An Original Didactic about Standard Model (Geometric Model of Particle: The Quarks)

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Abstract
This work shows a didactic model representative of the quarks described in the Standard Model (SM). In the model, particles are represented by structures corresponding to geometric shapes of coupled quantum oscillators (GMP). From these didactic hypotheses emerges an in-depth phenomenology of particles (quarks) fully compatible with that of SM, showing, besides, that the number of possible quarks is six.

Keywords
Golden Particle, Quark, Sub-Oscillator, Semi-Quanta, IQuO Geometric Structure, Golden Number, Massive Coupling, Interpenetration, IQuO, Pion, Meson

1. Introduction
Despite the clarity and systematic nature of Standard Model (SM), the greatest difficulty for students (and not only) in understanding the physical reality of particles is that of not being able to “see” the particles if they are represented as mere mathematical objects described by wave functions and equations. Since we cannot see a space “point”, we thought of giving the particle a representation in which it is not seen as point particle but becomes an “object” with a spatial dimension, identified in the Compton wavelength. Not only that, but to “see” a particle in “totus” we have also defined its fundamental properties in “visible” terms, such as mass, the various electric and color charges, and the various “savour” of quarks. Since the phenomena are strictly correlated and ordered in precise structured patterns, see SM, we therefore decided, for educational and speculative purposes, to assign a possible internal structure to a particle without
compromising its elementary nature. Starting then from some peculiar aspects, such as analogies and some indicative phenomena, we proceeded to formulate a representation in which the particles have structure with a “geometric” shape. The educational model (and not only) is defined as the Geometric Particle Model (GMP). In the Section 2.1, starting from a “golden” relation between the Compton lengths of the proton and that of Planck, the first didactic idea of a geometric representation of the quarks constituting a proton is formulated: the quarks \((u, d)\) are represented as two golden triangles \([\alpha(18˚, 18˚, 104˚), \delta(36˚, 36˚, 72˚)]\). The second teaching idea is to build triangles by coupling quantum oscillators \((IQuO)\) composed of elementary “sub-oscillators” with “semi-quanta” \((sq(\bullet))\) of energy moving along the sides. In this section, we talk about the color charge in the triangular structure of a quark. To adequately represent a particle, we formulate, Section 2.2, the didactic idea that triangular structures can propagate along a “guide” and rotate around this line, thus physically carrying out the spin. In the same section we formulate the hypothesis that the value of the electric charge is given by the probability of detecting the quantum \((\bullet)\) of energy \([\bullet = (\bullet, \bullet)]\) along the propagation side. We then proceed with the didactic ideas of representing the internal flow of the \(sq(\bullet)\) along the sides with oriented arrows and through them giving the sign of the electric charge of the particle. In sect. 2.3 we introduce the idea of IQuO (Intrinsic Quantum Oscillator), in which we highlight the double components of operators \((a, a^\dagger)\), and, thus, the 2-dim characteristic of the quantum oscillator which allows us to determine the direction of phase rotation, to which one associates the sign of electric charge. After, we give also the characteristic of an IQuO \((n = 0)\) on which one builds the representation of a gluon, and, thus, to highlight the color charge. In addition to the quarks \((u, d)\), the electron-triangle is also constructed, in Section 2.4. In Section 3.1, we construct the charged pions composed of two triangles \((u, d)\) and through the mass value of the charged pions we find the values of the bound masses of the quarks. In Section 3.2, the neutral pion is represented as a particular combination \((\otimes)\) of four quarks \((u, u, d, d)\) constituting so a first example of a “molecule” of quarks. Through this operation it is possible to obtain the mass of the neutral pion and the “electromagnetic” masses of the two quarks \((u, d)\), while by structure equation it is possible to define and calculate the spin and parity of the pion. In Section 3.3, the configurations of light mesons \((\eta, \rho, \omega)\) are presented, understood as combinations of pion molecules, and their structural equations with the calculation of their masses. Having thus defined a geometric structure of light quarks, we move on to show in Section 4, the structure of the remaining quarks, geometrically demonstrating that only “six structures” of quarks can exist. In Section 4.1, exploiting a geometric property of golden triangles, the geometric shapes of the strange quark and the charm quark are shown, introducing the didactic hypothesis of “sub-quarks”. In Section 4.2, we proceed in the construction of the remaining three quarks thus demonstrating that no more than six quarks exist.
2. The Hypothesis of Structure

2.1. The “Golden” Hypothesis of Quarks

As we indicate by \( n_{(pl,p)} \) the experimental numerical ratio between the Compton’s wavelength di Planck (\( \lambda_{pl} \)) and that of proton (\( \lambda_p \)):

\[
(n_{(pl,p)}) = \left( \frac{\lambda_{pl}}{\lambda_p} \right) = \left( \frac{m_{pl}}{m_p} \right) = \left[ \frac{(2.176450)(10)^{-8}}{(1.672623)(10)^{-27}} \right] \approx (1.301)(10)^{19}
\]

the power (10)^{19} can be a representative scale s-factor. If \( (\phi) \) is the “aureus” (golden) number and \( \phi^2 = (1,618)^2 = (2,618) \) then it is \( [n_{(pl,p)}/(10)^{19}] \sim [(\phi)^2/2] \) and \( [n_{(pl,p)} = (\phi)^2s/2] \). Between Compton’s wavelength (\( \lambda_{pl} \)) and (\( \lambda_p \)) there is so a golden relation [1], at less than a scale s-factor (10)^{19}. Recall the golden segments (see Figure 1).

By property of the golden segments it’s:

\[
\begin{align*}
\lambda_{\beta} &= \left( \frac{\lambda_{\gamma}}{\phi} \right) \\
\lambda_{\gamma} &= \phi^2 \lambda_{\alpha}
\end{align*}
\]

We find these relations in a pentagon between the side (base) and apothem. Recall the protons are composed of three quarks: three centres of diffusion (A, B, D) positioned in a triangular form in diffusion experiments with “bullet” electrons. These centres could indicate the proton having an internal “geometric structure”. The relation between the side and apothem and the golden relation between (\( \lambda_{pl} \equiv \lambda_p \)), see the Equation (2), pushes us to formulate the following first didactic idea:

**Didactic Idea (1):** “A proton has a ‘pentagonal geometric structure \( \Psi_{1p} \), where its three component quarks are coincident with three constituent triangles of a pentagon”.

The quarks (u, d) can be represented by “aureus triangles” (golden triangles) in the following representations, Figure 2.

(Physical Aspects) A possibility for having a physical sense could be to admit that **some quantum oscillators can couple to build a geometric figure.** This structural hypothesis [2] about quarks involves placing three quantum oscillators (I\(_A\), I\(_B\), I\(_C\)) at the vertices (A, B, D). To build this structure there are junction oscillators (I\(_{AB}\), I\(_{BC}\), I\(_{CA}\)). In this case, the quark-triangles are, see Figure 3.

We indicate by the acronym “IQuO” (Intrinsic Quantum Oscillator) the oscillators of coupling building a geometric structure. Note that the oscillations of IQuO are longitudinal along the sides. Nevertheless, since the structure is closed the positions of equilibrium of vertices appear being in rigid points. Recall in a set of three marble with springs there is an oscillation way in which the marble at centre always stays. In conclusions, only the Junction IQuO has the variations of oscillation length.

**Didactic Idea (2):** “A IQuO vertex must be made up of two oscillating parts, that is, two sub-oscillators”.

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To realize a structure of coupled quantum oscillators it needs to connect an IQuO-vertex to the two lateral IQuO of the junction, placed in two different directions, see Figure 3. In these cases, the IQuO-vertexes are “bent” along their oscillation amplitude and simultaneously are bound to two different oscillators, which oscillate along two different sides. Therefore, to physically admit a structure of coupled oscillators (IQuO) we must think that the IQuO-vertexes are “folded” in two parts: each part is a “sub-oscillator”. For induction, this last characteristic must be possessed by any IQuO, also the IQuO of junction: the IQuO oscillators [3] [4] are so decomposed into several oscillating or sub-oscillators, as in Figure 3. We also think the presence of two or more oscillating components in an oscillator causes the “splitting” of its energy quanta [1]:

**Didactic Idea (3):** “In a quantum oscillator at sub-oscillators there are ‘semi-quantum’ of energy”.

Individually we will speak of a “semi-quantum” (sq(•)). We will write:

\[ \text{1 quantum} \equiv (\bullet) = (\bullet, \bullet) \equiv 2 \text{sq} (\bullet) \quad (3) \]

**(Physical Aspects)** Recalling the elastic and kinetic (inertial) characteristics of
a quantum oscillator, we need to admit the same that also in any sub-oscillator there are two energetic components. In an IQuO at \((n = 0)\) we have only one sub-oscillator with energy \(\varepsilon = 1/2h\nu\) but two sq, one elastic and the other inertial, each having an energy of \([\varepsilon = 1/4h\nu]\). It follows for any \(n\) that:

\[
\begin{align*}
H_{(n)} &= U_{(n)} + K_{(n)} = \left[\left(U_{(n)}\right)_{el} + \left(K_{(n)}\right)_{in}\right] \\
&= \left[2(n+1)\left(\frac{1}{4}h\omega\right)_{el} + 2n+1\left(\frac{1}{4}h\omega\right)_{in}\right] \\
&= \left[2(n)\left(\frac{1}{4}h\omega\right)_{el} + 2n\left(\frac{1}{4}h\omega\right)_{in}\right] + \left[\frac{1}{4}h\omega\right]_{el} + \left\{\frac{1}{4}h\omega\right\}_{in}
\end{align*}
\]

From this equation, it derives conjecture that the energy value of \([\varepsilon = 1/4h\nu]\), indicated as an “empty” semi-quantum with the symbol \((o)\), while another value \([\varepsilon = (1/2h\nu)]\) indicates as “full” semi-quantum and symbol \((\bullet)\). So, an oscillator \(\varepsilon_n = (n + 1/2)h\nu\) will be represented by a set of empty sq\((o)\), with \((\varepsilon(o) = 1/4h\nu\), and full sq\((\bullet)\), with \((\varepsilon(\bullet) = (1/2h\nu)\); then it is:

\[
\begin{align*}
\varepsilon_n &= \left(n + \frac{1}{2}\right)h\nu = \left(n \frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4}\right)h\nu
\end{align*}
\]

Note IQuO\(a = 1\) = \([o, \bullet] + [o, o]\).

If the structure is composed by IQuO \((n = 1)\), we have the following geometric shape, see Figure 4.

The triangle is equilateral when all quantum oscillators are in an energy eigenstate with \((n = 1)\). Note: two contiguous sub-oscillators \((S_1, S_2)\) could be considered as “coupled” oscillators when the respective pairs of semi-quanta exchange a full semi-quantum \((\bullet): [(o, \bullet)_1 \leftrightarrow (o, o)_2] \rightarrow [(o, o)_1 \leftrightarrow (\bullet, o)_2]\).

Where the symbol \(\leftrightarrow\) indicates the elastic coupling, achieved through the exchange of a sq\((\bullet)\). To happen the exchange of sq\((\bullet)\), the two oscillators must be in phase “accordance”: in an equilateral triangle (where the oscillators are IQuO\(a = 1\)), this aspect happens in all the zones of overlap of the sub-oscillators. There are others two possibilities of have triangular structures of IQuO couplings, see Figure 5.

Figure 5(a) is a rectangular triangle isosceles while Figure 5(b) is a “golden” triangle. In the second case, to connect three vertices, the junction oscillators must be, according to the oscillators’ structure, IQuO in eigenstate \(n = 1\) \((I_{ab})\) or \(n = 2\) (all other IQuO). Recall the IQuO\(a = 2\) has three sub-oscillators; we use the following representation, Figure 6, where we highlight the presence of two sq \((o, \bullet)\) in each sub-oscillator:

Note the sub-oscillators \((S_{ab}, S_{ac}, S_{ca})\) are very important: we conjecture that they can originate the “gluons” from which emerge the hadronic jets (two or three jets) in the experiments to the CERN or the Fermilab. We conjecture that to have a “compact” triangular structure (golden) the sub-oscillators of junction oscillators and vertex ones they need to be “wedged” one inside the other. In this case, on each oblique side we must have two junction oscillators, with \(n = 2\),
At each junction there will be a phase shift during the couplings between the sub-oscillators; given the symmetry of the two oblique sides we will essentially have three phase shifts globally throughout the triangular structure: two phase shifts in one of the two oblique sides (equal to those of the other oblique side) and one in the base one). We will associate three degrees of freedom or three “colors” to these three phase shifts. Each phase shift (color) is an extra degree of freedom to be added within a “golden” structure of IQuO. It is understood that
the degrees of freedom are associated with transformations operating on the phase of the oscillation, that is they are “gauge” transformations. Therefore, each phase adaptation with relative reciprocal phase shifts of the overlapping IQuOs identifies a degree of freedom in the structure, i.e., a “color”, see Figure 7. Three phase shift “points” will thus be determined with mutual continuous adaptation and therefore three colours: (•,•,•). If we talk about color, it follows that the triangular structure built with double couplings and with golden geometry could very well be a quark. Note that the superposition of the sub-oscillators determines a “double” coupling or an additional coupling which will give the mass to quark. Besides, a double sub-oscillator in the state of an “excited vacuum” (one sq (•) in one only IQuO) could constitute a “gluon”. In conclusion, a quark is a structure of coupled gluons. A last note, the sub-oscillators (S_{A2}, S_{C2}) will perform a very important function: they will be available to be hooked by a line (guide rail) of B-type quantum oscillators (Bosons) which will allow the triangular structure of “roto-translate” along it. The “gluon boson” becomes so the gauge field that adapts the phase shifts in the junction for maintain the structure and to allow to this (in moment eigenstate) to propagate along an axis.

2.2. The Elementary Structures

If we want a structure of coupled quantum oscillators to represent a particle, it is necessary to establish the correspondence between the descriptive figures of the structure and the physical quantities of a particle.

Didactic Idea (4): “A ‘structure-particle’ can propagate along a line (X-axis) or a ‘guiderail’”.

(Physical Aspect) We recall in QM that a particle in an eigenstate of impulse (p_x) has propagates along an axis X, see Figure 3. In Field Theory the “guiderail” of a particle is given by line of oscillators of a base field (waveguide). For an electron the guide-field can be the electromagnetic field [5] [6]. It occurs to indicate in the triangle-structure of the particle the sign ± of the electric charge (q), the possible proper rotation of the structure or “spin” (s) and the direction of the (Φ) “flow” vector of the internal quanta sq (•) to the oscillators of the structure. Then, we formulate the following idea, see Figure 8.
Figure 8. Geometric Structure of quark ($u, d$).

**Didactic Idea (5):**

Note that a quark is represented by a golden isosceles triangle; in Figure 8 we identify: the “golden” angles—the two signs of the electric charge given by the “empty” arrow and the “full” arrow—the clockwise-counterclockwise direction of the arrows related to the $\Phi$—quantum flow within the structure (Flux Isospin)—the rotation of the structure around the X-axis with its spin vector $s$ (orientation symmetry of the structure)—the X-axis propagation axis.

(Physical Aspects) In physical terms the relation Structure $\Leftrightarrow$ Particle pushes us to say that the guiderail [5] [6] is a “waveguide” (see Wave-Particle duality) and the direction $\nu$ of propagation of the particle coincides with the direction of the flux $\Phi$ along the propagating side (the one lying along the X axis: $\Phi \Leftrightarrow \nu$).

**Didactic Idea (6):** “The value of the electric charge will be given by the probability $P(\bullet)$ of detecting the quantum (\bullet) along the propagating side”.

By the number of quanta (\bullet) in Figure 1, we find:

$$q(d) = P(\bullet)_d = -1/3, \quad q(u) = P(\bullet)_u = +2/3.$$  

It follows that d-quark contains only one quantum (\bullet) while u-quark contains 2(\bullet). Recall the gauge symmetry of the phase of the wave function correlated to the electric charge $q$ and its sign $\pm$.

### 2.3. The Intrinsic Quantum Oscillator

(Physical Aspects)

The doubling of the energy in a quantum oscillator with sub-oscillators determines the doubling of the components of each quantization operator ($a, a^\dagger$), that is

$$\begin{pmatrix} a_{\omega}, a_{\omega}^\dagger \end{pmatrix}, \begin{pmatrix} a_{\omega}^\dagger, a_{\omega} \end{pmatrix} :$$

$$\begin{aligned}
    a_{\omega} &= a(t)_{\text{elastic}} + a(t)_{\text{inertial}} = O\omega \\
    a_{\omega}^\dagger &= a^\dagger(t)_{\text{elastic}} + a^\dagger(t)_{\text{inertial}} = O\omega^\dagger
\end{aligned}$$

$$\Leftrightarrow \begin{pmatrix} a_{\omega}, a_{\omega}^\dagger \end{pmatrix} = a_{\omega}(e^{i\omega t} + a_{\omega}^\dagger e^{-i\omega t})$$

We will have, Figure 9.

Recall that the two sub-oscillators of the IQuO contain the $sq(\omega, \bullet)$ which move following the oscillation vectors ($O\omega, O\omega^\dagger$) and the corresponding components $\begin{pmatrix} a_{\omega}, a_{\omega}^\dagger \end{pmatrix}$. Let us then correspond to each component of the operators ($a, a^\dagger$) a $sq(\omega, \bullet)$ with their respective elastic and inertial characteristics; we will have the following representation of IQuO, Figure 10.
The IQuO with phase rotation opposites is said: B-IQuO (Boson type). Note that we can define this representation of an IQuO as its 2-dim representation. The aspect 2-dim of the oscillation determines an oscillator with an internal degree of freedom extra with two eigenvalues (the two direction of phase rotation), which could be correlated to two signs of the electric charge [3]. Coupling two B-IQuO, we can obtain two IQuOs with mono-direction of phase rotation, Figure 11.

These IQuO are F-IQuO, different from those of B-type.

**Didactic Idea (7):** “We associate the ± sign of electric charge to the direction of the phase rotation of a wave function of a particle represented by IQuO”.

Exactly, we state [3] [7]:

- Direction clockwise⇔ negative electric charge (−e)
- Direction anticlockwise⇔ positive electric charge (+e)

An IQuO of B-type coupling with a F-IQuO could read the direction of rotation of the phase of F-IQuO: this boson is the photon. The particles that probe the value and sign of the electric charge are photons. The probability \( P(\bullet) \) of detecting the quantum along the side of the propagation axis gives the value of electric charge. In the two quark (u, d) the probabilities are respectively (+2/3, −1/3). The geometric representation of an IQuO allows us to represent gluons. If we consider an IQuO in the vacuum state we will have, **Figure 12**.

The geometric representation can have the following configurations, see **Figure 13(a)** and **Figure 13(b)**.
Two IQuO with equal operators in the phase rotation but with opposite rotations.

Probability function in quantum oscillator with \( n = 0 \).

Where \( (I_n, E_l, I_n^+, E_l^+) \) are sq with direction of rotation anti-clock wise. To each configuration we associate a color charge, Figure 14.

With representation in matrices:

\[
G_R = \begin{pmatrix}
\hat{a}_{E_l} \\
\hat{a}_{I_n}^+
\end{pmatrix}_R = \begin{pmatrix} 0 & \hat{a}_{E_l}^+ \\
\hat{a}_{I_n}^- & 0 \end{pmatrix}_R
\]

\[
G_B = \begin{pmatrix}
\hat{a}_{E_l}^+ \\
\hat{a}_{I_n}^-
\end{pmatrix}_B = \begin{pmatrix} 0 & \hat{a}_{E_l}^- \\
\hat{a}_{I_n}^+ & 0 \end{pmatrix}_B
\]

\[
G_Y = \begin{pmatrix}
\hat{a}_{E_l}^- \\
\hat{a}_{I_n}^+
\end{pmatrix}_Y = \begin{pmatrix} 0 & \hat{a}_{E_l}^- \\
\hat{a}_{I_n}^+ & 0 \end{pmatrix}_Y
\]

A superpositions with a color and an anti-color will be, Figure 15.
Figure 14. Relations between the configurations of the IQuO, and color charge.

Figure 15. The representation of the gluon BY.

So, the gluon has two colors. These superpositions will constitute the “double” sub-oscillators (gluons) that we find along the sides of a quark. The representative matrix of gluons is:

\[
\begin{pmatrix}
RR & RB & RY \\
BR & BB & BY \\
YR & YB & YY
\end{pmatrix}
\]

The gluons of the guideline have two functions: the first is to operate phase shifts of adaption (to phase velocity) along the propagation side which after propagate along the other two sides to compensate for the phase shifts that occur between the IQuOs of the junctions. This phase-shifting “action” allows the structure to compact and for the two sq(●) to flow, see Figure 16, along the sides. The phase shift action occurs in a coordinated way in the junction points arranged on each side. In this case, the quark takes on a given color (one for each phase shift value). In one period we will thus have the alternation of three values of adaptation phase shift: the quark will vary in color three times per the rotation period of the two sq(●). In conclusion, the color change will be so induced by the guideline gluons. The second function is to allow the propagation of structure along the axis. Therefore, the gauge fields have a double function: to allow them to make the compact structure and propagate it along an axis (in a moment eigenstate). It follows that, see Figure 16.
2.4. The Electron Structure

**Didactic Idea (8):** “*The electron can be represented by a rectangular isosceles triangle*”.

The structure of the electron (positron) would be, see **Figure 17**.

The arrows are two on any side because the *sq (•)* are two on any side. Matter and antimatter are represented by the same geometric structures (the forms of arrows are changing). We can have the following possibilities of structures, **Figure 18**.

*(Physical Aspects)* Note the probability of detecting the quantum (•) along the propagating side is: \( \varphi(e) = P(\bullet) = 1 \). Asserting that the electric charge is the probability of detecting the quantum (•), makes us say that in GMP a quark is a “*quasi-particle*”. This is because there is the possibility that an observer “never” reveals the entire quantum through which to exchange energy with the quark. This makes a quark “unobservable” individually or not existing in the free state (see the quarks confinement problem). In this case a quark cannot exchange a quantum (•) with other particles because it has a semi-quanta number not sufficient.

3. Mesons

3.1. Pions

There is the possibility that two quarks (see **Figure 8**) can bond along the propagation side and generate a “real” particle: the pion \([1]\), see **Figure 19**.

**Didactic Idea (9):** *We bind two quarks (u, d) in the following way:*

The quarks must have, along the bonding and propagating side, concordant flux vectors and global value of “unity” of the electric charge.

We conjecture that the golden ratio is also present in bound quarks in a pion \([1]\): \( \{ [m(d)/m(u)] = \varphi, [m(d_1)/m(u_1)] = \varphi \} \). Where \( m \) is the bound mass. If it is \( m(\pi^\pm) \approx (139, 57) \text{ MeV} \), then we conjecture that:
Figure 17. The IQuO-structure with semi-quanta of a positive electron and its electromagnetic guideline.

![Image](https://example.com/figure17.png)

Figure 17. The IQuO-structure with semi-quanta of a positive electron and its electromagnetic guideline.

Figure 18. The configuration of the electron and positron, and \( \Phi \)-vector flow. Note: "the structure-particle propagates in the same direction of vector flow \( \Phi \) along the side AC".

![Image](https://example.com/figure18.png)

Figure 18. The configuration of the electron and positron, and \( \Phi \)-vector flow. Note: "the structure-particle propagates in the same direction of vector flow \( \Phi \) along the side AC".

Figure 19. Geometric form of pion.

![Image](https://example.com/figure19.png)

Figure 19. Geometric form of pion.

\[
\begin{align*}
\left\{ \begin{array}{l}
 m(u_x) + m(d_x) = m_n^x \\
 m(d_x) / m(u_x) = \phi
\end{array} \right. \\
\left\{ \begin{array}{l}
 m(u_x) = \frac{m_n^x}{(1 + \phi)} = \frac{m_n^x}{2.618} = (53.31)\text{MeV} \\
 m(d_x) = m(u_x) \phi = (86.26)\text{MeV}
\end{array} \right.
\]
We have \[ m(d)_π = (86.26) \text{MeV}/c^2, \] \[ m(u)_π = (53.21) \text{MeV}/c^2. \] We pose these mass values as the masses of quarks bound in a pion. The gluons \( (T_g) \) increase the internal elastic tension \( T_q \) of the “free” quarks: \[ T_q = (T_g + T_q) \] The gluons interacting with each other, in the gap between the two sides of the two quarks, increase the elastic bond tension which is transformed into mass. This aspect, in field theory, means that gluons are non-abelian gauge bosons. Finally, note, in Figure 19, that the flow vectors \( \Phi \) in two quarks \( (u, d) \) are opposite to allow the concordance of semi-quanta flows along the side BC of propagation of the pion: \( \Phi_x = v_x. \)

### 3.2. Neutral Pion

Since we introduced quarks as structures of coupled oscillators, we can describe a hadron by a “structure equation” built on the component quarks. A charged pion \( \pi^\pm \) will described by the following equation: \[ \pi^\pm = (u \otimes d). \] Where the sign \( \otimes \) indicates an operation of quantum superposition of the states relating to a “coupling” between structures (quarks). We find this aspect also in neutral pion \( \pi^0 \):

**Didactic Idea (10):** “The neutral pion is a combination \( (\otimes) \) of two charged pion \( (\pi^+, \pi^-) \) reciprocally interconnected and not quantum separated”.

The structure equation is:

\[
\pi^0 = \left( \left( \pi^+ \right) \otimes \left( \pi^- \right) \right)
\] (8)

(Physical Aspects) In literature [8] the wave function of pion is \[ \pi^0 = a(uu + dd) \] and it point out a no-separated state (or entanglement state) of quarks \( (u, u, d, d) \): \[ \pi^0 = a(uu + dd) = a'(ud + du) \]. To have so a neutral pion it is necessary to consider a combination of matter and antimatter couplings. Note this state \( \pi^0 = (ud + du) \) should have almost a mass value double that of a single charge pion. However, the QM doesn’t explain how it is possible that the neutral pion has almost the same mass as the charged pion. Something is missing from the QM. It needs to consider that the neutral pion is a unique elementary particle: it follows that its components \( (u, d) \) and \( (u, d) \) must reciprocally “interpenetrating” [1] themselves for originating a unique physical object. Then, we formulate:

**Didactic Idea (11):** “The superposition of configuration states implies a sort of ‘interpenetration’ of quarks”.

In GMP the possibility of more relative orientations in a charged pion \( \pi^\pm \) between two quarks implies a reciprocal rotation of two quarks \( (u \text{ and } d) \) around the X-axis (as orbital motions). Thus, these configurations, or orientations, can induce us to think of the “spin” of a pion. Nevertheless, the different relative orientations of quarks \( (u, d) \) induce us to admit interpenetrations between two quarks: in the rotation they could cross themselves but, being states of the wave-function, they interpenetrate themselves like the waves. This interpenetration could explain also the zero value of pion’s spin. In fact, the reciprocal interpenetration of quarks implies relative, opposite rotations, meaning \[ |s(u) = -s(d)|, \] \[ \rightarrow [s(u) + s(d)] = 0, \] that is the spin of a pion is zero. In fact, in nature [8] it is
$S(\pi^\pm) = [S(u) + S(d)] = 0$. Now, we consider the neutral pion. From its structure equation, in geometric terms we have, see **Figure 20**.

This could determine the decay [9] in two photons with opposite spins, see **Figure 21**.

The structure of **Figure 21** predicts the decay of neutral pion in two photons caused by annihilation of two quark pairs $[(u, u), (d, d)]$ along the side BD or axis X. Then we should admit a new combination of structures:

$$s(u) + s(d) = 0.$$  

Now, we consider the neutral pion. From its structure equation, in geometric terms we have, see Figure 20. This could determine the decay [9] in two photons with opposite spins, see Figure 21.

The structure of Figure 21 predicts the decay of neutral pion in two photons caused by annihilation of two quark pairs $[(u, u), (d, d)]$ along the side BD or axis X. Then we should admit a new combination of structures:

$$m(u, u) + m(d, d) = m(\pi^0) - m(c)$$

(Physical Aspect) Note in the annihilation process the quark “frees” itself and transforms its electromagnetic energy of bond into annihilation energy or photons. In this case, $[m(u, u), m(d, d)]$ represent the “electromagnetic mass” which could be considered as their “free” mass. We will then have the possibility, thanks to the structure equation, to obtain the mass of the free quarks or “naked” masses, $[m(u, u), m(d, d)]$. We will then find that:

$$\Delta m(\pi) = [m(\pi^0) - m(\pi^0)] = (4.59) \text{MeV}.$$  

We have found the masses of free quarks. The structure equation of pion helps us to find the spin of neutral pion:

$$s(\pi) = s[(\pi^0 \otimes (u^0)] = s[(u^0 \otimes (d^0)]$$
The spin would be given by: 
\[ s(\pi^0) = [s(u_1) + s(d_1)] + [s(u_2) + s(d_2)] = 0 \]
Since it is \([s(u_1) + s(d_2)] = 0, [s(d_1) + s(u_2)] = 0\)

We pass to “parity” concept. Recall the Electromagnetic Field has a parity value \(P(\gamma) = (-1)^J = -1\) with \(J = S + L\) (\(S\) spin and \(L\) angular moment), but since a photon has \(L = 0\) it follows: \(J = S = 1\). Then the photon parity is \(P(\gamma) = (-1)^1 = -1\).

So, a pair \((e^+, e^-)\) has parity \(P(e^+, e^-) = -1\), and thus, we have: \(P(e^+, e^-) = P(e^+)P(e^-) = -1\), or \(P(e^+) = (+1), P(e^-) = (-1)\). Then, the pair \((u, d)\) will also have the parity:

\[ P(u, d) = P(u)P(d) = -1 \rightarrow [P(u) = (+1), P(d) = (-1)] \]

Now, we can calculate the pion parity. We will have from structure equation of charged pion that:

\[ P(\pi^+) = P[(u \otimes d)] = P[(u \otimes d)] = P[(q \otimes q)] = P(q)P(q) = (-1)(+1) = -1 \]

For neutral pion we will have:

\[ P(\pi^0) = P[(\pi^+) \otimes (\pi^-)] = P[(u_1 \otimes d_1) \otimes (u_2 \otimes d_2)] = \]
\[ = [P(u_1)P(d_1) \otimes P(u_2)P(d_2)] = P(q)P(q) = -1 \]

Remember that the interpenetration operation \(\otimes\) makes the two component pions as if them are an only unique “object”.
3.3. The Light Mesons

The **Didactic Idea** (12) “Light mesons are structures composed of pion molecules”.

The $\eta$-meson has the structure following [10]:

$$\eta = \left(\pi^+ \oplus \pi^-\right) \otimes \left(\pi^+ \oplus \pi^-\right) = \left(\pi^0\right) \otimes \left(\pi^0\right)$$

where $\left(\pi^0\right) = \left(\pi^+ \oplus \pi^-\right)$ is a “molecule” of pions, composed by 4 quarks. The structure is, **Figure 22**.

There are four possible particles ($\pi^+$, $\pi^-$, $\pi^0$, $\gamma$) with the following combination or channels:

($\pi^+$, $\pi^-$, $\pi^0$, $\gamma$)$_{33\%}$, ($\pi^0$, $\pi^0$, $\pi^0$, $\gamma$)$_{33\%}$, with a virtual lattice $\pi^0$, ($\gamma$, $\gamma$)$_{39\%}$, ($\pi^+$, $\pi^-$, $\gamma$)$_{5\%}$, ($\gamma$, $\gamma$)$_{39\%}$.

Other mesons are [10] [11] as $\eta'$, while the $\rho$-meson and $\omega$ are with added the pair ($d$, $d^*$):

$$\eta' = \left(\pi^0\right) \otimes (\eta),$$

$$\rho = \left(d,d^*\right) \otimes \left[2\eta \otimes \left(\pi^0\right)\right] = \left(d,d^*\right) \otimes \left[2\eta \otimes \left(\pi^0\right)\right]$$

$$\omega = \left[2\left(d,d^*\right) \otimes \left[(\pi^+ \oplus \pi^-) \otimes (\pi^+ \oplus \pi^-)\right]\right]$$

4. The Heavy Quarks

4.1. The Construction If Heavy Quarks ($s$, $c$)

Equivalent ((Recall the geometric rule of golden triangles: in a golden triangle ($u$, $d$) it is possible to draw two other triangles (sub-triangles) inside, always of the type ($u$, $d$), which are indicated as ($u$, $d$) [2] [12].

**Didactic Idea** (13) “In each quark ($u^*$, $d^*$) one can insert another pair of golden triangles of type ($u$, $d$) which give origin to sub-quarks ($u$, $d$) inside the ‘tank’ quark ($u^*$, $d^*$)”

So, we will have two possible configurations, see **Figure 23**.

We see the composed quarks have two forms: [$d^*$, ($u$, $d$); $u^*$, ($u$, $d$)]. The quarks ($u^*$, $d^*$) are called “tank” quarks because contain the “sub-quarks” ($u$, $d$).

If we consider the rule that the charge of the composed quark is the sum of the charges of the quarks making, we note that: $q\left(d^*,(u,d)\right) = 0$, $q\left(u^*,(u,d)\right) = 1$. These combinations should be discarded: the charge values are integer, thus contradicting the hypothesis that the particle-structure is a quark. Then, we need to differentiate the sub-quarks from the tank quarks resorting also to anti-quarks. We find that:

$$q\left[u^*,(u,d)\right] = +1/3, \quad q\left[u^*,(u,d)\right] = -1/3;$$

$$q\left[d^*,(u,d)\right] = -2/3, \quad q\left[d^*,(u,d)\right] = +2/3$$

The first combination will give us the “strange” quark or $s$-quark [12]. The second will give us the “charm” quark or $c$-quark [13]. With these two
quarks the $K(s)$ mesons and the $D(c)$ mesons are formed. The structure of strange quark $[2] [12]$ is:

$$\begin{align*}
q(s) &= q\left[\left(\bar{u}^*, (u, d)\right)\right] = -1/3, \\
q(s) &= q\left[\left(\bar{u}^*, (u, d)\right)\right] = +1/3
\end{align*}$$

Therefore, matter has matter sub-quarks while antimatter has antimatter sub-quarks. Then, we will have, see Figure 24.

If we consider the internal flow of $sq(•)$, we will have Figure 25.

A note due: the couple $(u^*, u)$ does not annihilate because the bond energy is at gluons that is it is based on the color charge $(u^*, u)$, this means that a gluon $(g)$ will never transform into electromagnetic energy or photon $(γ)$ and vice versa. To the $s$-quark and $s$-quark we can associate two matrices:

$$s\text{-quark } \Leftrightarrow \begin{pmatrix}
(\bar{u}^*) \\
\# \\
\# \\
\# 
\end{pmatrix}, \quad \begin{pmatrix}
(\bar{u}^*) \\
\# \\
\# \\
\# 
\end{pmatrix}$$

(13)

Recall the “weak Isospin” in quarks, that is $T_z = \pm 1/2$, with $T_z = +1/2 = \rightarrow$ quark $u$-type, with $T_z = -1/2 = \rightarrow$ quark $d$-type. The $s$-quark is of $d$-type, because in its structure $[\bar{y}^*, (u, d)]$ if $(\bar{y}^*, u)$ annihilate then should remain only $d$-quark.

4.2. The Construction of the Quarks ($c$, $b$, $t$) Pion Mass

By Now we can derive the structures of the other quarks $[2] [12]$. Figure 23 represent the $c$-quark, that is, Figure 26.
the c-quark \[14\] has the structure: \[c = \left[ \frac{d^*}{u^*}, (u, d) \right]\]. The electric charge will be given by \[q(c) = q\left(\frac{d^*}{u^*}, (u, d)\right) = \left[ q(d^*) + q(u) + q(d) \right] = q(u) = +2/3\].

Note \(T_z(c) = +1/2\), the c-quark is type up \((u)\).

Another possibility with tank \(d\)-quarks and sub-quark can be, Figure 27.

The electric charge will be given by

\[q(b) = q\left(\left\{ \frac{d^*}{u^*}, [u, (u, d)] \right\}\right) = -1/3, \quad q(b) = q\left(\left\{ \frac{d^*}{u^*}, [u, (u, d)] \right\}\right) = +1/3\]

Note \(T_z(b) = -1/2\), the c-quark is type ups \((u)\).

Another possible structure with \(u^*\)-tank quark and sub-quarks is the following, Figure 28.

With electric charge: \[q(t) = q\left(\left\{ u^*, [u, (u, d)] \right\}\right) = +2/3\].

Note \(T_z(t) = +1/2\), the c-quark is type ups \((u)\).

In summary, we will only have “six” types of quarks \[12\] \[13\], see Figure 29.

5. Conclusion

As also can be seen in this article, GMP introduces, didactically and otherwise,
new descriptive paradigms of particle phenomenology as the interactions between particles through intermediary lattices (bosons) and the quantum oscillator at sub-oscillators and semi-quantas (IQuO). Thanks to these new paradigms, it is possible to see that mass, electrical charge, and color charge find “geometric” expressions that facilitate the understanding of these fundamental concepts of physics.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References


