Journal of Financial Risk Management, 2022, 11, 740-756
https://www.scirp.org/journal/jfrm
ISSN Online: 2167-9541
ISSN Print: 2167-9533

Recent Advances of Dynamic Model Averaging Theory and Its Application in Econometrics

Jiahui Li[1], Yuanying Jiang[1,2*]

[1]College of Science, Guilin University of Technology, Guilin, China
[2]Guangxi Colleges and Universities Key Laboratory of Applied Statistics, Guilin, China
Email: 1336243876@qq.com

How to cite this paper: Li, J. H., & Jiang, Y. Y. (2022). Recent Advances of Dynamic Model Averaging Theory and Its Application in Econometrics. *Journal of Financial Risk Management, 11,* 740-756.
https://doi.org/10.4236/jfrm.2022.114036

Received: December 2, 2022
Accepted: December 26, 2022
Published: December 29, 2022

Abstract

Dynamic Model Averaging (DMA) was first proposed by Raftery et al. (2010) when predicting the output strip thickness of cold rolling mills and is a recursive implementation of standard Bayesian model averaging, also known as recursive model averaging. Since Gary Koop and Dimitris Korobilis introduced DMA into the field of econometrics in 2012, dynamic model averaging has become a widely used estimation technique in macroeconomic applications because of its ability to adapt to the temporal change of parameters and the advantages of the specification of optimal prediction models, the method has good application prospects. This paper focuses on dynamic model averaging as a solution to model uncertainty problems, focusing on recent theoretical developments and their applications in econometrics. Discussions focused on uncertainties contained in covariates in regression models, such as normal linear regression and its extensions, and on advances in designing models to handle more challenging situations, such as time-dependent, spatially dependent, or endogenous data. The results show that the DMA method has good prediction accuracy, is a powerful tool for actual prediction, and provides important technical support for risk avoidance in management.

Keywords

Model Uncertainty, Parameter Instability, Model Averaging, Forecast Combination, Weight Choice

1. Introduction

Often, multiple models provide a proper description of the distribution of generated observational data. In such cases, it is very important to select a better model based on criteria such as matching the observed data set, predictive power, or likelihood penalties. After that, the inference is done and the conclusion is drawn, assuming that this model is a real model.

Of course, there are drawbacks to this approach. Choosing a particular model can cause overconfident riskier decisions because it ignores existing model uncertainties in favor of choosing very specific distributions. Hence, it is desirable to model this uncertainty by appropriately selecting or combining multiple models, and model averaging is a common and effective method. Model averaging, as its name implies, is to average estimates or predictions from different models by a certain weight. Some scholars named it model combination, which requires combination estimation and combination prediction. Model averaging is an international frontier problem in the field of statistics, it has a wide application prospect in the fields of economy, finance, biology, medicine, and so on.

Model averaging can avoid various defects that may exist in selecting a single model:

1) Model averaging uses a continuous weight to combine the estimation from different models, and model selection is a special case of model averaging whose weight is 0 or 1. Therefore, the conclusion obtained by the model averaging method is more robust and the selection process is more stable than the conclusion depending on the selected specific model. The selected model will not change the estimation greatly due to small fluctuations in observed data.

2) The model averaging method combines multiple models and does not eliminate any model easily, thus reducing the loss of unique information reflected by other models or variables by basing inferences on a single model. Therefore, model averaging may provide users with better estimates than traditional approaches that try to agree on an optimal model.

3) Model averaging provides an adequate safeguard against the possibility of choosing a poor model.

4) In fact, the model average results are robust to model selection. Generally, a selected model is not considered as the real data generation process, and several competing models are allowed to be added to the estimation process, so the uncertainty is not ignored.

5) More recently, the model averaging method is aimed at reducing the estimated or predicted risk directly.

In conclusion, model averaging can be seen as an extension of model selection, and with the improvement of computer technology, model averaging will be adopted more and more as a complex data mining method.

There are two main problems with forecasting time series, especially with financial and economic data. Problem 1: Model uncertainty in the selection of predictors. The forecasting procedure must minimize the choice of different models. Should each predictor be included? Problem 2: The parsimonious parameters in modeling the prediction of time-varying state vectors and achieving predictive performance improvements in terms of Mean Square Error (MSE) and Mean Absolute Deviation (MAD). The main goal of these questions is that, get better predictive performance, both theoretically and practically. Taking the forecast of China's annual Gross Domestic Product (GDP) as an example, many scientific research institutions have given their own forecasts, but due to the differences in the methods used and the information based on which they are based, they may overestimate or underestimate the real GDP value. So, choosing the appropriate weight to combine these forecasts can generally improve the accuracy of the forecast. Therefore, selecting appropriate weights to combine these forecasts can generally improve prediction accuracy. DMA is introduced in the research process of this paper. It can be seen from the analysis that this model is formed by improving the Bayesian model average and Kalman recursive methods. It shows good applicability and can deal with the above-mentioned related problems.

This paper proceeds as follows: we shall first briefly review the dynamic model averaging method and its research progress. The third part describes the extended DMA model and its application. The fourth part introduces the application of the DMA method in the construction of the Chinese financial condition index in detail. The last section summarizes the results and contains some conclusions.

2. DMA Methods and Research Progress

2.1. DMA Methods

Traditional model averaging methods can be used to provide simple estimates, but the disadvantages of not allowing the prediction variables and their coefficients used in the model to change over time are obvious. In addition, Koop and Korobilis (2012) showed that if the quantity of predictor variables in the model is large, it will obtain poor prediction results. Raftery et al. (2010) constructed a new method called Dynamic Model Averaging (DMA). The application of this method involves predicting the output strips that have just left a cold mill. Therefore, their utilization is basically a linear regression model, (1) the model of regression coefficient is affected by the time change, (2) a few variables in the model, and (3) the relevant model may vary with time.

To get a handle on (1) - (3), Raftery et al. (2010) propose a setup in which a (linear) state-space model of time-varying regression coefficients in the model set is combined. It lets the model of the control system to vary with time. Moreover, with the help of the forgetting factor, these two components are stated using reasonable approximation, which results in a highly parsimonious representation. Therefore, a DMA-based variable forecast is a weighted average of the predictions generated under each model, weighted by its model probability.

This method allows the prediction variables and their coefficients to change over time, just enough to solve the problem of fixed coefficients in the traditional method. So this method is often used in empirical macroeconomic studies. The dynamic model average model is as follows:

$$y_t = x_{t-1}^{(k)}\beta_t^{(k)} + \varepsilon_t^{(k)} \tag{1}$$

$$\beta_{t+1}^{(k)} = \beta_t^{(k)} + \eta_t^{(k)} \tag{2}$$

Including y_t is predicted variables, $x_{t-1}^{(k)}$ is predicted variable, and $\beta_t^{(k)}$ is predicted coefficient, $\varepsilon_t^{(k)} \sim N\left(0, H_t^{(k)}\right)$, $\eta_t^{(k)} \sim N\left(0, Q_t^{(k)}\right)$, is independent and identically distributed. The superscript $k = 1, \cdots, K$ represents the set of prediction models. If there are *m* predictor variables, it can be $K = 2^m$ various prediction models. Therefore, the predictor variable matrix $x_{t-1}^{(k)}$ and the coefficient matrix $\beta_t^{(k)}$ makes the predictor variable changes over time.

In *t* time the *k* model, let $L_t = k$, its evolution can be an excessive probability $k \times k$ matrix *P*, its elements $P_{i,j} = \Pr\left(L_t = i \mid L_{t-1} = j\right)$, type in the $i, j = 1, \cdots, K$. Koop and Korobilis (2012) show that unless *m* is very small, *P* would be a huge matrix, which will cause long computation time. To achieve feasible computations, the approximation method constructed by Raftery et al. (2010), involving factors α and λ.

Firstly, using coefficient prediction equation of Kalman filter, $\beta_t^{(k)}$ can be obtained by all the information before time *t*, the relevant equation is as follows:

$$\beta_t^{(k)} \mid y^{t-1} \sim N\left(\hat{\beta}_{t-1}^{(k)}, R_t^{(k)}\right) \tag{3}$$

$$R_t^{(k)} = \frac{1}{\lambda} \Sigma_{t-1|t-1}^{(k)} \tag{4}$$

The $\hat{\beta}_{t-1}^{(k)}$ and $\Sigma_{t-1|t-1}^{(k)}$ depends on the $H_t^{(k)}$ and $Q_t^{(k)}$, $y^{t-1} = \left\{y_1, \cdots, y_{t-1}\right\}$. According to Koop and Korobilis (2012), the factor λ ranges from 0.90 ≤ λ ≤ 0.99. Different values are taken to estimate different prediction models and parameters.

The parameter estimation equation is as follows:

$$\hat{\beta}_{t|t}^{(k)} = \hat{\beta}_{t-1|t-1}^{(k)} + R_t^{(k)} x_{t-1}^{(k)} \left(H_t^{(k)} x_{t-1}^{(k)} R_t^{(k)} x_{t-1}^{(k)}\right)^{-1} \left(y_t - x_{t-1}^{(k)} \hat{\beta}_{t-1}^{(k)}\right) \tag{5}$$

$$\Sigma_{t|t}^{(k)} = R_t^{(k)} - R_t^{(k)} x_{t-1}^{(k)} \left(H_t^{(k)} x_{t-1}^{(k)} R_t^{(k)} x_{t-1}^{(k)}\right)^{-1} x_{t-1}^{(k)} R_t^{(k)} \tag{6}$$

To achieve a feasible calculation, Equation (6) adds a factor λ, which enables the model to estimate $Q_t^{(k)}$, and in addition, weighted moving average is used to estimate $H_t^{(k)}$.

Secondly, the Kalman filter is used for model prediction equation to estimate variable probability. The specific equation is as follows:

$$\pi_{t|t-1,k} = \frac{\pi_{t-1|t-1,k}^{\alpha}}{\sum_1^k \pi_{t-1|t-1,1}^{\alpha}} \tag{7}$$

where α is a forgetting factor similar to λ. According to Koop and Korobilis (2012), the value is 0.90 ≤ α ≤ 0.99.

Combined with the prediction density, the updated Equation (8) is as follows:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,l} P_k\left(y_t \mid y^{t-1}\right)}{\sum_{l=1}^k \pi_{t|t-1,l} P_l\left(y_t \mid y^{t-1}\right)} \tag{8}$$

Type of $P_l\left(y_t \mid y^{t-1}\right)$ representation model *l* density estimate, y_t by $N\left(x_{t-1}^{(l)}\right)$. Equations (1)-(8) are all formulas for parameter estimation, model prediction equation and its update using Kalman filter.

Finally, we through the $\pi_{t|t-1} = \Pr\left(L_t = k \mid y^{t-1}\right)$, the probability that the estimation model of *k*. Dynamic model averaging based on the probability $\pi_{t|t-1,k}$, the weighted average of all possible model, each time point prediction results are obtained. The specific calculation equation is:

$$\hat{y}_t^{DMA} = \sum_{k=1}^{K} \pi_{t|t-1,k} x_{t-1}^{(k)} \beta_{t-1}^{(k)} \tag{9}$$

Dynamic model selection, choose each point in time with the highest probability model of $\pi_{t|t-1,k}^{*}$. Specific as follows:

$$\hat{y}_t^{DMs} = \sum_{k=1}^{K} \pi_{t|t-1,k}^{*} x_{t-1}^{(k)} \beta_{t-1}^{(k)} \tag{10}$$

The DMA method allows variables and their coefficients, thereby dynamically predicting outcomes. In addition to direct prediction, the DMA method can also estimate the prediction efficiency of a given predictor variable on the predicted variable in a certain period. In other words, it is possible to calculate the importance of factors at a given moment. Based on these advantages, the dynamic model averaging method performs well in all kinds of forecasting.

2.2. DMA Research Progress

Over the course of the past 10 years, DMA has emerged in macroeconomics research field by Koop. First, edition focused on using this method to forecast quarterly inflation rates in the United States using the generalized Phillips curve (i.e. forecast regression, in which current period inflation rate depends on macroeconomic forecasts), see Atkeson and Ohanian (2001). Inflation is evaluated by the GDP price deflator and CPI. The selection of macroeconomic forecasting indicators is driven by economic theory and previous research.

In essence, DMA has a good effect in solving the problems related to model uncertainty and parameter instability, so this idea has attracted widespread attention. In the research process of relevant economic issues, many theoretical studies on the above generalized Phillips curve have found that there is a close relationship between variables related to unemployment rate, economic activity level and house price. In this case, practitioners are faced with the situation that the model contains a large number of regression factors. Although it can be judged that some of the variables may be related based on prior information, the specific variables are not clear. Therefore, in the process of modeling and analysis, it is necessary to establish a model which contains all possibility elements. In this mode, the model may contain irrelevant variables, which will adversely affect the prediction accuracy and reduce the real-time performance of the model. The Bayesian response averaging method can effectively deal with the above problems. Based on the analysis of relevant economic theories, the Bayesian average value of all possible regression combinations meets the application requirements. The weight of each combination in the combination set can be determined based on certain maximum likelihood estimation, and this parameter is closely related to Bayesian quantity. This process is known as Bayesian Model Averaging (BMA).

According to the relevant data, with BMA method, any interest corresponds to the weighted average of the precise quantity of the model. But this model also has some defects, specifically, it can only be used statically. The main reasons why the model parameters change with time are as follows. In the process of changing over a certain period, the corresponding regulatory conditions, market sentiment and policy related factors will change. The models considered in the research process can be well applied to different data patterns, so that problems can be better solved through them. There are many studies related to the instability of parameters in time series econometrics. In this case, the problem is more complicated. For example, see the research data of cogley, Bauwens et al. (2015) and Pettenuzzo and Timmermann (2017). DMA model has obvious advantages over Bayesian model, which is based on dynamic combination of model and parameter instability. In the process of processing, linear state space and hidden Markov model methods are applied to dynamically update the probability and regression coefficient of each model in the set, to control the time variability of the identity of the best model. In addition, it does not require simulation, which means that the computation time will be significantly reduced.

The DMA applications introduced by Koop and Korobilis (2012) range from GDP growth rates, across countries to forecasting daily spot prices, stock return volatility, and commodity price in the European carbon market. Since then, many scholars have used the dynamic model averaging method to study. Examples include Dangl and Halling (2012) for predicting total equity returns; predicting the spot price of carbon permits; Buncic and Moretto (2014), Foreign Exchange Reserve forecasting; Wei and Cao (2017) predicted gold price returns; Koop and Korobilis (2015) on forecasting non-US inflation rate; Byrne et al. (2017), on predicting stock returns.

The topics in the above research are quite different, the conclusions are very similar, namely the advantages of DMA prompts improvements in relative forecasting. On various occasions, such proceeds make economic sense as well, for example Beckmann et al. (2020). More interestingly, following Koop and Korobilis (2012), later research has found that DMA method can be applied to extend existing models and treat computational bottlenecks. Koop and Onorante (2019) proposed a new DMA that could use Google search data to control switching between different models. Koop and Korobilis (2014) used DMA to predict Vector Autoregressive (VAR) models. The estimation methods in both above studies are easy to understand.

With these broad developments in mind, this article will provide a clear overview of: 1) the broad rationale behind DMA. Our goal here is to explain DMA, and 2) its various extensions DMA-based models are very helpful to understand especially among a range of possible alternatives why these extensions were proposed and why DMA technology was chosen.

3. Extended DMA Methods and Their Applications

The next nodes (four in all) contain topics, where either a new DMA-based model or an important extension of the framework suggested in Koop and Korobilis (2012) has been introduced.

3.1. DMA with Large Model Spaces

As mentioned above, model probability plays a significant role in DMA and DMS. However, these probabilities must be calculated for each model at each point in time. This in turn means that, in broad terms, each new additional explanatory variable added to the DMA increases the computation time triples. Thus, in the research process, if more than ten potential explanatory variables need to be processed and quarterly data are used, the processing capacity of DMA is not large, and ordinary desktop computers can meet the requirements. However, when the number of variables increases significantly, the computing capacity also increases sharply, and the corresponding real-time performance is significantly reduced.

As the research of Catania and Nonejad (2018) shows, the number of corresponding model combinations will reach a high level under certain circumstances, so ordinary personal computers cannot meet the processing performance requirements. High-performance computers or parallel computing methods cannot solve this problem. In addition, in the actual problem-solving process, not everyone can use high-performance servers for data calculation. For this reason, some scholars have proposed parallelization solutions. However, according to the practical application experience, the parallelization method cannot meet the application requirements when the quantity of variables is large.

The research of Onorante and Raftery (2016) shows that for typical problems in macroeconomic analysis, these assumptions are in line with the actual situation. To effectively deal with these troubles, the author proposes a new algorithm, which includes the operational steps related to prediction, expansion, evaluation, and reduction, and carries out continuous iteration: in the process of processing, this method also reduces the model set based on Occam window, which can well meet the requirements of practical applications. The algorithm flow is in the following way:

1) Divide the sample into in-sample part and out-of-sample part.

2) Start with an initial population of models and weights.

After that, for each observation during this period, do the following:

1) Prediction: Use the models and weights of the previous period. Model averaging was performed according to Raftery et al. (2010) and the required values were obtained.

2) Extension: The current set of models is extended to include all its neighbors (all models obtained by adding regressors).

3) Evaluation: Observe the y_t calculations of these models.

4) Reduction: The final population of a model is defined as the model in the current model set.

The DOW method is applied to the immediate forecast of GDP in the euro area. Here, y_t represents GDP growth rate, and $Z_{(t-1)}$ contains multiple predictors. But in contrast to Koop and Korobilis (2012) who rely on only 14 predictors. The out-of-sample period was from 2003 to 2014.

The results are robust. In general, the DOW method has good real-time forecasting property even under condition of turning points. In terms of point prediction accuracy, the comparative analysis shows that DMA using Dow Jones index is better than DMS and BMA related models. According to the practical application results, it is found that this model can also well describe the great recession in the euro area. When carried out the research, the purpose was not to determine the model with the greatest prediction ability, but to propose a calculation method with high feasibility to solve such problems. When the variables of the model are too large to be evaluated exhaustively, the DMA method can deal with the problem well. The author's research results show that the lag possibility of GDP growth rate is high. In the early indicators, industrial production variables are more likely to be included, showing up as significant predictors except for the period immediately following the Great Recession.

The method proposed by Onorante and Raftery (2016) has been used in many studies. Such as, Risse and Kern (2016) used the Dow Jones Index estimator to predict two widely studied time series in finance, namely gold price returns and the S&P 500 index. It can be seen from the analysis that the influencing factors of these two-time series mainly include financial market, monetary policy and macroeconomic conditions, which will have obvious changes. In the research process, the authors relied on 35 explanatory variables to predict the above series. It is necessary for investors, financial institutions, and government departments to closely monitor the overall situation of financial markets and warn of macroeconomic. In addition to simple specifications that consider all predictors, such as historical average benchmarks and OLS regressions, they also using LASSO. In term of statistical point of view, the authors find that the proposed method rarely leads to an improvement in out-of-sample prediction accuracy in terms of MSE relative to other models. But a portfolio investment strategy is used like that of Pesaran and Timmerman (2000) to carry out certain prediction analysis, the results show that when the prediction analysis is carried out according to the model established by Onorante and Raftery (2016), the wealth value of investors will obviously increase, and at the same time, it will also lead to the return of the buy and hold strategy.

3.2. DMA with Google Search Data

In the DMA method, the probability of a model is related to the predicted likelihood and the probability of the model in the previous period. In a study related to data statistics, Koop and Onorante (2019) combined this statistical index with the data of trends to better meet the statistical requirements. In this mode, the model switching process at each time point is significantly simplified, for example, it can be realized based on the Internet search query method, which shows high application performance advantages. The value range of Google trend data is 0 and 100. For example, in the search process, insert the term "oil price" into the search field, so that the time series within this value range can be observed at a certain sampling frequency. This method can be used to judge the degree of conformity of Google search for the word "oil price". Through comparative analysis, it can be seen that Internet search data has high convenience, which can easily determine macroeconomic and financial variables, and researchers can also expand the information set through it according to application requirements. Koop and Onorante (2019) proposed the hypothesis that Google search data can support researchers to obtain relevant and useful information and can judge which variables have higher weights at various time points. This also supports and helps Internet-based search variables, rather than being simply regarded as predictors.

Google search data was found that may not meet the prediction related requirements of model regression in practical applications, but it can reflect relevant turning points and some changes, which is important for establishing specific models and converting between models. Its research found that these data can be used to collect "collective intelligence" and reflect the changes of relevant macroeconomic variables at different time points. The fact that the number of oil price searches is increasing does not in itself indicate whether oil prices are going up or down. Similarly, if used as a regression in a model, it may not enhance the model predictive power. However, variables can be considered in obvious ways.

The univariate predictive regression of Koop and Onorante (2019) is as follows:

$$y_t = Z_{t-1}\beta_t + \tilde{Z}_t\gamma_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, V_t) \tag{11}$$

$$\theta_t = \theta_{t-1} + \omega_t, \quad \omega_t \sim N(0, W_t) \tag{12}$$

In which y_t represent major US macroeconomic variables, including inflation, industrial production index. For every variable in Z_{t-1}, create the relevant Google search variable, $\tilde{Z}_t$. The method of constructing $\tilde{Z}_t$ proposed by Koop is as follows: first the names of the variables of interest are searched and Google trend data are collected. In addition to these variables, the Google interface provides a cluster. These are the most popular searchers related in that field. The submitter takes these relevant searches and repeats the process for each search. To produce "Google variables" that correlate with the variables of interest, the authors then averaged the Google trend data over each time period and explained the time series convention in this way: Starting at $t + 1$, we suppose that y_t is unobserved and adhere to making an immediate prediction of it. Obviously, the publication of macroeconomic variables has a time lag, so only Z_{t-h} represents $h \geq 1$ available.

In this research process, there are still problems of model and parameter instability, and the corresponding treatment methods are basically the same Significantly, DMA can be performed on (11) and (12) in a similar manner as in Chapter 2. The most interesting contribution of this method is not only to use $\tilde{Z}_t$ directly as a regressor in (11), but also to provide information to determine the weights of macroeconomic variables at each time point in the model averaging process. Set $\tilde{Z}_t = \left(\tilde{Z}_t^{(1)}, \cdots, \tilde{Z}_t^{(p)}\right)$ is the vector of the variables of Google. Overall, the Google variable is between 0 and 100, so if you resize it, you can interpret it as a probability. In particular, we can define Pr^{Google} as the "Google probability":

$$\pi_{t,m}^{Google} = \prod_{s \in I^m} \tilde{Z}_t^{(s)} \prod_{s \in I^{\sim m}} \left(1 - \tilde{Z}_t^{(s)}\right) \tag{13}$$

where I^m represents the relevant variables in model *m*, and $I^{\sim m}$ represents that not in model *m*. We can see that $\sum_{n=1}^{k} \pi_{t,n}^{Google} = 1$ thus justifying the term "Google probability". The versions of DMA and DMS involve implementing the algorithm of Raftery et al. (2010), reflecting the Google model probabilities as follows:

$$\pi_{t|t-1,m} = \varpi \frac{\pi_{t-1|t-1,m}^{\alpha}}{\sum_{n=1}^{k} \pi_{t-1|t-1,n}^{\alpha}} + \left(1 - \varpi\right) \pi_{t,m}^{Google} \tag{14}$$

In which $0 \le \varpi \le 1$. If $\varpi = 1$, will back to traditional DMA. If $\varpi = 0$, Google model probability fully drives the model transformation process over time.

The empirical application considers immediate forecasts of nine major monthly macroeconomic variables in the United States, namely, inflation, unemployment, inflation, industrial production index and money supply. In the process of prediction and analysis in this regard, the applicable macroeconomic data range is from January 1973 to July 2012, and the available Google trend data start from January 2004. Therefore, two methods are used for estimation and analysis in the research process. First, according to the data analysis requirements, the data of 2004 for each variable was deleted and estimated through a short sample. Secondly, the data before 1973 is selected as the sample, and after certain processing, it is regarded as a macroeconomic variable, but the pre-2004 model does not use Google data because these are not available. For example, using $\varpi = 0.5$, the authors described in this way: for the data before 2004, according to the method of Raftery et al. (2010) for regular DMA, to define $\pi_{t|t,m}$. However, as of January 2004, $\pi_{t|t,m}$ use (14) definition. For the first, the assessment period for the near-term forecast starts in September 2005 and runs until July 2012. The near-term forecast evaluation period runs from January 2004 to July 2012 used MSE to evaluate the sum of point prediction accuracy and log-prediction likelihood to assess prediction accuracy between various specifications. In addition to various DMA, where ϖ = in 0, 0.5, and 1, in the study, the author chose a simple competition model for analysis, and used recursive OLS prediction and AR (2) model for related prediction research, and compared the results obtained. The latest available observation values of dependent variables are applied in the prediction of relevant change trends, which can improve the accuracy of the results and better suitable for practical applications. According to the predicted results, the advantages of DMA and DMS methods are obvious. Compared with other methods, the accuracy and reliability are significantly improved, and the applicability is stronger. The important contribution of the article shows that Google search variables showed high application value, and the effect was obvious when analyzing specific decomposition variables but can also be used in combination with DMA to improve instant forecasts of macroeconomic aggregates.

3.3. Adaptive Dynamic Model Averaging (ADMA)

DMA is essential for forecasting a series of macroeconomic events because it can take into account the temporal variability of the parameters and the specification of optimal forecasting models. To accommodate variations in the distribution of data generation, DMA includes two parameters named forgetting factors. The first one is part of the DLM formula while the second forgetting factor is related to the averaging phase of the model. These parameters accept a constant balance between estimation in a static environment and re-initiation of the estimation process, discarding all prior information which is suitable after a structural break. As a result, choosing the forgetting factor is crucial for the efficiency of DMA prediction.

Yusupova et al. (2019) developed an Adaptive Dynamic Model Averaging (ADMA) approach consisting of two components. One involves using random optimization to identify forgetting factors that minimize the one-step ahead squared prediction error for each DLM. This results in a completely web-based and data-driven algorithmic program, which called Adaptive Forgetting DLM (AF-DLM). As shown in experimental evidence, AF-DLM is effective for various changing types at data generation, covers the rate or type of change over time. AF-DLM is mathematically simpler than former methods as well because it does not include a lattice of forgotten factor values. Another handles model averaging. It is shown that the speed with which the DMA weights respond to recent observations depends on the choice of the appropriate forgetting factor as well as the mechanism that prevents underflow (the weight equals zero). Accordingly those ways to manage the scalability of the DMA by only adjusting the second forgetting factor is inherently limited. It is suggested that the ConfHedge model combination algorithm (also known as expert advice prediction) in the field of machine learning should be used to replace the current model average method and control the one-step squared prediction error on the finite time step within the known range of the one-step squared prediction error of the optimal sequence of the prediction model.

Expert advice prediction explores the following online learning problems: in the time step $t + 1$, K forecast model (called experts) of each provide a prediction, $\hat{y}_{t+1}^{(k)}$. Aggregation algorithm through experts predict convex combination to predict $\hat{y}_{t+1}$. After the observation of y_{t+1}, the weights of each expert are updated based on a measure of prediction error called the loss. In a static setting, the goal is to design the updates of the weights to ensure that the loss of the aggregation algorithm is never greater than the cumulative loss of the best expert or the convex combination of the best expert. In a dynamic setting, the expert (or the convex combination of experts) that achieves the smallest loss may vary across different segments of the time series, resulting in algorithms that perform worse than the best expert over the entire time series. To solve this problem, consider dividing the time series into at most $L + 1$ segments, $1 < t_{(1)} < t_{(2)} < \ldots < t_{(L)} < T$. And allow that the best experts (or convex combinations of experts) may differ between elements of the distribution.

The optimal segmentation of up to $L + 1$ segment is the segmentation of the optimal expert sequence to achieve the lowest cumulative loss. In this case, the learning problem is much more difficult. An ideal aggregation algorithm must achieve losses as close as possible to expert sequences that optimally segment the time series into at most $L + 1$ segments, with the maximum number of points of change L and the length of each segment unknown. Many algorithms have been proposed to achieve an optimal upper bound for this problem, but these algorithms usually assume that the loss function is uniformly bounded. Since each DLM expert in ADMA contains a Gaussian error term, this assumption is not satisfied in our case. ConfHedge is the first and only way to upper the losses of an aggregation algorithm for any sequence of experts when the loss function is unbounded.

To assess the effectiveness of the proposed methodology, Yusupova et al. have conducted an in-depth empirical evaluation of the ADMA to forecast house prices in the UK. The empirical application results show that ADMA provides significant forecasting benefits over linear Autoregressive (AR) benchmarks and competing dynamic and static forecasting models. It is also noted that the best housing projections vary significantly over time and across regions. The robustness test of the sample also supports the following result: there is structural instability in the process of generating UK regional house price data.

3.4. DMA with Factor Augmented TVP-VAR Model (DMA-TVP-FAVAR)

At present, due to the influence of the COVID-19 pandemic, there is great uncertainty in the global economy and increasing fragility in the financial market. Investors worried about the spread of the virus and the threat to world economic recovery, panic selling occurred in many financial markets around the world, which led to the financial crisis and caused a considerable impact on the real economy. An lesson of recent events is that the relationship between the macro economy and financial markets has taken on new behavioral characteristics that are not necessarily determined by monetary policy. Therefore, it is necessary for investors, financial institutions, and government departments to closely monitor the overall situation of financial markets and warn of macroeconomic and systemic financial risks. In response to that requirement, the recent literature has developed several empirical econometric methods to research the relevant indices. The FCI is essentially information on many financial variables in a single number to serve multiple purposes. In practice, they can be applied to distinguish periods of sudden deterioration in financial conditions (e.g. Duca & Peltonen, 2013), assess credit constraints. Recently, many institutions have created their own model. The results also show that with the extension of the simple VAR model, the prediction results will be improved. Adding heteroscedasticity tends to greatly improve predictions relative to homogeneous autoregressive models, and then adding time variation enhanced the results even better. In forecasting models, TVP-FAVAR using DMA tends to be the best, including TVP-FAVAR using DMS, VAR, TVP-VAR, while TVP-VAR may sometimes predict well, methods involving TVP-FAVAR consistently provide predictive improvements. Also, where $\alpha = \lambda = 1$ indicates the value of allowing the regression coefficient to change over time. The Kansas City Fed's Financial Stress Index (FSI), for example, is widely used by some scholar.

Kabundi and Mbelu (2021) suggested using DMA technology to construct FCI. In the author's view, there are four important questions in the compilation of the financial stability index: (1) selecting financial variables to enter the financial stability index; (2) to average these financial variables into the weights of the indices; (3) the relationship between the financial stability index and the macroeconomy; (4) It is perfectly reasonable to assume that (1) - (3) may change over time. Therefore, they have proposed a DMA method to handle (1) - (4). Taking into account the impact of different financial variables on FCI estimates at each point in time, it is possible to explicitly consider that each financial crisis has different causes and spreads with different intensity.

The model in this paper is an extension of the Time-Varying Parameter Factor weighted Autoregressive model (TVP-FAVAR), which jointly models of financial variables, regression coefficients and conditional innovation volatility are allowed to change over time. $t = 1, \cdots, T$ of x_t is the $j \times 1$ vector of financial variables used to construct FCI. Similarly, suppose y_t the vector of interest of $s \times 1$ macroeconomic variables. And $y_t = (\pi_t, \mu_t, g_t)'$, in which π_t is the GDP price deflator, g_t represents growth rate in real GDP of the United States. The TVP-FAVAR, with a lag of magnitude P, can take the following form:

$$x_t = \Lambda_t^y y_t + \Lambda_t^f f_t + \varepsilon_t, \sim N(0, V_t) \tag{15}$$

$$\begin{pmatrix} y_t \\ f_t \end{pmatrix} = c_t + B_{1,t} \begin{pmatrix} y_{t-1} \\ f_{t-1} \end{pmatrix} + \cdots + B_{p,t} \begin{pmatrix} y_{t-p} \\ f_{t-p} \end{pmatrix} + \omega_t \sim N(0, W_t) \tag{16}$$

And f_t represent the latent factor. It contains data shared by the x_t. The second equation models the interaction between the FCI and variables. In (15), y_t represents vector of regression coefficients, and f_t is a vector of factor loads. Like (16), they follow a random walk process. In the new statements, ε_t and ω_t are zero-mean Gaussian perturbations with covariances V_t and W_t. Here V_t is diagonal.

Kabundi and Mbelu (2021) estimate the regression coefficients by writing (15) and (16) and using forgetting factor method. Similarly, they estimate a conditional volatility measure based on EWMA. Besides, given the ability of the DMA to run simulations to handle a large number of TVP-FAVAR, these variables differ in the FCI estimates. Namely, a particular model is constructed by specifying that the effect of a particular combination at time t on f_t is zero. While the m th TVP-FAVAR can be expressed as:

$$x_t^{(m)} = \Lambda_t^{y(m)} y_t + \Lambda_t^{f(m)} f_t^{(m)} + \varepsilon_t^{(m)}, \sim N\left(0, V_t^{(m)}\right) \tag{17}$$

$$\begin{pmatrix} y_t \\ f_t^{(m)} \end{pmatrix} = c_t^{(m)} + B_{1,t}^{(m)} \begin{pmatrix} y_{t-1} \\ f_{t-1}^{(m)} \end{pmatrix} + \cdots + B_{p,t}^{(m)} \begin{pmatrix} y_{t-p} \\ f_{t-p}^{(m)} \end{pmatrix} + \omega_t^{(m)} \sim N\left(0, W_t^{(m)}\right) \tag{18}$$

Similar to Koop and Korobilis (2014), these different TVP-FAVARs can be averaged by DMA. In this process, in the process of DMS based model processing, the specifications related to the highest model probability are mainly selected at each time point, which can simplify the processing process and improve the reference of the obtained results. Through the prediction analysis, it is found that financial variables have a good predictability with respect to macroeconomic variables, namely GDP price deflator, unemployment rate and real GDP growth rate. The predictive power of financial variables is stronger for near-term forecasts and short-term projections. The results also show that with the extension of the simple VAR model, the prediction results will be improved. Adding heteroscedasticity tends to greatly improve predictions relative to homogeneous autoregressive models, and then adding time variation enhanced the results even better. In forecasting models, TVP-FAVAR using DMA tends to be the best, including TVP-FAVAR using DMS, VAR, TVP-VAR, while TVP-VAR may sometimes predict well, methods involving TVP-FAVAR consistently provide predictive improvements. Also, where $\alpha = \lambda = 1$ indicates the value of allowing the regression coefficient to change over time.

4. Discussion and Conclusion

Model averaging is one of the effective solutions to solve the uncertainty of models in economic research in recent years. It mainly averages the prediction results of each model based on a certain weight. Some scholars also call it model combination, and the operations needed in the practical application process include combination estimation and prediction. How to select the weight combination is the main problem of its research. The model averaging method avoids some defects of single model selection, such as unsteadiness, missing information, and target deviation. In addition, model averaging generally does not select a specific model, so the uncertainty generated in the process of model selection is not ignored, which reduces the information loss of useful information. At the same time, this method is not based on a single model for prediction or analysis, which reduces the possibility of selecting wrong or poor models in model screening. In this study, the dynamic model averaging method is systematically expounded, and the weights of different criteria and their asymptotic unbiased properties are comprehensively analyzed and summarized. The main contents and conclusions of this paper are as follows:

In this paper, we briefly explain the motivation, origin, development, and application of dynamic model averaging. The DMA method generally has a good prediction effect, but it can improve the prediction accuracy and greatly reduce the computational burden if the model can be properly screened before combining the models.

We specifically introduce several new extended dynamic model averaging methods. The first part is the application in large-scale model space. The advantage of DMA is that it extends the model average, which has a good effect on model variables and form uncertainty. However, in macroeconomics, there may be many candidates' explanatory variables available in some cases, and the quantity of related models is too large to directly apply DMA in the process. For this reason, this paper proposes a new method, so that the analysis does not need to analyze the whole model space, and based on this, the model analysis can be carried out efficiently, instead uses a subset of the model meanwhile optimizes the model selection dynamically at every time intervals. This gives rise to the dynamic form of Occam's window. The second part is the combination with Google search data. In this paper, the author constructs variables and applies them to regression models. Google variables are allowed to probabilistically control model switching through Google models by using DMS methods that allow model switching between time-varying parametric regression models. Rather than using Google variables as regressors, they are allowed to decide which immediate forecasting model to use at each point in time. Thus, confirming Google data would help enhance immediate and temporary prediction results. The third part is adaptive dynamic model averaging. To overcome the restraints of present DMA stipulations, a new DMA adaptive approach is introduced to estimate the optimal forgetting factor and model average process for each Dynamic Linear Model (DLM). Simulation results show that the method can powerfully approximate the optimal forgetting factors under various change types (including change speed or change type with time) in the process of data generation. Another advantage of this approach is that it requires less computation than other DMA stipulations, which updates the DLM forgetting factor in turn by taking into account the lattice of values for this parameter.

The last part is the combination with factor augmented TVP-VAR model. Based on the TVP-FAVAR, a Financial Condition Index (FCI), which can accurately show the future trend of inflation and economic conditions, is constructed by using the techniques of Dynamic Model Selection and Dynamic Model Averaging (DMS and DMA). Compared with constant coefficient VAR, TVP-VAR, and FAVAR, the prediction performance of the TVP-FAVAR-DMA model is optimal, that is, the macro prediction ability of FCI measured by the DMA method is stronger than other methods.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Atkeson, A., & Ohanian, L. E. (2001). Are Phillips Curves Useful for Forecasting Inflation? *Federal Reserve Bank of Minneapolis Quarterly Review, 25,* 2-11. https://doi.org/10.21034/qr.2511

Bauwens, L., Koop, G., Korobilis, D., & Rombout, J. (2015). A Comparison of Forecasting Models for Macroeconomics Series: The Contribution of Structural Break Models. *Journal of Applied Econometrics, 30,* 596-620. https://doi.org/10.1002/jae.2387

Beckmann, J., Koop, G., Korobilis, D., & Schüssler, R. (2020). Exchange Rate Predictability and Dynamic Bayesian Learning. *Journal of Applied Econometrics, 35,* 410-421. https://doi.org/10.1002/jae.2761

Buncic, D., & Moretto, C. (2015). Forecasting Copper Prices with Dynamic Averaging and Selection Models. *The North American Journal of Economics and Finance, 33,* 1-38. https://doi.org/10.1016/j.najef.2015.03.002

Byrne, J. P., Cao, S., & Korobilis, D. (2017). Forecasting the Term Structure of Government Bond Yields in Unstable Environments. *Journal of Empirical Finance, 44,* 209-225. https://doi.org/10.1016/j.jempfin.2017.09.004

Catania, L., & Nonejad, N. (2018). Dynamic Model Averaging for Practitioners in Economics and Finance: The eDMA Package. *Journal of Statistical Software, 84,* 1-39. https://doi.org/10.18637/jss.v084.i11

Dangl, T., & Halling, M. (2012). Predictive Regressions with Time-Varying Coefficients. *Journal of Financial Economics, 106,* 157-181. https://doi.org/10.1016/j.jfineco.2012.04.003

Duca, M. L., & Peltonen, T. A. (2013). Assessing Systemic Risks and Predicting Systemic Events. *Journal of Banking & Finance, 37,* 2183-2195. https://doi.org/10.1016/j.jbankfin.2012.06.010

Kabundi, A., & Mbelu, A. (2021). Estimating a Time-Varying Financial Conditions Index for South Africa. *Empirical Economics, 60,* 1817-1844. https://doi.org/10.1007/s00181-020-01844-0

Koop, G., & Korobilis, D. (2012) Forecasting Inflation Using Dynamic Model Averaging. *International Economic Review, 53,* 867-886. https://doi.org/10.1111/j.1468-2354.2012.00704.x

Koop, G., & Korobilis, D. (2014). Large Time-Varying Parameter VARs. *Journal of Econometrics, 177,* 185-198. https://doi.org/10.1016/j.jeconom.2013.04.007

Koop, G., & Korobilis, D. (2015). A New Index of Financial Conditions. *European Economic Review, 71,* 101-116. https://doi.org/10.1016/j.euroecorev.2014.07.002

Koop, G., & Onorante, L. (2019). Macroeconomic Nowcasting Using Google Probabilities. In I. Jeliazkov, & J. L. Tobias (Eds.), *Topics in Identification, Limited Dependent Variables, Partial Observability, Experimentation, and Flexible Modeling: Part A* (Vol. 40A, pp. 17-40). Emerald Publishing Ltd. https://doi.org/10.1108/S0731-90532019000040A003

Onorante, L., & Raftery, A. (2016). Dynamic Model Averaging in Large Model Spaces Using Dynamic Occam's Window. *European Economic Review, 81,* 2-14. https://doi.org/10.1016/j.euroecorev.2015.07.013

Pesaran, M. H., & Timmermann, A. (2000). A Recursive Modelling Approach to Predicting UK Stock Returns. *The Economic Journal, 110,* 159-191. https://doi.org/10.1111/1468-0297.00495

Pettenuzzo, D., & Timmermann, A. (2017). Forecasting Macroeconomic Variables under Model Instability. *Journal of Business & Economic Statistics, 35,* 183-201. https://doi.org/10.1080/07350015.2015.1051183

Raftery, A. E., Kárný, M., & Ettler, P. (2010). Online Prediction under Model Uncertainty via Dynamic Model Averaging: Application to a Cold Rolling Mill. *Technometrics: A Journal of Statistics for the Physical, Chemical, and Engineering Sciences, 52,* 52-66. https://doi.org/10.1198/TECH.2009.08104

Risse, M., & Kern, M. (2016). Forecasting House-Price Growth in the Euro Area with Dynamic Model Averaging. *The North American Journal of Economics and Finance, 38,* 70-85. https://doi.org/10.1016/j.najef.2016.08.001

Wei, Y., & Cao, Y. (2017). Forecasting House Prices Using Dynamic Model Averaging Approach: Evidence from China. *Economic Modelling, 61,* 147-155. https://doi.org/10.1016/j.econmod.2016.12.002

Yusupova, A., Pavlidis, N. G., & Pavlidis, E. G. (2019). *Adaptive Dynamic Model Averaging with an Application to House Price Forecasting*. ArXiv, 1912.04661.