2}/\mathrm{s}^2$

The formula (3.1) for direct mass now becomes:

$$M_{DIRECT}(<r)=\frac{a^2\cdot r^{2b+1}}{G}=\frac{a^2\cdot\sqrt{r}}{G} \tag{8.5}$$

8.3. Bernoulli Solution for E and DM Density in M31 Halo

In the chapter 6 was got the solution for field in the halo region, now thanks dimensional analysis it is possible to simply the Bernoulli formulas, using the Table 8. Namely:

The formula (6.10) for field $E(r)=\left(Cr^{\alpha}+Dr\right)^{\beta}$ being $\alpha=2B-2=\frac{4}{3}$ and $\beta=\frac{1}{1-B}=\frac{-3}{2}$ becomes

$$E(r)=\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-3}{2}} \tag{8.6}$$

The formula (6.11) for the initial condition $C=\frac{E_0^{1/\beta}-D\cdot R_0}{R_0^{\alpha}}$ becomes

$$C=\frac{E_0^{\frac{-2}{3}}-D\cdot R_0}{R_0^{\frac{4}{3}}} \tag{8.7}$$

The parameter D becomes $D=\frac{4\cdot\pi\cdot G\cdot A(1-B)}{3-2B}=8\cdot\pi\cdot G\cdot A$ and as

$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$ (8.1) then

$$D = a^{\frac{-4}{3}} = 5.85\times10^{-15}\ \mathrm{m}^{-5/3}\cdot \mathrm{s}^{4/3} \tag{8.8}$$

Bernoulli Solution for Dm Density in Halo Region

As the formula for DM density is $D_{DM} = A\cdot E^{B}$, formula (4.2), being $B = 5/3$ it is right to get the new formula for DM density.

$$D_{BERNI}(r) = A\cdot E^{\frac{5}{3}} = A\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}} = \frac{D}{8\pi G}\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}} \tag{8.9}$$

8.4. Dark Matter at a Spherical Corona by Bernoulli Solution in Halo Region

Formula below expresses the dark matter contained inside a spherical corona defined by

R_1 and R_2 belonging at halo.

$$M_{BERNI} = \int_{R_1}^{R_2} 4\pi\cdot r^2\cdot D_{BERNI}\mathrm{d}r = \int_{R_1}^{R_2} 4\pi\cdot r^2 AE^{B}\mathrm{d}r$$
$$= 4\pi A\int_{R_1}^{R_2} r^2\left[C\cdot r^{4/3} + D\cdot r\right]^{\frac{-5}{2}}\cdot \mathrm{d}r$$

The indefinite integral

$$I = 4\pi A\cdot\int\frac{r^2}{\left(C\cdot r^{4/3} + D\cdot r\right)^{2.5}} = \frac{8\pi A\sqrt{r}}{D\cdot\left(C\cdot\sqrt[3]{r} + D\right)^{\frac{3}{2}}} = \frac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r} + D\right)^{\frac{3}{2}}} \tag{8.10}$$

as $\frac{8\pi A}{D} = \frac{1}{G}$

By the Barrow's rule over the primitive (8.10) is got the DM contained into the spherical corona defined by R_2 and R_1.

$$M_{BERNI}{}_{R_1}^{R_2} = M_{BERNI}\left(R_2\right) - M_{BERNI}\left(R_1\right) \tag{8.11}$$

8.5. Newton's Theorem with Bernoulli Mass Formula

The name for this theorem has been chosen because the relation between field E and total mass $M(<r)$ is the same that in Newton's theory.

From Bernoulli field (8.6) $E_{BERNI}(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}} = \frac{1}{r^{3/2}\cdot\left(C\cdot r^{1/3} + D\right)^{3/2}}$

Bernoulli mass formula (8.10) $M_{BERNI}(r) = \frac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ so

$$G \cdot M_{BER}(r) = \frac{\sqrt{r}}{\left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} \quad \text{and}$$

$$\frac{G \cdot M_{BER}(r)}{r^2} = \frac{\sqrt{r}}{r^2\left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} = \frac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D\right)^{3/2}} = E(r)$$

Therefore

$$E_{BERNI}(r) = \frac{G \cdot M_{BER}(r)}{r^2} \tag{8.12}$$

This identity shows how the DMbQG theory, adding an extra of mass depending on radius, is able to explain the DM in galaxies and clusters in the Newtonian framework.

Corollary

According to the Newtonian framework, the function of mass included in the formula of field E (8.12) means the total mass included inside the sphere with radius r. Therefore $M_{BER}(r)$ must be renamed as $M_{BER}(<r) = M_{TOTAL}(<r)$ where r ranges in the halo region *i.e.* $M_{BER}(r)$ gives the total mass enclosed by the sphere with radius r.

In conclusion, the total mass enclosed by the sphere with radius r is given by the formula:

$$M_{TOTAL}(<r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} \tag{8.13}$$

where r belong to the halo region. *i.e.* $r > 40$ kpc for M31 galaxy.

Put in brief, the formula (8.13) is a corollary of the identity (8.12)

Notice how in the formula the variable $M(r)$ has been changed by $M(<r)$ and the subscript **Berni** has been changed by **Total**.

8.6. Calculus of Parameter *C*

This parameter, see formula (8.7), may be calculated by the data set of rotation curve in halo region, see Table 1.

Theoretically any point belonging to the data set may be used as initial condition, and the value got must be the same.

E_0 is the gravitational field at R_O radius. Considering that data measures are in dynamical equilibrium, it is possible to estimate the field by $E_0 = v_0^2 / r_0$ using data in Table 1.

Finally the parameter D = 5.85E−15 was calculated in epigraph 8.3 using the value B = 5/3 got by dimensional analysis.

It is clear that the data set are not in perfect dynamical equilibrium, but data selected are quite close to the dynamical equilibrium because the celestial bodies in the halo region are linked gravitationally to M31 since billions years ago.

In Table 9 it is calculated C_0 for every point of data set.

Table 9. The different parameters C_0 calculated by the different initial points from 1 to 12.

Points	Radius	Velocity	Field Eo	Parameters
	kpc	m/s	m/s^2	C_0
1	40.5	2.299E+05	4.23E-11	6.8830E−23
2	49.1	2.374E+05	3.72E-11	6.3753E−24
3	58.4	2.505E+05	3.48E-11	−5.3082E−23
4	70.1	2.192E+05	2.22E-11	4.2804E−26
5	84.2	2.069E+05	1.65E-11	6.8683E−24
6	101.1	2.135E+05	1.46E-11	−3.3317E−23
7	121.4	1.978E+05	1.04E-11	−1.6934E−23
8	145.7	1.788E+05	7.11E-12	9.9988E−24
9	175	1.656E+05	5.08E-12	2.3822E−23
10	210.1	1.656E+05	4.23E-12	2.5446E−24
11	252.3	1.607E+05	3.32E-12	−3.7771E−24
12	302.9	1.508E+05	2.43E-12	3.0572E−24

It is remarkable that all the values C_0 are close all of them and they are close to 0 as well.

Some values of parameters C_0 are negatives because these points are above the fitted curve, see Figure 2, its points plotted are numbered from left to the right.

The points of Table 1 plotted in Figure 2 verify that the more close to the curve the point is, the smaller, in absolute value, the parameter C_0 is. The parameter C_0 corresponding to point 12 is the smaller. This close oscillation around the fitted curve suggests strongly that the data of Table 1 belong to celestial bodies very close to dynamical equilibrium.

This property of data set has inspired the following theorems of epigraph 8.7 and a surprising corollary in the 8.8 one.

8.7. Parameter *C* Equal Zero Theorems

Taking into account that by the dimensional analysis the exponent of rotation curve has to be −1/4 and the results got in previous epigraph about the values of parameter C, it is natural to consider that exist an ideal rotation curve whose power is −1/4 and the data set of Table 1 are very close to this curve. Notice that the fitted curve in Figure 2 has a power very close to −1/4, in fact differ 18 ten-thousandth only.

Definition. Hereafter, it will be named *Buckingham halo curve* to the points (r, v) r belonging to the halo region and its velocity is $v = a \cdot r^{-1/4}$ being a, the parameter associated to the galactic halo.

As the exponent −1/4 was got by the Buckingham theorem, it has been select such name for that curve.

In Table 10, the columns first, second and fourth come from Table 2, the third column is got with the formula $V_0 = a \cdot R_0^{-1/4}$. The parameter a_{M31} comes from Table 8.

Table 10. Comparison between data velocity and Buckingham curve.

Radius kpc	Radius m	Buckingham Velocity m/s	Data velocity m/s	Relative Diff. %
40.5	1.25E+21	2.5144E+05	2.2990E+05	8.57E+00
49.1	1.52E+21	2.3962E+05	2.3740E+05	9.27E−01
58.4	1.80E+21	2.2945E+05	2.5050E+05	−9.17E+00
70.1	2.16E+21	2.1921E+05	2.1920E+05	5.58E−03
84.2	2.60E+21	2.0939E+05	2.0690E+05	1.19E+00
101.1	3.12E+21	2.0004E+05	2.1350E+05	−6.73E+00
121.4	3.75E+21	1.9109E+05	1.9780E+05	−3.51E+00
145.7	4.50E+21	1.8257E+05	1.7880E+05	2.06E+00
175	5.40E+21	1.7440E+05	1.6560E+05	5.04E+00
210.1	6.48E+21	1.6660E+05	1.6560E+05	6.03E−01
252.3	7.79E+21	1.5915E+05	1.6070E+05	−9.72E−01
302.9	9.35E+21	1.5204E+05	1.5080E+05	8.18E−01

The fifth column shows the relative difference of velocities between the data and the ideal curve.

Velocity at radius 302.9 kpc has the lowest relative difference, 0.8% only.

In Figure 3 is plotted the ideal curve called *Buckingham halo curve*.

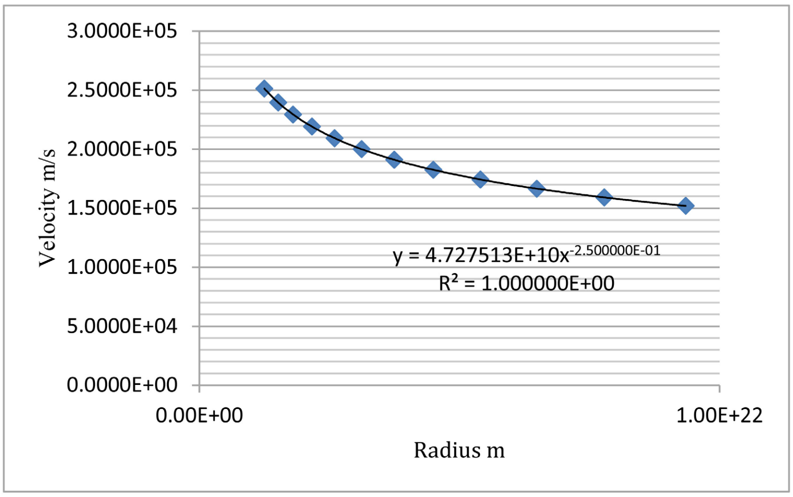

Figure 3. Buckingham halo curve.

Direct Theorem: If it is supposed that a point belonging to Buckingham halo curve is in dynamical equilibrium and if it is selected such point as initial point to calculate *C*, formula (8.7), then such parameter is zero.

***Proof*:** Suppose a point (R_0, V_0) belonging to Buckingham halo curve, then $V_0 = a \cdot R_0^{-1/4}$ As dynamical equilibrium leads to $E_0 = \frac{GM(<r)}{r^2} = \frac{V_0^2}{r}$ then

$E_0 = a^2 \cdot R_0^{-3/2}$ and $E_0^{\frac{-2}{3}} = a^{-4/3} \cdot R_0 = D \cdot R_0$ because $D = a^{\frac{-4}{3}}$, see (8.8). Therefore $C = 0$ because its numerator is zero, formula (8.7).

It is important to highlight that it is considered the hypothesis of dynamical equilibrium for the Buckingham halo curve. However this is a very plausible supposition by the reasons explained in this epigraph and the previous one.

Reverse Theorem

If it is selected a point (R_0, V_0) which is supposed to be in dynamical equilibrium and its parameter $C = 0$ then such point belong to Buckingham halo curve.

***Proof*:** Suppose that $V_0 = a \cdot R_0^b$ being the exponent b unknown.

If $C = 0$ then $E_0^{-2/3} = D \cdot R_0$ and as there is dynamical equilibrium $E_0 = \dfrac{V_0^2}{r}$

then $E = \dfrac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ and $E_0^{-2/3} = a^{-4/3} \cdot R_0^{\frac{2-4b}{3}}$ so $a^{-4/3} \cdot R_0^{\frac{2-4b}{3}} = D \cdot R_0$

using the formula (8.8) it is got: $D \cdot R_0^{\frac{2-4b}{3}} = D \cdot R_0$ which leads to $R_0 = R_0^{\frac{2-4b}{3}}$

So $b = -1/4$

Final comments

The data of rotation curve do not belong to Buckingham halo curve by two reasons:

The first one it is simple: measures have experimental errors. The second one is more subtle: the celestial bodies are not in perfect dynamical equilibrium. It is right to think that celestial bodies which belong to M31 gravitational system from its formation times, more than ten billions years ago, will be closer to dynamical equilibrium regarding other ones that were captivated by the gravitational field of M31 afterwards.

Watching the Figure 2, it is clear that point 1 and point 3 at 40.5 kpc and 58.4 kpc are the points more distant regarding Buckingham halo curve. This important difference regarding dynamical equilibrium curve may be explained by the asymmetries of gravitational field during the history of dynamic evolution. Anyway it is undeniable that in general data are very close to Buckingham halo curve, see Table 10.

8.8. Bernoulli Formulas Become Direct Formulas When Parameter *C* = 0

The reader can check that in the paper [8] Abarca, M.2019 the DMbQG theory is essentially developed except the formula for total mass that is the Bernoulli mass instead of Direct mass. In practice, the relative differences between both formulas are totally negligible. However be able to demonstrate that the Direct mass is the Bernoulli formula when parameter C is zero means a better understanding of DM phenomenon. In addition allows continuing the development of the theory more easily, for example when the theory is extended to cluster of galaxies. See chapter 12.

Thanks the results got in the epigraphs 8.6 and 8.7 is natural to consider parameter $C = 0$ into the Bernoulli formulas.

For the Field *E*

When in formula $E(r)=\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-3}{2}}$ (8.6) $C=0$ then it is got $E=a^2\cdot r^{\frac{-3}{2}}$ because $D^{3/2}=a^2$ which is precisely direct formula for E, (8.4).

For the D.M. Density

As $D_{DM}=A\cdot E^{5/3}$ Using field got by Bernoulli solution it is right to get

$D_{DM}(r)=A\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-5}{2}}$ (8.9) Being $A=\frac{D}{8\pi G}$ and $D=a^{\frac{-4}{3}}$ if $C=0$ then formula becomes $D_{DM}(r)=A\cdot D^{\frac{-5}{2}}\cdot r^{\frac{-5}{2}}=L\cdot r^{\frac{-5}{2}}$ Being $L=\frac{a^2}{8\cdot\pi\cdot G}$ which is the direct DM density formula, (8.3).

For the Direct Mass Formula

If $C=0$ then $M_{BERNI}(<r)=\frac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}$ (8.13) becomes

$M_{DIRECT}(<r)=\frac{a^2\cdot\sqrt{r}}{G}$ (8.5) because $D^{3/2}=a^2$

As the Direct mass formula are a particular case of Bernoulli formulas when parameter C=0 and in the epigraph 8.5 the corollary of Newton's theorem claims that the Bernoulli mass is the total mass, see formula (8.13) then it is concluded that the Direct mass is the total mass at a specific radius.

$$M_{DIRECT}=M_{TOTAL}(<r)=\frac{a^2\cdot\sqrt{r}}{G} \tag{8.14}$$

Final comment

According to findings got in this chapter it is clear that the DMbQG claims that the total mass is unbounded and grows up with the square root of radius.

In addition the total mass formula depends on the parameter a solely, instead two parameters C and D associated to Bernoulli formulas, which is a magnificent simplification of the theory.

Finally it is very important to highlight that from now on, the direct formulas have unbounded dominion as they are a particular case of Bernoulli formulas. Notice that before this epigraph the dominion of direct formulas was associated to the data, *i.e.* from 40 kpc up to 300 kpc.

9. Masses in M31

In this chapter, it will be calculated and compared three types of masses related to M31.

9.1. Dynamical Mass versus Direct Mass

As it is known, dynamical mass represents the total mass enclosed by a sphere with a radius r in order to produce a balanced rotation with a specific velocity at such radius, so it is right to consider dynamical mass as the total mass, baryonic and DM mass, enclosed at radius R. Ranging the radius in the interval of radius measured.

The formula of dynamical mass is $M_{DYN}(<r) = \frac{V^2 \cdot r}{G}$ and $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $a^2/G = 3.35 \times 10^{31}$ (I.S. units)

Using the data of Table 2, radius and velocities into the I.S. of units, it is got the results shown in Table 11.

Table 11. Comparison between dynamical and direct masses.

Radius	Dyn Mass	Direct mass	Relative diff.
kpc	M_Θ	M_Θ	%
40.5	4.974E+11	5.95E+11	1.639E+01
49.1	6.429E+11	6.55E+11	1.849E+00
58.4	8.514E+11	7.14E+11	−1.919E+01
70.1	7.825E+11	7.83E+11	1.419E−02
84.2	8.373E+11	8.58E+11	2.373E+00
101.1	1.071E+12	9.40E+11	−1.392E+01
121.4	1.103E+12	1.03E+12	−7.143E+00
145.7	1.082E+12	1.13E+12	4.087E+00
175	1.115E+12	1.24E+12	9.833E+00
210.1	1.339E+12	1.35E+12	1.204E+00
252.3	1.514E+12	1.48E+12	−1.951E+00
302.9	1.600E+12	1.63E+12	1.629E+00

In Figure 4 are plotted both masses, Points over the line represent the direct masses and the other ones the dynamical masses.

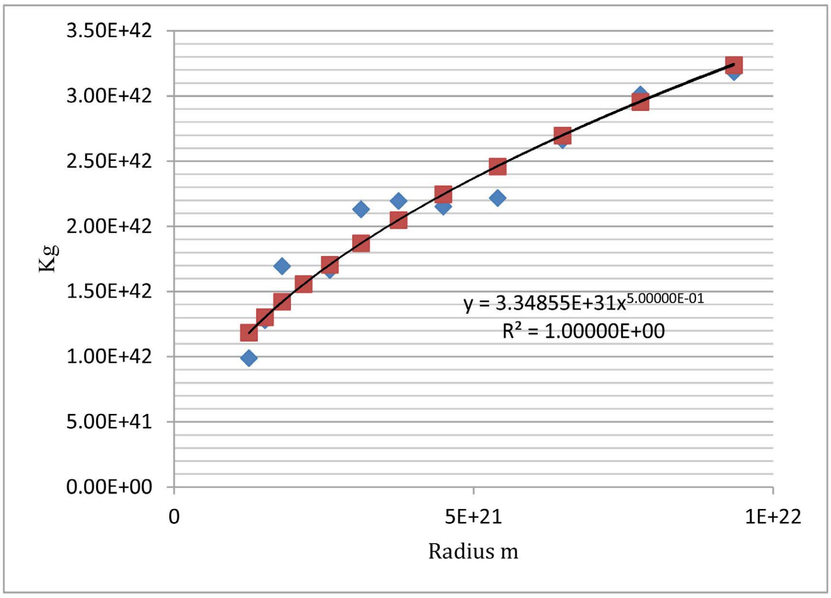

Figure 4. Mdynamic vs Mdirect.

The first and third points have the maximum difference regarding fitted curve whereas relative differences decreased as radius increased. Namely, relative differences are below 10% for radius bigger than 120 kpc and are below 2% for radius bigger than 210 kpc.

So direct mass is a very good approximation for dynamical mass enclosed at radius R, ranging the radius in the interval of radius measured.

9.2. Bernoulli Mass versus Direct Mass

In this epigraph will be shown that relative difference between both kinds of formulas is negligible through an unbounded dominion. Remember that in epigraph 8.8 was shown that the direct mass (8.5) is a particular case of Bernoulli mass (8.13)

In the corollary of Newton's theorem, epigraph 8.5, was demonstrated that the Bernoulli mass is the total mass enclosed by a sphere with radius r. Now it will be shown that relative differences between Bernoulli and direct mass are quite small, even for extended haloes.

In the paper [9] Abarca, M.2023, epigraph 9.6 it is developed a method to calculate the optimal parameter C

[9] Abarca, M. 2023	
$C_{M31} = -3.777 \times 10^{-24}$	This selected value C_{M31} belong to the initial point radius equal to 252. kpc.
$D_{M31} = a^{\frac{-4}{3}} = 5.85 \times 10^{-15}$	

In **Table 12** are tabulated both functions and its relative difference. It is remarkable that even at 2 Mpc its difference is only 3.8%, despite the fact that its dominion has been extended 7 times.

Table 12. Comparison between direct and Bernoulli masses.

		Direct mass $C = 0$	Bernoulli mass $C \neq 0$	Relative diff.
kpc	m	Msun	Msun	%
40.5	1.250E+21	5.949E+11	6.011E+11	1.04E+00
60	1.851E+21	7.240E+11	7.327E+11	1.19E+00
80	2.469E+21	8.361E+11	8.471E+11	1.31E+00
100	3.086E+21	9.347E+11	9.481E+11	1.41E+00
200	6.171E+21	1.322E+12	1.346E+12	1.77E+00
385	1.188E+22	1.834E+12	1.875E+12	2.20E+00
500	1.543E+22	2.090E+12	2.142E+12	2.40E+00
770	2.376E+22	2.594E+12	2.668E+12	2.77E+00
1000	3.086E+22	2.956E+12	3.048E+12	3.02E+00
1500	4.629E+22	3.620E+12	3.750E+12	3.46E+00
2000	6.171E+22	4.180E+12	4.346E+12	3.80E+00

Final comment

It is clear that the epigraphs 8.7 and 8.8 have stated that the direct mass is the most suitable formula to calculate the total mass in the halo region. Despite this fact this epigraph has been made to show that the difference between both kinds of masses is negligible. In addition, the reader interested, may read in [9] Abarca, M. 2023 the process to get the optimal parameter C for the Bernoulli mass formula.

10. Dark Matter by Gravitation Theory in Milky Way

In the last rotation curve for MW published by [3] Sofue.2020, the radius of data range from 0.1 kpc up to 95.5 kpc whereas in the previous rotation curve [1] Sofue. 2015 the radius range up to 300 kpc. Afterwards will be discussed the importance to reduce the dominion up to 95 kpc. However firstly it is needed to calculate the lowest radius for the halo region, where the baryonic density is negligible versus DM density.

10.1. An Estimation for the Halo Radius

As it is known, Bernoulli solution is only right into the halo region, where the baryonic mass is negligible.

To calculate baryonic volume density has been used the model provided by Sofue for the baryonic disc of MW.

The data below comes from [3] Sofue. They are the parameters for baryonic matter at disc in Milky Way.

Sofue data	Parameter	Fitted value
Expo.disk	a_d	4.38 ± 0.35 kpc
	Σ_0	(1.28 ± 0.09) × $10^3 M_\odot$ pc^{-2}

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents the superficial density at disc region. Similarly as it was done for M31 galaxy, to convert superficial baryonic density to volume density it is right to get the formula $D_{BARYONIC}^{VOLUME} = \dfrac{\Sigma(r)}{2r}$.

So $D_{BARYONIC}^{VOLUME}(30.5\ \text{kpc}) = 1.34 \times 10^{-24}\ \text{kg/m}^3$, the formula of Direct Dark matter density was got in epigraph 8.8. $D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ Being $L_{MILKY\text{-}WAY} = 9.1\text{E}+29$, according with parameter a got in epigraph 11.3, see **Table 14**.

For example D_{DM} (30.5 kpc) = 3.35E−23 kg/m^3. So the ratio of both volume density (baryonic versus DM) at 30.5 kpc is 0.04.

In conclusion: it is right to consider negligible the baryonic density for radius bigger than 30.5 kpc, therefore it is possible to estate that halo dominion begins at 30.5 kpc for Milky Way.

10.2. Rotation Curve of Milky Way by Sofue 2020 Data

The data of rotation curve of Milky Way, **Table 13**, comes from [3] Sofue.2020, and there have been selected data with radius bigger than 30 kpc. *i.e.* its halo region.

Table 13. Rotation curve in halo region for MW.

Radius	Veloc.	Radius	Veloc.
kpc	km/s	m	m/s
30.448	229.6	9.40E+20	229,600
33.493	222.5	1.03E+21	222,500
36.842	215	1.14E+21	215,000
40.527	207.1	1.25E+21	207,100
44.579	200.3	1.38E+21	200,300
49.037	194.7	1.51E+21	194,700
53.941	189.8	1.66E+21	189,800
59.335	186.2	1.83E+21	186,200
65.268	184.7	2.01E+21	184,700
71.795	183.9	2.22E+21	183,900
78.975	181.4	2.44E+21	181,400
86.872	175.5	2.68E+21	175,500
95.56	167.7	2.95E+21	167,700

This new set of Sofue data is very important for the theory of DMbQG theory because gives a rotation curve at halo region with a power for radius, whose exponent is very close to −1/4, which is the same for M31.

This fact backs strongly the hypothesis of this theory.

In his previous paper [1] Sofue, Y.2015, the author gave an extended dominion up to 300 kpc, However data with radius bigger than 100 kpc have too high velocity and the fitted power function did not fit properly with exponent −1/4.

The logical explanation about the "bad" behaviour of these data is to consider that the celestial bodies more far away than 100 kpc are not in dynamical equilibrium. Perhaps they came from the outskirts of MW and were captivated by MW gravitational field afterwards so it is right to consider that these data are far away to be in dynamical equilibrium, whereas celestial bodies below 100 kpc of radius are properly in dynamical equilibrium with Milky Way.

Anyway, the important data are those closer, because it is right to think that celestial objects with lower radius belong to MW halo from times of MW formation so these objects may have a better dynamic equilibrium with MW.

10.3. Fitted Function Velocity versus Radius at Halo Region

Figure 5 plotted the data of **Table 13** and the statistical fitted function. According to the statistical power regression $v = a \cdot r^b$ being a = 3.68918E+10 and b = −0.248717 It is remarkable its high correlation coefficient.

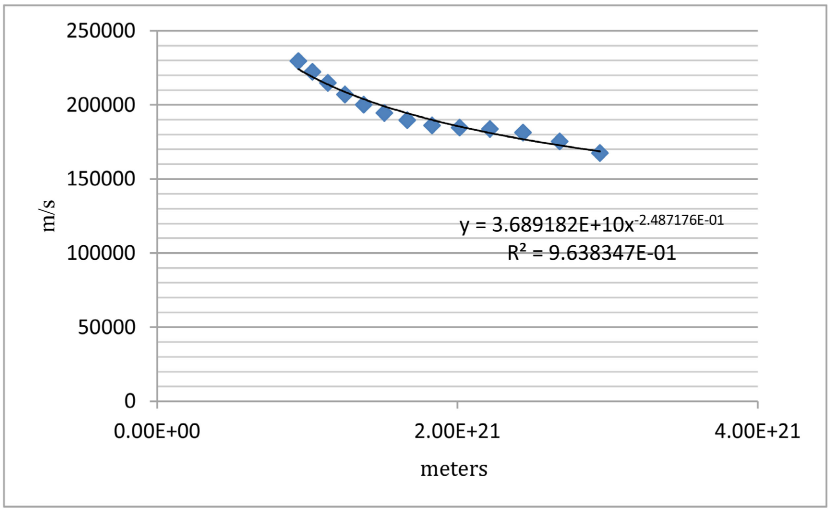

Figure 5. Velocity vs Radius in MW halo.

In addition the exponent b is almost identical to the one associated to M31 galaxy and it is very close to −1/4

Using the method developed in chapter 8, it has been stated $b = -1/4$ so it is needed to recalculate the parameter a, using the formula for a optimal, see epigraph 8.1

$$a_{OPTIMAL} = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 3.90787373 \times 10^{10} \approx 3.9\text{E}10\ \mathrm{m}^{5/4}/\mathrm{s}$$

So the optimal parameter $a_{M\text{-}W} = 3.9\text{E+}10\ \mathrm{m}^{5/4}/\mathrm{s}$

Which is lightly bigger compared with the one associated to $b = -0.248717$, see the formula written in the **Figure 5**.

According to the hypothesis of DMbQG theory, the parameter a is similar for similar galaxies, for example $a_{M31} = 4.7275\text{E+}10\ \mathrm{m}^{5/4}/\mathrm{s}$

However parameter b is the same, so it is right to consider that parameter b is constant for different types of galaxies.

Table 14 shows the most important parameters for MW at the halo region.

Table 14. New parameters a & b – A & B for MW.

B	5/3
$b = \frac{B-2}{2B-2}$	−1/4
a optimal	$3.90787373 \times 10^{10}\ \mathrm{m}^{5/4}/\mathrm{s}$
$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A 4.496262×10^{-6}

10.4. Masses Ssociated to Milky Way up to 2 Mpc

As it has been demonstrated in the chapter 8, the direct mass, formula (8.14) is the total mass enclosed by a sphere with a radius r.

In Table 15, the total mass is tabulated form 30 kpc up to 2 Mpc.

Table 15. Direct mass at different radii in MW.

Radius	Radius	Direct Mass
kpc	m	M_Θ
30.448	9.3953E+20	3.524E+11
40.527	1.251E+21	4.066E+11
53.941	1.664E+21	4.691E+11
78.975	2.437E+21	5.676E+11
95.56	2.949E+21	6.244E+11
770	2.38E+22	1.772E+12
1000	3.09E+22	2.020E+12
2000	6.17E+22	2.856E+12

$$M_{TOTAL}\left(<r\right)=\frac{a^2\cdot\sqrt{r}}{G} \quad \text{Being} \quad a^2/G=2.2885\times10^{31}\ \mathrm{kg/m^{1/2}} \tag{10.1}$$

10.5. Comparing Direct Mass with Results from Gaia DR2 Published in JCAP 2020

In this section will be compared the results got by direct mass, formula (10.1) with the results published in the prestigious Journal of Cosmology and Astroparticle Physics by [10] E.V. Karukes *et al.* 2020 in the paper *A robust estimate of the Milky Way mass from rotation curve data*.

Results come from GAIA DR2 and others remarkable sources.

Table 16 made the comparison only with the four radiuses bigger than 30 kpc, as DMbQG theory only works in the halo region.

Table 16. Comparison between direct mass and measures published.

Radius kpc	Radius m	Direct Mass M_Θ	Karukes M_Θ	Relative difference %
45.79	1.4129E+21	4.305E+11	4.27E+11	8.23E−01
74	2.2834E+21	5.473E+11	5.68E+11	−3.78E+00
119.57	3.6896E+21	6.957E+11	7.26E+11	−4.35E+00
193.24	5.9628E+21	8.845E+11	8.95E+11	−1.19E+00

In the last column is shown the relative difference between both results of mass, being quite small indeed.

It is awesome how a simple theory which associates only one parameter *a* to the galactic halo, which has been calculated with a data set from 30 kpc up to 95 kpc, is able to give results so close with results got by GAIA DR2 which have been got with the highest current technology and processed through sophisticated software.

Notice how the relative difference at 193 kpc is even lower that the ones at 74 kpc or 119 kpc.

Table 17 comes from [10] E.V. Karukes *et al.* 2020.

Table 17. Total mass in MW at some radius published by [10]. E.V. Karukes *et al.* 2020 in page 25.

Radius kpc	Total mass × $10^{11} M_{\odot}$
45.79	4.27
74.0	5.68
119.57	7.26
193.24	8.95

Notice that there is a perfect concordance, between direct mass and measures if it is considered the interval of errors.

Notice that although the parameter a was got using a data set ranging from 30 kpc up to 95 kpc the Direct mass formula gives accuracy calculus for mass not only at 193 kpc but up to 770 kpc and beyond as it will show in the next chapter.

10.6. Results Got by Jeff Shen *et al.* APJ. 2022 versus Direct Mass at MW Halo

In this epigraph will be compared the results published in [11] Jeff Shen.2022 with the results calculated by the Direct mass formula in Milky Way, formula (10.1).

Below is copied a fragment of the abstract of the paper [11] J. Shen. 2022, where it is possible to see two masses results at different radii.

fragment of the abstract of the paper [11] J. Shen. 2022
We report a median mass enclosed within 100 *kpc of* $M(<100\ kpc) = (0.69 \pm 0.04) \times 10^{12} M_{\odot}$ (68% *Bayesian credible interval*), *or a virial mass of* $M_{200} = M(216.2 \pm 7.5\ kpc) = (1.08 \pm 0.11) \times 10^{12} M_{\odot}$, *in good agreement with other recent estimates.*

Now it will be calculated the total mass at the same radius with the direct mass formula (10.1). In Table 18, both kind of masses and the last column shows its relative difference are shown.

Table 18. Comparison between direct mass and measures published.

Radius kpc	Total mass $\times 10^{12} M_{\odot}$ (Direct formula)	Total mass $\times 10^{12} M_{\odot}$ (Jeff Shen measures)	Relative difference %
100	0.64	0.69 ± 0.04	7
216 ± 7.5			
216 + 7.5	0.96	1.08 ± 0.11	11

As it is shown the relative difference is small especially at 100 kpc. In addition both results match if it is considered the range of the errors measures.

It is important to remark that results by the direct mass formula at 216 kpc is calculated through the direct mass whose parameter a^2 was got using a data set whose dominion ranges between 30 kpc and 95 kpc.

11. The Mass Calculus for the Local Group of Galaxies

According to [4] Azadeh Fattahi, Julio F. Navarro. 2020. The pair MW, M31 has a mass around $5\times10^{12} M_\Theta$. In addition, the authors claim that to suppose there is dynamical equilibrium in the Local Group of galaxies is a plausible hypothesis supported by the fact that only one third of dwarf galaxies belonging to the L.G. are satellite associated rightly to M31 or MW, whereas the other two thirds are linked to global gravitational field of LG. See [12] A. Fattahi, *et al.* 2020.

According to [1] Sofue, (see epigraph 4.6 of his paper) the mutual velocity MW-M31 is 170 km/s, that supposing dynamical equilibrium correspond to a dynamical mass equal to $5\text{E}12 M_\Theta$, considering 770 kpc as its mutual distance. However using the current models of DM, the total mass of M31 and Milky Way is approximately $3\text{E}12 M_\Theta$.

In conclusion, there is a scientific consensus about a lack of mass equal to $2\times10^{12} M_\Theta$ in the L.G. that the current DM models are not able to explain.

In this epigraph will be demonstrated by DMbQG theory that the total mass MW-M31 system is $5\times10^{12} M_\Theta$, so this result is a remarkable success of the theory.

Up to now, in order to do calculus with data of rotation curve, the halo border of M31 has been 300 kpc. However when it is considered the gravitational interaction between both giant galaxies it is needed to extend their haloes up to 770 kpc, because according to the DMbQG theory the phenomenon of Dark matter is linked to gravitational field, which is unlimited. Therefore the M31 halo is extend up to 770 kpc and reciprocally the MW halo is extend up to 770 kpc as well, when it is calculated the total mass accumulated as consequence of their mutual gravitational interaction.

Even the dominion may be extended to bigger distances when it is calculated the total mass of the Local Group as it will be made in the following epigraph.

Stimating the Total Mass for the L.G. at Different Radii

In Table 19, the parameter a^2 belonging to the four most important galaxies in the L.G are tabulated.

Table 19. Parameter a^2 for M31, MW, M33 and LMC.

Galaxies	Parameter a^2 $m^{5/2}/s^2$
M31	2.235E+21
MW	1.527E+21
M33	4E+20
LMC	1.1881E+20
Parameter a^2 Local Group	4.28E+21

In order to estimate the total mass of Local Group only it will be considered M31, MWand their main galaxy satellites: M33 and LMC. The rest of galaxies have a mass negligible to estimate the total mass of Local Group. In fact M33 add only a 9% of total mass and LMC add only a 2.8%.

The values of parameters a^2 for the LMC and M33 have been got in the paper [9] Abarca, M.2023, chapter 14, using its rotation curves.

By the formula (8.14) of the Direct mass, it is right to get the table of masses at different distances using parameters a^2 associated to galaxies.

Calculus written in Table 20 are only an estimation, as the gravitational interaction between the four galaxies is quite complex. The masses calculated below are in M_{Θ} units.

Table 20. Masses in the Local Group at different radii.

Radius kpc	MW M_{Θ}	LMC M_{Θ}	M31 M_{Θ}	M33 M_{Θ}	Local Group Total Mass M_{Θ}
770	1.772E+12	1.379E+11	2.594E+12	4.63E+11	4.967E+12
1000	2.020E+12	1.571E+11	2.956E+12	5.27E+11	5.660E+12
1500	2.474E+12	1.924E+11	3.620E+12	6.46E+11	6.932E+12
2000	2.856E+12	2.222E+11	4.180E+12	7.46E+11	8.004E+12

In conclusion adding MW + LMC + M31 + M33 at 770 kpc the mass is 4.97E12M_{Θ} that match fully with the stated in [4] Azadeh Fattahi, Julio F. Navarro. *et al.* 2020 and [1] Sofue, Y.2015.

As DMbQG theory stated an unbounded dominion of DM, it is possible to extend the radius. For example at 2 Mpc the total mass is $8\times 10^{12} M_{\Theta}$.

These results are a magnificent success of DMbQG theory.

12. Virial Theorem as a Method to Calculate the Direct Mass in Clusters

In the paper [5] Abarca, M.2024, it is developed fully the DMbQG theory in cluster of galaxies, but in this chapter it will be shown a method to calculate the parameter a^2 associate to the cluster, solely.

As the Direct mass formula contains only the parameter a^2 then is enough to know the data pair: virial radius and virial mass associated to the cluster.

The property of dynamical equilibrium is crucial to be able to calculate the parameter a^2 with a formula so simple.

If it is considered that the virial radius is the border of the halo cluster where galaxies are in dynamical equilibrium and at the same time is negligible the amount of Baryonic matter outside the sphere with such radius, then it is possible to apply the equation $M_{VIRIAL} = M_{TOTAL}$ ($< R_{VIRIAL}$). Then from this equation it will be possible to clear up a^2.

$$M_{VIRIAL} = M_{TOTAL}\left(< R_{VIRIAL}\right) = \frac{a^2 \cdot \sqrt{R_{VIRIAL}}}{G}$$

Getting the value for

$$a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}} \quad (12.1).$$

This formula is only a way to estimate parameter a^2 because outside the virial radius always there will be a fraction of the galaxies belonging to the cluster. Anyway, this method may estimate a lower bound of parameter a^2 associated to the cluster.

12.1. Parameter a^2 Associated to the Local Group

For example, in the Local Group of galaxies, the dynamical data according to [1] Sofue, Y. 2015, are 770 kpc for distance between M31 and MW and 170 km/s for its relative velocity, so using $M_{DYNAMIC}(<r) = \frac{v^2 \cdot R}{G}$ it is got a $M_{DYNAMIC}$ = 1.03E+43 kg. Supposing that there is dynamical equilibrium between MW and M31, see [12] A. Fattahi *et al.* 2020, we have the equation:

$M_{DYNAMIC} = M_{TOTAL} = \frac{a^2 \cdot \sqrt{r}}{G} = 1.03 \times 10^{43}$ kg, that for a radius = 770 kpc leads to the value a^2 = 4.45E+21, which is very close to the parameter a^2 = 4.28E+21 shown in **Table 19**. Even these values would be closer if it was considered other galaxies such as the Small Cloud of Magellan and the others dwarfs satellite galaxies of MW and M31. Anyway its relative difference is lower than 4%.

It is remarkable the fact that the parameters a^2 of M31 and MW were calculated with data in the halo whose radius range from 40 kpc up to 300 kpc in M31 case and range from 35 kpc up to 100 kpc in MW case, whereas calculus for parameter a^2 associated to the Local Group has been made with only one data radius 770 kpc and velocity 170 km/s. So, despite the fact that both methods are independent they give a value to the parameter a^2 whose relative difference is only 4%.

Clearly, this result is another validation for the DMbQG theory.

12.2. Paramerer a^2 Associated to Coma and Virgo Clusters

In **Table 21** there are recent data for the Coma and Virgo cluster. With such data will be calculated its parameter a^2 with the formula (12.1).

Table 21. Virial radius and mass for the Virgo and Coma clusters.

	Virial Radius	Virial mass
	Mpc	$\times 10^{14} M_{\Theta}$
Virgo [13] Kashibadze 2020	1.7	6.3 ± 0.9
Coma [14] Seong-A. 2023	2.8	27

Using the data [13] Kashibadze 2020, it is right to get the parameter a^2 = 3.65E+23 $m^{5/2}/s^2$ for the Virgo cluster, which is 17 Mpc far away from MW.

As it has been comment, this value is a lower bound, because always there will be a fraction of baryonic mass outside from the virial radius. Also, it is clear that parameter a^2 error depend on the measure errors for virial mass and radius.

Similarly using the data for the Coma cluster [14] Seong-A.2023, it is right to get the parameter $a^2 = 1.22\text{E}+24\ \text{m}^{5/2}/\text{s}^2$

13. Dark Matter Is Counter Balanced by Dark Energy

The paper [5] Abarca, M. 2024 may be considered as an extension of this chapter. In that paper the DMbQG theory co working with the Dark energy, DE, finds unexpected theoretical findings which are able to explain some important open problems for the current cosmology.

The basic concepts about the DE on the current cosmology, used in this paper, can be studied in [15] Chernin, A.D. 2013.

As currently there is a tension regarding the experimental value of Hubble constant, in this paper will be used $H = 70$ Km/s/Mpc and $\Omega_{DE} = 0.7$ as the fraction of Universal density of DE.

13.1. Zero Gravity Radius Depending on Parameter a^2 Formula

According to [15] Chernin, A.D. in the current cosmologic model ΛCDM, dark energy has an effect equivalent to antigravity *i.e.* the mass associated to dark energy is negative and the dark energy have a constant density for all the Universe equal to $\varphi_{DE} = \varphi_C \cdot \Omega_{DE} = 6.444 \times 10^{-27}\ \text{kg/m}^3$ being $\Omega_{DE} = 0.7$ and $\rho_C = \dfrac{3H^2}{8\pi G} = 9.205\text{E}-27\ \text{kg/m}^3$ the critic density of the Universe.

As DE density is constant, the total DE mass is proportional to Radius with power 3, whereas DM mass grows with radius power 0.5 so it is right to get a radius where DM is counter balanced by DE.

According to [15] Chernin, A.D. The mass associated to DE is

$$M_{DE}(<R) = -\frac{\rho_{DE} 8\pi R^3}{3}, \tag{13.1}$$

or equivalently

$$M_{DE}(<R) = -\varphi_{DE}\frac{8\pi R^3}{3} = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R^3 \quad (13.2).$$

Notice that the author multiplies by two the volume of a sphere by reasons explained in his work.

[15] Chernin.2013 defines gravitating mass

$$M_G(<R) = M_{DE}(<R) + M_{TOTAL}(<R) \tag{13.3}$$

where M_{TOTAL} is baryonic plus dark matter mass, and defines R_{ZG}, Radius at zero Gravity as the radius where $M_{DE}(<R_{ZG}) + M_{TOTAL}(<R_{ZG}) = 0$. *i.e.* where the gravitating mass is zero.

The definition of R_{ZG} leads rightly to the equation

$$M_{TOTAL}(<R_{ZG}) = \varphi_{DE}\frac{8\pi R_{ZG}^3}{3}, \quad (13.4).$$

Using the formula for total mass (8.14), the expression (13.4) leads to

$$\frac{a^2 \cdot \sqrt{R_{ZG}}}{G} = \rho_{DE} \cdot \frac{8\pi \cdot R_{ZG}^3}{3}, \tag{13.5}$$

where it is possible to clear up

$$R_{ZG} = \left[\frac{3a^2}{8\pi G \rho_{DE}}\right]^{2/5} \tag{13.6}$$

and as

$$\varphi_{DE} = \frac{3 \cdot H^2}{8\pi G}\Omega_{DE} \tag{13.7}$$

then by substitution results

$$R_{ZG} = \left[\frac{a^2}{H^2 \cdot \Omega_{DE}}\right]^{2/5} \tag{13.8}$$

This formula will be called R_{ZG} (parameter a^2) because depend on the parameter a^2.

As the radius R_{ZG} is the distance to cluster centre where is zero the gravitating mass, it is right to consider R_{ZG} as the halo radius and its sphere defined as the halo cluster.

13.2. Zero Gravity Radius for Some Important Clusters of Galaxies

According to the parameter a^2 value got in chapter 11 for the Local group, $a_{L-G}^2 = 4.28 \times 10^{21}$, by (13.8) it is right to get R_{ZG} = 2.19 Mpc. So at that radius the gravitating mass is zero, in other words, for radius under 2,19 Mpc dark matter dominates and for bigger radius dark energy dominates and it is not possible for the Local Group to have any dwarf galaxy linked gravitationally beyond this radius.

According to the parameter a^2 value got in chapter 12 for the Coma cluster it is right to get R_{ZG} = 21 Mpc. In other words 21 Mpc is the radius of region where the DM of Coma Cluster dominates versus dark energy.

Similarly for the Virgo cluster the parameter a^2 = 3.65E+23 leads to R_{ZG} = 12.97 Mpc.

In Table 22, the previous calculus is summarized.

Table 22. Parameter a^2 and Zero gravity radius for these clusters.

	parameter a^2 Units $m^{5/2}/s^2$	Zero Gravity Radius
Local Group	4.28E+21	2.19 Mpc
Virgo Cluster	3.65E+23	12.97 Mpc
Coma Cluster	1.22E+24	21 Mpc

With these three important clusters of galaxies, it has been illustrated how the total mass, formula (8.14), is counter balanced by the dark energy at mega parsecs scale, and precisely this Radius at zero gravity determines the regional size where the cluster has gravitational influence.

14. Concluding Remarks

As it has been outlined at the introduction, this work is the consequence of the new data set for rotation curve of Milky Way published by [3] Sofue, 2020. With these new data, it is possible to state that the rotation curve of MW at halo is governed by the ideal curve named Buckingham halo curve, which has the same exponent for M31 and Milky Way galaxies in the framework of DMbQG theory.

This fact back strongly the main hypothesis of Dark gravitation theory *i.e.* Dark matter is generated according to an unknown quantum gravitational mechanism, which depend on the gravitational field, so it is a Universal law.

Through the first nine chapters is developed the theory using M31 rotation curve. These chapters are identical to the previous paper [6] Abarca, M. 2019, excepting the process of getting the parameter C because in the current work, it has been found that the value of parameter C is zero and consequently the Bernoulli mass becomes direct mass. Another important theoretical finding is the called Newton's theorem, because as a corollary it is deduced that the called Bernoulli mass determines the total mass at a specific radius. See epigraph (8.5).

In Chapter 10, it has been calculated parameter a^2 associated to Milky Way halo using the [3] Sofue, 2020 data set of rotation curve into the halo region and it is calculated the Direct mass at different radii. These results got by the Direct mass are compared with data published by two prestigious astrophysics teams in a dominion radius which ranges from 45 kpc up to 220 kpc. The relative differences are below 4% regarding a team and below 11% regarding the other team, so these tests are a successful experimental validation of the Direct mass, the main formula of DMbQG theory.

Chapter 11 is calculated the direct mass associated to the Local Group, L.G, considering the parameter a^2 associated to MW and its satellite galaxy, the LMC, as well as M31 and its satellite, M33. The total mass calculated for the Local Group is $5\times10^{12} M_{\Theta}$ at 770 kpc that match with the mass published by [4] Fattahi *et al.* 2020. As far I know, there is no any other theory of DM able to justify theoretically such amount of mass, so this calculus is a big success of DMbQG theory.

In Chapter 12, it is shown a method to estimate the Direct mass formula for a cluster of galaxies, using only its Virial Mass and radius. Through this method it is estimated the parameter a^2 for the L.G. which match with the one calculated in previous chapter using a different method, *i.e.* the DMbQG theory is able to calculate the total mass of L.G. by two different methods. Also are calculated the parameters a^2 associated to Virgo and Coma clusters *i.e.* it is determined the Direct mass formula associated to such cluster of galaxies.

Chapter 13 is crucial to demonstrate how the Dark energy is able to counterbalance the DM at cluster scale, because the Direct mass grows up with the square root of radius whereas the DE grows up with the cubic power.

To achieve this goal, it is introduced the Zero gravity Radius, R_{ZG}, defined by [15] Chernin, A.D. *et al.* (2013) and it is calculated its formula in the framework of DMbQG theory. Using such formula are calculated the radius, R_{ZG}, for the L.G, for Virgo and Coma clusters.

In the paper [5] Abarca, M.2024, it is fully developed the DMbQG theory for cluster of galaxies and there are found remarkable theoretical results validated with measures in cluster of galaxies published by well known researchers.

This paper develops the best version of DMbQG theory in the Newtonian framework.

In the papers [16] Corda, C. 2009 and [17] Corda, C. 2012, the author claims that an extension of General Relativity theory, *i.e.* an extended gravity theory, would be an intermediate step previous to develop the definitive Quantum Gravity theory. I propose a natural way to study the DM phenomenon in the framework of an extended gravity theory using the *DMbQG theory.* Namely, it is right to get the density of energy associated to DM, multiplying the density of dark matter by c^2, *i.e.* multiplying the formula (8.3) by c^2 would be got the density of energy associated to the density of DM. So this density of energy would be a new term to consider into the energy tensor of Einstein's gravitational field equations.

The DMbQG theory introduces a powerful method to study DM in the halo region of galaxies and cluster of galaxies and conversely the current studies in galaxies and clusters offer the possibility to validate the theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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