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Empirical Analysis of Potential Put-Call Parity Arbitrage Opportunities with Particular Focus on the Shanghai Stock Exchange 50 Index

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Abstract

Put-Call-Parity is a major cornerstone of the option pricing theory. The equation provides an answer to the equilibrium of the option market. It tells us what the right call option price should be assuming put price, actual stock price, risk free rate and maturity. The call price depends on these parameters. No arbitrage opportunities are possible if the equilibrium equation is met. In financially well developed countries and regions the put-call-parity holds and allows no arbitrage opportunities except in abnormal market conditions. This paper aims to analyse the put-call-parity in China for a certain period of time. It reviews if arbitrage opportunities can be identified. It shows that the put-call-parity dominates the option market in China as well despite shorter periods in the development of the financial markets and allows no arbitrage opportunities.

Keywords

Put-Call-Parity, Put-Call-Arbitrage, Volatility Indices, Volatility Smile, Volatility Trading

1. Literature Review

When investors hold a financial asset and are worried about the risks caused by asset price fluctuations, they can buy and sell options in advance to hedge the risk. For example, when investors predict that the market price will fall, they can purchase put options to hedge, the only cost accruing to them consisting of the premium for buying the option. Therefore, option pricing is of great significance in risk assessment and risk control.

Bachelier (1900) gives a strict mathematical description of the trend of stock prices and assumes that stock price change follows a standard Brownian motion, the key insight being the following prediction: If, there is an identifiable pattern of asset prices in the short run, investors will find it and exploit it, thereby eliminating it. Supposing that investors do not care about risk, Boness (1964) assumes a fixed logarithmic distribution of stock returns. Samuelson and Merton (1969) test the option pricing theory with a simple equilibrium model of product portfolio selection and prove that the public probability can present an option problem. Black and Scholes (1973) develop the Black-Scholes-Merton pricing model (BSM model) for European stock options. Considering the BSM model, the volatility σ of a stock is a measure of uncertainty about the returns provided by the stock and cannot be observed directly. However, the BSM model assumes that stock prices are log-normally distributed, which "implies that stock log-prices are normally distributed (Corrado & Su, 1997)". Therefore, considering the development trend of the relationship between strike prices and corresponding implied volatilities, a flat curve can be drawn, which does not represent a volatility smile effect or leverage ratio.

In actual practice, however, the model often prices deep-in-the-money and deep-out-of-the-money options inconsistently. When drawing a curve for the implied volatilities in a range of strike prices, a U-shaped curve resembles a smile appears. This anomalous pattern is called volatility smile or skew. The smile effect is systematically related to the degree to which the options are in or out of the money.

Assuming call and put options have the same expiration date and strike price, Stoll (1969) demonstrates a variety of combinations of European call and put, long and short positions in the stock market which may bring about positions of varying degrees of risk and expected return referring to arbitrage opportunities.

Kamara and Miller (1995) demonstrate that the arbitrage opportunities of index call and put options in the US market may not be easily exploited due to liquidity risk. Given the short-selling restrictions in Germany, Mittnik and Rieken (2000) conclude that arbitrage opportunities are severely limited in practice by examining the data for options in the German stock index (DAX). Due to the strict rules applying to short positions in the Chinese market, Wang (2006) concludes that there exists arbitrage space but no arbitrage opportunities. Zhao and Gu (2015) examine the efficiency of the CSI 300 index options market by using a strategy of Put-Call Parity and conclude that the CSI 300 index options market is not efficient. Therefore, the ex-post and ex-ante arbitrage may earn significant profits, which shows that it is limited rationality of investors that explains the low pricing efficiency of CSI 300 index options market.

Xian and Liu (2016) empirically analyze the option arbitrage path and risk strategy of the SSE 50 ETF in China and point out that there exist arbitrage opportunities by examining the option pricing difference under conditions of incomplete information, thereby avoiding interest rate fluctuations and increasing the yield of the option portfolio.

Li and Zhang (2016) emphasize the transaction risk of SSE 50 ETF options, including the huge number of margins based on the change of price of SSE 50 ETF and the significant fluctuation influenced by the change in intrinsic value.

Shan and Zheng (2017) demonstrate that risk-free arbitrage opportunities based on price convexity can be used to make a profit in the Chinese market and that arbitrage transactions are conducive to the formation of fairer prices in the market, which, in turn, can improve market liquidity and curb excessive speculation.

Based on the transaction data of SSE 50 ETF in China from 2015 to 2016, Lei and Wu (2017) construct a put-call parity strategy and empirically examine the arbitrage space of options. By using the Tobit model, analysing the relationship between the arbitrage space and the transaction behavior of the stock market and option market, the pricing of options was not yet effective in the Chinese market in the period between 2015 and 2016. Deng (2017) constructs the implied volatility surface (IVS) of SSE 50 ETF options and finds that there is a pronounced implied volatility smile for short-term contracts and that the characteristics of the volatility smile also explain a key model for the variation of the whole IVS of the SSE 50 ETF.

Considering the parity relationship of European options, Xia, Gao and Yang (2018) research the pricing efficiency of the SSE 50 ETF option and empirically conclude that the deviation of the option contract price gives the SSE 50 ETF a lot of arbitrage opportunities. The observational deviation phenomenon is caused by the following three factors: a high transaction cost, a lack of effective short-selling mechanism in the Chinese market, and short-term market sentiment factors.

Based on the data of the SSE 50 ETF option during the sample period (February 2015-April 2017), Zhang and Watada (2019) reveal that arbitrage opportunities exist but are infrequent when transaction costs are accounted for when put-call parity, box spread, and boundary arbitrage are used to analyze the market.

2. Data and Methodology

This paper chooses the Shanghai Stock Exchange 50 ETF (SSE50ETF) options as its research objective. The SSE 50 ETF is a European-type option and the first stock options product in China's stock market which started trading on the SSE in February 2015.

The closing price of the SSE 50 ETF on March 19, 2021 was locked and three sets of options with different expiration dates were selected. The experimental data in this paper come from the Wind-Economic Database. Headquartered in Shanghai, China, Wind Information Co., Ltd. (Wind) provides timely and accurate and complete Chinese financial data on a 24 × 7 × 365 basis. The main source of Chinese market data is purchases from stock exchanges, including the Shanghai Stock Exchange, Shenzhen Stock Exchange, and Shanghai Futures Exchange, etc. The data covers stocks, bonds, funds, index, warrants, commodity futures, foreign exchange. It is also popular with Qualified Foreign Institutional Investors (QFII) certified by the China Securities Regulatory Commission.

The first set includes 27 groups with 5 days to maturity; the second one includes 12 options with 40 days to maturity; the last set includes 14 options with 187 days to maturity.

The empirical research objective is to test the relationship between the implied volatility and the strike price of three sets of SSE 50 ETF options and draw volatility smile graphs, respectively. The implied volatilities are calculated by the Black-Scholes-Merton option pricing model given below (Hull, 2018):

$$C_t = S_0 N(d_1) - K\mathrm{e}^{-rT} N(d_2) \tag{1}$$

$$P_t = K\mathrm{e}^{-rT} N(-d_2) - S_0 N(-d_1) \tag{2}$$

with

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{4}$$

where $N(x)$ is the cumulative distribution function of the standard normal distribution. C_t and P_t are the market price of a call and put option at time t, respectively. S_0 is the current share price of the underlying asset. K is the strike price. T is the time to maturity expressed in years, and r is the risk-free rate, which is the annual rate expressed in terms of continuous compounding. σ is the volatility of return of the underlying asset.

Assuming that, under the conditions of no-arbitrage and no dividend, the current price of a stock is S_0, and the call option with C_t in value and put option with P_t in value, the stock as the underlying asset has time T to the expiration day, the execution price is the same, both are K, the future stock price is S_0, and the one-year risk-free interest rate is r. Two portfolios are considered. The first one, portfolio A, includes a call option and a cash asset with K in value at the maturity date. The other one, portfolio B, includes a put option and a stock. The values of the two portfolios for $S_T > K$ and $S_T < K$ are shown below (**Table 1**).

According to this table, the values of Portfolio A and B are always the same no matter how high the share price at the maturity date is. The maximum value is max (S_T, K). Therefore, the theoretical price at the current moment should also be the same. Otherwise, there would be opportunities for a risk-free arbitrage. Assuming cash with the value of K at time T, the current value of K is $K\mathrm{e}^{-rT}$ discounted by compound interest. So the equation of put-call parity is:

$$C_t + K\mathrm{e}^{-rT} = P_t + S_0 \tag{5}$$

The Equation (5) can be rearranged, and a synthetic call option can be created, which is shown below

Table 1. The value of the two portfolios at time T.

Portfolio		$S_T > K$	$S_T < K$
A	A Call Option	$S_T - K$	0
	A Cash Asset	K	K
	Portfolio A	S_T	K
B	A Put Option	0	$K - S_T$
	A Stock	S_T	S_T
	Portfolio B	S_T	K

Source: Hull (2018).

$$C_t = P_t + S_0 - Ke^{-rT} \tag{6}$$

The right side of Equation (6) is a synthetic call. When traders currently hold the underlying asset and except the stocks' future prices to increase, they will use the synthetic call option strategy to limit the associated downside risk. Stoll (1969) describes the conversion mechanism: It states that a call option can be converted into a put, or a put option can be converted into a call option with no risk involved to the converter and with no capital investment (Mittnik & Rieken, 2000).

If the above relationships (Equation (6)) do not hold, then that leaves arbitrage opportunities. The first condition follows from buying a put option and a stock and borrowing the debt (Ke^{-rT}) at a risk-free rate. If $S_0 > K$, by selling the stock at S_0 and liquidating the debt, the cash inflow is $S_0 - K > 0$. If $S_0 < K$, the call option will be worthless at maturity. In the meantime, the payoff from liquidating the portfolio is also zero, since the put value is $K - S_0$. For this situation, a call can be converted into a put. Similarly, a long put can be converted into a call by buying the call ($-C_t$), shorting the stock ($-S_0$), and lending debt ($-Ke^{-rT}$). When $S_0 > K$, the put expires worthlessly. When $S_0 < K$, the cash inflow is $K - S_0 > 0$.

Considering that put and call prices are out of equilibrium, two arbitrage strategies can be applied. If the call price is higher than the put price according to the Put-Call-Parity, then the conversion strategy or short-call strategy can be used. This means to sell the overpriced call and to buy the underlying share and the corresponding put, in order to protect the investor from the loss of the declining price of a share. This leads to an immediate cash inflow ($C_t - P_t - S_0 + Ke^{-rT} > 0$) and a zero cash flow at terminal time. The opposite strategy of this is the reverse conversion or short-put strategy: If the put price is too high according to the Put-Call-Parity, investors can sell puts as well stocks, and buy calls. The immediate cash inflow will be $P_t - C_t + S_0 - Ke^{-rT} > 0$.

Based on Equation (5), the risk-free rate r is the annual rate expressed in terms of continuous compounding. However, in the real market, interest rates are compounded over multiple periods. Therefore, the options spread can be defined by

$$d = C_t + K(1+r)^{-T} - P_t - S_0 \tag{7}$$

If $d > 0$, the arbitrage opportunity is the converse strategy by selling call options, buying put options and underlying assets and holding the position until maturity. The arbitrage amount reaches:

$$AR_1 = d(1+r)^T = K + (C_t - P_t - S_0)(1+r)^T \tag{8}$$

If $d < 0$, investors can apply the reverse converse strategy by buying call options, selling put options and underlying assets until maturity. The final arbitrage amount reaches:

$$AR_2 = -d(1+r)^T = (P_t + S_0 - C_t)(1+r)^T - K \tag{9}$$

In terms of transaction costs, the current commissions paid to brokerage firms are significantly different, ranging from CNY7 to CNY10, and CNY20. Besides, the clearing corporation charges CNY0.3 for an option, the handling fee is CNY2, and the exercise fee is CNY0.6. Therefore, this paper assumes that the transaction cost of buying SSE 50 ETF is set at 0.05%, and the cost of option exercise is CNY15. Furthermore, the contract size of an SSE 50 ETF is 10.000 shares. Therefore, if investors want to obtain profits by long arbitrage, Equation (10) should be achieved

$$10000 \cdot d(1+r)^T - 0.05\% S_0 - 15 > 0 \tag{10}$$

For short arbitrage with SSE 50 ETF, in actual practice, it is necessary to consider the securities lending interest rate and margin ratio. The implementation of short arbitrage requires the purchase of call options, the sale of put options, and the short-selling of stocks, and the short-selling needs to consider the initial margin, maintaining the margin and the interest rate of the short-selling. The margin is the sum of the put option short *Mp* and the securities lending margin *Ms*. According to the official documents from the Shanghai Stock Exchange, *Mp* and *Ms* are defined as Equation (11) and Equation (12), respectively (SSE, 2015).

$$Mp = \min\left[P_t + \max(12\% \cdot S_0 - K, 7\% \cdot K), K\right] \tag{11}$$

$$Ms = 50\% S_0 \cdot I_{S_0 \le \frac{150}{130} S_0} + 50\% S_0 \cdot I_{S_0 > \frac{150}{130} S_0} \tag{12}$$

The sum is the total margin. The securities lending interest rate is set to 8.6%, and the programming will automatically find short arbitrage opportunities based on the formula below (Qian, 2016)

$$-10000 \cdot d(1+r)^T - 0.05\% S_0 - S_0(1+8.6\%)^T - 15 > 0 \tag{13}$$

Assuming that the final arbitrage yield is π, the lowest expected rate of return π_e should be higher than the current risk-free rate of return. This paper assumes that the risk-free rate is 3.27% which is the 10-year Chinese Government Treasury bond yield shown on the Wind Database. So, when the expected rate of return is higher than 3.27%, the corresponding arbitrage strategy can be implemented.

3. Data Analysis

3.1. The Volatility Smile of SSE 50 ETF

The sample under investigation is based on the current strike price of CNY3.496 on March 19th 2021, covering three sets of SSE 50 ETF with different maturity. The three sets are SSE 50 ETF with 5 days, 40 days and 187 days to the maturity, respectively. The annual risk-free rate of 3.27% is the 10-year Treasury bond yield shown on the Wind Database on March 19th 2021. The results are summarized in Figure 1 where the horizontal axis shows the strike price, and the vertical axis shows the implied volatility.

For the set of SSE 50 ETF with 5 days to maturity, the graph for the relationship between implied volatility and strike price looks like a smile. Implied volatility rises when the underlying asset of an option is further out of the money or in the money, compared to at the money. It shows that the SSE 50 ETF that is furthest in- or out-of-the-money has the highest implied volatility. The index option with the lowest implied volatility has a strike price near-the-money of around CNY3.6. Its implied volatility is around 18%. However, the other two sets, SSE 50 ETF with 40 days to maturity and SSE 50 ETF with 187 days to maturity, show a smirk phenomenon exhibiting a forward skew tilt to the right. The implied volatilities at the lower strike prices are lower than the implied volatility at higher strike prices, suggesting that out-of-the-money calls and in-the-money puts are met with greater demand in the Chinese market. The option set of SSE 50 ETF with 40 days to maturity shows a higher slope than the set with 187 days to maturity.

The graphical difference between these three sets is in line with the findings of Na, Jin and Liu (2007) according to which volatilities of short-term options usually show a smile shape while volatilities of long-term options exhibit a smirk. Surging and slumping with a short period could bring about extremely high volatility. Selling short-term deep out-the-money options could lead to a larger short gamma position after these transactions since these out-the-money options will be close to at-the-money, but the extremely high volatility generates a large loss on the delta change. Therefore, the price of selling options goes up, which is reflected in the higher implied volatility.

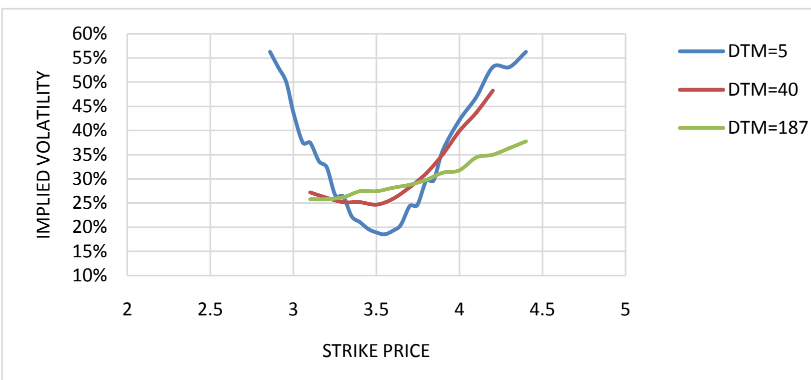

Figure 1. SSE 50 ETFs relationship between implied volatility and strike prices.

On the other hand, the implied volatility level of the medium- and long-term options will be largely affected by the supply and demand of options. Shi (2015) explains that the implied volatility level of the underlying asset determines the extrinsic value in the option quotation, that is, the non-intrinsic value, while the price of the underlying asset determines the intrinsic value. If the market demand for each option is rising, the implied volatility level of the option will also rise. For long-run options, due to the natural long property of stock markets, many funds sell the medium- and long-term covered call and buy the protective put (Ding, Granger, & Engle, 1993). A demand-supply imbalance can immediately drive the prices up or down. Moreover, the pattern is also common for options in the commodities market, such as agricultural items and oil (Soini & Lorentzen, 2019).

3.2. The Put-Call Parity Arbitrage Strategy of SSE 50 ETF

Table 2 summarizes the outcome of the Put-Call-Parity for the three selected data sets as of 19 March 2021.

Table 2. Test of Put-Call parity arbitrage opportunities of SSE 50 ETF (CNY).

DTM-5							
K	C_t	P_t	short call	short put	Arbitrage after transaction costs	Initial margin	Maximum yield p.a.
4.400	0.0001	0.9024	−0.03%	**0.03%**	−71.58	43983	−11.7%
4.300	0.0001	0.8113	−0.91%	**0.91%**	17.02	60480	2.0%
4.200	0.0003	0.7078	−0.54%	**0.54%**	−20.45	59480	−2.5%
4.100	0.0003	0.6028	−0.03%	**0.03%**	−70.92	58480	−8.7%
4.000	0.0004	0.5046	−0.20%	**0.20%**	−54.36	57480	−6.8%
3.900	0.0005	0.4060	−0.32%	**0.32%**	−41.80	56480	−5.3%
3.844	0.0003	0.3500	−0.34%	**0.34%**	−40.05	55920	−5.2%
3.800	0.0007	0.3010	**0.20%**	−0.20%	−12.45	37963	−2.4%
3.746	0.0006	0.2492	−0.03%	**0.03%**	−71.50	54940	−9.4%
3.700	0.0016	0.2023	0.16%	−0.16%	−16.00	36967	−3.1%
3.647	0.0022	0.1501	0.15%	−0.15%	−17.77	36439	−3.5%
3.600	0.0053	0.1065	0.12%	−0.12%	−20.56	35972	−4.1%
3.549	0.0138	0.0654	−0.02%	**0.02%**	−72.38	52970	−9.8%
3.500	0.0326	0.0340	**0.10%**	−0.10%	−22.11	34974	−4.6%
3.450	0.0635	0.0164	−0.04%	**0.04%**	−69.82	51980	−9.7%
3.400	0.1045	0.0074	−0.04%	**0.04%**	−70.04	51480	−9.8%
3.351	0.1475	0.0029	−0.19%	**0.19%**	−55.26	50990	−7.8%
3.300	0.1960	0.0020	−0.35%	**0.35%**	−39.48	50480	−5.6%
3.253	0.2436	0.0007	−0.16%	**0.16%**	−58.70	50010	−8.5%
3.200	0.2953	0.0008	−0.29%	**0.29%**	−44.93	49480	−6.5%
3.154	0.3421	0.0004	−0.17%	**0.17%**	−57.14	49020	−8.4%

Continued

3.100	0.3963	0.0003	−0.14%	**0.14%**	−60.38	48480	−9.0%
3.056	0.4459	0.0001	**0.44%**	−0.44%	11.89	30502	2.8%
3.000	0.4937	0.0002	−0.38%	**0.38%**	−35.82	47480	−5.4%
2.957	0.5481	0.0003	**0.75%**	−0.75%	42.34	29482	10.3%
2.908	0.5865	0.0002	−0.30%	**0.30%**	−44.23	46560	−6.8%
2.859	0.6047	0.0002	−3.38%	**3.38%**	263.69	46070	41.2%
DTM−40							
4.20	0.0027	0.7335	−4.18%	**4.18%**	52.84	42268	1.1%
4.10	0.0036	0.6332	−4.02%	**4.02%**	37.22	41256	0.8%
4.00	0.0051	0.5358	−4.10%	**4.10%**	44.68	40267	1.0%
3.90	0.0077	0.4365	−3.87%	**3.87%**	22.03	39248	0.5%
3.80	0.0104	0.3417	−4.09%	**4.09%**	43.54	38273	1.0%
3.70	0.0257	0.2543	−3.78%	**3.78%**	12.86	37246	0.3%
3.60	0.0480	0.1755	−3.63%	**3.63%**	−1.76	36235	0.0%
3.50	0.0871	0.1150	−3.64%	**3.64%**	−1.33	35239	0.0%
3.40	0.1373	0.0686	−3.94%	**3.94%**	29.21	34273	0.8%
3.30	0.2110	0.0398	−3.66%	**3.66%**	0.54	33248	0.0%
3.20	0.2942	0.0224	−3.56%	**3.56%**	−9.06	32242	−0.3%
3.10	0.3799	0.0123	−3.95%	**3.95%**	29.5	31284	0.8%
DTM−187							
4.4	0.0319	0.9756	−11.26%	**11.26%**	−448.93	44397	−1.9%
4.3	0.0372	0.8834	−11.35%	**11.35%**	−440.37	43422	−2.0%
4.2	0.047	0.7927	−11.13%	**11.13%**	−462.30	42417	−2.1%
4.1	0.0573	0.7112	−11.79%	**11.79%**	−395.78	41499	−1.8%
4	0.071	0.6124	−10.37%	**10.37%**	−539.74	40374	−2.6%
3.9	0.0848	0.5365	−11.23%	**11.23%**	−451.86	39477	−2.2%
3.8	0.1087	0.4541	−10.44%	**10.44%**	−532.77	38414	−2.7%
3.7	0.1353	0.3801	−10.21%	**10.21%**	−555.73	37408	−2.9%
3.6	0.166	0.315	−10.47%	**10.47%**	−529.87	36450	−2.8%
3.5	0.2045	0.2536	−10.31%	**10.31%**	−545.71	35451	−3.0%
3.4	0.2455	0.1985	−10.54%	**10.54%**	−522.91	34490	−2.9%
3.3	0.2975	0.1522	−10.54%	**10.54%**	−522.48	33507	−3.0%
3.2	0.3582	0.1132	−10.40%	**10.40%**	−536.38	32510	−3.2%
3.1	0.423	0.0828	−10.72%	**10.72%**	−504.33	31558	−3.1%

There are three sets of SSE50 ETF with different expiration days in total. The first set will be exercised after five days on March 24 (data gathered on March 19, 2021), DTM-40 can be transacted on April 28, the third set of options with 187 days to maturity will be exercised on September 22. The shown data are the strike price, call option price, and put option price, respectively. The short call and short put, respectively, is calculated by Equation (7). The arbitrage results are calculated on the maximum of short call or short put including transaction costs with Equation (11) and represented by the percentage per annum. Results above 0 indicate that the arbitrage is successful.

The first set of data shows the strike prices of the March expiration options, their option spread, and the corresponding arbitrage results. The closing price of the SSE 50 ETF on March 19th 2021 was CNY3.496. The first arbitrage strategy as the converse strategy is to short call options and long put options. When $d > 0$, the call options are overvalued. So investors can sell call options, borrow risk-free assets, and buy undervalued put options and underlying assets at the same time. The current cash inflow is worth $C_t + K\left(1+r\right)^{-T} - P_t - S_0 > 0$ and maintains until the expiration date. For example, investors can choose an option with a strike price of CNY3.056, and the option spread is 0.004453. The transactions include a short call option and a long put option, as well as buying 10,000 shares of SSE 50 ETFs; the initial capital investment is $10000 \cdot \left(S_0 + P_t - C_t\right) = \text{CNY}30502$, and the lowest profit is CNY12.07. The investment period is 5 days, and the rate of return is 0.04% for the five-day investment; the annual yield is 2.89%. Comparing with the expected rate of return of 3.27% (the risk-free interest rate), the arbitrage rate is lower than the expected one. So the option is worthless to arbitrage. For this group, only one option with a strike price of CNY2.957 can generate profit by using the long arbitrage strategy. The theoretical annual return is 10.53% during the 5-day investment period, higher than the risk-free rate.

The alternative is the reverse converse strategy by having short put options and long call options. In this case, the put options are overvalued. Investors can sell the put options and underlying assets, buy undervalued call options, and lend the risk-free asset. The cash inflow is $P_t + S_0 - C_t - K\left(1+r\right)^{-T} > 0$. But for this set, there is no chance of generating profit by using a short arbitrage strategy as all annual yields are negative.

Both arbitrage methods are also applicable to the other two sets of options. For the second set of SSE 50 ETF with 40 days to maturity, the call options that are below CNY0.0871 in value are undervalued. There is no short arbitrage opportunity. When the prices of call options are higher than 0.1373, they are overvalued and should be sold. In the meantime, investors can borrow risk-free assets with $K\left(1+r\right)^{-T}$ in value. Every option in this team can exceed the expected return. One with a strike price of CNY3.4 can provide an expected annual return of 17.49%. The highest annual return can reach 106.65%, providing a great profit.

For the third set of options with 187 days to maturity, there are only short put arbitrage opportunities, i.e. the reverse converse strategy. An option with a strike price of CNY4.0 is selected. Its option spread is negative with 0.102799 in value. Therefore a short arbitrage strategy should be considered, including buying a call option, selling a put option, and lending debt to buy 10,000 50 ETFs. Considering the lending interest of 8.6%, the lowest profit is CNY 1012.6. The initial investment is CNY29,991. The six-monthly rate of return is 1.852%. The annual return is 6.59%, higher than the risk-free rate.

It should be noted that there are regulatory constraints on short-selling and market restrictions in China's stock market (Zhang & Tian, 2020). Therefore, investors should consider a situation where the option price exceeds a given threshold or is lower than a given threshold and adopt long-arbitrage and short-arbitrage strategies. Qian (2016) maintains that the participation of arbitrage investors would increase the liquidity of options trading, which could let the options price return to their proper value.

4. Conclusion

China's market is an important emerging market, and its financial market development is still in a development stage.

The empirical evidence in this paper shows that the volatility smile and smirk in China's stock index market were proven to exist. Particularly the relationship between the strike price and implied volatility represents a smile for short-term options, while there is a volatility smirk for mid-term and long-term options. This skew is influenced by the investors' financial innovation; they hedge risks by buying out-of-the-money calls and in-the-money puts to protect themselves from a sudden price drop. A possible explanation could be that many investors are continuously afraid of being exposed to a bubble. Particularly since the US housing crash in 2007 and the dotcom bust in 2001 hardly a month goes by without someone warning of the burst of the next bubble. Thus, hedging a diversified stock market portfolio with out of the money puts on the stock market index would be an interesting alternative to be protected versus a stock market crash initiated by the burst of a bubble.

Important to note is, that based on this empirical study of three data sets of the index SSE 50 ETF, arbitrage opportunities are literally not existent. Long and short arbitrage opportunities are negligible before transaction costs. After regarding transaction costs including options margin, there are not any arbitrage opportunities of the options regarded. These findings support the statement that Chinese stock markets work efficiently on an index level.

However it has to be stated that this paper covers only a short review period from March 2021 to September 2021. When it comes to further research an appealing extension would be to have evidence of a longer time period as the last 10 years for example. Finally an extension to other financially well developed countries as the US, UK or EU capital markets should not be missed.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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