An Empirical Evaluation of Alternative Asset Allocation Policies for Emerging and Frontier Market Investors in Africa

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Abstract

Despite forming an integral part of literature and practitioner knowledge, Markowitz-based optimization has been shown to suffer severe drawbacks of estimation errors and sensitivity to input parameters when implemented in practice. The best diversification methods from the perspective of a private investor in real-life situations still remain largely unsolved. Most of the potential diversification benefits so far have primarily been analyzed for internationally diversified stock portfolios, with a focus on the special viewpoint of U.S investors. Studies have suggested that the Mean-Variance optimization can be robustified by the use of robust covariance estimators other than the sample covariance that relies on the classical Maximum Likelihood Estimator. Using a portfolio formed from 2 Emerging Market and 5 Frontier Market indices in Africa, this study sought to compare the performance of the traditional Mean-Variance model against the performance of the Mean-Variance optimization model robustified with the Orthogonalized Gnanadesikan-Kettenring, Minimum Covariance Determinant, Minimum Volume Ellipsoid and shrink estimators, with an aim of recommending the best model applicable to the African emerging and frontier markets investors. The robustified models were found to indeed have better characteristics in terms of gross returns, annualized returns and net portfolio returns over time compared to the traditional Mean-Variance optimization model.

1. Introduction

1.1. Background

Harry Markowitz's mean-variance portfolio theory laid the ground for the con-

cept of asset pricing and portfolio diversification. The theory provides a framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk. It is a formalization and extension of diversification in investing, where it suggests that it is riskier to hold only one type of financial asset than it is to hold different kinds of financial assets. It proposes that an asset's risk-return properties should be assessed by how it contributes to the overall risk and return of the portfolio.

1.1.1. Early History of Portfolio Theory

Prior to Markowitz's MV theory, the practice of diversification of investments was very much present. For example, Markowitz (1999) noted that Wiesenberger's annual reports in Investment Companies prior to 1952 (beginning 1941) showed that these firms held large numbers of securities. They were neither the first to provide diversification for their customers, nor were diversification new then. According to Markowitz (1999), what was lacking prior to 1952 was an adequate theory of investment that covered the effects of diversification when risks are correlated, distinguished between efficient and inefficient portfolios, and analyzed risk-return trade-offs on the portfolio as a whole.

Markowitz (1999) proposed expected (mean) return, E, and variance of return, V, of the portfolio as a whole as criteria for portfolio selection, both as a possible hypothesis about actual behavior and as a maxim for how investors ought to act. The assumptions in the Markowitz article imply that: the expected return on the portfolio is a weighted average of the expected returns on individual securities, and the variance of return on the portfolio is a particular function of the variances of, and the covariance between, securities and their weights in the portfolio. Markowitz distinguished between efficient and inefficient portfolios with regard to the mean and variance. The "*set of efficient mean-variance combinations*" would then be termed as the "*efficient frontier*". This frontier would be proposed to investors for the choice of the desired risk-return combination.

According to Markowitz (1999), Roy proposed making choices on the basis of the mean and variance of the portfolio as a whole. Specifically, he proposed choosing the portfolio that maximizes portfolio $(E-d)/\sigma$, where d is a fixed disastrous return and σ is the standard deviation of return. Roy's formula for the variance of the portfolio, like Markowitz's, included the covariance of returns among securities. A major difference between Roy's approach to Markowitz's approach is that while Markowitz's approach allowed investors to choose a desired portfolio from the efficient frontier, Roy's approach recommends the choice of a specific portfolio.

Markowitz identified some problems with his 1952 model. According to Markowitz, even though his article noted that the same portfolios that minimize standard deviation for given E also minimize variance for given E, it failed to point out that standard deviation (rather than variance) is the intuitively meaningful measure of dispersion. For example, "*Tchebychev's inequality*" says that

75 percent of any probability distribution lies between the mean and ±2 standard deviations, not two variances. The most serious problem between Markowitz and the views that came later concerned questions about "*why mean and variance?*" and "*mean and variance of what?*"

Later in 1956, Markowitz attempted to solve some of the issues in his 1952 paper. According to Markowitz (1999), a portfolio in Markowitz was considered feasible if it satisfied one equation (the budget constraint) and its values (investments) were not negative. Markowitz in 1956, however, solved the (single-period mean-variance) portfolio selection problem for a wide variety of possible feasible sets, including the Markowitz and Roy feasible sets as special cases. Markowitz's work in 1956 allowed the portfolio analyst to designate none, some, or all variables to be subject to non-negativity constraints and the remaining variables to not be thus constrained (as in Roy). Markowitz presented a computing procedure, the "*critical line algorithm*", that computes each corner portfolio in turn and the efficient line segment between them, perhaps ending with an efficient line "*segment*" on which feasible E increases without end.

Unlike Markowitz's paper in 1952 that made an assumption sufficient to ensure that a unique feasible portfolio would minimize variance for any given level of expected return, Markowitz made no such assumption, rather it demonstrated that the critical line algorithm will work for any covariance matrix. Markowitz argued that analysis of a large portfolio consisting of many different assets has too many covariances for a security analysis team to carefully consider them individually, but such a team can carefully consider and estimate the parameters of a model of covariance. Markowitz considered what happens to the variance of an equally weighted portfolio as the number of investments increases. It showed that the existence of correlated returns has major implications for the efficiency of diversification. As per Markowitz, with uncorrelated returns, portfolio risk approaches zero as diversification increases. With correlated returns, even with unlimited diversification, risk can remain substantial. Markowitz would later define semi-variance and presented a 3-security geometric analysis showing how the critical line algorithm can be modified to trace out mean-semi-deviation efficient sets.

Markowitz in his work acknowledges that he got his first views on the portfolio theory while reading William's paper in 1938. Williams asserted that the value of a stock is the expected present value of its future dividends. Markowitz however proposed that if an investor is only interested in some kind of expected value for securities, he/she must be only interested in that expected value for the portfolio, but the maximization of an expected value of a portfolio does not imply the desirability of diversification. This implied that diversification made sense as well as being common practice. "*What was missing from the analysis, I thought, was a measure of risk. Standard deviation or variance came to mind*". The presence of covariances in the formula for the variance of a weighted sum of random variables led Markowitz to infer that effective diversification required

avoiding securities with high covariance. The two quantities-mean and variance-helped draw a trade-off curve. The dominated mean-variance combinations were labeled "*inefficient*" while the un-dominated ones were labeled "*efficient*".

Tobin's Separation Theorem proposed in 1958 was concerned with the demand for money as distinguished from other "*monetary assets*". Monetary assets, including cash, were defined by Tobin as "*marketable, fixed in money value, free of default risk*". Tobin assumed that the investor seeks a mean-variance efficient combination of monetary assets. According to Markowitz, Tobin justified the use of expected return and standard deviation as criteria on either of two bases: Utility functions are quadratic, or probability distributions are from some two-parameter family of return distributions. Tobin's work resulted in the now called Tobin Separation Theorem. "*Tobin assumed a portfolio selection model with n risky assets and one riskless asset, cash. Holdings had to be non-negative and borrowing was not permitted. Implicitly, Tobin assumed that the covariance matrix for risky assets is non-singular*". The primary purpose of Tobin's analysis was to provide an improved theory of the holding of cash. Tobin's work drew similarities to William Sharpe's work in terms of suggesting a model with n risky assets and one riskless security. The difference between them is that Tobin's theory did not allow for risk-free borrowing, while Sharpe's work permitted borrowing and lending at the risk-free rate. Sharpe also suggested that his model could be applied to all securities, while Tobin restricted his model to "*monetary assets*".

Hicks (1962) noted that the risk factor affected the expected period and the expected net yield of the investment. He however did not designate standard deviation or any other specific measure of dispersion as the measure he meant when speaking of risk, therefore he could not show a formula relating risk on the portfolio to risk on individual assets. In this regard, Hicks therefore contained no distinguishing between efficient or inefficient portfolios, no drawing of an efficient frontier and had no hint of any kind of theorem to the effect that all efficient portfolios that include cash have the same proportions among risky assets. Hicks (1962) derived the Tobin conclusion that among portfolios that include cash, there is a linear relationship between portfolio mean and standard deviation and that proportions of risky assets remain constant along this linear portion of the efficient frontier as documented by Markowitz (1999). The difference between the Hicks and Tobin models is that Hicks assumed that all correlations are zero whereas Tobin permitted any non-singular covariance matrix.

According to Markowitz (1999), Hicks was a forerunner of Tobin in seeking to explain the demand for money as a consequence of the investor's desire for low risk as well as high return. On investment theory, Hicks summarizes that: "*It is one of the peculiarities of risk that the total risk incurred when more than one risky investment is undertaken does not bear any simple relation to the risk involved in each of the particular investments taken separately...Now, in a world where cost of investment was negligible, everyone would be able to take considerable advantage of this sort of risk reduction. By dividing up his*

capital into small portions, and spreading his risks, he would be able to insure himself against any large total risk on the whole amount. But in actuality, the cost of investment closes the possibility of risk reduction along these lines to all those who do not possess the command over considerable quantities of capital, making it unprofitable to invest less than a certain minimum amount in any particular direction,.... By investing only a proportion of total assets in risky enterprises, and investing the remainder in ways which are considered more safe, it will be possible for the individual to adjust his whole risk situation to that which he most prefers, more closely than he could do by investing in any single enterprise".

Kenneth Arrow said that Jacob Marschak made some efforts to construct an ordinal theory of choice under uncertainty. He assumed a preference ordering in the space of parameters of probability distributions (mean and variance). G.M. Constantinides and A.G. Malliaris said that Marschak expressed preferences for investments by indifference curves in the mean-variance space. As per Markowitz (1999), Marschak noted that people usually like high mean and low standard deviation (i.e. expected return as well as correlation coefficient in the preceding quotation "*are positive utilities*" as opposed to standard deviation which is "*a disutility*"). He also noted that people "like 'long odds' (i.e. high positive skewness of yields)". However, it "*is sufficiently realistic... to confine ourselves, for each yield, to two parameters only: the mathematical expectation ('lucrativity') and the coefficient of variation ('risk')*".

Markowitz acknowledges that his views on the portfolio theory were greatly influenced by Williams' work. Williams observed that the future dividends of a stock or the interest and principal of a bond may be uncertain. He said that, in this case, probabilities should be assigned to various possible values of the security and the mean of these values used as the value of the security. Finally, he assured readers that by investing in sufficiently many securities, risk can be virtually eliminated. Williams advised investors to diversify their funds among securities which give maximum expected return since the law of large numbers will ensure that the actual yield of the portfolio will be almost the same as the expected yield. However, this presumption that the law of large numbers applies to a portfolio of securities cannot be accepted. This is because the returns from securities are too intercorrelated, hence diversification cannot eliminate all variance. Despite this drawback, Markowitz (1999) believed that Williams's "*dividend discount model*" remains one of the standard ways to estimate the security means needed for a mean-variance analyses.

Leavens (1945) illustrated the benefits of diversification on the assumption that risks are independent. However, he cautions that "...*The assumption that each security is acted upon by independent causes, is important, although it cannot always be fully met in practice. Diversification among companies in one industry cannot protect against unfavorable factors that may affect the whole industry; additional diversification among industries is needed for that purpose. Nor can diversification among industries protect against cyclical factors that*

may depress all industries at the same time". Thus Leavens implies that some kind of covariance influences how an investor should invest.

1.1.2. The Expected Utility Maxim and Portfolio Analysis

One of the assumptions underlying the Markowitz MV theory is that investors are rational and risk averse. The rational investors have to make decisions, such as the selection of a portfolio in the face of uncertainty. Since their information is limited, they may sometimes make less than perfect decisions. Their actions are perfectly thought out and their risks perfectly calculated. It is however unrealistic to expect to find rational investors in the real world. It would therefore not be prudent to use the rational behavior theory to explain human behavior.

Early work on the portfolio theory was based on the assumption that the objective of investors was to maximize the expected money return on an investment. This objective was later found to be bad. An investor seeking to maximize only the expected return would never prefer a diversified portfolio. Such an investor would always place all his funds on the security that had a higher expected return compared to other securities. He would be indifferent between portfolios, diversified or not, provided the portfolios are of only the securities with the highest expected returns. If we consider diversification as a sound principle of investment, then we shouldn't only consider the objective to maximize expected returns alone.

The expected utility theorem was proposed as a substitute for the expected return rule. Authors came up with a curve relating utility to different levels of return. They argued that a return of 20% was not necessarily twice as good as a 10% return; while a loss of 20% is not necessarily twice as bad as a 10 % loss. Instead of maximizing the expected return, the authors argued that a rational investor would maximize the expected value of the utility of return. The expected utility maxim says that individuals should act as if they: Attach numbers, called their utility to each possible outcome, and select the outcome with the greatest expected value of utility when faced with chance alternatives. The expected utility maxim avoided the difficulties which condemned the expected return maxim. An individual whose utility curve is such that increasingly great returns add less and less to utility, will generally prefer a diversified portfolio.

An individual who maximizes expected utility may opt to buy insurance rather than take a 50 - 50 chance of winning or losing. Individuals' propensity to assume risks depends on their utility curves. This affects their choice between a diversified and undiversified portfolio. An individual with a concave utility function will prefer to pay a small premium rather than incur a small chance of a large loss. Such individuals would prefer to insure against large losses even if the insurance company makes some profit. If an individual's utility curve is strictly concave, diversification between two equally good portfolios cannot produce a worse portfolio and will generally produce a better one.

Concave utility functions are quite conservative. An individual with a convex utility curve would not buy insurance even if the insurance company made no

profits. Action based on a convex utility curve is even more reckless than the maximization of expected returns. Utility curves with convex segments raise some serious computing problems in portfolio selection. Therefore, in the case of allocation of "important money", it seems reasonable to use a strictly concave utility function.

1.1.3. Criticisms to the Markowitz Theory

The Markowitz MV portfolio theory has since formed an integral component of research with regard to finance. According to Marakbi (2016), a strand of criticism has however emerged that points to the phenomenon that MVO suffers from the severe drawback of estimation errors contained in the expected return vector and the covariance matrix when implemented in practice, resulting in portfolios that may significantly deviate from the true optimal portfolio. While a substantial amount of effort has been devoted to estimating the expected return vector in this context, much less is written about the covariance matrix input. In recent times, however, research that points to the importance of the covariance matrix in MVO has emerged. As a result, there has been a growing interest whether MVO can be enhanced by improving the estimate of the covariance matrix.

The empirical findings in Marakbi's study suggest one dominant estimator: the covariance matrix estimator implied by the Gerber Statistic (GS). Specifically, by using this covariance matrix estimator in lieu of the traditional sample covariance matrix, the MVO rendered more efficient portfolios in terms of higher Sharpe ratios, higher risk-adjusted returns and lower maximum draw-downs. The out-performance was protruding during recessionary times. This suggests that an investor that employs traditional MVO in quantitative asset allocation can improve their asset picking abilities by changing to the, in theory, more robust GS covariance matrix estimator in times of volatile financial markets.

Rosadi et al. (2020) in their study seem to back up Marakbi's findings. Their results suggest that the classical mean and covariance matrix estimators used in the Markowitz framework rely on the assumption of multivariate normal distributed returns, which is rarely fulfilled in real applications. Lauprete (2003) showed that many empirical portfolio returns have the sample skewness and the sample kurtosis which exhibit fat tails, follow a non-symmetric distribution and have multivariate tail dependence. They then suggested that the use of robust estimators would help handle data that contained outliers and deviated from the assumption of multivariate normality.

Over the last decades, returns within the stock universe have become increasingly correlated as illustrated by Goetzmann (2005), leading to decreasing diversification gains (Driessen & Laeven, 2007). Erb et al. (1994) and Longin and Solnik (2001) show that return correlations also tend to be higher during periods of poor performance. This means that the benefits from global diversification tend to be smallest when they are most needed. By exclusively focusing on stocks, most studies on portfolio optimization neglect the additional potential

offered by other asset classes. Since asset allocation has been shown to be the main determinant of portfolio performance (Ibbotson, 2000) and (Brinson et al., 1986) limiting portfolios to the stock universe seems harmful. There is need to explore other asset classes.

1.1.4. Focus on Emerging and Frontier Markets

The term 'emerging market' arises from the description of emerging economies applied by the World Bank to low and middle income economies. If a country's GNP per capita did not achieve the World Bank's threshold for a high-income country, the stock market in that country was said to be emerging. Distinction between emerging markets and the developed markets should be based on the economic differences between the two. However, in the literature the distinction between the two has been adapted from the term as used by the World Bank. More recently, this definition has proved to be less than satisfactory due to wide fluctuations in dollar-based GNP per capita figures. Dollar based figures have been significantly affected by swings in exchange rates, especially in Asia. Also reported GNP figures are often out-of-date by the time they are released, since they take a significant time to prepare.

According to Gupta (2006), major capital markets of the world are considered nearly efficient and the correlations between these markets during the past years appear to have risen. Consequently, the expected gains from diversifying across these major markets are assumed to be minimal. To gain diversification benefits it would appear necessary to invest in the emerging markets, which are still assumed less efficient. The correlations between these emerging markets and the major markets also appear to be lower. The argument that the investors should increase the proportion of their portfolios committed to emerging country equities is developed by Divecha, Drach and Stefek, Wilcox, and Speidell and Sappenfield (1992).

Emerging and frontier markets have recently been seen as an avenue for additional diversification by global investors. These markets, according to Gupta (2006), exhibit high expected returns as well as high volatility. Importantly, Harvey (1995) shows the low correlations with developed countries' equity markets significantly reduces the unconditional portfolio risk of a world investor. However, standard global asset pricing models, which assume complete integration of capital markets, fail to explain the cross section of average returns in emerging and frontier countries. A recent article written by the University of Toronto Beaulieu (2015) sheds light on the need for Canada to diversify its trade beyond the United States and increase its links to rapidly emerging market economies. It documents that growth has pivoted to these emerging markets over the last 15 years, singling out economies in Asia specifically for their rapid growth.

Many studies so far have analyzed potential diversification benefits for internationally diversified stock portfolios. U.S investors have received special attention in literature so far while the non-U.S perspectives have received far less as

De Santis and Gerard (1997) and Harvey (1995) point out. However, Bekaert and Urias (1996) find significant diversification benefits for the U.K country funds, but not for the U.S funds. De Roon et al. (2001) and Driessen and Laeven (2007) show that the additional benefit from investing abroad is economically small. In their study, De Roon et al. (2001) test whether it was possible for U.S investors to extend their efficient set by investing in emerging markets when accounting for frictions such as short sale constraints and transaction costs. They found strong evidence for diversification benefits when market frictions are excluded, but this evidence disappears when investors face short sale constraints or small transaction costs. Their spanning tests however show that for five out of the nine emerging markets that they study, direct investments in the emerging markets provide significant diversification benefits beyond diversified portfolios created from U.S-traded securities.

1.1.5. Robust Covariance Estimators

According to Rosadi et al. (2020), the Markowitz Mean-Variance framework adopts the use of the classical mean and covariance estimators. These estimators generally assume that returns are multivariate normally distributed; an assumption which is rarely the case in real life. Indeed, Lauprete (2003) showed evidence that portfolio returns have the sample skewness and the sample kurtosis which exhibit fat tails, follow a non-symmetric distribution and have multivariate tail dependence. It is in light of this anomaly in returns that the use of robust estimators is proposed.

Robust estimators can be applied to handle data that contain outliers and deviate from the assumption of multivariate normality. According to Huber (2009), these robust estimators are characterized by their sensitivity to small deviations from the assumptions of the underlying distribution of the data. Lauprete (2003) begins the usage of robust estimators in portfolio optimization. Their work proves that robust alternatives to the classical variance estimator have lower risk of loss than the non-robust ones. Welsch and Zhou (2007) in their work, consider several robust covariance estimators in the mean-variance portfolio optimization, such as FAST-MCD, Iterated Bivariate Winsorization, and Fast 2DWinsorization. DeMiguel and Nogales (2009a) prove that certain robust estimators produce more stable and less sensitive portfolios than the traditional portfolio. Rosadi et al. (2020) tested four robust estimators namely the Minimum Covariance Determinant (MCD) estimator, the Tukey's Biweight S-estimator, the Orthogonalized Gnanadesikan-Kettenring (OGK) estimator, and the MM-estimator as an improvement to the mean-variance portfolio optimization model.

1.2. Problem Statement

The Markowitz Mean-Variance model has long been considered to be the ideal optimization model for investors. Despite its prominence however, further studies have however found it to be prone to the problem of estimation errors that

arise in the variance-covariance matrix that it employs. This is especially the case as the number of securities making up the portfolio increases, making computation more complex. There has been substantial amount of study done on investment by various authors. A close examination however reveals that these studies have predominantly focused on the investment perspectives of investors based in developed markets. Emerging and frontier markets have in the recent past become a vital contributor to the world economy. Despite their significant contribution, many studies on investment have not given these markets the much needed attention.

There have been several propositions in literature aiming to tackle the estimation error associated with the classical MLE of the variance-covariance employed by the Markowitz framework. One suggestion is the use of more robust covariance estimators in the optimization process. These robust estimators can handle data that has outliers and do not necessarily assume multivariate normality as in the case of the classical mean-variance framework. They in turn produce more stable and less sensitive portfolios than the traditional mean-variance model. Using price indices of 2 emerging and 5 frontier market countries in Africa, this study hoped to evaluate the effectiveness of four robust covariance estimators against the traditional variance-covariance estimator of the Markowitz framework.

1.3. Objectives

1.3.1. Main Objective

This study hoped to evaluate the effectiveness of various extensions of Markowitz Mean-Variance framework which are aimed at improving on the estimation error trait characterizing the traditional MVO by using robust estimators. By using portfolios formed from emerging and frontier markets in Africa, this evaluation would in turn benefit investors in the African emerging and frontier markets by providing them with a basis for making their investment decisions.

1.3.2. Specific Objectives

The specific objectives of the study were:
- To analyze the portfolio performance of 5 recently proposed extensions of the Markowitz mean-variance framework.
- To compare the performance of the 5 models using various portfolio performance criteria to find the best performing optimization model.

1.4. Significance of the Study

The findings of the study will generally benefit private investors, fund managers and portfolio managers from the emerging market economies by highlighting on the best way of getting the benefits of diversification at the least cost. This knowledge may help them reap maximum from their investments while at the same time mitigating against risks. The academies and researchers would use the findings of the study as a basis for further research in determining whether in-

vesting in emerging and frontier markets could help investors benefit more from diversification. It would also add to the literature on the growing importance of emerging and frontier markets in finance as an additional avenue for portfolio diversification. It would also add to the literature on methods of improving the traditional Markowitz framework by the use of robust estimators to tackle the inherent problem of estimation error linked to the mean-variance framework.

2. Literature Review

2.1. Theoretical Literature

The Markowitz Portfolio Theory

The theory of portfolio optimization is traced back to the path-breaking work of Harry Markowitz: The MV Portfolio Theory. In the theory, Markowitz argues that investors are interested not only in the expected return of the portfolio, but also in the risk associated with the portfolio as a whole. As such, the investors choose assets that maximize their end period returns. Some of the underlying assumptions of this framework are that there are that investors are risk averse, there are no transaction costs, there are no short sales and assets are infinitely divisible. Markowitz portfolio optimisation can be stated mathematically as follows:

$$\min_{w_i} \sigma_p^2$$

subject to

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

$$r_p = r^*$$

$$r_p = \sum_{i=1}^{N} w_i r_i$$

$$\sum_{i=1}^{N} w_i = 1$$

$$0 \le w_i \le 1 \quad \forall i$$

where:

    σ_{ij} is covariance between asset i and j, if $i = j$, it is variance of asset i,

    σ_p^2 is variance of the portfolio of assets,

    r_i is expected return of asset i,

    r_p is the expected return of the portfolio,

    r^* is a predefined level of return,

    w_i is weight or proportion of asset i in the portfolio p.

Portfolio variance (or equivalently standard deviation) is touted as the measure of risk, and the risk-adjusted portfolio performance is measured by the Sharpe ratio, which the investor wants to maximize. Since investors are concerned about expected return as well as risk, it results in an efficient frontier, which is typically a set of pareto-optimal expected return, variance combinations identical to each investor.

Markowitz advanced that investing in a "single security" did not make sense.

The MV theory emphasizes that investors tend to form portfolios based on their different expected return-risk preferences. The Markowitz efficient set; the optimal risk-return combination of a portfolio lies on the efficient frontier of maximum returns for a given level of risk. Therefore, as per the theory, a portfolio will be inefficient if it gives an expected return that is too low for the level of risk taken. Prudent investors (as per the theory) therefore would prefer portfolios that give the highest expected return for a given level of risk.

According to Mao (1970), portfolio diversification is the process of allocating capital in a way that reduces the exposure to any particular asset or risk. A common way to diversify a portfolio is by reducing risk or volatility by investing in a variety of assets. Diversification and hedging are two general techniques that can be used to reduce investment risk. The MV theory places strong emphasis on the idea that diversification is the only "free lunch" in investment. This is backed by the fact that portfolio variance incorporates covariances. This implies portfolio risk is affected by how much each individual asset contributes to the overall portfolio risk. Investors should be concerned not only with the contribution of an asset to the overall portfolio risk but also with the manner in which the assets forming the portfolio correlate.

Markowitz portfolio theory places huge emphasis on portfolio diversification as the only sure way to mitigate (not eliminate) portfolio risk. According to Jacobs et al. (2009), the concept of diversification as "the only free lunch in investment" has become part of the accepted wisdom among practitioners and motivated extensive research. A key question however remains unsolved: What is the best way to diversify in real-life situations from the perspective of a private investor? Diversification benefits have so far been primarily analyzed for internationally diversified stock portfolios. Special focus has been given to US investors and non-US investors have received less attention in the literature so far as indicated by Jacobs et al. (2009).

2.2. Empirical Literature

2.2.1. Asset Allocation

Perold and Sharpe (1995) state that fluctuations in the values of the risky assets contained in many portfolios will lead to the portfolio values fluctuating too. This will consequently lead to the asset allocation of the portfolio changing, hence the need to rebalance the portfolio frequently. According to Markowitz (2002), the asset classes fall into three broad categories: *equities*, *fixed-income*, and *cash and equivalents*. These three are generally referred to as the *traditional asset classes*. Anything outside these three categories (e.g. real estate, commodities, art) is often referred to as *non-traditional* or *alternative assets*. When making investment decisions, an investor's portfolio distribution is influenced by factors such as personal goals, level of risk tolerance and investment horizon.

Financial advisers usually advise that to reduce the level of volatility of portfolios, investors must diversify their investment into various asset classes. Such ba-

sic reasoning is what makes asset allocation popular in portfolio management because different asset classes will always provide different returns. Thus, investors will receive a shield to guard against the deterioration of their investments.

Importance of asset allocation

How important is asset allocation policy in determining performance (Ibbotson, 2010)? The first attempt to answer this question was made by Brinson et al. (1986) more than two decades ago in their article, "*Determinants of Portfolio Performance*". They regressed the time-series returns of each fund on a weighted combination of benchmark indices reflecting each fund's policy. They found that the policy mix explained 93.6% of the average fund's return variation over time (as measured by the R^2).Unfortunately, their time-series results were not very sensitive to each fund's asset allocation policy because most of the high R^2 came from aggregate market movement. Ibbotson and Kaplan (2000), and Hensel, Ezra, and Ilkiw pointed out that most of the variation in a typical fund's return comes from market movement. The funds differ by asset allocation, but almost all of them participate in the general market instead of just holding cash. Nevertheless, the idea that asset allocation policy explains more than 90% of performance has become accepted folklore.

Ibbotson and Kaplan (2000) presented a cross-sectional regression on annualized cumulative returns across a large universe of balanced funds over a 10-year period and found that about 40% of the variation of returns across funds was explained by policy. Vardharaj and Fabozzi applied Ibbotson and Kaplan by using similar techniques for equity funds and found that the $R2$s were time-period sensitive and that approximately 33% to 75% of the variance in fund returns across funds was attributable to differences in asset allocation policy. As Xiong, Ibbotson, Idzorek, and Chen demonstrated, the actual percentage of the variation of returns among funds that is explained by policy is sample specific. It is not necessarily 40%, as in Ibbotson and Kaplan (2000), but has been measured across a wide range of values. For any given portfolio, the importance of asset allocation policy (the passive return) versus the active return (i.e. timing, security selection, and fees) depends on the preferences of the fund manager. For a true market-neutral hedge fund that has hedged away all possible beta risk exposures, the active performance dominates.

For a long-only passive index product, asset allocation policy dominates. According to (Ibbotson, 2010), in general (after controlling for interaction effects), about three-quarters of a typical fund's variation in time-series returns comes from general market movement, with the remaining portion split roughly evenly between the specific asset allocation and active management. The time has come for folklore to be replaced with reality. Asset allocation is very important, but nowhere near 90% of the variation in returns is caused by the specific asset allocation mix. Instead, most time-series variation comes from general market movement, and Xiong, Ibbotson, Idzorek, and Chen showed that active management has about the same impact on performance as a fund's specific asset al-

location policy. Many private investors will not be in a position to control the market movements. As such, it is of great importance to ensure that these investors are able to make maximum gain from their investment by having the most appropriate asset allocation policy.

2.2.2. Discussion on the Benefits of Diversification

According to Goetzmann et al. (2001), there is considerable academic research that documents the benefits of international diversification. Grubel and Herbert found that between 1959 and 1966, U.S. investors could have achieved better risk and return opportunities by investing part of their portfolio in foreign equity markets. Levy and Sarnat analyze international correlations in the 1951-1967 period, and show the diversification benefits from investing in both developed and developing equity markets. Grubel and Fadner show that between 1965 and 1967 industry correlations within countries exceeded industry correlations across countries. These early studies marked the beginning of an extensive literature in financial economics on international diversification

Goetzmann et al. (2001) found that international equity correlations change dramatically through time, with peaks in the late 19th century, the Great Depression and the late 20th Century. This is despite the limitations of their data. They therefore suggest that the diversification benefits to global investing are not constant and that the most important thing for the investor of the early 21st Century is that the international diversification potential today is very low compared to the rest of capital market history.

Goetzmann et al. (2001) attempt to tackle the important question on whether diversification works when it is most needed. This issue has been of interest in recent years due to the high correlations in global markets conditional upon negative shocks. Evidence from capital market history suggests that periods of poor market performance, most notably the Great Depression, were associated with high correlations, rather than low correlations. Wars were associated with high benefits to diversification; however these are precisely the periods in which international ownership claims may be abrogated, and international investing in general may be difficult. Indeed, investors in the past who have apparently relied upon diversification to protect them against extreme swings of the market have been occasionally disappointed.

Throughout the last 150 years, literature has been able to identify two related sources of the benefits to diversification, both of which have affected investor risk. The first source is the variation in the average correlation in equity markets through time (the average covariance or correlation between markets). A lower covariance rotates the diversification curve downwards. The second source is the variation in the investment opportunity set (the number of markets that are available to investors). An increase in the available markets allows investors to move down along a given diversification-curve. Goetzmann et al. (2001) say that, "*For example, in the last two decades, the opportunity set expanded dramatically at the same time correlations of the major markets have increased. As a*

result, the benefits to international diversification have recently been driven by the existence of emerging capital markets, smaller markets on the margin of the world economy where the costs and risks of international investing are potentially high. For other periods, such as the two decades following the era of World War II, risk reduction derived from low correlations among the major national markets. From this, we infer that periods of globalization have both benefits and drawbacks for the international investor. They expand the opportunity set, but the diversification benefits of cross-border investing during these periods rely increasingly on investment in emerging markets".

The main motive for international diversification has been to take advantage of the low correlation between stocks in different national markets. Solnik (1977), for example, shows that an internationally diversified portfolio has only half the risk of a diversified portfolio of U.S stocks. In his study, the variance of a diversified portfolio of U.S. stocks approaches 27% of the variance of a typical security, as compared to 11.7% for a globally diversified portfolio. The steady increase in the number of equity markets over the past century has provided additional diversification opportunities to investors.

Based on emerging market country funds, Bekaert and Urias (1996) find only mixed evidence for the diversification benefits of emerging markets. Using industry portfolios, multinational corporation stocks, closed-end country funds, and American depository receipts, Errunza et al. (1999) show that U.S investors can create mimicking portfolios from U.S-traded securities that are highly correlated with the IFC emerging markets indices. Their spanning tests show that for five out of the nine emerging markets that they studied, direct investments in the emerging markets provide significant diversification benefits beyond diversified portfolios created from U.S.-traded securities.

De Roon et al. (2001) found that if frictions were ignored, there were significant diversification benefits from adding emerging markets to an international stock portfolio that invests in the United States, Europe, and Japan. The evidence in favor of these diversification benefits disappeared when short sales constraints and investability restrictions were taken into account. The results in De Roon et al. (2001) can be summarized as: "*There is substantial evidence available in the literature that suggests that, in the absence of market frictions, U.S. investors can benefit from including emerging markets assets in their well-diversified international portfolio of developed market assets.... When accounting for short sales constraints and investability restrictions, the evidence in favor of diversification benefits of the emerging markets disappears, that is, for the three geographical regions, we can no longer reject the hypothesis of spanning. This is mainly due to the short sales constraints on the emerging markets*".

2.2.3. Alternative Methods of Asset Allocation

Markowitz's MVO framework has over the years become the asset allocation model of choice. While the maximization of return per unit of risk is a logical and worthwhile objective, according to Idzorek (2006), the Markowitz frame-

work may be too powerful for its own good. Common issues arising from the use of Mean Variance Optimization (MVO) are that: It leads to asset allocations in which the majority of the holdings are concentrated in a small number of asset classes that make up the opportunity set, contradicting the common-sense notion of diversification. Also, basing one's decision solely on an asset allocation's mean and variance is insufficient especially in a world in which asset class returns are not normally distributed. The MV framework is a single period model yet most investors have multi-period objectives.

These potential shortcomings are the likely reasons that practitioners especially private investors have not fully embraced MVO. For them, MVO creates an illusion of being sophisticated; yet, in practice, asset allocations are developed using judgmental, ad hoc approaches. Recent advances however significantly improve the quality of typical MVO based asset allocations that should allow a far wider audience to realize the benefits of the Markowitz paradigm, or at least the intent of the paradigm.

Idzorek (2006) shows in his article that the traditional MVO often led to concentrated, and undiversified portfolios. The article indicates that when using the traditional MVO, its outputs are very sensitive to changes in inputs (capital market assumptions). The extreme asset allocations were shown in the article using efficient frontier graphs and efficient frontier asset allocation area graphs. Out of an opportunity set containing 9 asset classes, the efficient frontier asset allocation area graph from the traditional MVO only contained 5 asset classes. Nearly half of the asset classes are excluded from the asset allocation! Important asset classes are also excluded from the asset allocation. In each of the sections of the graph, the asset allocations are dominated by allocations to one or two particular asset classes. Other important takeaways from the traditional MVO were that; different inputs led to significantly different asset allocations and that the allocations that were optimal in one period were not always optimal in the other periods. Idzorek (2006) suggests that the allocations of traditional MVO are concentrated because the MVO is sensitive to market assumptions (input sensitivity). Of these market assumptions, returns are the most important yet they are the least stable.

The Black-Litterman Model and resampled MVO attempt to overcome some of the challenges inherent in the traditional MVO. The Black-Litterman Model by Fischer Black and Robert Litterman enables investors to combine their unique views regarding the performance of various assets with CAPM market equilibrium returns in a manner that results in intuitive, diversified portfolios. According to Idzorek (2006), the Black-Litterman Model uses a Bayesian approach to combine the subjective views of an investor regarding the expected returns of one or more assets with the CAPM market equilibrium expected returns (the prior distribution) to form a new, mixed estimate of expected returns (the posterior distribution). The model combines the distribution of CAPM equilibrium returns and the distribution of view returns to form a mixed estimate of expected returns anchored by the CAPMreturns but also reflects the

opinion/view of returns. As a result, the model produces return estimates that produce well diversified asset allocations when used either in traditional MVO or resampled MVO.

Resampled MVO, according to Idzorek (2006), combines traditional MVO with Monte Carlo simulation to account for the uncertainty in forward-looking capital market assumptions. It accounts for input uncertainty and addresses the input sensitivity, diversification issues and estimation error of the traditional MVO. Resampled MVO is computationally intensive and, depending upon the number of asset classes, can take several minutes to complete. A Monte-Carlo Simulation produces a set of capital market assumptions based on either parametric or non-parametric approaches. This simulated set is fed into an MVO resulting in an intermediate frontier called a simulated frontier and the resulting asset allocations from these simulated frontiers are saved. After repeating the process many times, the asset allocations from the simulated frontiers are averaged. The averaged asset allocations are then linked to the original inputs to plot the resampled efficient frontier.

When comparing the asset allocations created by the traditional MVO to those created using Black-Litterman returns, Idzorek found that there was substantial increase in the number of asset classes in the efficient asset allocations from the Black-Litterman returns. Unlike the 5 asset classes included by the traditional MVO, all 9 assets in the opportunity set are included in the allocation using the Black-Litterman model. Allocations were also diversified and intuitive. The allocations from Resampled MVO with historical returns evolved more smoothly across the efficient frontier asset allocation area graph and were significantly more diversified than the traditional MVO based allocations. The allocations also included the assets that were absent from the traditional MVO. Resampled MVO with Black-Litterman returns produced the most diversified asset allocations, incorporating all of the 9 asset classes in the investment set. Higher risk asset allocations were also significantly more diversified than the traditional MVO and the resampled MVO with historical returns. Idzorek's findings point that the Black-Litterman model and the resampled MVO independently perform better than the traditional MVO in terms of asset allocations not being concentrated and that a combination of the two optimization techniques would be the best advice for investors.

Farrelly (2006) sought to overcome the problem of instability in the Markowitz MV models using the Robust Frontier Model. The model was found to produce more stable and intuitive results, with minimal sacrifice to efficiency. The robust frontier model aims to identify over a wide range of scenarios, the most robust of the portfolios just below the efficient frontier that may have different asset weightings but very similar risk-return characteristics. These portfolios are believed to be the drivers of the instability of the MV model, since small changes in assumptions could cause a portfolio that was close to the efficient frontier to move to the frontier, displacing the old efficient portfolio.

Using a methodology almost similar to Idzorek's resampled MVO, the Ro-

bust Frontier Model uses Monte-Carlo process plus assumptions (expected return, standard deviation and correlation matrices for assets) to simulate scenarios involving a return for each asset class under consideration. From the simulated returns at different scenarios, a simulated ex-post efficient frontier (SEPE frontier) is created. For each scenario, the difference in return between each candidate portfolio and the SEPE frontier is calculated and the difference raised to the power of in order to find the robust portfolios. The λ is a coefficient of sensitivity to under-performance, hence the higher the value of λ the more portfolios are penalized for producing returns that are distant from the SEPE frontier. The robust portfolios therefore are those that for a given level of risk, have the lowest sum of differences raised to the power of λ across the entire range of scenarios.

Farrelly (2006) found that the robust portfolios tended to draw more from the broader set of investment alternatives and less concentration in the most favored sectors. It therefore pointed out that a more even spread produces few extreme results, and the extreme results are penalized more by the Robust approach. The Robust approach produced portfolios that were more stable to swings in the return assumptions compared to MVO which was too volatile. Changing the allocations of two assets in the portfolio didn't affect the Robust portfolio as much as it did for the MV model. The expected returns from the Robust portfolios do not differ greatly from the those of the efficient portfolios. It is thus possible to create far more diversified portfolios without giving up much in the way of expected returns (Robust portfolios entail modest sacrifices in the expected returns). The Robust portfolios were also found to be less likely to produce extreme results, since they began producing few outliers at the one to five percentile levels. The Robust portfolio approach therefore provides a logical methodology for practitioners to trade off risk and return when making asset allocation decisions, since it produces portfolios with more diversification and less sensitivity to small changes in input assumptions.

Coggeshall and Wu (2005) describe a heuristic empirical approach that uses concepts of shortfall risk as an objective and actual data as a direct model of stochastic model evolution. Their alternative approach uses data directly to obtain actual historical distributions and use these empirical distributions to stochastically simulate performance, without using a theoretical structure. Stephen and Guowei propose the use of overlapping over non-overlapping windows, since they improve the statistical measures. Non-overlapping windows also have the potential to miss potential important events which can be captured by overlapping windows.

By comparing the cumulative distributions of long term equity returns to the Random Walk projections, Coggeshall and Wu (2005) found that for shorter holding periods, stock return tails were heavy while for longer holding periods, the stock return tails were skinny (stocks are less riskier for longer holding periods). The risks associated with stocks were also substantially less than what theory suggests for holding periods greater than 20 years. Bonds on the other

hand were shown to be riskier than stocks for holding periods greater than 20 years. The stock distributions were always to the right of the bond distribution for the holding period greater than 20 years, hence would always outperform the bonds (making bonds riskier to invest in for the long term). For the one year holding (short term), the stocks were riskier than the bonds since their distributions showed substantial likelihood to underperform bonds.

Coggeshall and Wu (2005) sought to find out: "What would be the optimal initial asset allocation if we were to hold a mixture of stocks and bonds for different holding periods without rebalancing?" To achieve this, they ran 9 analysis of mixed portfolios of 10%, 20%, 30%, ...% stocks and 90%, 80%, 70%, ...% bonds in addition to the 100% stocks and 100% bond portfolios. They assumed a moderate risk tolerance (that requires 90% probability to meet or exceed the target), and always selected the higher distribution curve for the selected holding period. At 90% confidence, it was found that for holding periods less than a year, portfolios of pure bonds were best performing. For holding periods between 1 year to 15 years, a mix of stocks and bonds was the most optimal. The optimal mixes would smoothly increase and decrease respectively for stocks and bond during this holding period. For holding periods greater than 15 years, pure stock portfolios were the best performers. This study provided a basis for further study on heuristic asset allocation strategies between asset classes.

DeMiguel et al. (2009b) evaluate the out-of-sample performance of the sample-based MV portfolio rule-and its various extensions designed to reduce the effect of estimation error-relative to the performance of the naive portfolio diversification rule. The naive rule is defined to be one in which a fraction 1/N of wealth is allocated to each of the N assets available for investment at each rebalancing date. The naive rule is chosen as a benchmark because first, it is easy to implement because it does not rely either on estimation of the moments of asset returns or on optimization. Second, despite the sophisticated theoretical models developed in the last 50 years and the advances in methods for estimating the parameters of these models, investors continue to use such simple allocation rules for allocating their wealth across assets. They compared the out-of-sample performance of 14 different portfolio models relative to that of the 1/N policy across seven empirical datasets of monthly returns, using three performance criteria: the out-of-sample Sharpe ratio; the certainty-equivalent (CEQ) return for the expected utility of a mean-variance investor; and the turnover (trading volume) for each portfolio strategy.

Of the 14 models evaluated by DeMiguel et al. (2009b), none was consistently better than the naive 1/N benchmark in terms of Sharpe ratio, certainty-equivalent return, or turnover. In general, the unconstrained policies that try to incorporate estimation error perform much worse than any of the strategies that constrain shortsales, and also perform much worse than the 1/N strategy. Imposing constraints on the sample-based mean-variance and Bayesian portfolio strategies led to only a modest improvement in Sharpe ratios and CEQ returns, although it showed a substantial reduction in turnover. Of all the optimizing

models studied, the minimum-variance portfolio with constraints performs best in terms of Sharpe ratio. But even this model delivers a Sharpe ratio that is statistically superior to that of the 1/N strategy in only one of the seven empirical datasets, a CEQ return that is not statistically superior to that of the 1/N strategy in any of these datasets, and a turnover that is always higher than that of the 1/N policy. This points to some need for practitioners to move away from sophisticated optimization models to simple asset allocation rules/heuristics to better their portfolio performances.

Rosadi et al. (2020) in their paper, presented an improvement to the mean-variance framework with the integer transaction lots constraint, by considering robust estimators of the covariance matrices to deal with the presence of outliers in the data. They tested four robust estimators comparing them to the classical MLE. Based on their simulation studies and empirical results, their study showed that the robust estimators outperformed the classical MLE when data contained outliers. The study recommended that further research be done using other robust estimators as well as adding more constraints to represent the real condition of the stock markets.

2.3. Research Gap

Over the last couple of years, the literature on ways of improving the issues inherent in the MVO has been growing. This study seeks to continue on this literature by analyzing portfolio performance of 5 extensions of the Markowitz optimization model, that have been recently proposed to improve the traditional MVO's estimation error and sensitivity to input parameters by use of robust estimators. Most studies however have focused on the implementation of these Markowitz based models to U.S and Euro developed market investors. Many studies focus on how U.S and Euro investors can improve their diversification by incorporating asset classes from the emerging markets. There are few studies that exclusively focus on how the investors from emerging and frontier markets can best diversify. Emerging and Frontier markets especially in Africa have received less attention. This study hopes to localize its scope to African emerging and Frontier markets.

3. Methodology

3.1. Research Design

This was an empirical research that intended to evaluate portfolio performance of 5 Markowitz based models using the stock market indices of 2 emerging (Egypt and South Africa) and 5 frontier market (Kenya, Mauritius, Morocco, Nigeria and Tunisia) countries in Africa.

3.2. Asset Classes and Data

The study used the Morgan Stanley Capital International (MSCI) index family. Specifically, the study relied on the MSCI Emerging and Frontier Markets Africa

Index (USD). The MSCI Emerging and Frontier Markets Africa Index capture large and mid-cap representation across 2 Emerging Market countries and 13 Frontier Markets (FM) countries. The index includes 69 constituents, covering about 85% of the free float-adjusted market capitalization in each country. The study considered South Africa and Egypt from the Emerging Markets in Africa and Kenya, Mauritius, Morocco, Nigeria and Tunisia from the frontier markets in Africa. The frontier markets are an important component as recent studies have shown that they have increased returns and created risk diversification opportunities for global investors (Marshall et al., 2013). The study used monthly data from 26th December 2009 to 26th March 2021.

3.3. Asset Allocation Models

Markowitz-Based Portfolio Optimization Models from the Existing Literature

Various studies have suggested extensions of Harry Markowitz's theory, to deal with the problem of estimation error which is ignored in the traditional MV model of Markowitz. The models either impose additional constraints in the optimization process or shrink the estimated parameters in order to mitigate the effect of estimation error, or both. By imposing short-sale constraints, one prevents the optimization model from taking extreme long and short positions to exploit even small differences in the return structure of the assets. Shrinkage models correct the estimated parameters toward a common value.

Another of these solutions is the use of robust covariance estimators aside from the covariance estimator of the traditional MVO. The study considered four robust estimators for optimization namely: the Minimum Covariance Determinant (MCD) Estimator, the Minimum Volume Ellipsoid (MVE) Estimator, the Orthogonalized Gnanadesikan-Kettenring (OGK) Estimator and the shrinkage estimator. The Markowitz-based models that the study used therefore were:

1) The minimum risk mean-variance model (*Min-Risk MV*).

2) The Minimum Covariance Determinant (MCD) robustified mean-variance model (*MCD-MV*).

3) The Minimum Volume Ellipsoid (MVE) robustified mean-variance model (*MVE-MV*).

4) The Orthogonalized Gnanadesikan-Kettenring (OGK) robustified mean-variance model (*OGK-MV*).

5) The shrinked mean-variance model (*Shrinked-MV*).

The minimum risk mean-variance model, also the traditional mean-variance model, minimizes the risk for a given level of return when optimizing. It uses the classical MLE in the sample covariance in its optimization. The traditional MV model can be robustified by using alternative covariance estimators, aside from the sample covariance estimator used in the standard MVO. The traditional MVO's performance was compared against 4 other robustified MVO models. The *MCD-MV* model uses the *minimum covariance determinant* estimator of

location and scatter to look for the h > n/2 observations out of n data records whose classical covariance matrix has the lowest possible determinant. The raw *MCD* estimate of location is then the average of these h points, whereas the raw *MCD* estimate of scatter is their covariance matrix, multiplied by a consistency factor and a finite sample correction factor. The *MVE-MV* model picks, from samples from a multivariate normal distribution that form ellipsoid-shaped 'clouds' of data points, the smallest point cloud containing at least half of the observations; the uncontaminated portion of the data. These 'clean' observations are used for preliminary estimates of the mean vector and the covariance matrix. Using these estimates, the program computes a robust Mahalanobis distance for every observation vector in the sample. The *OGK-MV* model computes the orthogonalized pairwise covariance matrix estimate. The shrinkage estimator of the *Shrinked-MV* model computes the empirical variance of each considered random variable, and shrinks them towards their median.

3.4. Portfolio Performance Evaluation Procedure

The study used the stock market price index data from 2 emerging and 5 frontier market countries in Africa as the investment set. The stock market price indices were considered as the assets forming the portfolio. These indices with the currencies denoting them are shown in Table 1.

From the prices of the indices, returns were generated and the returns used to perform MVO. There were 5 different optimization types, differing based on the type of covariance estimator used in each. The optimizations were done using a rolling window approach with overlapping windows of 36 months and monthly rebalancing. The rebalanced portfolio weights were later smoothed every 6 months. Based on the results from the 5 optimization types, comparisons and analysis of performance of the optimization models were made.

For robustness checks, each optimization's performance was compared against two indices as benchmarks. These indices were the MSCI World Index and the MSCI Emerging and Frontier Market Index. These indices have a comprehensive and consistent index construction approach that allows for meaningful global views and cross regional comparisons across all market capitalization

Table 1. Stock price indices with their currency denotations.

COUNTRY	STOCK PRICE INDEX	CURRENCY
Egypt	EGX30	Egyptian Rand
Kenya	NSE20	Ksh
Mauritius	SEMDEX	Mrupee
Nigeria	SE30	NG Naira
South Africa	JSE TOP 40	Rand
Tunisia	TUNINDEX	Dinar
Morocco	MADEX	Dirham

size, sector and style segments and combinations.

4. Results and Discussions

4.1. Process of Obtaining the Results

For portfolio back testing, the study used the "fportfolio" package. The Rmetrics "fPortfolio" package is a very powerful collection of functions to optimize portfolios and to analyze them from different points of view. The package implements portfolio models like the traditional mean-variance Markowitz portfolio, robust variants of the Markowitz portfolio, the mean-LPM (lower partial moment) portfolio, and the mean-CVaR portfolio. The package was run in R-studio and used to generate the results and graphical displays.

4.2. Analysis of Portfolio Weights Recommendation

The portfolio weight recommendations from each optimization were analyzed in order to assess the recommended weights for investment into the portfolio under study. Figure 1 shows the weight recommendation from the traditional MVO (*Min-Risk MV*):

From Figure 1, it is noted that in the early years of investment, the JSETOP40 takes up the larger proportion of the allocation in the portfolio. Over time however, its allocation keeps decreasing. However, towards the end of the study period, the JSETOP40 is the only index whose allocations show an increasing trend. TUNINDEX's allocation shows a decline from the year 2012 to 2013. It showed a short spike between 2013 and 2014 before beginning to steadily rise from 2014 to 2017. It then declines between the year 2017 and 2018. Its trend then shows an upward trajectory from the year 2018 to 2020 before it starts to decline towards the end of the period. SEMDEX starts among the indices with the least allocation from the year 2012 to 2014. It then begins an upward trend from 2014 to 2015 before declining all through to 2017. From 2017 to 2019 it shows a sharp rise before declining up to the end of the study period. MADEX
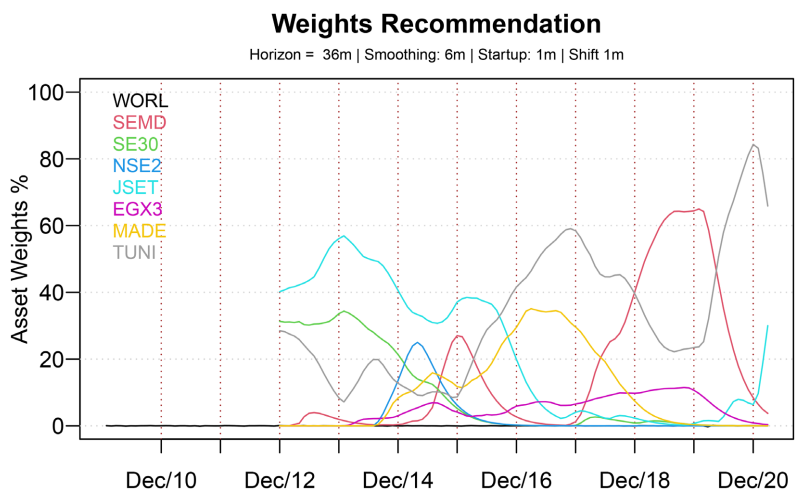
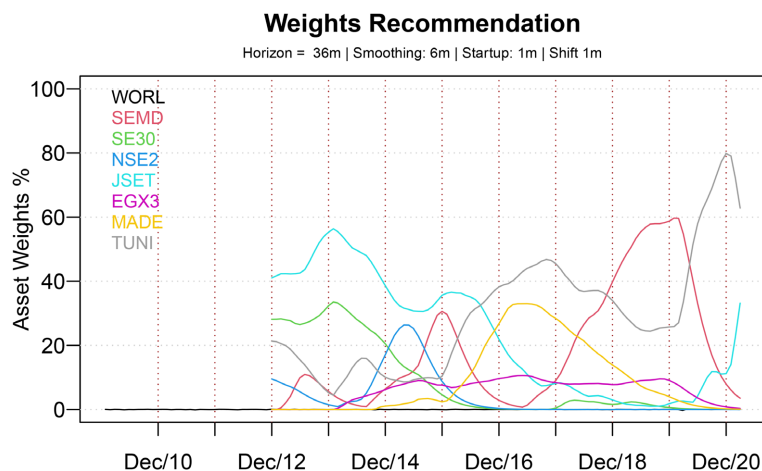**Figure 1.** Min-Risk MV portfolio weights recommendations.

shows a steady rise from the year 2013 to 2017 before declining up to the end of the study period. EGX30 shows a gentle rise between 2012 and 2019 before declining up to the end of the study period. The SE30's allocation showed steady decline from 2012 to 2016 and had close to zero allocation thereafter up to the end of the study period. NSE20 showed a short spike between 2013 and 2015 and generally had close to zero allocation thereafter.

Figure 2 shows the weights recommendation from the *Shrinked-MV*.

From **Figure 2**, the JSETOP40 takes up the largest proportion of investment in the early years and declines toward the end of the investment period. However, it is again the only index showing an upward trajectory in the years toward the end of the investment period. TUNINDEX's allocation declines from 2012 to 2013, and then shows a short spike from 2013 to 2014, before steadily increasing up to 2017. From 2017 to 2019 it steadily decreases before showing a steep rise within the year 2020 before starting to decline towards the end of the study period. SEMDEX showed a spike between the years 2012 and 2014, and 2014 and 2016. It then rose steadily from 2016 up to 2019 before starting to decline up to the end of the study period. MADEX showed a steady rise between 2015 and 2017 before declining up to the end of the study period. EGX30 showed a gentle rise between 2013 and 2018 before declining up to the end of the study period. NSE20 showed a short spike between 2013 and 2015 and generally had close to zero allocation thereafter. The SE30's allocation showed steady decline from 2012 to 2016 and had close to zero allocation thereafter up to the end of the study period.

Figure 3 shows the weights recommendation from the *MVE-MV*.

The JSETOP40's allocations show an upward trajectory between 2012 and 2013 before declining up to 2014. The allocation then rises slightly in the year 2015 before declining towards the end of the investment period. JSETOP40 still remains the only index showing an upward trajectory in the later years of the study period. TUNINDEX's allocation shows a general decline from 2012 up to 2015 before steadily rising up to 2017. It again declined up to 2019 before rising
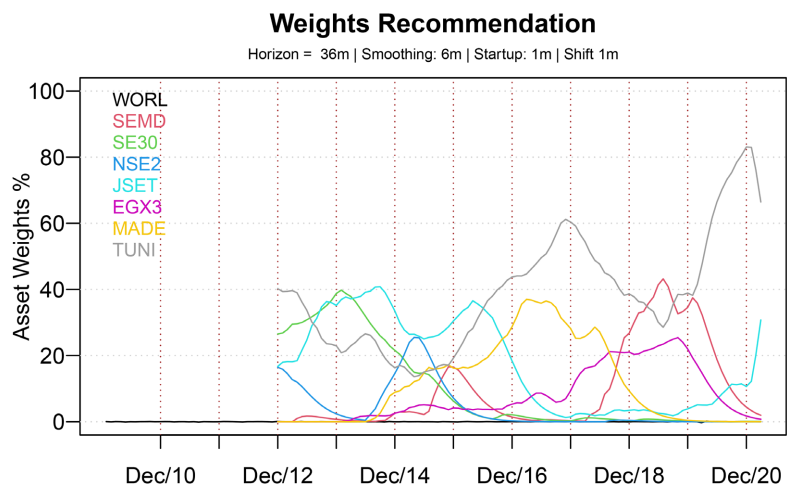

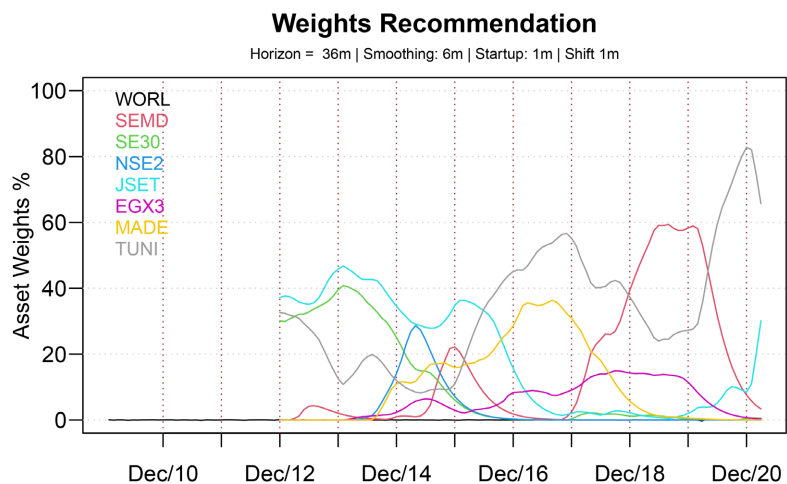**Figure 2.** Shrinked-MV portfolio weights recommendations.

sharply between 2019 and late 2020 and then began a downward trend as the study period ended. The SE30 showed a short rise between 2012 and 2013 before declining up to 2016. Thereafter it had close to zero in its weight recommendation. NSE20 shows a declining trend from 2012 to 2013 before shortly spiking between the year 2014 and 2015. Thereafter, it had close to zero weight in its allocation. EGX30 shows a steady gentle rise from 2012 to 2019 before declining towards the end of the study period. SEMDEX had close to zero allocation between the years 2012 and 2015. It shortly spikes between 2015 and end of 2016 before having close to zero allocation up to the year 2018. It shows a huge spike between 2018 and early 2020 before declining up to the end of the study period. MADEX showed a steady rise from the year 2014 to the year 2017 before declining up to the end of the study period.

Figure 4 shows the weights recommendation from the *OGK-MV*.
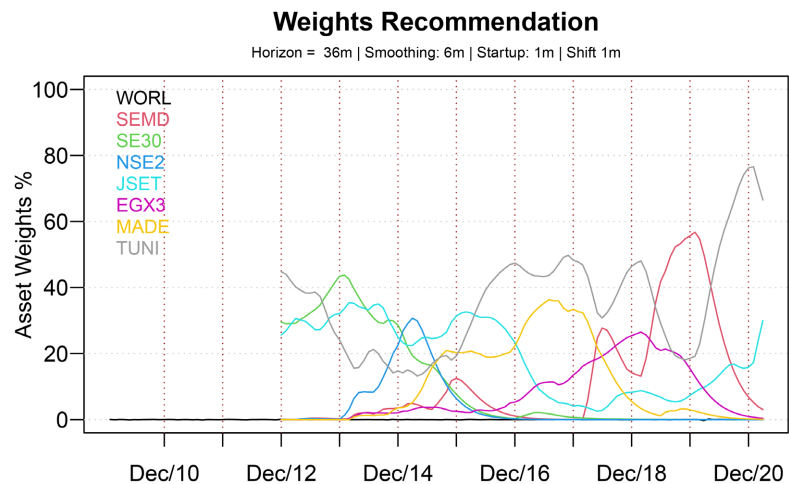
Figure 4 shows a lot of similarity to that of Figure 3 in terms the trends shown by the indices across the investment period. Figure 5 shows the weight recommendation from the *MCD-MV*.

Figure 3. MVE-MV portfolio weights recommendations.

Figure 4. OGK-MV portfolio weights recommendations.

Weights Recommendation

Horizon = 36m | Smoothing: 6m | Startup: 1m | Shift 1m

Figure 5. MCD-MV portfolio weights recommendations.

Figure 5 shows a similar trend to the one of Figure 3 too.

Discussion on the Weight Recommendations

From the charts on the weight recommendations from the optimization models, the study gained insight into the relative attractiveness of different markets to investors. At a glance, one would quickly say that investing in the Tunisian market would be more profitable since the TUNINDEX got more of the allocation across time from all the 5 optimizations. It implies that the Tunisian market gave better returns when the level of risk over time is minimized. One could also infer that over most of the period of investment, the Tunisian market must have been doing well hence the optimizations allocating more weight to it over time. The Tunisian market has been more consistent over time.

A look at the JSETOP40's allocations across the 5 optimizations showed that its proportions in the investment have generally been declining across the investment period. This could at a glance point out that the South African market has been more risky over the investment period, accounting for the low weight allocated to its index. On the brighter side, the JSETOP40 was the only index whose allocations in the later years of the investment showed an upward trajectory. It could imply that the South African market would be ideal for longer-term investments.

From the weights recommendations, the other indices like NSE20, EGX30 and the SE30 in the investment set had relatively lower allocations over time compared to indices like the TUNINDEX and JSETOP40. It could be a pointer of the high risk and lower returns associated with these markets. The indices also showed alternating periods of high allocations and periods of low allocation. The possibility of regime switches within their markets could be the reason for the occasional spikes in allocation. This also highlights the need for investors to always constantly rebalance portfolios over time as frequently as possible. This will ensure they place more of their investments on the assets that offer better return or lower risk for the period under consideration. However, this is subjective

judgment!

The study sought to compare the returns and risk profiles from the optimization models for better analysis of portfolio optimization performance. Also the net performances of the optimization models were compared among the optimization models and against two benchmarks.

4.3. Analysis and Comparison of the Optimization Models

4.3.1. Analysis of Return and Risk Profiles of the Optimizations

Table 2 gives a summary of the total return, standard deviation and Sharpe ratios of the 5 optimization models. The total return is the actual rate of return the portfolio generates over the entire period of back testing. The Sharpe ratios were calculated relative to a zero risk free rate as the total return risk adjusted by the standard deviation:

From Table 2, the *shrinked-MV* had the highest total return and the highest sharpe ratio taken relative to a zero risk-free rate. The *OGK-MV* had the least total return and hence sharpe ratio. The *shrinked-MV* was the only robustified model that outperformed the traditional MVO (*Min-Risk MV*) in terms of sharpe ratio. The *MVE-MV* is in per with the *Min-Risk MV* in terms of total returns and sharpe ratio.

4.3.2. Analysis of Optimization Net Performance

Table 3 shows the net performance of the optimization models in terms of the 1-month, 3-month, 6-month, 1-year, 3-year and 5-year gross returns and the 3-years and 5-year annualized returns:

From the findings in Table 3, it is noted that the 3-month gross return of all the robustified models was higher than the traditional MVO, except for the *OGK-MV* whose 3-month gross return was same as that of the *Min-Risk MV* (0.03). Two of the robustified optimization models have a higher 6-month gross return than that of the *Min-Risk MV* except for the *shrinked-MV* and *OGK-MV*, whose returns are per with the *Min-Risk MV* (0.06). All the robustified models have a higher 1 year gross return than the traditional MVO. The 3-year gross returns were all negative. The robustified models however, all had less negative 3-year gross returns compared to the traditional MVO. All the robustified models had higher 5-year gross returns compared to the *Min-Risk MV*. For the 3-year annualized return, the robustified optimization models performed at least as

Table 2. A summary of the return and risk characteristics from the optimization models.

	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV
Total Return	0.22	0.24	0.22	0.20	0.21
Mean Return	0.00	0.00	0.00	0.00	0.00
StdDev Return	0.03	0.03	0.03	0.03	0.03
Max. Loss	−0.22	−0.02	−0.21	−0.22	−0.22
Sharpe Ratio	7.33	8.00	7.33	6.66	7.00

much the traditional MVO, since the *MVE-MV* and the *MCD-MV* has less negative returns and the *shrinked-MV* and *OGK-MV* had the same level of return as the *Min-Risk MV*.

Table 4 shows the net portfolio performance per calender year for each of the optimization models.

From Table 4, in the year 2013, only the *shrinked-MV* has a higher gross return than the *Min-Risk MV*. In the years 2014, 2015 and 2018, none of the robustified models is able to outperform the traditional MVO in terms of gross return. It is however important to note that even though the robustified models do not outperform the *Min-Risk MV* in these years, their returns are never lower than those of the *Min-Risk MV*. In 2016, all the robustified models have higher returns compared to the gross return of the *Min-Risk MV* except for the *MVE-MV*, whose gross return is the same as that of the *Min-Risk MV*. In 2017, only the *MCD-MV* has a higher return than the *Min-Risk MV*. All the other robustified models have

Table 3. A comparison of per period gross returns and annualized returns.

	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV
1-month	0.05	0.05	0.05	0.05	0.05
3-month	0.03	0.04	0.04	0.03	0.04
6-month	0.06	0.06	0.07	0.06	0.07
1-year	0.10	0.11	0.12	0.11	0.14
3-year	−0.13	−0.11	−0.09	−0.11	−0.09
5-year	0.08	0.10	0.11	0.10	0.12
3 years p.a	−0.04	−0.04	−0.03	−0.04	−0.03
5 years p.a	0.02	0.02	0.02	0.02	0.02

Table 4. A comparison of the optimization net portfolio gross return performance per calendar year.

	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV
2013	0.18	0.19	0.17	0.17	0.15
2014	0.03	0.03	0.02	0.02	0.01
2015	−0.10	−0.11	−0.11	−0.11	−0.11
2016	0.08	0.09	0.08	0.09	0.09
2017	0.12	0.12	0.12	0.12	0.13
2018	0.04	0.02	0.01	0.03	0.01
2019	−0.01	−0.01	0.01	0.00	0.03
2020	−0.16	−0.15	−0.12	−0.14	−0.14
YTD	0.03	0.04	0.04	0.03	0.04
Total	0.22	0.24	0.22	0.20	0.21

a return in per with the traditional MV in 2017. In 2019, the *MVE-MV*, *OGK-MV* and *MCD-MV* models have higher gross return than the *Min-Risk MV*. In 2020 when the returns from the optimizations were all negative, the robustified optimization models all had lesser negative returns than the *Min-Risk MV*. Three of the robustified models had a Year-To-Date return higher than the traditional MVO, with the *OGK-MV*'s YTD return being in per with the *Min-Risk MV*.

4.3.3. Discussion on the Optimization Models

The *shrinked-MV* is the only model with a higher total return and hence sharpe ratio compared to the *Min-Risk MV*. The *MVE-MV* performs in per with the *Min-Risk MV*, while the *OGK-MV* and *MCD-MV* under-perform the traditional MVO in terms of sharpe ratio. These results imply that for an investor seeking to maximize their risk-adjusted returns, the *shrinked-MV* would be the ideal optimization model to use. The investor would be indifferent between the *MVE-MV* and the *Min-Risk MV* in terms of sharpe ratio.

The robustified models always performed better than the *Min-Risk MV*, and in the instances when they did not outperform the *Min-Risk MV* in terms of per period gross returns and annualized returns, they performed in per with it. This implied that across the investment horizon, the robustified optimization models showed more certainty to outperform the traditional MVO and in the worst case would always give same returns as the traditional MVO. There is assurance that using robust estimators in optimization will always give returns higher than the *Min-Risk MV* and never lower. Prudent investors would prefer the robustified optimization to the traditional MVO.

Except for the years 2014, 2015 and 2018 when none of the robustified models outperformed the traditional MVO in terms of gross return, there was always atleast a robustified model that outperformed the *Min-Risk MV* in all the other years. All the robustified models outperform the traditional MVO in 2020, and three of the four models outperform the *Min-Risk MV* in 2019. Three of the robustified models outperform the traditional MVO in terms of the Year-To-Date returns from the entire investment period, while the remaining one (the *MVE-MV*) performs in per with the *Min-Risk MV*. This is evidence of the superior performance of the robustified optimization models across time, further vouching in favour of the robustified optimization models. Robust estimators showed that they would always give higher returns than the classical MLE of the *Min-Risk MV* and even in the worst case, their returns will never be lower than those of the *Min-Risk MV*.

4.4. Comparison of the Optimizations against Benchmarks

As robustness checks, the 5 rebalanced optimization models were compared against two benchmarks; the MSCI World Index (benchmark 1) and the MSCI Emerging and Frontier Market Index (benchmark 2). These indices were chosen since they are world-widely acceptable indices to be used as benchmarks. These

comparisons will help assess the performance of the portfolio in the study relative to recognized benchmark indices.

4.4.1. Comparison against the MSCI World Index

Table 5 gives a comparison between the MSCI World Index and the optimizations in terms of the total return, standard deviation and sharpe ratio.

The MSCI World Index significantly outperformed the 5 optimization models in terms of the total return and the Sharpe ratio. It however has higher risk as measured by the standard deviation of returns.

Table 6 gives a comparison between per period gross and annualized returns of the 5 optimization models and those of the MSCI World Index.

It is again noted that all the period returns from the MSCI World Index are higher than those from the optimization models. Table 7 shows the net portfolio performance of the optimization models per calendar year against the net performance of the MSCI World Index.

Table 7 shows that the MSCI World Index outperformed the gross returns and YTDs from the optimizations in all the years when performance was evaluated. The index was significantly superior to the portfolio regardless of the optimization model used. This is illustrated in Figure 6 with a summary of each optimization's performance over time against the MSCI World Index.

Table 5. A comparison of the return and risk characteristics from the optimizations with the risk and return characteristics of the MSCI World Index.

|	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV	Benchmark 1
Total return	0.22	0.24	0.22	0.20	0.21	0.85
Mean return	0.00	0.00	0.00	0.00	0.00	0.01
StdDev return	0.03	0.03	0.03	0.03	0.03	0.05
Max. loss	−0.22	−0.02	−0.21	−0.22	−0.22	−0.21
Sharpe ratio	7.33	8.00	7.33	6.66	7.00	17.00

**Table 6.** A comparison of per period gross returns and annualized returns against the MSCI World Index.

	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV	Benchmark 1
1-month	0.05	0.05	0.05	0.05	0.05	0.00
3-month	0.03	0.04	0.04	0.05	0.04	0.05
6-month	0.06	0.06	0.07	0.06	0.07	0.17
1-year	0.10	0.11	0.12	0.11	0.14	0.54
3-year	−0.13	−0.11	−0.09	−0.11	−0.09	0.37
5-year	0.08	0.10	0.11	0.10	0.12	0.63
3 years p.a	−0.04	−0.04	−0.03	−0.04	−0.03	0.12
5 years p.a	0.02	0.02	0.02	0.02	0.02	0.13

A drawdown is a peak-to-trough decline during a specific period for an investment, trading account, or fund, usually quoted as the percentage between the peak and the subsequent trough. According to Marakbi (2016), it measures the maximum loss from a peak to a nadir over a period of time of a portfolio, and

Table 7. A comparison of the optimization net portfolio performance per calendar year against the MSCI World Index.

	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV	Benchmark 1
2013	0.18	0.19	0.17	0.17	0.15	0.18
2014	0.03	0.03	0.02	0.02	0.01	0.06
2015	−0.10	−0.11	−0.11	−0.11	−0.11	−0.04
2016	0.08	0.09	0.08	0.09	0.09	0.08
2017	0.12	0.12	0.12	0.12	0.13	0.18
2018	0.04	0.02	0.01	0.03	0.01	−0.09
2019	−0.01	−0.01	0.01	0.00	0.03	0.25
2020	−0.16	−0.15	−0.12	−0.14	−0.14	0.22
YTD	0.03	0.04	0.04	0.03	0.04	0.05
Total	0.22	0.24	0.22	0.20	0.21	0.85

(a)

(b)

(c)

(d)
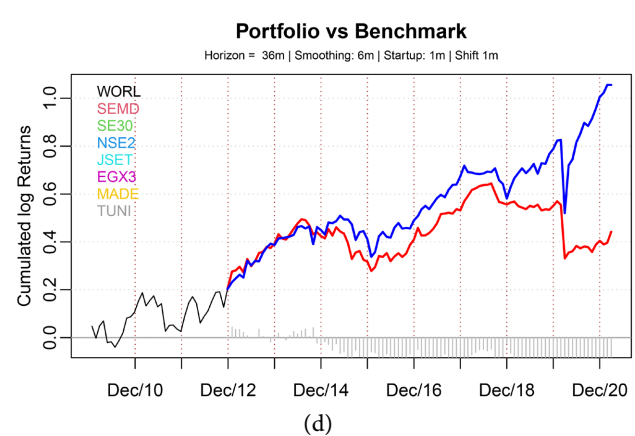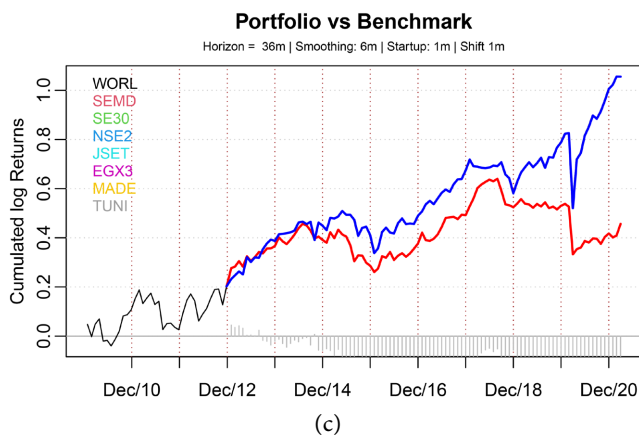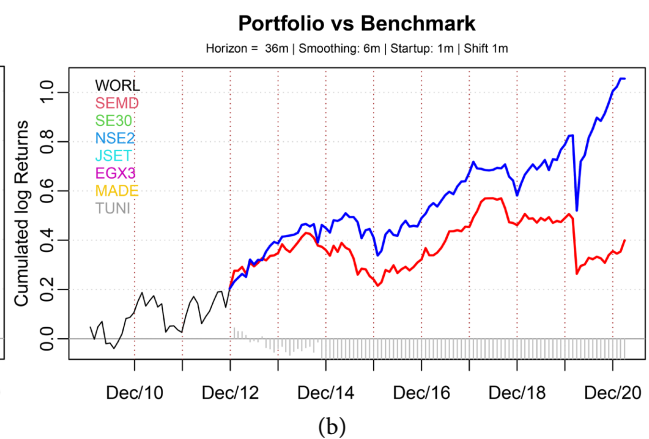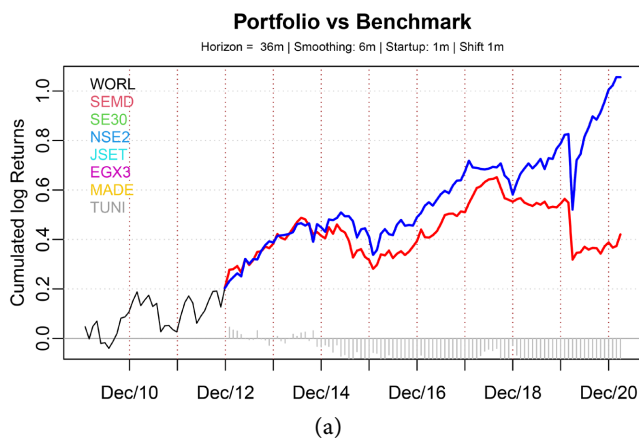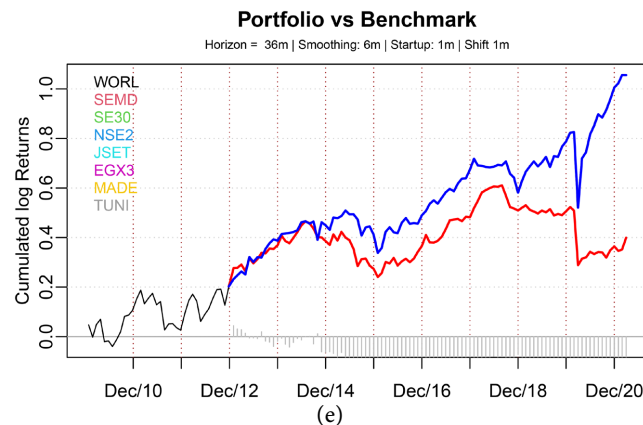
Figure 6. A comparison of the net portfolio performance from the 5 optimization models with the MSCI World Index. (a) A comparison of the net portfolio performance per calendar year of the Min-Risk MV with the MSCI world Index; (b) A comparison of the net portfolio performance per calendar year of the MCD-MV with the MSCI world Index; (c) A comparison of the net portfolio performance per calendar year of the MVE-MV with the MSCI world Index; (d) A comparison of the net portfolio performance per calendar year of the Shrinked-MV with the MSCI world Index; (e) A comparison of the net portfolio performance per calendar year of the OGK-MV with the MSCI world Index

complements the notion of using volatility well as it is an indicator of downside risk. In other words, it measures the maximum accumulated loss that an investor may suffer from buying high and selling low. **Figure 7** shows a comparison of the drawdowns from the optimization models against the MSCI World Index drawdowns.

All the optimizations performed worse than the MSCI World Index with their drawdown returns being more negative compared to the MSCI world Index. This implies that impacts from peak-to-low movements in the portfolio investment's value are more severe for the optimization models compared to the MSCI World Index.
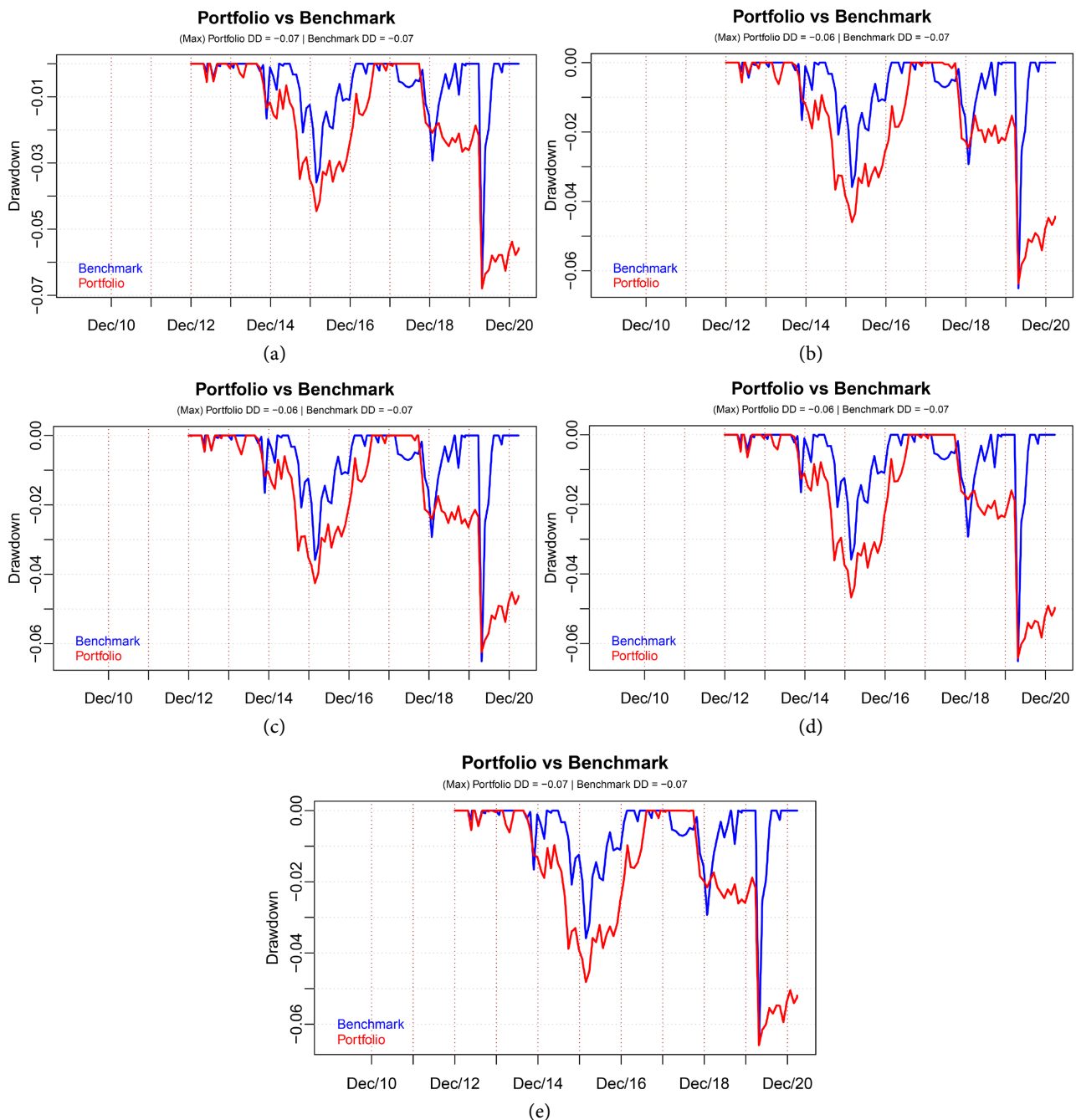
4.4.2. Comparison against the MSCI Emerging and Frontier Market Index
Table 8 gives a comparison between the MSCI Emerging and Frontier Market Index and the optimizations in terms of the total return, standard deviation and sharpe ratio.

The MSCI Emerging and Frontier Market Index also significantly outperformed the 5 optimizations in terms of the total return and the Sharpe ratio. It however has higher risk as measured by the standard deviation of returns.

Table 9 gives a comparison between per period gross and annualized returns of the 5 optimizations and those of the MSCI Emerging and Frontier Market Index.

It was noted that all the period returns from the MSCI Emerging and Frontier Market Index were higher than those from the optimizations, except the one month gross return which was lower. Still, the index significantly outperformed all the optimizations in terms of per period returns. **Table 10** shows the net portfolio performance of the optimizations per calendar year against the net performance of the MSCI Emerging and Frontier Market Index.

Figure 7. A comparison of the net portfolio drawdown performance from the 5 optimization models with the MSCI World Index. (a) A comparison of the net portfolio drawdown performance per calendar year of the Min-Risk MV with the MSCI World Index; (b) A comparison of the net portfolio drawdown performance per calendar year of the MCD-MV with the MSCI World Index; (c) A comparison of the net portfolio drawdown performance per calendar year of the MVE-MV with the MSCI World Index; (d) A comparison of the net portfolio drawdown performance per calendar year of the Shrinked-MV with the MSCI World Index; (e) A comparison of the net portfolio drawdown performance per calendar year of the OGK-MV with the MSCI World index

Table 10 shows that the MSCI Emerging and Frontier Market Index under performed in 2013 since it had the least gross return compared to all the optimization models. In 2014, the *Min-Risk MV* and *shrinked-MV* outperformed the index's gross return, while the *MVE-MV* and *OGK-MV* performed in per with

Table 8. A comparison of the return and risk characteristics from the optimizations with the risk and return characteristics of the MSCI Emerging and Frontier Market Index.

	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV	Benchmark 2
Total return	0.22	0.24	0.22	0.20	0.21	0.60
Mean return	0.00	0.00	0.00	0.00	0.00	0.01
StdDev return	0.03	0.03	0.03	0.03	0.03	0.05
Max. loss	−0.22	−0.02	−0.21	−0.22	−0.22	−0.25
Sharpe ratio	7.33	8.00	7.33	6.66	7.00	12.00

Table 9. A comparison of per period gross returns performance and annualized returns performance against the MSCI Emerging and Frontier Market Index.

	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV	Benchmark 2
1-month	0.05	0.05	0.05	0.05	0.05	−0.05
3-month	0.03	0.04	0.04	0.05	0.04	0.07
6-month	0.06	0.06	0.07	0.06	0.07	0.19
1 year	0.10	0.11	0.12	0.11	0.14	0.52
3-year	−0.13	−0.11	−0.09	−0.11	−0.09	0.32
5-year	0.08	0.10	0.11	0.10	0.12	0.59
3 years p.a	−0.04	−0.04	−0.03	−0.04	−0.03	0.11
5 years p.a	0.02	0.02	0.02	0.02	0.02	0.12

Table 10. A comparison of the optimization net portfolio performance per calendar year against the MSCI Emerging and Frontier Market Index.

|	Min-Risk MV	shrinked-MV	MVE-MV	OGK-MV	MCD-MV	Benchmark 2
2013	0.18	0.19	0.17	0.17	0.15	0.01
2014	0.03	0.03	0.02	0.02	0.01	0.02
2015	−0.10	−0.11	−0.11	−0.11	−0.11	−0.06
2016	0.08	0.09	0.08	0.09	0.09	0.08
2017	0.12	0.12	0.12	0.12	0.13	0.23
2018	0.04	0.02	0.01	0.03	0.01	−0.11
2019	−0.01	−0.01	0.01	0.00	0.03	0.15
2020	−0.16	−0.15	−0.12	−0.14	−0.14	0.19
YTD	0.03	0.04	0.04	0.03	0.04	0.07
Total	0.22	0.24	0.22	0.20	0.21	0.60

the index. In 2016, none of the optimization models under-performed the index. In 2018, all the optimization models outperformed the MSCI Emerging and Frontier Market Index! In 2019 and 2020, the index outperformed all the opti-

mization models. The YTDs from the all the optimization models were lower than the index's YTD. Figure 8 shows a summary of each optimization's performance over time against the MSCI Emerging and Frontier Market Index.
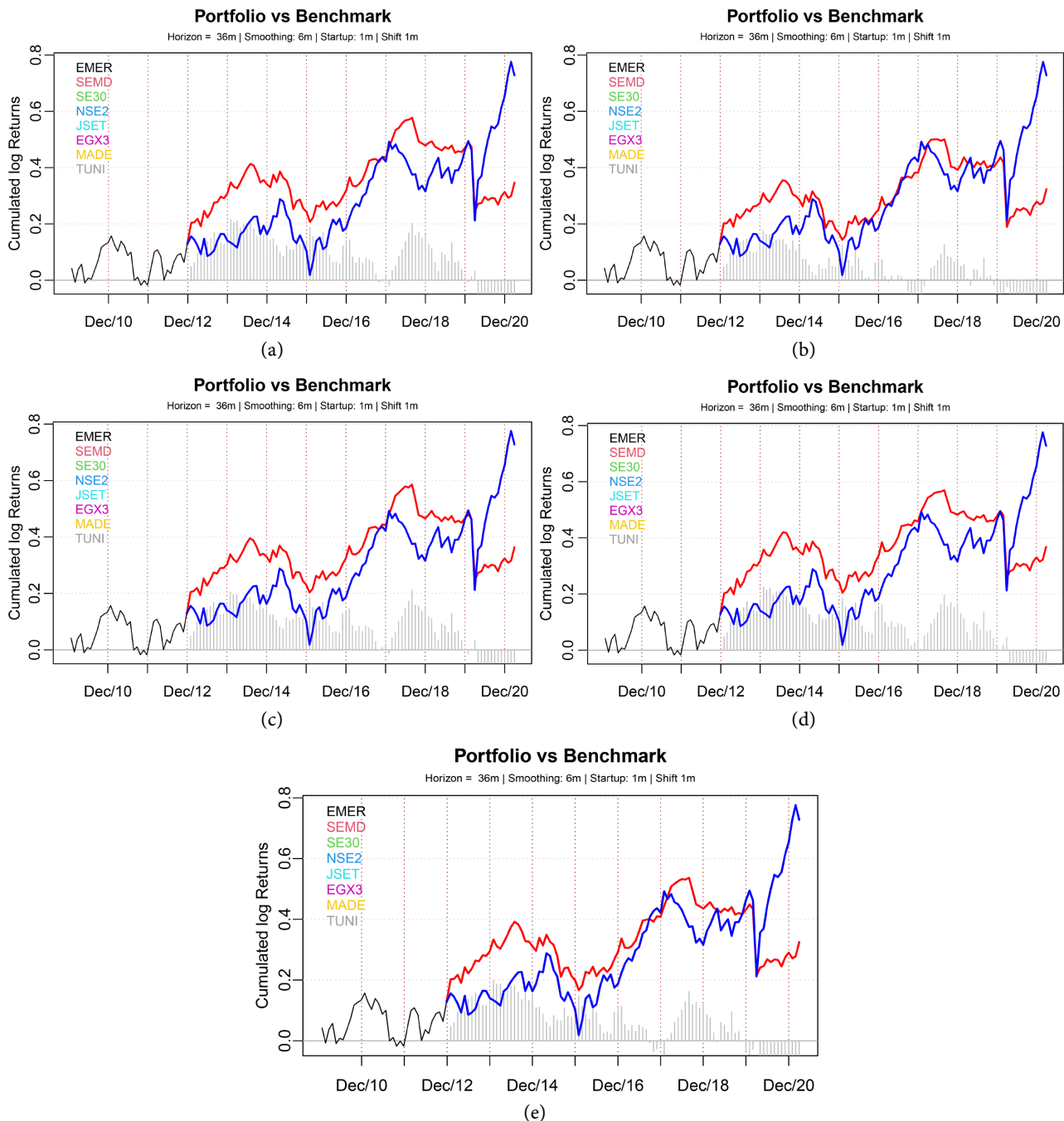
Figure 8. A comparison of the net portfolio performance from the 5 optimization models with the MSCI Emerging and Frontier Market Index. (a) A comparison of the net portfolio performance per calendar year of the Min-Risk MV with the MSCI Emerging and Frontier World Index; (b) A comparison of the net portfolio performance per calendar year of the MCD-MV with the MSCI Emerging and Frontier World Index; (c) A comparison of the net portfolio performance per calendar year of the MVE-MV with the MSCI Emerging and Frontier World Index; (d) A comparison of the net portfolio performance per calendar year of the Shrinked-MV with the MSCI Emerging and Frontier World Index; (e) A comparison of the net portfolio performance per calendar year of the OGK-MV with the MSCI Emerging and Frontier World Index.

Figure 8 shows some periods when the optimization models perform better than the MSCI Emerging and Frontier Market Index in terms of portfolio returns. This is good news for investors especially in Africa. Figure 9 shows the
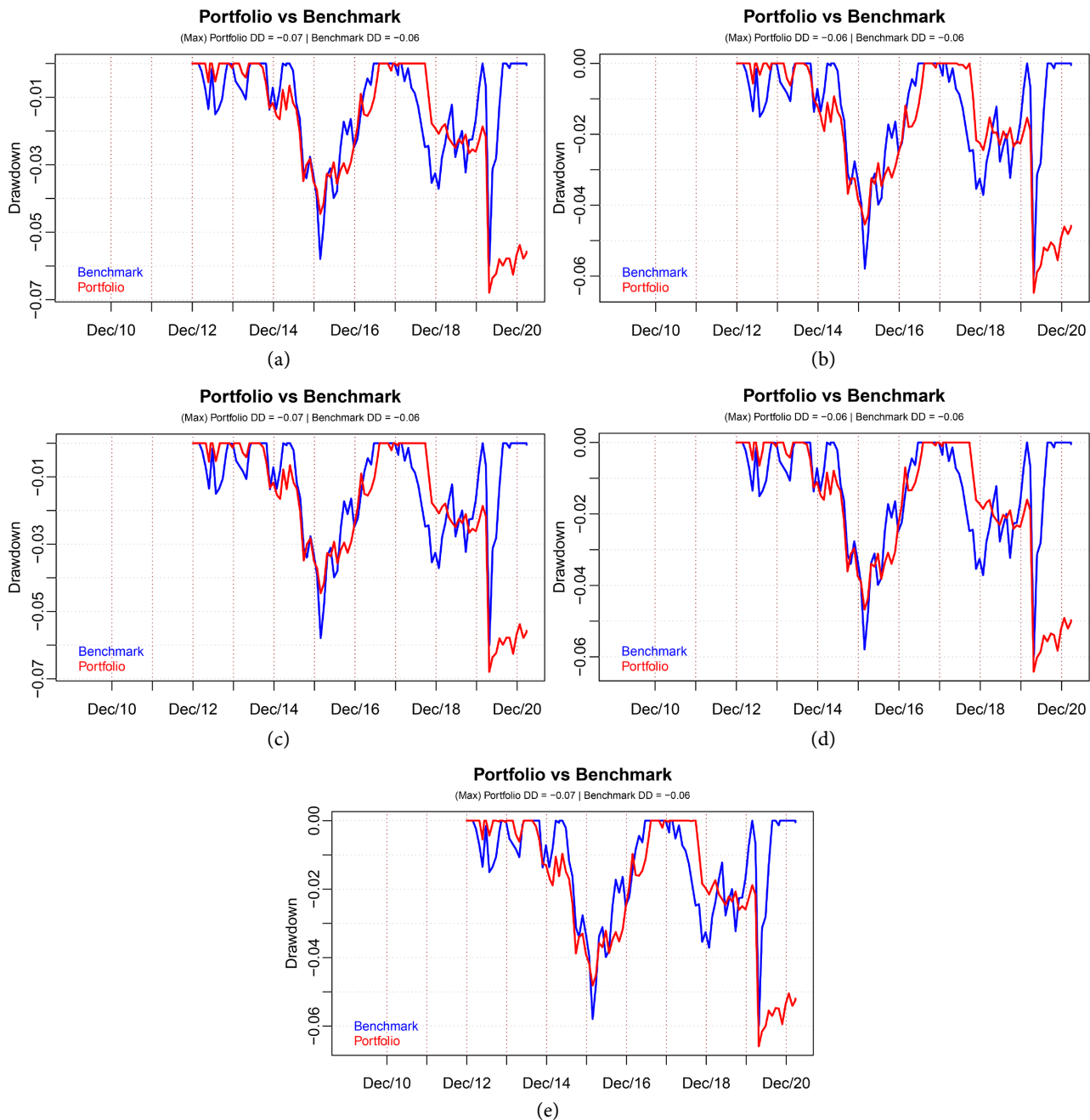
Figure 9. A comparison of the net portfolio drawdown performance from the 5 optimization models with the MSCI Emerging and Frontier Market Index. (a) A comparison of the net portfoliodrawdown performance per calendar year of the Min-Risk MV with the MSCI Emerging and Frontier World Index; (b) A comparison of the net portfolio drawdown performance per calendar year of the MCD with the MSCI Emerging and Frontier World Index; (c) A comparison of the net portfolio drawdown performance per calendar year of the MVE-MV with the MSCI Emerging and Frontier World Index; (d) A comparison of the net portfolio drawdown performance per calendar year of the Shrinked-MV with the MSCI Emerging and Frontier World Index; (e) A comparison of the net portfolio drawdown performance per calendar year of the OGK-MV with the MSCI Emerging and Frontier World Index

comparison in drawdowns between the optimization models and the index.

Figure 9 shows that the optimization models perform better than the MSCI Emerging and Frontier Market Index on numerous occasions. This means that the impacts of peak-to-trough declines would be felt less when using the optimization models as compared to the MSCI Emerging and Frontier Market Index.

4.4.3. Discussion on Performance against the Benchmark Indices

It was noted that there was general under-performance of the optimization models in terms of total return and sharpe ratio when compared to both indices. The MSCI World Index's performance in terms of the period returns and returns across the investment back testing years was dominant over the optimization models' performance. The index's drawdowns were also less severe compared to the optimization models. These findings point out that the portfolio formed from the seven indices in the investment set had a very poor record relative to the World Index.

Against the MSCI Emerging and Frontier Market Index, the portfolio had mixed fortunes. The index outperformed the optimization models in terms of the total return and sharpe ratios. The index's one-month gross return however was the least when compared to all the optimization models, although it still outperforms the optimizations in the rest of the period returns. The index's gross return in 2013 is the least when compared to all the models. In 2014, two of the optimization models outperform the index. All the optimization models perform at least as much as the MSCI Emerging and Frontier Market Index, and in 2018, they all outperform the index. The index however outperforms all the models in terms of YTD. On numerous occasions, the portfolio optimization models' drawdowns were less than those of the index.

The results from the optimization models' performance against the MSCI Emerging and Frontier Market Index give investors some belief that the Emerging and Frontier markets in Africa could compete favorably against other emerging markets across other continents. Since the study also found ground to suggest that robustified MVO models show better qualities than the traditional MVO, it would be interesting to see how future portfolios formed from more African economies and optimized using robustified models would perform against other developed economies.

5. Conclusion and Recommendations

5.1. Conclusion

Markowitz's mean-variance optimization framework has become the asset allocation model of choice over the last 50 years. Unfortunately, studies have shown that the model often suffers the problem of estimation error in its sample covariance estimator, and that is why practitioners haven't fully embraced it. Diversification benefits have so far been analyzed for internationally diversified portfolios from the perspectives of investors based in developed markets. Studies rec-

ommend robustifying MVOs by using different covariance estimators. These estimators are able to handle data containing outliers with much ease. This study used the *Minimum Covariance Determinant estimator*, the *Minimum Volume Ellipsoid estimator*, and the *Orthogonalized Gnanadesikan-Kettenring estimator* for large covariance matrices and the *shrinkage* estimator to assess their influence on the portfolio formed from its investment set. The investment set was formed from 2 emerging markets and 5 frontier market economies in Africa.

The study found that in terms of the one-month, three-month, six-month, one-year, three-year and five-year gross returns, the three-year and 5-year annualized returns and YTDs, the robustified models gave higher returns than the traditional MV model. The robust estimators ensured the optimization always gave higher returns and in the worst-case scenario, these returns were in per with the *Min-RiskMV*'s returns. These returns were never lower than the traditional MV model. Portfolio net performance per calendar year was also examined in the study. The portfolio performance was evaluated between the years 2013 to 2020. In 5 out of the 8 years when net portfolio performance was evaluated, the robustified models always had higher returns compared to the *Min-Risk MV*. In the years that the returns were not higher than those of the *Min-Risk MV*, they were in per with the *Min-Risk MV*'s returns. They never went lower than the returns from the traditional MV. From these findings, the study concluded that robust estimators gave assurance of better performance compared to the classical MLE of the MV framework.

When compared against the MSCI World Index, the traditional, as well as the robustified models, seem to significantly under-perform the index in terms of gross returns, YTDs and drawdowns. However, against the MSCI Emerging and Frontier Market, the optimizations show instances when they outperform the gross return and drawdown characteristics of the index. The portfolio considered in the study showed that it could compete favorably against other emerging and frontier market economies not necessarily found within Africa. This is encouraging for the African investors.

5.2. Recommendations

From the findings in the study, it is seen that the development of a robust investment set is a critical step in the strategic asset allocation process; yet, all too often, its importance is overlooked. In general, investors should be encouraged to expand their investment sets so as to benefit more from diversification. A similar study could be carried out to include other African, Latin American and Asian emerging and frontier markets. It would also be of interest to conduct a similar study under more constrained optimization to assess whether there would be a significant difference in their results. It would be interesting to assess the effect of the addition of transaction costs to the optimization processes considered in this study when doing further research.

Since asset allocation has been shown to be the main determinant of portfolio performance, this study recommends that a similar study is extended to other

asset allocation heuristics that are not necessarily Markowitz based. The portfolio performance of these simple rules of thumb in asset allocation should then be compared with the performance of the Markowitz-based models, so as to find a better asset allocation strategy. The study recommends that priority be given to the African emerging and frontier markets with an increased investment set.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Beaulieu, E. A. (2015). *What Dependency Issues? Re-Examining Assumptions about Canada's Reliance on the US Export Market.* School of Public Policy Research Papers, University of Calgary.

Bekaert, G. A., & Urias, M. S. (1996). Diversification, Integration and Emerging Market Closed-End Funds. *The Journal of Finance, 51,* 835-869. https://doi.org/10.1111/j.1540-6261.1996.tb02709.x

Brinson, G. P., Randolph Hood, L., & Beebower, G. L. (1986). Determinants of Portfolio Performance. *Financial Analysts Journal, 42,* 39-44. https://doi.org/10.2469/faj.v42.n4.39

Coggeshall, S. A., & Wu, G. (2005). Asset Allocation and Long-Term Returns: An Empirical Approach. *SSRN*, Article ID: 873184. https://doi.org/10.2139/ssrn.873184

De Roon, F. A., Nijman, T. E., & Werker, B. J. M. (2001). Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets. *The Journal of Finance, 56,* 721-742. https://doi.org/10.1111/0022-1082.00343

De Santis, G. A., & Gerard, B. (1997). International Asset Pricing and Portfolio Diversification with Time-Varying Risk. *The Journal of Finance, 52,* 1881-1912. https://doi.org/10.1111/j.1540-6261.1997.tb02745.x

DeMiguel, V. A., & Nogales, F. J. (2009b). Portfolio Selection with Robust Estimation. *Operations Research, 57,* 560-577. https://doi.org/10.1287/opre.1080.0566

DeMiguel, V. A., Garlappi, L., & Uppal, R. (2009a). Optimal versus Naive Diversification: How Inefficient Is the $1/N$ Portfolio Strategy? *The Review of Financial Studies, 22,* 1915-1953. https://doi.org/10.1093/rfs/hhm075

Driessen, J. A., & Laeven, L. (2007). International Portfolio Diversification Benefits: Cross-Country Evidence from a Local Perspective. *Journal of Banking & Finance, 31,* 1693-1712. https://doi.org/10.1016/j.jbankfin.2006.11.006

Erb, C. B., Harvey, C. R., & Viskanta, T. E. (1994). Forecasting International Equity Correlations. *Financial Analysts Journal, 20,* 32-45. https://doi.org/10.2469/faj.v50.n6.32

Errunza, V., Hogan, K., & Hung, M.-W. (1999). Can the Gains from International Diversification Be Achieved without Trading Abroad. *The Journal of Finance, 54,* 2075-2107. https://doi.org/10.1111/0022-1082.00182

Farrelly, T. (2006). Asset Allocation for Robust Portfolios. *The Journal of Investing, 15,* 53-63. https://doi.org/10.3905/joi.2006.669100

Goetzmann, W. N. (2005). *Why Do Individual Investors Hold Under-Diversified Portfolios?* Yale School of Management.

Goetzmann, W. N., Li, L., & Geert Rouwenhorst, K. (2001). *Long-Term Global Market Correlations* (Working Paper No. 8612). National Bureau of Economic Research.

https://doi.org/10.3386/w8612

Gupta, R. (2006). Benefits of Diversification into Emerging Equity Markets with Changing Correlations: An Australian Perspective. *International Review of Business Research Papers, 2,* 22-38.

Harvey, C. R. (1995). Predictable Risk and Returns in Emerging Markets. *The Review of Financial Studies, 8,* 773-816. https://doi.org/10.1093/rfs/8.3.773

Hicks, J. R. (1962). Liquidity. *The Economic Journal, 72,* 787-802. https://doi.org/10.2307/2228351

Huber, P. J. (2009). *Robust Statistics* (2nd ed.). John Wiley & Sons. https://doi.org/10.1002/9780470434697

Ibbotson, R. G. (2010). The Importance of Asset Allocation. *Financial Analysts Journal, 66,* 18-20. https://doi.org/10.2469/faj.v66.n2.4

Ibbotson, R. G., & Kaplan, P. D. (2000). Does Asset Allocation Policy Explain 40, 90, or 100 Percent of Performance? *Financial Analysts Journal, 56,* 26-33. https://doi.org/10.2469/faj.v56.n1.2327

Idzorek, T. M. (2006). *Developing Robust Asset Allocations.* Citeseer.

Jacobs, H. A., Müller, S., & Weber, M. (2009). How Should Private Investors Diversify? An Empirical Evaluation of Alternative Asset Allocation Policies. *Journal of Financial Markets, 19,* 62-85. https://doi.org/10.2139/ssrn.1471955

Lauprete, G. A. (2003). Robust Portfolio Optimization. In R. Dutter, P. Filzmoser, U. Gather, & P. J. Rousseeuw (Eds.), *Developments in Robust Statistics* (pp. 235-245). Springer. https://doi.org/10.1007/978-3-642-57338-5_20

Leavens, D. H. (1945). Diversification of Investments. *Trusts and Estates, 80,* 469-473.

Longin, F. A., & Solnik, B. (2001). Extreme Correlation of International Equity Markets. *The Journal of Finance, 56,* 649-676. https://doi.org/10.1111/0022-1082.00340

Mao, J. C. (1970). Essentials of Portfolio Diversification Strategy. *Journal of Finance, 25,* 1109-1121. https://doi.org/10.1111/j.1540-6261.1970.tb00871.x

Marakbi, Z. (2016). *Mean-Variance Portfolio Optimization: Challenging the Role of Traditional Covariance Estimation.* Master's Thesis, KTH Royal Institute of Technology Stockholm, Stockholm.

Markowitz, H. M. (1999). The Early History of Portfolio Theory: 1600-1960. *Financial Analysts Journal, 55,* 5-16. https://doi.org/10.2469/faj.v55.n4.2281

Markowitz, H. M. (2002). *The Theory and Practice of Investment Management.* John Wiley & Sons.

Marshall, B. R., Nguyen, N. H., & Visaltanachoti, N. (2013). Liquidity Commonality in Commodities. *Journal of Banking & Finance, 37,* 11-20. https://doi.org/10.1016/j.jbankfin.2012.08.013

Perold, A. F., & Sharpe, W. F. (1995). Dynamic Strategies for Asset Allocation. *Financial Analyst Journal, 51,* 149-160. https://doi.org/10.2469/faj.v51.n1.1871

Rosadi, D. A., Putranda Setiawan, E., Templ, M., & Filzmoser, P. (2020). Robust Covariance Estimators for Mean-Variance Portfolio Optimization with Transaction Lots. *Operations Research Perspectives, 7,* Article ID: 100154. https://doi.org/10.1016/j.orp.2020.100154

Solnik, B. H. (1977). Testing International Asset Pricing: Some Pessimistic Views. *The Journal of Finance, 32,* 503-512. https://doi.org/10.1111/j.1540-6261.1977.tb03288.x

Welsch, R. E., & Zhou, X. (2007). Application of Robust Statistics to Asset Allocation Models. *REVSTAT-Statistical Journal, 5,* 97-114.

List of Abbreviations

MV	Mean Variance
GNP	Gross National Product
CFI	Corporate Finance Institute
IFC	International Finance Corporation
MVO	Mean Variance Optimization
CAPM	Capital Asset Pricing Model
SEPE	frontier-Simulated Ex-Post Efficient frontier
CEQ	Certainty Equivalent
MSCI	Morgan Stanley Capital International
MLE	Maximum Likelihood Estimator
Min-Risk MV	The minimum risk mean-variance model
MCD-MV	The Minimum Covariance Determinant robustified mean-variance model
MVE-MV	The Minimum Volume Ellipsoid (MVE) robustified mean-variance model
OGK-MV	The Orthogonalized Gnanadesikan-Kettenring (OGK) robustified Mean-Variance model
Shrinked-MV	The Shrinked Mean-Variance model
VaR	Value at Risk
CVaR	Conditional Value at Risk
ES	Expected Shortfall
StdDev	Standard Deviation
YTD	Year-To-Date