Scientific Research Publishing

Journal of High Energy Physics, Gravitation and Cosmology, 2024, 10, 1045-1053
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

Dark Matter and Dark Energy from Lattice Model of Universe

Branislav Majerník

Faculty of Mathematics, Physics and Informatics Comenius University, Bratislava, Slovakia
Email: branislav.majernik@gmail.com

How to cite this paper: Majerník, B. (2024) Dark Matter and Dark Energy from Lattice Model of Universe. *Journal of High Energy Physics, Gravitation and Cosmology*, **10**, 1045-1053.
https://doi.org/10.4236/jhepgc.2024.103064

Received: April 24, 2024
Accepted: July 16, 2024
Published: July 19, 2024

Abstract

The article considers a conceptual universe model as a periodic lattice (network) with nodes defined by the wave function in a background-independent Hamiltonian based on their relations and interactions. This model gives rise to energy bands, similar to those in semiconductor solid-state models. In this context, valence band holes are described as dark matter particles with a heavy effective mass. The conducting band, with a spontaneously symmetry-breaking energy profile, contains particles with several times lighter effective mass, which can represent luminous matter. Some possible analogies with solid-state physics, such as the comparison between dark and luminous matter, are discussed. Additionally, tiny dark energy, as intrinsic lattice Casimir energy, is calculated for a lattice with a large number of lattice nodes.

Keywords

Dark Energy, Dark Matter, Lattice Universe Model

1. Introduction

Existence of significant amount of cca 27% of dark matter (DM) in energy budget of the Universe is supported by many astrophysical and cosmological observation. Dark matter is a label for all matter that does not interact with the electromagnetic force as normal matter does. Astrophysical evidences for DM are based on gravitational effects which cannot be explained by just observed luminous normal matter. Astrophysical evidence for dark matter is based on gravitational effects that cannot be explained solely by observed luminous normal matter. On a galactic scale, dark matter (DM) is observed through the rotation curves of spiral galaxies, the peculiar motions of galaxies within the Coma cluster, and the dynamics of the well-known Bullet Cluster. Numerical simulations of DM explain the observed filamentary distribution of luminous matter at large scales, where DM acts as gravitational traps, creating such a filamentary network. On a cosmological scale, DM has a footprint in the anisotropies of the cosmic microwave background. Dark matter is electrically neutral, non-relativistic (cold), and non-baryonic. This last feature is concluded from the predictions of Big Bang nucleosynthesis as well as cosmic microwave background measurements. From other observations and simulations of DM halos we can conclude that DM is a long-lived particle with self-interaction and cross section $0.1 < \frac{\sigma}{m} < 1\,\text{cm}^2 \cdot \text{g}^{-1}$.

The Standard Model (SM) of particle physics does not contain any viable candidates for dark matter (DM). Many extensions of the SM propose potential candidates, such as supersymmetric theories with superpartners of ordinary matter. However, there has been no experimental evidence from the Large Hadron Collider (LHC) or other devices to support these theories.

Measurements of the Cosmic Microwave Background (CMB) performed by the PLANCK satellite in 2018 indicate that DM accounts for 27% of the universe's energy budget, compared to 5% for ordinary matter, with the remaining 68% attributed to dark energy. This suggests that if DM particles are approximately five times heavier than the particles of luminous matter described by the SM, then the particle abundance between ordinary and dark matter would be approximately similar.

$$m_{DM} \propto 5m_{SM} \rightarrow n_{DM} \sim n_{SM}$$

Here is a proposed model where dark matter has similar aspects to holes in the valence band of semiconductors, and luminous matter resembles electrons in the conducting band. Depending on the semiconductor material and lattice structure, holes can have different effective masses compared to electrons. For example, in GaAs semiconductors, the effective mass of holes is five times greater than the effective mass of electrons.

The idea that elementary particles of the Standard Model are akin to the collective excitations in solid-state physics was introduced by P. W. Anderson [1] and has been developed further in subsequent works. The original and distinctive feature of the conceptual model proposed in this article is the formulation where Standard Model particles are excitations in the conducting energy band of such a solid-state universe model, while dark matter particles are described as vacancies, or holes, in the valence band, similar to semiconductors. In other words, our universe is akin to a semiconductor fabric where excitations in the conducting energy band are described by the Standard Model. The valence or other energy bands constitute the dark sector, and dark matter is the "missing" particle matter, or vacancies, in these bands. During the Big Bang heating, these missing particles jumped the energy gap between the valence and conducting bands. What turns particles in the conducting band into luminous matter, is the spontaneous symmetry breaking that occurs within this band. This breaks the original $SU(2)\times U(1)$ to rezidual electromagnetic $U_{EM}(1)$ symmetry.

The next section demonstrates how a periodic lattice model with two energy bands separated by an energy gap can mimic our universe, distinguishing between the Standard Model (SM) and Dark Matter (DM) sectors. Section three presents calculations of the lattice Casimir energy and explores the resulting energy relations that can be interpreted as the dark energy component of the model. In the concluding section four, various interactions between DM and SM are discussed, including DM/SM excitons and the Universe Hall effect. Additionally, it is noted how a 1 + 1 dimensional model of dark energy can potentially be extended to 3 + 1 spacetime dimensions with an additional compactified dimension.

2. Conceptual Model

Let's consider simple periodic lattice model with wave function $\Psi_n \in C$ describing node n in lattice with Hamiltonian in so called tight binding model, where fermion particles can sit only on the location nodes of lattice and has some probability g to hop to a neighbouring node

$$H = E_0 \Psi_n - g\left(\Psi_{n+1} + \Psi_{n-1}\right) \tag{1}$$

Using $H|\Psi\rangle = E|\Psi\rangle$ give us

$$E_0 \Psi_n - g\left(\Psi_{n+1} + \Psi_{n-1}\right) = E\Psi_n \tag{2}$$

This can be solved with ansatz $\Psi_n = \mathrm{e}^{ikna}/\sqrt{N}$ with wave number or momentum k, where

$$k \in \left[-\frac{\pi}{a}, \frac{\pi}{a}\right)$$

Than $\Psi_{n\pm1} = \mathrm{e}^{\pm ika}\Psi_n$ and

$$E = E_0^i - 2g\cos(ka) \tag{3}$$

Due to periodic function cos in expression for energy is energy limited in band $\left[E_0^i - 2g, E_0^i + 2g\right]$. For small momentum $k \ll \pi/a$ is possible expand energy

$$E(k) \sim \left(E_0^i - 2g\right) + ga^2k^2 \sim \frac{h^2k^2}{2m} \tag{4}$$

Which give a well known formula for particle with effective mass inherited from properties of the lattice

$$m^* = \frac{h^2}{2ga^2}$$

Or another effective mass definition from energy dispersion relations is

$$m^* = \frac{h^2}{\left(\dfrac{\partial^2 E}{\partial k^2}\right)} \tag{5}$$

With standard energy dispersion relations like (3), see **Figure 1**, in solid states one can gets different effective masses for valence and conduction band depending on sharpness of flatness energy dispersion curve for small k.

When we extend our simple model with some hoping probability g' between second neighbouring nodes

$$E_0^i \Psi_n - g\left(\Psi_{n+1} + \Psi_{n-1}\right) + g'\left(\Psi_{n+2} + \Psi_{n-2}\right) = E\Psi_n \tag{6}$$

If $\Psi_{n\pm1} = \mathrm{e}^{\pm ika}\Psi_n, \Psi_{n\pm2} = \mathrm{e}^{\pm i2ka}\Psi_n$ and

$$E = E_0^i - 2g\cos(ka) + 2g'\cos(2ka) \tag{7}$$

With wave number or momentum k, where

$$k \in \left[-\frac{\pi}{a}, \frac{\pi}{a}\right)$$

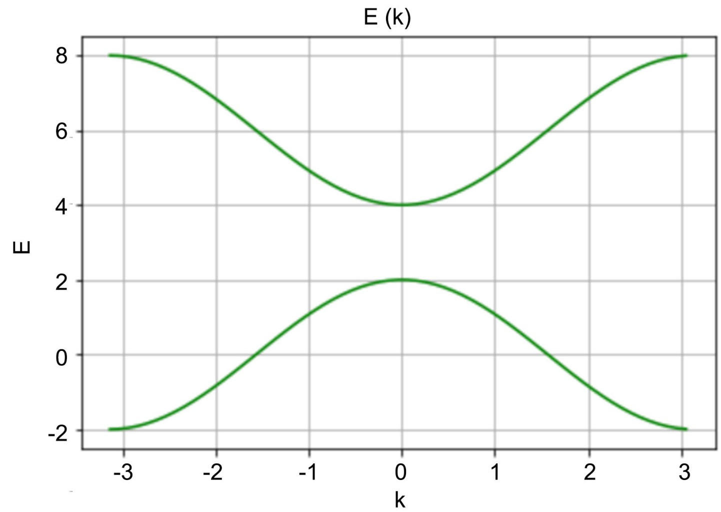

Figure 1. The standard energy dispersion relation in solids with energy gap separting conducting and valence band.

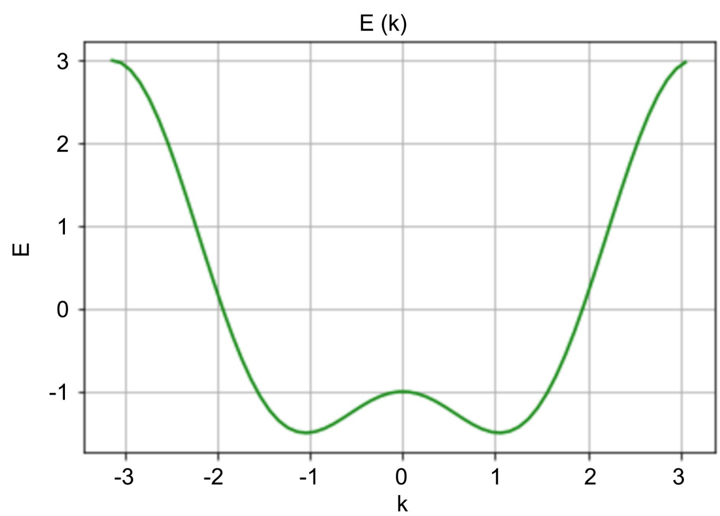

Figure 2. The energy dispersion curve with different hoping parameters.

One obtains an energy dispersion curve, as shown in Figure 2, formally similar to a double-well potential in the Standard Model (SM). However, this is not a plot of vacuum potential energy dependent on a scalar field. The figure illustrates that the minimum energy of the conducting band does not occur at zero lattice momentum. Additional hopping terms to term (6)

$$g''(\Psi_{n+3}+\Psi_{n-3}), g'''(\Psi_{n+4}+\Psi_{n-4})$$

Can tune energy dispersion to more sharp curves around minimum. Here is not some apriori constrain on relations between values of hoping parameter, but one can expect $g > g' > g'' > g'''$. Similar to the concept of tunneling probability (instantons) between several minima, for more widely separated minima, the probability is lower compared to directly neighboring minima. If we consider a situation similar to solid-state physics, where non-orthogonal or different atom basis lattices are involved, we can obtain an energy band picture as depicted here (see Figure 3).

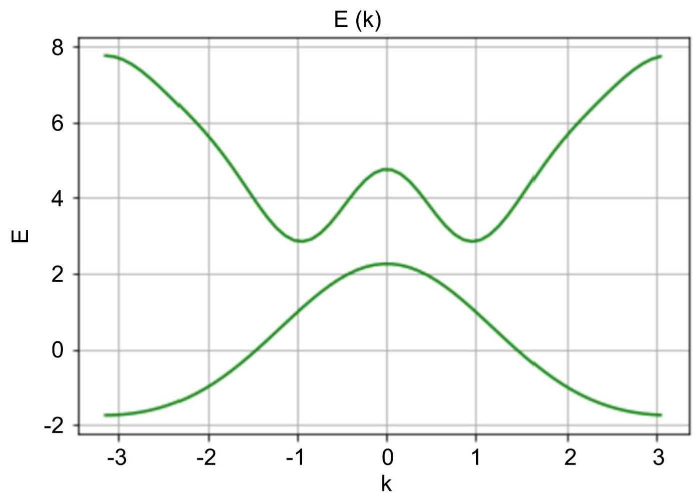

Figure 3. Energy dispersion curves with minimal energy of conducting band in different value of lattice momentum comparing with maximum energy of valence band.

The diagram illustrates that the minimum energy of the conducting band occurs at a different lattice momentum value compared to the maximum energy of the valence band. This phenomenon is known as an indirect gap, which means a particle cannot transition from the highest energy state in the valence band to the lowest energy state in the conduction band without changing its momentum, mediated by a phonon. In solid-state physics, a phonon is a quantum or particle of crystal lattice oscillation. In this context, entities like the Higgs boson or dilaton [2] could play a similar role, assuming that crystal lattice nodes represent chunks of spacetime with quantized volume, akin to quantum loop gravity. If the energy gap between the valence and conduction bands equals the mass/energy of the Higgs boson, the Higgs boson can propel a fermion particle from the valence band to the conduction Standard Model (SM) band, where the kicked fermion manifests as an SM particle with an effective mass given by (5). Is simple to see that effective mass of particle in conducting band in $k = \pm 1$ is smaller comparing effective mass of hole particle in valence band in $k = 0$. This give possible explaining reason why $n_{DM} \sim n_{SM}$, $m_{DM} \propto 5m_{SM}$, $\Omega_{DM} = 5\Omega_{SM}$.

3. Dark Energy as Lattice Casimir Energy

Observational data from the type Ia supernovae, cosmic microwave background, and baryon acoustic oscillations continue to confirm that about 68% of the energy density today consists of dark energy responsible for the accelerated expansion of the Universe [3].

The simplest candidate for dark energy is the so-called cosmological constant Λ. If the cosmological constant originates from a vacuum energy of particle physics, zero point energy of some scalar field, its energy scale $k_{\max}$ to be Planck mass

$$\rho_{vac} = \int_0^{k_{\max}} \frac{\mathrm{d}^3 k}{2(2\Pi)^3} \sqrt{k^2 + m^2}$$

Give $\rho_{vac} \propto 10^{74}\ \mathrm{GeV}^4$, which is significantly larger as the dark energy density today $\rho_{DE} \propto 10^{-47}\ \mathrm{GeV}^4$.

It is therefore necessary to find a mechanism that can explain the small value of Λ consistently with observations. One reason why Λ is so small could be related to t'Hooft's technical naturalness of the cosmological constant. A parameter is considered technically natural if setting it to zero would enhance symmetry. In supersymmetry, when unbroken, an equal number of bosonic and fermionic degrees of freedom ensures that the total vacuum energy cancels out. However, it is well-known that supersymmetry is broken at sufficiently high energies, leading to a generally non-zero vacuum energy. Achieving a tiny Λ value in broken supersymmetry theories is possible in models involving certain classes of Kähler potentials in the 10-dimensional action, particularly with modulus fields from 6-dimensional T after compactification into the 4-dimensional space [4].

Another example in recent developments of string theory demonstrates the possibility of constructing de Sitter vacua with a small positive Λ by compactifying extra dimensions in the presence of fluxes, taking into account non-perturbative corrections. [5].

Closed membranes, quantized fluxes are considered in general effective cosmological constant schemas where large negative bare cosmological constant from scalar field in anti-deSitter metric is neutralized to effective tiny value with contributions from quantized fluxes of energy density from 4-form field

$$\frac{F^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma}}{2 \cdot 4!} = \frac{(nq)^2}{2}$$

where n is natural number and q is some charge. Than the effective vacuum constant is given as

$$\Lambda = -\Lambda_b + \sum_{i=1}^{J} n_i^2 q_i^2 / 2$$

Each of these approaches calculate the difference between sum of discrete values and an integral of a function of vacuum fields. Such relation is generalised in Abel-Plana formula and is used for Casimir energy calculation.

Here lattice model can offer good regularization as well as Casimir energy environment. Casimir energy is know as zero point energy in special boundary condition, usually know as source of attractive force acting beetween two uncharged parallel plates due to change in zero point energy of the vacuum extending between the plates with respect to the vacuum contained in the same region in the absence of plates. From condition of periodicity one have lattice discrete parameter, number of nodes $N = \frac{L}{a}$ where L is lenght of lattice and a is nodes displacement parameter, lattice spacing. Than we can define k in discrete values with periodic boundary condition $k = \frac{2\pi a n}{L} = \frac{2\pi n}{N}$. Using Abel-Plana formula

$$\sum_{n=0}^{\infty} f(n) - \int_0^{\infty} f(x)\,\mathrm{d}x = \frac{f(0)}{2} + i\int_0^{\infty} \frac{f(it) - f(-it)}{\mathrm{e}^{2\pi t}}\,\mathrm{d}t$$

For Casimir effect or simple quantitative analyse for 1 + 1 dimensional lattice with energy dispersion relation E(k) given by term (3)

$$E_{Casimir} = \sum_n E_n - a\int_0^{\pi/a} E(k)\,\mathrm{d}^2 k = i\sum_n \mathrm{e}^{i2\pi n/N} - a\int_0^{\pi/a} \frac{k\mathrm{d}k}{2\pi}\cos(ka)$$

One get for lattice Casimir energy

$$E_{Casimir} \approx N - \frac{\mathrm{e}^{-\frac{i}{N}}}{\sin\left(\frac{1}{N}\right)} \tag{8}$$

Casimir energy graph for real part is on **Figure 4**.

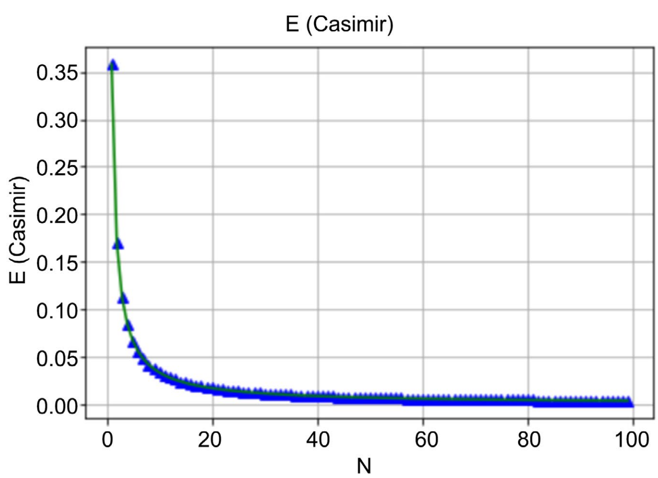

Figure 4. Casimir energy graph for 1 + 1 dimensional lattice for different numbers of lattice nodes.

For large lattice model with many nodes (large N), the Casimir energy can manifest as a tiny positive value, potentially leading to a repulsive effect akin to dark energy. As the number of nodes (N) in the lattice universe expands, dark energy becomes very small but not zero, possessing intrinsic attributes tied to the fundamental construction of the lattice nodes themselves.

4. Conclusions

This article presents conceptual ideas on how a "semiconductor" scheme of collective excitation can describe SM and DM particles in their respective bands/sectors. Here, we do not investigate the nature of such lattice nodes or the binding forces of DM and SM. However, if this analogy with solid-state physics holds, similar effects as those observed in solid-state physics may be detectable. One well-known example from solid-state physics is excitons, which are bound states of holes and electrons. Such states have been observed through optical absorption in crystals, resembling the energy spectrum of hydrogen type with $E \sim -\frac{1}{n^2}$ where $n = 1, 2, 3, \cdots$.

If collider experiments can detect energy states where energy (from collisions or other sources) is absorbed within such a spectrum, it could resemble something like a DM/SM exciton. Another analogy could be seen in the Hall effect in semiconductors, where under an external magnetic field, holes and electrons in an electric current are deflected to the appropriate side of the material, generating an additional voltage bias in the transverse direction against the electric current. In our speculative analogy, the current of DM/SM particles on a cosmological scale could be analogous to the large-scale peculiar velocities of clusters of galaxies, known as the dark flow [6], and a cosmic Hall effect similar to the Bullet Cluster's deflection of DM versus SM matter. Here, the components of the cluster—stars, gas as luminous matter, and dark matter—behave differently and are spatially separated. In the case of dark energy, it has been demonstrated that lattice Casimir energy can function similarly to dark energy with a very low density. But in 3 + 1 dimensional spacetime universe all dimensions are continuous. Where could the source of discretization needed for term (8) be found? Here can help theory $R^D \times S^1$ with additional compactified spatial dimension S^1, where boundary periodic conditions produce the same effect as a lattice. Thus, our Universe could harbor a hidden fourth compactified dimension, acting as the source of dark energy that drives the expansion of 3D space.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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