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A Dark Energy Hypothesis I

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Abstract

The cosmological constant, Λ, represents dark energy. The dark energy hypothesis (DEH) replaces Λ with a variable quantity, the cosmological parameter:

$$\Lambda = \frac{1}{a^2\eta^2}$$

In this formula, "*a*" is the scale factor and η the conformal time: $a\mathrm{d}\eta = c\mathrm{d}t$. A companion paper (DEH II) develops and explores a cosmological model with this variable parameter. This paper portrays the origin of the cosmological parameter in the uncoupling of time and space in the early universe from a prior state in which the comoving coordinates $x^0 = \eta$ and $x^1 = \chi$, the cosmic latitude, are coupled. In this hypothesis dark matter is a co-product of the decoupling, but its nature remains mysterious.

Keywords

Dark Energy, Dark Matter, Cosmological Constant, Tensor Calculus

1. Derivation

The cosine rule for any triangle in Euclidean space is

$$B^2 = A^2 + C^2 - 2AC\cos(b)$$

A, *B*, and *C* are the lengths of the three sides and b is the angle between *A* and *C*. If *A* is an imaginary quantity, $A = iD$, then the cosine rule becomes

$$B^2 = -D^2 + C^2 - 2iDC\cos(b)$$

Use this rule to construct a line element in which the specified comoving coordinates are coupled: $B \to \mathrm{d}s$, $C \to a\mathrm{d}\eta$, $D \to a\mathrm{d}\chi$, and add the angular terms for isotropy, although they play no role in the subsequent analysis:

$$\mathrm{d}s^2 = a^2\left[\mathrm{d}\eta^2 - \mathrm{d}\chi^2 - 2i\mathrm{d}\eta\mathrm{d}\chi\cos(b) - f(\chi)\left(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2\right)\right]$$

Consequently, $g_{01} = g_{10} = -i\cos(b)$ in the metric tensor [1], and $f(\chi) = \sinh(\chi)$, χ, $\sin(\chi)$ depending on the curvature constant $k = -1, 0, +1$, resp. Versions of the cosine rule exist for curved spaces, but in the "infinitesimal domain" of differentials revert to the Euclidean form [2].

When the angle $b = 0$ the vectors $\mathrm{d}\eta$ and $\mathrm{d}\chi$ lie parallel to one another meaning that space and time are not independent quantities. When $b = \pi/2$ they are independent and orthogonal, which is the Weyl postulate, $g_{10} = g_{01} = 0$: the worldline of a particle at rest in its coordinate frame χ = constant must be orthogonal to a surface η = constant, which is the locus of all events occurring simultaneously in that frame. That defines the Hubble flow. The tensor calculus is the mathematical formalism to analyze the decoupling process that gives rise to the cosmological parameter.

The starting point is the metric tensor for the coupled state. The matrix is 4 × 4 in block form, but only the upper 2 × 2 block is needed:

$$\boldsymbol{g} = a^2 \begin{bmatrix} 1 & -i\cos(b) \\ -i\cos(b) & -1 \end{bmatrix}$$

The determinant of the matrix is $g = -a^4\sin^2(b)$. The matrix is singular for $b = 0$, but otherwise the inverse exists; the initial coupled state will be that the angle is a small positive number. The inverse is

$$\boldsymbol{g}^{-1} = \frac{1}{a^2\sin^2(b)} \begin{bmatrix} 1 & -i\cos(b) \\ -i\cos(b) & -1 \end{bmatrix}$$

The next step in the analysis is to calculate Christoffel symbols:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left[g_{\rho\nu,\mu} + g_{\mu\rho,\nu} - g_{\mu\nu,\rho} \right]$$

The covariant quantities on the right are from the matrix tensor and the contravariant is from its inverse. The convention is to sum over the index repeated upstairs/downstairs, ρ in this case. The comma means to take the derivative with respect to the index on the comma's right.

The analysis begins with the vectors $\mathrm{d}\eta$ and $\mathrm{d}\chi$ separated by a small angle b. Fix the latter to a horizontal axis and let the angle increase by rotating the former counterclockwise while maintaining a constant magnitude. Hence the only change in η is due to the changing angle, so by the chain rule

$$\frac{\mathrm{d}}{\mathrm{d}\eta} = \frac{\mathrm{d}b}{\mathrm{d}\eta}\frac{\mathrm{d}}{\mathrm{d}b}$$

This gives only two non-zero Christoffel symbols:

$$\Gamma^0_{00} = \frac{2\cot(b)}{\eta} \quad \text{and} \quad \Gamma^1_{00} = -\frac{i}{\eta\sin(b)}$$

The contribution of the second symbol will disappear in the subsequent analysis, which is to calculate the Ricci tensor. For $\mu = \nu = 0$

$$R_{00} = \Gamma^{\beta}_{0\alpha}\Gamma^{\alpha}_{0\beta} - \Gamma^{\alpha}_{00}\frac{\partial \ln\sqrt{-g}}{\partial x^{\alpha}} + \frac{\partial^2 \ln\sqrt{-g}}{\partial \eta^2} - \Gamma^{\beta}_{00,\beta}$$

The result is

$$R_{00} = \frac{1}{\eta^2}\left[3\cot^2(b) + \cot(b) + 1\right]$$

Now raise an index:

$$g^{00}R_{00} = \frac{1}{a^2\sin^2(b)}R_{00} = R_0^0(b)$$

The decoupling occurs when $b = \pi/2$:

$$R_0^0\left(\frac{\pi}{2}\right) = \frac{1}{\eta^2 a^2}$$

The Dark Energy Hypothesis is that the cosmological parameter is the raised Ricci tensor for π/2.

$$\Lambda = \frac{1}{\eta^2 a^2} = \kappa\varepsilon$$

The right-hand side of course is the product of the Einstein gravitational constant and the dark energy density. When $b = \pi/2$, the line element becomes

$$\mathrm{d}s^2 = a^2\left[\mathrm{d}\eta^2 - \mathrm{d}\chi^2 - f(\chi)\left(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2\right)\right]$$

This is a line element for curved spacetime, curved spacetime is gravity, and gravity is associated with matter, so by hypothesis this is the line element of dark matter, whatever that is.

2. Summary

In summary, the DEH is two-fold, that dark energy and dark matter are products of a kind of phase change in the early universe, viz., the uncoupling of comoving coordinates of space and time. They are products of metric or geometric cosmology in contrast to baryonic matter, which is a product of physical cosmology. An aside is that the scale factor is non-zero and constant throughout the decoupling meaning that there is no singularity.

Addendum

Gödel [3] provides a precedent for coupling space and time coordinates. Oks [4] reviews recent contributions to the subject.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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