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Risk Budgeting: A Tactical Asset Allocation Approach for Retirement Reserve Funds in Morocco

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Abstract

This paper aims to suggest a risk budgeting approach that can enhance the asset allocation process of retirement reserve funds. The approach takes into consideration the importance of incorporating risk into tactical decisions to improve fund performance. The significance of this study lies in the need to adopt a strong approach that enables asset managers to outperform strategic asset allocation while avoiding excessive risk. The article proposes a methodology for measuring and allocating market risk in investment decisions, through the computation and decomposition of Value at Risk and Expected Shortfall. Additionally, it introduces a novel scientific method of using Expected Shortfall and its decomposition into marginal units to guide short-term tactical decision-making.

Keywords

Risk Budgeting, Strategic Asset Allocation, Tactical Asset Allocation, Value at Risk, Expected Shortfall, Retirement Reserve Funds

1. Introduction

Financial crises have been notorious for their devastating impact on the economy, employment, and the financial system. In recent decades, several crises have arisen, causing severe economic problems both at the national and global levels. Nonetheless, these crises have also paved the way for improving the practices of risk management and assessment (Artus, 2008). They have brought to the fore the significance of risk management in investment strategies and helped fortify its pivotal role (Haugh et al., 2015). On the one hand, authorities are taking measures to introduce more rigorous regulations for financial institutions. They require them to establish more rigorous internal risk measures designed to anticipate failures and coordinate them to prevent the possibility of losses and bankruptcies. On the other hand, financial institutions often reconsider their understanding of risk by adopting new approaches grounded in risk quantification.

Defining the concept of risk is not as straightforward as defining return. The complexity of the risk definition has been explored by various researchers (Lee & Lam, 2001; Hansson, 2004; Hansson, 2010; Kermisch, 2012; Unger, 2015). In the realm of finance, the concept of risk gained importance with the emergence of Markowitz's theory in the 1950s, which was followed by other measures such as Beta, the Capital Asset Pricing Model (CAPM), and multifactor models including the Arbitrage Pricing Theory (APT) and the three-factor profitability model (Pierandrei, 2019). Financial institutions have also adopted techniques like risk budgeting (Chow et al., 2001; Yildirim, 2015), which helps define the acceptable level of risk beforehand and facilitates better risk management at all levels of strategic, tactical, and operational decision-making.

Banks were the pioneers in adopting the risk budgeting framework for managing their funds. This framework aims to define a risk-based investment strategy by establishing boundaries within which performance objectives can be achieved. The Basel Committee on Banking Supervision (BCBS) recommends defining risk acceptance and risk appetite for all managed risks to achieve this objective (Manganelli & Engle, 2001). On the other hand, the Solvency II prudential framework requires insurers to integrate risk management into the managerial function and align it with their previously defined risk appetite (Kouwenberg, 2017). Finally, managers of defined benefit pension funds have also shown interest in using risk budgeting and control techniques to manage their funds (Urwin et al., 2001; Sharpe, 2002).

Risk budgeting involves assessing the risk of exposure and interaction between assets to achieve better risk allocation. It can address three essential questions: What is the current level of risk? Which assets generate the most risk? And how can risk be allocated more efficiently in the future? Risk budgeting, as a dynamic process that combines measurement, contribution, and risk allocation, complements strategic asset allocation (SAA) decisions in the investment process (Berkelaar, Kobor, & Kouwenberg, 2006; Kouwenberg, 2017). It can be viewed as a strategy for directing investments to the most value-generating classes and as an approach to adjusting tactical deviations through the prior definition of a risk budget (RB) (Lewis et al., 2007), as we apply in our study.

Our study was conducted just prior to the reform of the pension system in Morocco, which aims to unify existing schemes and provide old age risk coverage to all Moroccan households. As a result of this merger, previously separate reserves and provisions will be consolidated into a single fund to enhance the sustainability and solvency of the new system. Accordingly, investment policies must be revised to account for the new asset/liability matching constraints. Reflection on asset management should incorporate past experiences as well as new methods and adapted management techniques.

The purpose of our paper is to present a risk budgeting methodology for a diversified investment portfolio consisting of four asset classes: stocks, bonds, money market investments, and real estate. Firstly, we elaborate on the current literature regarding risk budgeting in the context of strategic asset allocation for pension plans. Secondly, we analyze the characteristics of Value-at-Risk (VaR) and Expected Shortfall (ES) as consistent measures of market risk. Lastly, we implement our risk-based approach to tactical adjustment of strategic asset allocation.

This article is structured as follows: after the introduction, the first section summarizes the literature review on risk budgeting and SAA for pension plans. Then, Section 2 describes Value-at-Risk and Expected Shortfall as risk budgeting tools. Section 3 presents the data and methodology used in the study. Finally, empirical results and their interpretations are summarized in Section 4.

2. Literature Review

2.1. Risk Budgeting and Asset Allocation for Pension Plans

As time passes, pension plans build up reserves and provisions that are invested in the stock market. These investments play a crucial role in funding pension benefits, with the aim of balancing asset and liability constraints and improving the balance sheet by boosting the funding ratio (Urwin et al., 2001).

Pension funds invest their funds in various traditional asset classes such as stocks, bonds, and cash, as well as other instruments such as commodities, venture capital, and hedge funds. The initial step in the investment process is to allocate the funds to different asset classes and group assets with similar risks and returns, which defines the primary risk limits and generates most of the performance (Brinson et al., 1991; Berkelaar, Kobor, & Tsumagari, 2006). In addition, tactical asset allocation (TAA) and security selection are often used to exploit short-term opportunities, complementing the SAA.

Asset managers are always searching for new methods to enhance returns, starting with a thorough examination of asset allocation, which is based on retiree liabilities (Sharpe, 2002). The strategic asset allocation (SAA) aims to construct an optimal portfolio that meets a specific long-term return objective and risk tolerance (Lhabitant, 2004). Tactical asset allocation (TAA) is then employed, with deviations from strategic weights defined by certain margins, in order to take advantage of short-term market fluctuations and potentially improve performance (Bouyé, 2012). These margins are expressed as a percentage, allowing managers to deviate from strategic weights within a given range (e.g., a strategic allocation of 20% for stocks with a margin of ±25% would allow the actual share of stocks to vary between 15% and 25%). The method of using fixed margins of deviation from strategic weights in tactical asset allocation has some drawbacks. Firstly, the margins are predetermined and do not adapt to market conditions. Secondly, this approach does not take into account the contribution of each asset class to overall risk, nor the correlation between asset classes. Consequently, the portfolio may be exposed to high levels of risk if all margins are exploited simultaneously.

Risk budgeting is a method that focuses on evaluating and breaking down risk into smaller components. This technique usually involves a shorter time frame compared to asset-liability modeling and aims to allocate funds among different assets effectively to enhance returns and make appropriate asset allocation decisions (Urwin et al., 2001; Pearson, 2002; Schneider & Sams, 2009). It has gained popularity in investment management after significant losses in recent years (Da Silva et al., 2008). To establish asset allocations based on their contributions to overall risk, risk budgeting necessitates a comprehensive measure to quantify risk (Pearson, 2002).

Urwin et al. (2001) suggested the information ratio (IR) as a measure to evaluate the performance of a portfolio. The IR is calculated as the excess return of a portfolio over its benchmark divided by the tracking error (TE). According to Urwin et al. (2001), the risk budget should be allocated based on the marginal contribution of different risk sources. Berkelaar, Kobor & Kouwenberg (2006), Lee & Lam (2001), and Kouwenberg (2017), have also supported the use of TE as a risk measure. On the other hand, Jorion (2003), Berkelaar, Kobor & Kouwenberg (2006), Da Silva et al. (2008), and Gourieroux & Jasiak (2010) have argued that VaR and ES are widely used as tools for risk budgeting due to their effectiveness in measuring risk.

Asset allocation and risk budgeting are two strategies that aim to balance the trade-off between return and risk in investment management (Schneeweis et al., 2010). SAA involves allocating funds across asset classes based on their return and risk, while risk allocation models convert capital allocations into risk allocations by exploiting the marginal contributions of each asset to overall risk (Berkelaar, Kobor, & Tsumagari, 2006). Asset allocation adjustments can be made by calculating the contribution of each risk point to additional value creation (Darolles et al., 2012).

2.2. Value at Risk and Expected Shortfall: Risk Budgeting Tools

Financial institutions use mathematical models to manage their risk, which involves theorizing the market's equilibrium state, gauging potential gains, and assessing the associated levels of risk. VaR, a concise measure of risk represented as a single number, was developed by major derivatives traders in the late 1980s and later widely adopted by banks, asset managers, and corporate treasurers (Jorion, 1996; Glasserman et al., 2000; Al Janabi, 2012). Its popularity grew significantly following its inclusion in the July 1993 Group of Thirty report and the release of the initial version of RiskMetrics by JP Morgan in the same month (Da Silva et al., 2008). In 1995, new regulations based on VaR were introduced, mandating that financial institutions maintain capital commensurate with the risks associated with their liabilities (Glasserman et al., 2000; Gourieroux & Jasiak, 2010).

VaR is a metric used to quantify the market risk of an investment portfolio, measuring the potential loss in value in the event of adverse changes in asset prices (Culp et al., 1998; Yildirim, 2015; Pearson, 2002). Unlike other standard measures such as standard deviation or beta (Racicot & Théoret, 2006: p. 470), VaR is expressed in monetary units. It can also be defined as the maximum potential loss that may occur with a given probability over a specific time horizon (Manganelli & Engle, 2001). In essence, VaR provides an answer to the question, "What is the maximum loss we will not exceed with (1 − α) confidence (where α is the risk of being incorrect) over the next N days?" However, it should be noted that VaR does not account for the α% worst-case scenario (Christoffersen, 2012). VaR is evaluated by the following equation:

$$P\left[R > \text{VaR}_\alpha\right] = 1 - \alpha, \tag{1}$$

where R is the result of the portfolio observed over the time horizon N.

VaR is estimated using two approaches: statistical and parametric. The first is based on the frequency distribution of the result (profit or loss). For a given confidence level, the VaR corresponds to a quantile of the distribution calculated according to random scenarios, on historical or generated data. The second approach, also called "delta-normal", is based on a model of the distribution of risk factors ac-cording to mathematical laws and assumes the normality of the distribution of results (Jorion, 1996). Based on the assumption of a normal distribution of returns characterized by its mean ($E(R)$) and its standard deviation (σ_R), Equation (1) becomes:

$$P\left[\frac{R - E(R)}{\sigma_R} > \frac{\text{VaR}_\alpha - E(R)}{\sigma_R}\right] = 1 - \alpha, \tag{2}$$

with $z_\alpha = \left(\text{VaR}_\alpha - E(R)\right)/\sigma_R$, the quantile of the normal distribution N(0,1) of order α. We thus obtain:

$$\text{VaR}_\alpha = E(R) + \sigma_R * z_\alpha. \tag{3}$$

VaR is determined by three main factors. The first one is the confidence level, typically 95% or 99%, which indicates the acceptable margin of error in the estimate and reflects the investor's tolerance for uncertainty. This is aligned with the investment objective of accepting potential losses due to market price fluctuations. The second factor is the management horizon, which depends on the frequency of portfolio reallocation and the liquidity of assets (Jorion, 1996). The third factor is the assumed distribution of portfolio returns, which is usually normal and facilitates model parameter determination (Vlaar, 2000).

Despite its ease of calculation and interpretation, VaR has limitations that have been identified by several researchers (Artzner et al., 1999; Nagy et al., 2019; Al Janabi, 2012). Consistent risk measures should satisfy four properties: monotonicity, translation invariance, positive homogeneity, and subadditivity. VaR fails to satisfy the last property and may not accurately reflect extreme crisis situations (Artzner et al., 1999). Consequently, a new measure called Expected

Shortfall (ES) has been proposed, which represents the average of losses that exceed the VaR. It is calculated using a specific formula:

$$\text{ES} = E\left[R \mid R < \text{VaR}_\alpha\left(R\right)\right]. \quad (4)$$

The ES is considered as a coherent risk measure which, unlike the VaR, allows to verify the fourth property of subadditivity and to consider extreme values with a very low probability of occurrence not considered by the VaR (Sarykalin et al., 2008; Bargès et al., 2009; Fan et al., 2012). Hence, ES complements VaR to better capture the level of risk in the portfolio.

Risk budgeting requires the decomposition of the risk measure into elementary contributions. The objective is to estimate the loss that the portfolio will incur on each of its assets. The difficulty is to decompose the VaR and the ES into units because of the correlation between the assets. To overcome this problem, we use partial VaR (PVaR_i) and partial ES (PES_i) to measure the contribution of each asset i to the total risk of a portfolio (Muromachi, 2009). We denote, for a portfolio (P) consisting of n assets, by:

- $x\left(x_1,\cdots,x_n\right)$: the vector of weights of the n assets constituting the portfolio such that $x'e = 1$;
- $e\left(1,\cdots,1\right)$: the n-dimensional vector of 1;
- $R\left(r_1,\cdots,r_n\right)$: the vector of returns of the n assets;
- $\sigma\left(x\right)$: the portfolio standard deviation which depend on the positions or weights x_i;
- $\text{VaR}_\alpha\left(x\right)$: the portfolio value-at-risk, which depend on the positions or weights x_i;
- $\sigma\left(r_i, r_j\right)$: the covariance between the returns of assets i and j.

We start by looking for the formulation of PVaR_i so as to have: $\text{VaR}\left(x\right)_\alpha = \sum_{i=1}^{n} \text{PVaR}_i$. In Equation (3), we replace $E(R)$ by $\sum_{i=1}^{n} x_i E\left(r_i\right)$ and we obtain:

$$\text{VaR}_\alpha\left(x\right) = \sum_{i=1}^{n} x_i E\left(r_i\right) + z_\alpha * \sigma\left(x\right). \quad (5)$$

On the other hand, the standard deviation is a positively homogeneous function of degree 1 since $\sigma\left(\lambda x\right) = \lambda\sigma\left(x\right) \quad \forall\lambda > 0$. It verifies Euler's law which allows to write it under the following equality: $\sigma\left(x\right) = \sum_{i=1}^{n} x_i\, \partial\sigma\left(x\right) / \partial x_i$ (Pearson, 2002: p. 161).

Replacing in Equation (5), we obtain:

$$\text{VaR}_\alpha\left(x\right) = \sum_{i=1}^{n}\left[x_i E\left(r_i\right) + z_\alpha x_i\, \partial\sigma\left(x\right) / \partial x_i\right]. \quad (6)$$

$\partial\sigma\left(x\right) / \partial x_i$ is interpreted as the effect of a variation, of one unit, of the weight x_i on the risk measured by the standard deviation. It can be deconstructed according to Mina & Xiao (2001) and Unger (2015) in the following form:

$$\partial\sigma\left(x\right) / \partial x_i = \sum_{j=1}^{n} x_i \sigma\left(r_i, r_j\right) / \sigma\left(x\right). \quad (7)$$

By replacing it in Equality (6), we arrive at the following result:

$$\text{VaR}_\alpha = \sum_{i=1}^{n}\left[x_i E\left(r_i\right) + z_\alpha x_i \sum_{j=1}^{n} x_j \sigma\left(r_i, r_j\right) / \sigma\left(x\right)\right]. \quad (8)$$

And, by posing: $\text{PVaR}_i = x_i E(r_i) + z_\alpha x_i \sum_{j=1}^{n} x_j \sigma(r_i, r_j) / \sigma(x)$, we obtain a decomposition of the VaR_α into PVaR_i for i: $1, \cdots, n$.

The risk contribution can be positive or negative depending on the direction and magnitude of the correlation. Assets with a positive PVaR serve as risk hedging instruments (Pearson, 2002). In the same way as for the marginal variations of the standard deviation, we can deduce those of the VaR under the following formulation:

$$\partial \text{VaR}(x) / \partial x_i = E(r_i) + z_\alpha \sum_{j=1}^{n} x_j \sigma(r_i, r_j) / \sigma(x) = \text{PVaR}_i / x_i . \quad (9)$$

After decomposing the VaR, we proceed to decompose the ES. Equation (4) defines the ES as the average of the returns below the VaR. Since the mean positively homogeneous function of degree 1, we can write: (Fan et al., 2012)

$$\text{ES}(x) = \sum_{i=1}^{n} \text{PES}_i = \sum_{i=1}^{n} x_i \, \partial \text{ES}(x) / \partial x_i . \quad (10)$$

From Equations (4) and (10), we obtain:

$$\text{PES}_i = x_i E\left[r_i \mid R < \text{VaR}_\alpha(R) \right] . \quad (11)$$

PES_i is the average of the returns of class i under the condition that the daily loss of the overall portfolio exceeds its overall VaR.

Finally, we derive the marginal variations of the ES from Equations (10) and (11) as follows:

$$\partial \text{ES}(x) / \partial x_i = \text{PES}_i / x_i . \quad (12)$$

During the literature review, it emerged that most previous theoretical and empirical studies have focused solely on describing risk budgeting as a risk management technique, without proposing a concrete approach integrated into the investment decision-making process of pension funds, which have a dual concern of ensuring funding for benefits and seeking profitability. Furthermore, no study has explained how to tactically manage the strategic asset allocation based on the consumption of the authorized risk budget. These gaps are particularly concerning as practitioners need a clear approach to adjust their tactical positions based on realized performance and consumption of the previously allocated risk budget.

In this study, we aim to address these gaps by proposing a method based on Expected Shortfall to measure the risk and contribution of each asset to the overall risk. We use the performances of the Moroccan markets to outline the steps for implementing risk budgeting based on the calculation, decomposition of Expected Shortfall, and allocation of unused risk to the most rewarding asset classes.

3. Data and Methodology

Our research focused on pension plans in Morocco, specifically their reserve funds. At the end of 2020, the pension provisions invested amounted to 328.4 billion MAD (ACAPS, 2020) and were primarily allocated among four asset classes: bonds, stocks, real estate, and money market investments. These pension plans are subject to specific legislation and management rules. For the purpose of our study, we applied the same management framework to the portfolios analyzed.

To obtain our data, we computed the daily returns vector (R_i: i = 1, 2, 3, 4) using the recent and available indices of the four compartments of the Moroccan financial market, published between January 1, 2018, and December 31, 2022. The Moroccan All Stocks Index Return (MASI R), published by the Casablanca Stock Exchange (CSE), was used as the benchmark for the stocks class. For bonds, we relied on the Moroccan Bond Index (MBI) calculated by Bank of Africa. For money market investments, we constructed an index based on the Moroccan overnight index average (MONIA) provided by the Central Bank of Morocco (BAM). In the absence of an official reference index for real estate, the real estate benchmark was assimilated to the index of real estate investment companies (SPI) listed on the CSE.

To better implement our approach, we analyzed the data in two sub-phases. The first phase, from January 1, 2018, to December 31, 2021, was used to calculate VaR, ES, and risk allocation. The data for the year 2022 was then utilized for implementing risk budgeting.

We present the main trend indicators and the correlation matrix calculated over the 2018-2021 period for each asset class in Table 1.

Table 1. Descriptive statistics for the four asset classes (2018-2021).

		Stocks	Bonds	Real Estate	Monetary
Daily	Number of observations	1492	1492	1492	1492
	Average return	0.03%	0.011%	0.012%	0.005%
	Standard deviation	0.51%	0.05%	0.92%	0.001%
	Min	−3.34%	−0.29%	−5.32%	0.004%
	Max	3.58%	1.63%	6.09%	0.01%
Annualized	Average return	10.11%	4.18%	4.38%	1.92%
	Standard deviation	9.77%	1.05%	17.62%	0.02%
	Kurtosis	10.4	21.0	7.2	−1.7
	Skewness	0.8	0.9	0.3	−0.3
Correlation	Stocks	100%	−2.2%	22.0%	−6.2%
	Bonds	−2.2%	100%	−7.7%	1,0%
	Real estate	22.0%	−7,7%	100%	−1.1%
	Monetary	−6.2%	1.0%	−1.1%	100%

Note: The authors calculate the performance based on official published indices. Data for stocks and real estate are available on the Casablanca Stock Exchange website: https://www.casablanca-bourse.com/en. For money market investments, the Moroccan overnight index average is published daily by the Central Bank of Morocco: https://www.bkam.ma/. Finally, the Moroccan Bond Index can be downloaded from the website: http://www.bmcecapitalgestion.com/glossaire/indice-mbi-moroccan-bond-index.

Based on our initial findings, we observed that equities have an attractive average annual return of 10.1% with a volatility of 9.77%, while real estate investments offer higher returns with higher risk and money market investments provide steady returns with minimized risk.

To calculate VaR and ES, the study aligned with BCBS guidelines on confidence levels, accepting a certain number of exceptions over a 100-day test period. Backtesting was performed for various confidence levels, and the maximum level that yielded satisfactory results was retained. However, the backtesting showed discrepancies between theoretical confidence levels and actual losses above VaR, which was attributed to the overreaction of financial markets to information or events (Scott, 1991). The results in **Table 2** indicated the number of exceptions for a given confidence level, with higher levels leading to more exceptions. Ultimately, a confidence level of 95% was used in the study, corresponding to a risk of being wrong one day out of twenty working days.

For the management horizon, we retain a sufficient period to implement the tactical choices. In asset management practice, monthly or quarterly committees are held to analyze the market and portfolio realizations and recommend adjustments to the tactical allocation. For the calculation of periodic ES and VaR, we use the approximation $\text{VaR}_{(\alpha,T)} \approx \text{VaR}_{(\alpha,1\,\text{day})} \times \sqrt{T}$ and $\text{ES}_{(\alpha,T)} \approx \text{ES}_{(\alpha,1\,\text{day})} \times \sqrt{T}$ with T the number of working days. We choose a quarterly monitoring frequency to unwind our tactical allocation steering approach.

Finally, to fit the distribution to the normal distribution, we resort to the Cornish Fisher expansion to approximate the quantiles using the real skewness and kurtosis coefficients to correct for non-normality (Amédée-Manesme et al., 2019). The transformation formula is as follows:

$$\widetilde{z_\alpha} = z_\alpha + \left(z_\alpha^2 - 1\right)S/6 + \left(z_\alpha^3 - 3z_\alpha\right)\left(K-3\right)/24 - \left(2z_\alpha^3 - 5z_\alpha\right)S^2/36. \quad (13)$$

with:

- $\widetilde{z_\alpha}$: adjusted quantile of the normal distribution;
- z_α: quantile of the normal distribution N(0,1);
- S: skewness coefficient of the real distribution;
- K: kurtosis coefficient of the real distribution.

Table 2. Comparison of theoretical and observed confidence levels.

	Theoretical confidence levels		
	99%	97.5%	95%
	Observed confidence levels		
Stocks	98.8%	97.8%	96.5%
Bonds	98.6%	97.9%	96.9%
Real estate	97.1%	95.9%	95.3%
Monetary	100.0%	100.0%	96.8%
Portfolio (S)	97.6%	96.6%	95.4%

Source: The authors.

The skewness and kurtosis coefficients, calculated for the four asset classes, differ from those of a normal distribution (Skewness = 3 and Kurtosis = 0). The results of the Shapiro-Wilk and Kolmogorov-Smirnov tests (Razali & Wah, 2011), presented in **Table 3**, confirm that the vectors of returns are not normal, hence the need to adjust the quantiles.

In order to test our study, we constructed three theoretical portfolios (S), (R) and (P) from the indices, whose structures are presented in **Table 4**.

The portfolio (R) is designed to use all available tactical margins within the regulatory limits for each asset class, resulting in maximum risk exposure that the manager cannot surpass. In contrast, the portfolio (S) represents the strategic asset allocation (SAA) that we assume to be established for at least one year, with a focus on risk budgeting. Passive management involves replicating this portfolio to achieve its performance, while active management seeks to enhance returns by making bets on each asset class based on the manager's convictions and market predictions.

The construction of the portfolio, in line with the pension plan's liabilities, implicitly involves taking risks, which we refer to as ES_{SAA}. We analyze the profile of this portfolio to answer the question of whether ES is an appropriate measure of risk and use PES to determine risk allocation. We then investigate the impact of tactical decisions on overall return and risk by examining marginal changes in return and risk. Finally, we conduct backtesting to ensure that the observed quarterly returns do not exceed the ES.

Table 3. Normality test results.

	Kurtosis	Skewness	Kolmogorov-Smirnov[a]		Shapiro-Wilk	
			Statistiques	Sig.[b]	Statistiques	Sig.[b]
Stocks	10.4	0.837	0.18	0.00	0.84	0.00
Bonds	21.0	0.862	0.21	0.00	0.74	0.00
Real Estate	7.2	0.306	0.29	0.00	0.73	0.00
Monetary	−1.7	−0.327	0.29	0.00	0.73	0.00

Source: The authors' computation on IBM SPSS. a. Correction de signification de Lilliefors; b. The normality hypothesis is rejected for all series (Sig. < 0.05).

Table 4. Allocations of the studied portfolios.

Asset Class	Portfolio (R)	Portfolio (S)	Portfolio (P)
Stocks	30%	20%	23%
Bonds	55% (*)	65%	61%
Monetary		5%	5%
Real Estate	15%	10%	11%

Source: Information published on the website of the Supervisory Authority of Insurance and Social Welfare can be found at https://www.acaps.ma/en. (*) The regulatory allocation combines bonds and monetary assets in the same class.

To address the second question, we take a two-step approach. First, we establish a risk budget that quantifies the amount of potential loss that the retirement fund can bear. This budget can be defined in monetary units or as a percentage of total assets. Our study links the potential loss to the investment objective of outperforming the strategic portfolio, which requires managers to increase their exposure to risky assets and manage their risk appetite within a predefined budget. To achieve this, we assume that the manager is willing to accept a potential loss equal to the outperformance achieved relative to the funding rate of the pension reserves or the expected return on the strategic asset allocation over the viability horizon of the pension plan. In addition to the risks taken by replicating the strategic allocation portfolio (ES_{SAA}), it must be recognized that managers are required to take on additional risks and adjust strategic weightings. We define our maximum risk budget ($RB_{TAA\ max}$) as follows:

$$-ES_p + ES_{SAA} \leq RB_{TAA\ max}. \tag{14}$$

- ES_p: maximum potential portfolio (P) loss;
- ES_{SAA}: maximum potential loss incurred by the strategic portfolio (S);
- $RB_{TAA\ max}$: the maximum risk budget allocated to managers to create outperformance.

Once the risk budget is established, the second step is to calculate the ES_{SAA} of the strategic portfolio (S), which is the loss required to replicate it. The manager can then construct a portfolio by deviating from the strategic weights, depending on the expected return and the likely contribution of each asset to overall risk. We perform backtesting on the portfolio's (P) return and risk in 2022 to verify whether the ES provides correct guidance to the manager in his tactical decisions.

Finally, we apply our approach to real data for the year 2022 by testing three investment strategies. Table 5 shows the asset allocations of the three portfolios constructed according to various market expectations. The managers of portfolios (P) and (P') anticipate a positive stock market performance with a moderate pace for the former and very favorable for the latter. On the other hand, portfolio (P") underweights the equity market and real estate and adopts a defensive strategy focused on strengthening money market investments.

Table 5. Profiles of the three strategies.

Asset Class	Portfolio (P) Moderate		Portfolio (P') Aggressive		Portfolio (P") Defensive	
	Capital allocation	Tactical deviations	Capital allocation	Tactical deviations	Capital allocation	Tactical deviations
Stocks	23%	+3%	25%	+5%	18%	−2%
Bonds	61%	−4%	59%	−6%	66%	+1%
Real Estate	11%	+1%	11%	+1%	9%	−1%
Monetary	5%	0%	5%	0%	7%	+2%

Source: The authors.

4. Results and Interpretations

4.1. Expected Shortfall (ES) Is a Consistent Measure of Market Risk

Table 6 summarizes the initial indicator calculation results for the four asset classes and the portfolio (S) for the 2018-2021 period.

The strategic portfolio (S) shows an annual return of 5.27 percent and a potential annual loss as measured by the $VaR_{(SAA,1year)}$ of 3.22 percent. For example, for a portfolio worth one billion MAD, the manager is 95 percent certain that his portfolio will not lose more than 32.2 million MAD in one year. However, there are 5 "chances" out of 100 that the loss will exceed this amount. Of these 5 percent, the $ES_{(SAA,1year)}$ tells us the average loss beyond the $VaR_{(SAA,1year)}$, which in our example is 57.9 million MAD.

For this portfolio, the weighted sum of the individually calculated VaR (ES) is greater than the overall VaR (ES) of the portfolio. Diversification reduces the risk of exposure to asset classes. On the other hand, the use of PVaR and ES helps to accurately allocate the overall risk across asset classes. Comparing VaR and ES with realized quarterly returns confirms our choice of ES as a consistent risk measure. Indeed, as shown in Figure 1, the ES was not exceeded during the study period while the VaR did not correctly assess the risk of potential loss in the third quarter of 2022.

Table 6 illustrates the shift from capital allocation to risk allocation for the strategic portfolio (S), where the calculation of partial ES enables the measurement of the contribution of each asset class to the overall risk. With a comprehensive understanding of the portfolio's exposure to risky asset classes, the manager can quantify the impact of any tactical decision on the return and risk taken.

Table 6. Capital allocation and risk allocation for the portfolio (S) (2018-2021).

	Stocks	Bonds	Real Estate	Monetary	Portfolio (S)
Capital Allocation	20%	65%	10%	5%	100%
$VaR_{(1year)}$	−9.59%	−0.70%	−21.18%	0.05%	−3.22%
$PVaR_{(1year)}$	−1.70%	−0.03%	−1.50%	0.01%	
$\partial VaR(x)/\partial x_i$	−8.52%	−0.08%	−15.01%	0.08%	
$ES_{(1year)}$	−16.69%	−1.25%	−36.82%	0.00%	−5.79%
$PES_{(1year)}$	−2.67%	−0.10%	−3.03%	0.00%	
$\partial ES(x)/\partial x_i$	−13.35%	−0.15%	−30.28%	0.08%	
Allocation Risk	46.1%	1.7%	52.3%	−0.1%	100.0%
Annual performance	10.11%	4.18%	4.38%	1.92%	5.27%
Contribution to performance	2.02%	2.72%	0.44%	0.10%	

Source: The authors.

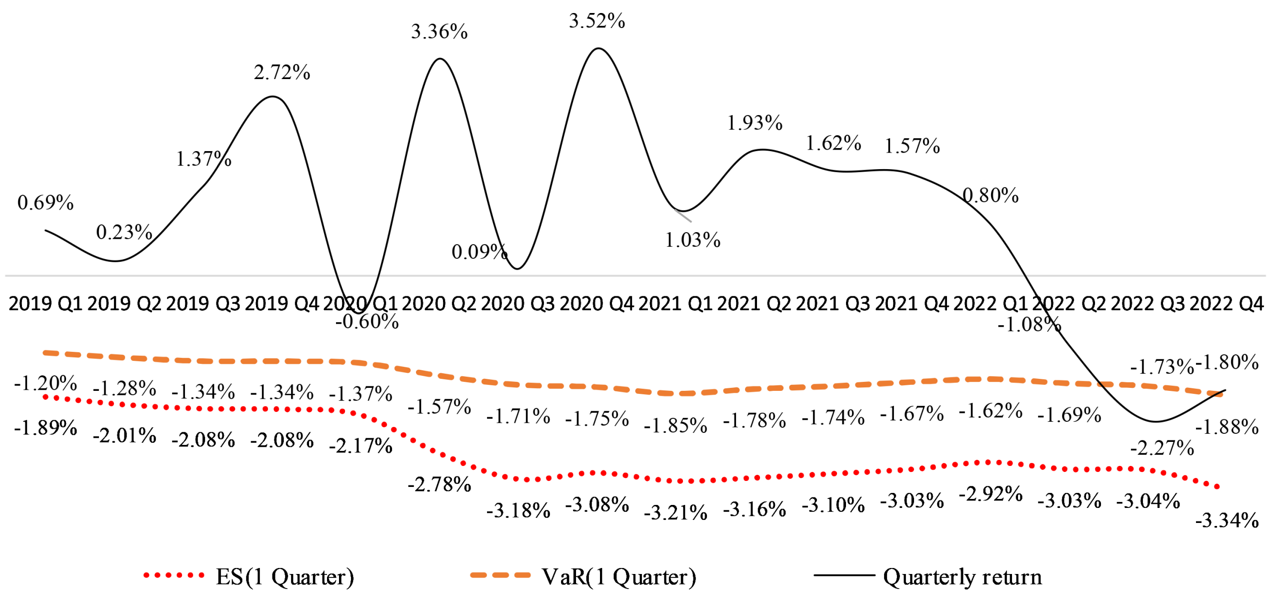

Figure 1. Comparison between $VaR_{(1\ Quarter)}$, $ES_{(1\ Quarter)}$ and quarterly performance. Source: The authors.

Using the partial ES, **Figure 2** presents a risk allocation that is entirely different from the capital allocation. Notably, although stocks only represent 20% of the investment, they contribute 46.1% to the overall risk. Similarly, a 10% investment in real estate generates 52.3% of the risk. These findings highlight the value of ES as a risk budgeting measure in identifying risk-generating assets and hedging assets.

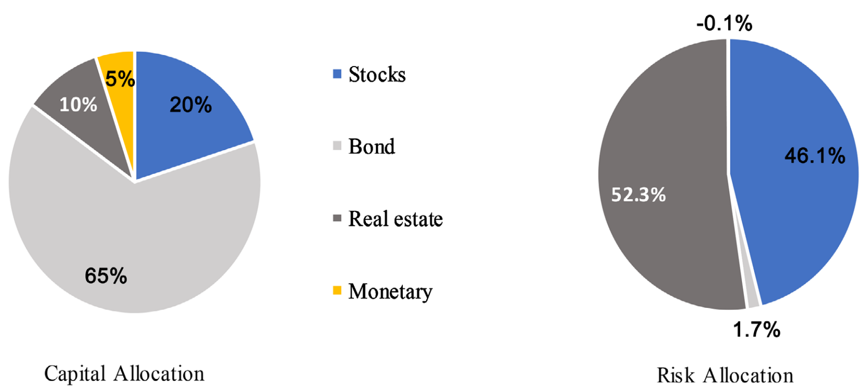

Figure 2. Capital allocation and risk allocation for the portfolio (S). Source: The authors.

To use ES as a measure of risk is to assess the impact of adjusting the weight of one asset class relative to others on the overall risk. For instance, if we increase or decrease our allocation to stocks by 1%, what will be the effect on the risk? Our findings, shown in the last two columns of **Table 7**, indicate that adding a position in stocks would increase the annual return by 0.10%, but worsen the potential annual loss by 0.13%. Conversely, consolidating a position in the money market reduces risk. In summary, any tactical investment decision affects potential losses positively or negatively.

By computing ES and VaR, we can address the first question about an appropriate risk measure and its decomposition into marginal units for each asset class. Hence, we have allocated risk differently from the capital structure, providing guidance to managers in their investment choices. They can gauge the impact of a 1% change in exposure to each asset class on return and risk.

4.2. Implementation of Risk Budgeting for Tactical Adjustment

We then turn to the second question in our study, which focuses on the use of ES to tactically manage the SAA. First, we set the maximum accepted tactical budget (RB_{TAA} max) to 0.77 percent = (5.27% − 4.50%), 5.27 percent is the average return generated in the study period and 4.50 percent is the funding rate of the retirement reserve fund.

Table 7 displays the results of our analysis, indicating that the strategic portfolio (S) generates an annual return of 5.27 percent with a potential annual loss of 5.79 percent. The manager aims to enhance the portfolio's performance by exploiting the marginal contributions of each asset class. We simplify the impact of the manager's tactical decisions by assuming that they made the following deviations: +3 percent, −4 percent, and +1 percent on stocks, bonds, and real estate, respectively. As a result, the target portfolio (P) is expected to generate a return of 5.45 percent and an ES of −6.49 percent.

Table 7. Portfolio Indicators (R), (S) and (P) (2018-2021).

	Portfolio (R)	Portfolio (S)		Portfolio (P)			
						Δ	Δ
	Capital allocation	Capital allocation	∂ES(x)/∂xi	Tactical deviations	Capital allocation	Expected $ES_{(1year)}$	Expected return
Stocks	30%	20%	−13.4%	3%	23%	−0.40%	0.30%
Bonds	50%	65%	−0.1%	−4%	61%	0.004%	−0.17%
Real estate	15%	10%	−30.3%	1%	11%	−0.30%	0.04%
Monetary	5%	5%	0.1%	0%	5%	0.0%	0.00%
Total	100%	100%	-	-	100%	−0.70%	0.18%
Annual performance	5.88%	5.27%			5.45%		
Performance gap	0.61%	-			0.18%		
$ES_{(1year)}$	−8.65%	−5.79%			−6.49%		
ES_{SAA}				5.79%			
$RB_{TAA\ max}$				0.77%			
RB_{TAA} used	2.86%	0.00%			0.70%		
RB_{TAA} residual	-	0.77%			0.07%		

Source: The authors. Performance gap = portfolio (P) performance − portfolio (S) performance. $RB_{TAA}\text{ used} = -ES_{R\text{ or }P} + ES_{SAA}$, $RB_{TAA}\text{ residual} = RB_{TAA\ max} - RB_{TAA}\text{ used}$.

Out of a global budget of 6.49 percent, the portfolio manager (P) used a part to align with the benchmark (ES_{SAA} = 5.79 percent) and a second part to implement his tactical choices. The decisions made on the basis of the contribution to the return and the risk give identical results to those expected. We obtained an expected return of 5.45 percent (5.27 percent from passive management +0.18 percent due to tactical choices) and a probable annual loss of −6.49 percent (−5.79 percent due to the replication of the strategic benchmark + (−0.70 percent) due to tactical risk). The convictions of active management can, therefore, be translated in terms of performance gain or loss and risk budget. We conclude that managers can take advantage of the risk budgeting approach to improve the efficiency of SAA.

The portfolio (P) was implemented with 98.9 percent of the allowed risk budget (6.49 percent of 6.56 percent): 5.49 percent to replicate the benchmark and 0.70 percent to implement tactical decisions. The manager still has a budget of 0.07 percent that he can use to improve his performance.

To understand how the investment oversight committee can control the tactical allocation through risk budgeting, we simulate the realizations of the three strategies identified in **Table 5**. The committee sets an annual return target of 4.5 percent with a potential loss of −5.79 percent. In addition to this risk, the committee allows a maximum tactical risk budget of 0.77 percent. Otherwise, it expects the managers to achieve a return of 5.27 percent without exceeding the allocated risk budget.

Table 8 summarizes the three proposed strategies at the beginning of 2022 and their quarterly evolution under price movements. We assume that no rebalancing is carried out, in order to be able to identify any overshooting of the risk budget.

The Moroccan stock market ended the year 2022 on a negative note of −17 percent. This had a negative impact to varying degrees on the performance of the three portfolios. The bond market also posted a negative performance of −2 percent, which is explained by the rise in interest rates across all maturities of the curve. After recovering in the first quarter, the real estate market fell into the red, ending the year with a −3 percent underperformance. In 2022, only cash investments were profitable with a limited performance of 1.6 percent.

Table 9 tracks the quarterly performance of the three portfolios in terms of return and risk budget consumption.

The sensitivity of the ES to negative stock market fluctuations can be observed from the first quarter, when the stocks class recorded a negative performance of −4.8 percent. This result is captured by the ES, going from −5.79 percent estimated at the end of 2021 to −6.85 percent observed at the end of 2022. The third quarter recorded an underperformance of less than −4 percent for all portfolios due to the impact of rising interest rates on bond returns.

The comparison between ES and monthly returns shows that the maximum loss was not exceeded which confirms that the ES did anticipate the maximum potential loss. The realized return of the strategic portfolio is negative and the

Table 8. Quarterly evolution of strategies and asset class performances due to market fluctuations (2022).

		Stocks	Bonds	Real estate	Monetary
Quarterly market performance	Q1	−4.8%	0.1%	11.0%	0.3%
	Q2	−4.7%	0.0%	−1.4%	0.4%
	Q3	−0.9%	−2.2%	−7.3%	0.4%
	Q4	−7.6%	0.1%	−4.3%	0.5%
Portfolio profile (P)	End of 2021	23.0%	61.0%	11.%	5.0%
	Q1	21.9%	60.9%	12.2%	5.0%
	Q2	21.1%	61.7%	12.2%	5.1%
	Q3	21.4%	61.8%	11.5%	5.2%
	Q4	20.2%	63.2%	11.3%	5.4%
Portfolio profile (P’)	End of 2021	25.0%	59.0%	11.0%	5.0%
	Q1	23.8%	59.0%	12.2%	5.0%
	Q2	23.0%	59.8%	12.2%	5.1%
	Q3	23.3%	59.9%	11.6%	5.2%
	Q4	22.0%	61.3%	11.3%	5.4%
Portfolio profile (P”)	End of 2021	18.0%	66.%	9.%	7.0%
	Q1	17.1%	65.9%	10.0%	7.0%
	Q2	16.4%	66.5%	9.9%	7.1%
	Q3	16.7%	66.6%	9.4%	7.3%
	Q4	15.6%	67.8%	9.1%	7.4%

Source: The authors.

Table 9. Quarterly balances of achievements for the three portfolios in 2022.

Quarters 2022	Indicator	Strategic (S)	Risk profile		
			Moderate (P)	Aggressive (P’)	Conservative (P”)
Q1	Performance	1.14%	1.04%	0.67%	1.12%
	Performance gap	-	−0.10%	−0.47%	−0.02%
	$ES_{(1year)}$	−6.01%	−6.83%	−7.14%	−5.38%
	RB_{TAA} used		0.82%	1.13%	0.00%
	RB_{TAA} residual		-	-	1.40%
Q2	Performance	−1.60%	−1.98%	−2.36%	−1.35%
	Performance gap	-	−0.38%	−0.76%	0.25%
	$ES_{(1year)}$	−6.03%	−6.89%	−7.34%	−5.51%
	RB_{TAA} used		0.86%	1.31%	0.00%
	RB_{TAA} residual		-	-	1.29%

Continued

Q3	Performance	−4.07%	−4.33%	−4.55%	−3.71%
	Performance gap	-	−0.26%	−0.47%	0.36%
	$ES_{(1year)}$	−6.62%	−7.24%	−7.77%	−5.93%
	RB_{TAA} used		0.62%	1.15%	0.00%
	RB_{TAA} residual		0.15%	-	1.46%
Q4	Performance	−4.80%	−5.26%	−5.57%	−4.33%
	Performance gap	-	−0.46%	−0.76%	0.47%
	$ES_{(1year)}$	−6.85%	−7.36%	−7.80%	−6.03%
	RB_{TAA} used		0.50%	0.95%	0.00%
	RB_{TAA} residual		0.27%	-	1.59%

Source: The authors. Performance gap = portfolio performance (P), (P') or (P'')-portfolio performance (S). $RB_{TAA}\ \text{used} = -ES_{P,P'\text{ or }P''} + ES_{SAA}$, $RB_{TAA}\ \text{residual} = RB_{TAA\max} - RB_{TAA}\ \text{used}$.

potential loss has increased significantly due to the poor performance of all asset classes. This is reflected in the weightings of the risky asset classes adjusted downward in favor of the less risky assets. In such a situation, the defensive strategy, which focuses on strengthening monetary assets, is proving to be a winner.

At the end of the first quarter, the investment oversight committee notes the achievements of each risk profile. The strategy (P") is the most efficient with the highest return and lowest risk compared to the strategic portfolio. The managers of portfolios (P) and (P') have misjudged the behavior of the stock market. As a result, their portfolios exceeded the allocated risk budget. Portfolio (P) saw its stocks exposure decrease by 2.8 percent due to market effects, allowing it to approach portfolio (S). On the other hand, the (P') portfolio, which is heavily exposed to stocks, has exploded the risk budget. The committee intervenes to frame the tactical choices and recommends reducing the exposure to stocks.

The portfolio manager (P") dominates by underweighting risky asset classes. His strategy limited the underperformance to −4.33 percent, i.e. 47 basis points more than the strategic portfolio (S), while controlling the risk budget.

5. Conclusion

Risk budgeting is an approach that allows for tactical allocation decisions and provides a global model that explicitly considers the interaction of different sources of risk. Combined with strategic asset allocation, it offers a management framework that enables pension fund managers to allocate their risk budget appropriately, based on the characteristics of each risk factor. This approach also enables them to fulfill their mission of paying pensions while capturing performance opportunities beyond their strategic return objectives.

Our results are not limited to providing a theoretical framework and approach for Moroccan pension funds, but they can also be applied to a wide range of financial organizations. For example, pension funds, insurance companies, particularly in the life insurance sector, and central banks can use our methodology to manage their currency portfolios by integrating predefined risk into their strategic investment choices. In addition, our approach can be used to manage risk across multiple risk factors such as asset classes, management styles, and asset managers. This allows fund managers to allocate their risk budget appropriately, based on the specific objectives and constraints of each risk factor.

Risk budgeting relies heavily on assumptions that are subject to considerable estimation errors. Three of the four asset classes considered in our study benefit from a lower level of estimation error, due to the existence of substantial historical data and representative indices. In the absence of reliable and comprehensive data, the real estate class has a higher level of error that affects the quality of risk estimates.

VaR is a market risk indicator that, despite its popularity, has limitations that should be kept in mind. These limitations lie in the models used and in the treatment of extreme losses, which are based on assumptions that do not always coincide with reality. However, VaR has the advantage of being easier to calculate and interpret. The use of ES and partial ES for the short-term steering of strategic asset allocation has allowed us to overcome the limitations of VaR as a consistent risk measure.

In this respect, it is crucial to note that, under the assumption of normality, the ES is an estimate of market risk that should be complemented by stress testing scenarios, aimed at estimating the losses that could be incurred in extreme or catastrophic scenarios. Similarly, the approach proposed in this study focused only on the risks generated by fluctuations in the prices of the financial instruments studied. It is important to consider other types of risk, mainly liquidity risk and credit risk.

Finally, it should be noted that risk budgeting should be considered over a longer period of time, considering the economic cycles of the markets. Indeed, market conditions may be more or less favorable to risk-taking and to the identification of attractive investment opportunities, justifying a variable use of risk budgets over time.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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