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Modeling Ultimate Loss-Given-Default and Time-to-Resolution on Corporate Debt

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Abstract

Loss-given-default ("LGD") is a critical parameter in modeling the credit risk of instruments subject to default risk, in addition to various other facets of credit risk modeling. However, another source of uncertainty in addition to LGD is the *time-to-resolution* ("TTR") of the default event, which has been given limited attention in the literature. LGD and TTR are likely to be correlated with each other and both are likely to vary significantly with various recovery modeling risk factors such as collateral characteristics and the macroeconomic environment. As the TTR is often right censored due to a cut-off in the data sample underlying the estimator of the LGD, such estimators not accounting for this may suffer from what is known in the statistics literature as *censoring*, which in the credit risk modeling literature is known as LGD *resolution bias.* LGD models not adjusting for resolution bias through omitting a consideration of the distribution of TTR, the standard variety prevalent in the industry, will result in biased estimates when applied to non-defaulted performing instrument. In this study, we propose to address this issue through the simultaneous modeling of the LGD on resolved cases and TTR on both resolved (non-censored) and unresolved (censored) cases. This study empirically investigates the determinants of LGD and TTR through building alternative econometric models on bonds and loans using an extensive sample of most major U.S. defaults in the period 1985-2022. The key finding is that when compared with standard approaches that do not account for resolution bias, our approach has superior fit to the data in terms of out-of-sample performance where the LGD is unresolved at the point of model development. This study extends prior work by modeling LGD by an advanced (yet practical to implement) econometric technique, incorporating the TTR as well the obligor's complete capital structure characteristics, for a large corporate asset class and rigorously testing the proposed model on an out-of-sample basis.

Keywords

Loss Given Default, Resolution Bias, Default Risk, Credit Risk, Model Validation

1. Introduction

Loss-given-default (LGD), the loss severity on defaulted debt obligations, is a critical component in the risk management, pricing and portfolio modeling of credit[1]. LGD is among the three primary determinants of credit risk, the other two being *probability-of-default* ("PD") and *exposure-at-default* ("EAD"). However, LGD has not been as extensively studied and is considered a much greater modeling challenge as compared to PD. Traditional credit models, such as quantitative PD scorecards (Altman, 1968), have focused on systematic components of credit risk that attract risk premia. Unlike PD, determinants of LGD have typically been ascribed to idiosyncratic, borrower-specific factors. However, there is now an ongoing debate about whether the risk premium on defaulted debt should reflect systematic risk, and in particular whether the intuition that LGDs would rise in worse states of the world is correct; and how this could be refuted empirically given limited and noisy data. This heightened focus on LGD has been motivated by the large number of defaults and nearly simultaneous decline in recovery values observed through the prior downturns in the credit cycle, as well as what is expected to occur in the current economic slowdown following the distortions of the COVID crisis, the resurgence in inflation, the response of monetary authorities to the latter and a higher interest rate regime. We may add to this past banking supervisory responses to credit downturns that lead to developments such as the Basel prudential regulations (Basel Committee on Banking Supervision, 2003, 2017) and the supervisory stress testing exercises (Board of Governors of the Federal Reserve System, 2009, 2016), as well as the continued growth in credit markets beyond the traditional domains of supervisory purview (i.e., private credit and Fintech). However, obstacles to better understanding and predicting LGD endure, including a dearth of relevant data in many asset classes and the lack of a coherent theoretical underpinning, that result in continuing challenges to researchers.

We add to the aforementioned challenges issues around demonstrating the conceptual soundness and adequacy of performance for LGD models, as doing so is critical in the model validation process that lies at the heart of programs to measure and manage model risk. The latter activities are core supervisory expectations as documented in the guidance for model risk management issued by prudential supervisory in the previous decade (Board of Governors of the Federal Reserve System, 2011). It is well-known in the industry that models for LGD

[1]LGD is equivalent to one minus the recovery rate, the dollar recovery as a proportion of par or EAD, assuming that all debt becomes due at default. We focus on LGD as opposed to the recovery rate with a view toward credit risk management applications.

suffer from a lack of depth, breadth and quality of data for model development, contributing to a proliferation of model validation issues handed down from independent validation groups and other 3rd party reviewers. We contribute to the resolution of these issues by studying the *time-to-resolution* ("TTR"), which allows institution to leverage the information on recoveries embedded in non-resolved instances of default, which may lead to more robust models and greater accuracy in computing portfolio credit risk measures. Aside from the possibility of producing more accurate models of LGD, we also address a conceptual gap, since while it is recognized that uncertainty in the TTR is the 2nd main source of uncertainty in modeling recoveries on defaulted debt, most models of LGD seen in the industry do not account for this aspect. In addressing the violation of the assumption that TTR does not influence LGD, this study demonstrates the measurement of model risk more broadly, as opposed to the narrower concept of parameter uncertainty in a statistical model of LGD[2].

A model for LGD may consider many factors, such as the obligor's characteristics (e.g., financial ratios, industry, etc.), collateral types, seniority rank, product features, the macroeconomic environment and the TTR. We define the TTR as the time period where all recoveries have been realized. In the case of private bank loans, this is the time from default (i.e., the determination of unlikeliness to repay or non-payment) to the point when there is a determination that no further workout recoveries are collectable ("certified LGDs"). In the case of the large corporate asset class and rated bonds or loans the TTR is the time from default (i.e., a bankruptcy filing or distressed exchange) until public resolution of the default event by the entity emerging from default in a reorganization or being liquidated.

LGD can be defined variously depending upon the institutional setting, the type of instrument (e.g., traded bonds or bank loans) or the credit risk model (e.g., pricing debt instruments subject to the risk of default, expected loss calculation or credit risk capital). The ultimate LGD represents eventual discounted loss per dollar of outstanding balance at default. When considering loans that may not be traded, and taking into consideration when cash was received as well as other losses incurred in the collection process, ultimate LGD is the relevant measure for an input into a regulatory or economic credit capital model. In the case of bonds or marketable loans, one can measure the prices of traded debt at the initial credit event, or the discounted market values of instruments received at the resolution of default. The latter is potentially a proxy for the ultimate LGD, the focus of our study for two purposes. First, our primary objective is to provide results of use to agents invested in defaulted securities having time horizons that span the resolution period, who wish to assess expected value upon emergence relative to some benchmark available at default, such as trading or model-based prices. Agents who would benefit include bank workout specialists, risk managers, or vulture hedge fund investors. Second, our results would be

[2]See Jacobs, Jr. (2022) for an analogous exercise in measuring the model risk arising from violation of modeling assumptions in the context of PD models and the omission of a factor related to the equity markets.

relevant for financial institutions attempting to quantify economic LGD for purposes of the *Basel advanced internal rating based* ("AIRB") approach to regulatory capital, or the stress testing exercises mandated by supervisors as well as for internal capital management purposes, which requires estimation of the ultimate LGD.

However, a challenge to meeting these objectives arises, in that for many cases the length of the data sample used for estimating LGD may be relatively short, while the TTR may be relatively long, resulting in a right censoring of recovery cash flows due to the end date of the sample. Arguably, it is likely that censored observations may have different characteristics from those of non-censored or closed counterparts. Furthermore, if the TTR is correlated with LGD but omitted from the specification of the LGD model, then such model will suffer from a specification error due to both an omitted variable bias as well as a sample selection bias.

Another important aspect of considering TTR in the context of LGD modeling is the implication for the stress testing exercises previously alluded to, which is very important in assessing the credit risk of bank loan portfolios, the importance of which has grown over time. Currently, these exercises are accepted as the primary means of supporting capital planning, business strategy and portfolio management decision making (Financial Services Authority UK, 2008). Such analysis gives us insight into the likely magnitude of losses in an extreme but plausible economic environment conditional on varied drivers of loss. It follows that such activity enables the computation of unexpected losses that can inform a regulatory or economic capital according to the Basel III guidance (Basel Committee on Banking Supervision, 2017). In light of this, a model for LGD that explicitly accounts for the time dimension is naturally suited to the development of forward-looking forecast scenarios in stress testing exercises.

Aspects of this modeling exercise deserving of special attention include the distributional properties of LGD. While the available theory and empirical evidence suggest it to be stochastic and predictable with respect to other variables, in most extant credit models LGD has been treated as either deterministic or as an exogenous stochastic process. Another aspect to consider is that many practitioner models of LGD are in the linear class, while it is well-known that LGD has a bounded and multi-model distribution (Jacobs, Jr. & Karagozoglu, 2011). The quest for tractability gives rise to such assumptions, but in practical applications this results in understated capital, mispricing, and unrealistic dynamics of model outputs. Our research helps to resolve such deficiencies by modeling ex ante the distribution of LGD—as a function of empirical determinants such as contractual features, firm capital structure, borrower characteristics and systematic factors—utilizing a modeling approach that accommodates the non-normality of LGD. In order to empirically investigate the determinants of and build predictive econometric models for ultimate LGD and TTR we use the Moody's *Ultimate Recovery Database*™ ("MURD") (Emery et al., 2007), an extensive sample of 6094 defaulted firms covering the period 1985 to 2022, and a dataset unique in that it contains the complete capital structures of each obligor. Our sample is highly representative of the U.S. large corporate loss experience over the last two decades, and therefore is relevant for financial institutions and other credit market participants active in this footprint.

This study contributes to the research on LGD in several respects. The methodology involves construction of internally consistent models of LGD in line with prior empirical findings and theoretical expectations, which has favorable features in line with industry and supervisory expectations regarding model risk management (Board of Governors of the Federal Reserve System, 2011). Regarding the latter, we demonstrate how in a rigorous model validation process the impact of mis-specifying a model of LGD by omitting the TTR. We extend the prior literature on considering LGD and TTR (Gürtler & Hibbeln, 2011; Yashkir & Yashkir, 2013; Carey & Gordy, 2007, 2016; Chen, 2018) through using an extensive sample of corporate bond and loan defaults with the consideration of both obligor complete capital structure characteristics and macroeconomic factors. In the process we not only address several academic aspects of LGD, but also offer an actionable approach that may be applied by bank portfolio and risk managers, as well as by supervisors and market participants in this space. In the latter domain, distressed debt traders may use this model in forecasting ultimate LGD or as an input into *credit Value-at-Risk* ("C-VaR") models, bank portfolio managers may deploy it in early warning systems, and bank risk managers or supervisors may consider it for regulatory capital calculations.

Our principal findings are as follows. In the MURD database sample, LGD and TTR are positively correlated, covary significantly and intuitively with a set of standard risk factors (the ranking of collateral, a collateral tangibility index, the proportion of debt in the capital structure above and below the instrument, the defaulted balance and the change in industrial production macroeconomic factor), where some of these factors are common to each. We compare linear regression models for both LGD and TTR to a champion approach, the *fractional response logit model* ("FRLM") for LGD and a survival regression model for TTR, showing that the chosen approach has better qualitative aspects (including increased sensitivity to macroeconomic conditions) and superior predictive performance. The challenger approach considered is a simultaneous linear regression model, a 3-*stage least squares regression model* ("3 SLSRM"), and we demonstrate that the favored approach outperforms the challenger approach an out-of-sample basis in predicting unresolved LGDs.

This paper is organized as follows. In Section 2 we review the literature. Then we present the econometric models in Section 3, the summary statistics in Section 4, the estimation and main empirical results in Section 5. Finally, in Section 6 we provide conclusions and directions for future research.

2. Review for the Literature

It may be argued LGD has been a relatively neglected aspect of credit risk research[3]. Starting with the seminal work by Altman (1968), modeling of PD is currently in a relatively mature stage as compared to LGD. The heightened focus on LGD is evidenced by the recent flurry of research into the application of credit models and estimation of LGD from available data. This literature ranges from simple quantification of LGD, to calibration of credit models embedding LGD assumptions, to empirical or vendor models of LGD. The Zeta model of Altman, Haldeman, and Narayanan (1977) was a second generation of the Altman Z-Score PD estimation model. In this model, loan LGD estimates were based on a workout department survey (1971-1975), which yielded conclusions regarding the magnitude of discounted post-restructuring recoveries on unsecured bank loans. Bank studies focusing on internal loan data included JP Morgan Chase (Araten, Jacobs, Jr., & Varshney, 2004), where the authors studied ultimate workout LGD for wholesale loans during 1982-2000.

Among early studies relying almost exclusively on secondary market prices of bonds or loans soon after the time of default, Altman and Kishore (1996) estimated LGDs for defaulted senior secured and senior unsecured bonds in the period 1978-1995, yielding estimates that could be statistically distinguished among various industry groups. Altman and Eberhart (1994) and Fridson, Garman, and Okashima (2000) provided evidence that the more senior bonds significantly outperformed the more junior bonds in the post-default period, results confirmed by Hamilton, Gupton, and Berthault (2001) for secondary market loan prices a month after default. Emery, et al. (2007) and Altman & Fanjul (2004) compared LGDs on bank loans and bonds, respectively, as inferred from the prices of the traded instruments at default in a Moody's database, revealing that loans experience lower loss severities when controlling for seniority. Cantor, Hamilton, and Varma (2003) showed similar findings for corporate bonds as Altman and Fanjul (2004), and additionally found differential LGD by rating at origination, such that "fallen angels" of the same seniority had significantly lower LGDs[4].

Among studies that looked at ultimate LGD, Standard and Poors (Keisman & van de Castle, 2000) presented empirical results from the LossStats™ database in the period 1987-1996 for marketable bonds and loans. This study also showed that the influence of position in the capital structure (i.e., the proportion of debt above or below a claimant in bankruptcy) was independent of collateral and seniority in determining loss severities. A more recent rating agency study by Moody's (Cantor & Varma, 2004) examined the determinants of ultimate LGD for North American corporate issuers over a period of 21 years (1983-2003), looking at many of the variables considered herein (e.g., seniority and security; firm-specific, and macroeconomic factors).

Several recent empirical studies of LGD attempting to measure LGD-PD cor-

[3]Acharya, Bharath, & Srinivasan (2007); Altman, Resti, & Sironi (2001); Altman & Fanjul (2004); Altman & Ramayanam (2006); Araten, Jacobs, Jr., & Varshney (2004); Carey & Gordy (2007); Frye (2000a, 2000b, 2000c); Jarrow (2001).

[4]Median LGDs of 49.5% and 66.5% for defaulted issuers originally investment and speculative grade, respectively.

relation have put more structure around this exercise, either by building predictive econometric models, or by attempting to directly test models. Frye (2000a, 2000b, 2000c) examined the LGD-PD correlation in extensions of the Merton (1974) framework allowing for systematic recovery risk, finding a significant positive relationship between PD and LGD at various levels of aggregation. Other studies examined this by looking at LGD as implied from the prices of traded debt at or prior to default, as opposed to ultimate LGD, or the "reduced form" approach. Among these, Jarrow (2001) developed a hybrid structural-reduced model, in which PDs and LGDs were functions of the underlying state of the macro-economy. Hu and Perraudin (2002) also examined this relationship and found LGD-PD correlations on the order of 20%. Jokivuolle and Peura (2003) presented an option theoretic model for bank loans and were able to produce a positive correlation between PD and LGD. Bakshi et al. (2001) extended the reduced-form approach by allowing a flexible correlation between PD, LGD and the risk-free rate. Imposing a negative correlation between PD and LGD, they found that a 4% increase in the (risk-neutral) hazard rate was associated with a 1% increase in the expected LGD. In related work on the resolution of default focusing on high-yield debt portfolios, Parnes (2009) developed a theoretical model that explicitly incorporated LGD assumptions.

Several studies showed that LGDs tend to rise more in periods of recession than they fall during expansions, suggesting that more is at play than a macroeconomic factor influencing the value of collateral. Keisman and van de Castle (2000) found that during the earlier stress period at the beginning of the previous decade, LGDs of all seniorities rose in the S&P LossStats™ database for the period 1982-1999. Altman, Resti, and Sironi (2001) also found that LGDs increase as the credit cycle worsens and as default rates increase above the cycle's long-run average[5]. Araten, Jacobs, Jr., and Varshney (2004) related unsecured U.S. large corporate borrower-level LGDs to the average Moody's All-Corporate default rate and reported similar behavior. However, Altman, Brooks, Resti, and Sironi (2005) found that a systematic variable had no effect on LGD when controlling for bond market conditions (i.e., supply-demand imbalances). Acharya, Bharath, and Srinivasan (2007) examined the same data and time period as Keisman and van de Castle (2000), and while they verified that seniority and security are key determinants of LGD, in addition they found industry-specific factors influencing LGD independently of the macroeconomic state and bond market conditions analyzed in Altman et al. (2005). In particular, after controlling for firm-specific, contractual, and systematic factors they found elevated LGDs in distressed industries, defined as those sectors having significantly lower profitability than the overall market. They argued that in these cases fewer redeployable assets, greater leverage and lower liquidity is driving lower average recoveries; and that their results support a test of Shleifer & Vishny (1992) "fire-sale" hypothesis, an industry equilibrium phenomenon in which macroe-

[5]Altman (2006) reported that his model over predicts LGD in recent years, which he speculated may be due to bubble conditions in the high-yield market.

conomic and bond market variables are spuriously significant due to omitting an industry factor.

We also make note of this evidence regarding the PD-LGD correlation influencing the Basel II guidelines: paragraph 468 on downturn LGD in the Bank for International Settlements (BIS) Accord (Basel Committee on Banking Supervision, 2003, 2004), and the additional guidance offered by the BIS (Basel Committee on Banking Supervision, 2005, 2017). Basel II required AIRB banks to not only capture all relevant risks regarding possible cyclical variability in LGD, but at the same time states that bank estimates of long-term ultimate LGDs having no such systematic variations may be acceptable. Miu and Ozdemir (2006) argued that banks can incorporate conservatism into cyclical LGDs estimated in a point-in-time framework, without an LGD-PD correlation; however, they estimated commensurate increases in credit capital to compensate for this.

Serval studies argued for a two-stage approach to measuring LGD, first estimating an "estate LGD" at the obligor level, and then treating instrument-level LGDs according to a contingent claims approach, as under the *absolute priority rule* ("APR") as such recoveries can be viewed as collar options on residual value of the firm. However, it has been argued that the endogeneity of the bankruptcy decision would result in a measurement problem in the first-stage borrower level, and furthermore an extensive literature on violations of APR suggested a similar problem in the second-stage instrument level (Eberhart, Moore, & Rosenfeldt, 1990; Hotchkiss, 1993; Weiss, 1990; Carey & Gordy, 2007, 2016). Jacobs and Karagozoglu (2011) followed in this line of research by building a simultaneous equation model in the *beta-link generalized linear model* ("BLGLM") class for an extensive sample of 871 defaulted firms in the period 1985-2008 from the MURD dataset used herein, containing the complete capital structures of each obligor. In a departure from the then extant literature the authors found the economic and statistical significance of firm-specific, debt and equity-market variables; and further a then new finding that larger firms have significantly lower LGDs while larger loans have higher LGDs.

Several empirical LGD studies have shown that TTR can play an important role in LGD recoveries (see, for example, Gürtler & Hibbeln (2011), Yashkir & Yashkir (2013), and Carey & Gordy (2016)). Chen (2018), one of the key studies that we are extending, addressed this topic through a novel approach modeling LGD and TTR simultaneously, adopting the FRLM of Papke and Wooldridge (1996). The author empirically demonstrated the applicability of this model with a banking workout LGD dataset having both censored and non-censored LGDs for five model specifications, finding that as compared with a more standard approach there is superior performance, as well as a more conservative prediction of LGD and greater marginal sensitivity to macroeconomic factors. An advantage of this approach that explains these favorable results is that the model of Papke & Wooldridge (1996) predicted a LGD with bounded domain of between 0 and 1 that tends to outperform more complex models such as linear models with LGD (or log-odds transformed LGD) as the dependent variable; and other alternatives including logistic growth, censored Tobit, zero-inflated beta models, decision trees (Grunert & Weber, 2009; Bastos, 2010; Qi & Zhao, 2011; Yashkir & Yashkir, 2013). That said, these studies and others found that for any given dataset, model performance is mainly driven by the proper choice of covariates to model LGD.

Finally, we will mention the stress testing context in this domain, in satisfaction of the *Current Expected Credit Loss* ("CECL") (Financial Accounting Standards Board, 2016) accounting standards or compliance with the Federal Reserve's *Dodd-Frank Act stress testing* ("DFAST") program (Board of Governors of the Federal Reserve System, 2016) program. We observe that the predominant types of models used in the industry differ slightly from the earlier context of the financial crisis, as the applications must meet particular capital adequacy and accounting requirements not previously a consideration (Global Credit Data (GCD), 2019), which speaks to the importance of loan level modeling which is naturally the level at which LGD analytics is performed. Dwyer et al. (2014) developed a model that forecasts losses under stressed scenarios by combining separate PD, LGD, and EAD models that can serve as the LGD component of such a model suite. Their stressed LGD model projected recovery rates based upon stressed macroeconomic variable scenarios, debt type, industrial sector and stressed PD levels. The model used macroeconomic variables from the Federal Reserve Board's CCAR scenarios to facilitate the use for CCAR post-stress capital analysis. Pineau (2023) proposed a simple approach to include a modelled LGD as part of a stress test using the PD-LGD dependency, studied three such PD-LGD models and obtained an assessment of capital requirements that depends only on the PD. This approach was illustrated by conducting a carbon price stress test on the Stoxx 600 index.

Our approach extends the existing research along several dimensions. First, we contribute to prior work by modeling LGD jointly with the TTR at the instrument level. In particular, a simultaneous equation estimation of LGD and TTR is advantageous in that we can coherently address the aforementioned issues resolution bias and censoring. Second, as compared to extant work in the stream of research addressing LGD and TTR, using the MURD database we integrate new variables such as contractual, capital structure and obligor specific factors, and address the large corporate asset class not previously considered in this line of research. Third, in addition to these we confirm many of the findings of the literature in regard to determinants of LGD such as macroeconomic considerations. Finally, we test our model rigorously on unresolved LGD on an out-of-sample basis, contributing to the literature on the validation of LGD models.

3. Econometric Modeling of LGD and TTR

A useful taxonomy to characterize *stochastic data generating processes* ("SDGPs") consists of four dimensions (Chen, 2018) according to whether the state and time are discrete or continuous. An example of a process in credit risk management where both state and time are discrete is modeling account delinquency status (e.g., if 60 or 90 days past due is true or false) as measured over fixed time intervals (e.g., monthly, quarterly or annually). Typically, *binary choice models* (e.g., logistic regression) are used to model the probability of account delinquency or default in a fixed time interval. In the case where time is continuous and the state is discrete, commonly *survival models* are applied to predict the of time to event, examples kindred to default and recovery credit risk applications being analysis of duration of unemployment in labor economics or longevity in population demographics. In the case of a continuous response variable where time is a fixed interval, we see the application of regression models such as OLS or time series, examples in credit risk including spending or balance analysis within a predefined interval. We could classify in this category standard models of LGD that do not account for TTR and are thus subject to model specification error and resolution bias. Finally, where we have both continuous state and time that are not predetermined is how we can classify the LGD model considered herein, having a variable performance duration window of TTR. In this case a measurement of LGD is considered to have occurred when the all the recoveries have been received, the last workout cash flow in the case of bank loans, else the instruments received in settlement or reorganization following a default in the case of traded loans or bonds. This is in contrast to LGD mortality rate analysis which requires accurate measurements of the recovery discounted cash flows for all time periods (Dermine & Neto de Carvalho, 2006). As the task herein is forecasting of LGD starting at the default date, we need not convert the continuous TTR into discrete intervals as in the mortality rate analysis, which implies that the cumulative recovery workout recoveries need only be observable at the end of the workout period. In the case of the MURD database of loans and bonds, the recovery value is represented by the prices of the debt at the resolution of the default event, so that the LGD equals 1 one minus this amount discounted to the default date as a percent of par value.

The latent variable system of equations for LGD and TTR may be written as

$$\mathrm{LGD}^{*}=\begin{cases}\mathrm{LGD} & \text{for resolved instances,}\\ \text{missing} & \text{for unresolved instances,}\end{cases} \tag{1}$$

and

$$t^{*}=\begin{cases}t & \text{for resolved instances,}\\ \bar{t} & \text{for unresolved instances,}\end{cases} \tag{2}$$

where LGD is the latent variable for LGD^{*}. We may represent the dependence of LGD upon TTR in the following form,

$$\mathrm{LGD}^{*}=\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta t\right)+\upsilon, \tag{3}$$

where $\boldsymbol{x}$ is a vector of explanatory variables as of the default date, $\boldsymbol{\beta}$ is a vector of coefficients, $\upsilon \sim g(\bullet)$ is a random error term and t is the latent TTR variable (with coefficient δ) where we admit that the latter may covary with the LGD. $\bar{t}$ represents the TTR as of the LGD sample end date for the censored episodes. In Equation (1) the LGD will be missing if the default has not been resolved or the TTR is right censored. Following a property of the linear exponential family of distributions, the model in Equation (3) can be consistently estimated using a FRLM (Papke & Wooldridge, 1996), where $\Lambda(\boldsymbol{x},t)\in(0,1)$ for the set of explanatory variables $\boldsymbol{x}$ and the TTR $t\in\mathbb{R}^{+}$ assuming that the mean LGD $\Lambda(\boldsymbol{x},t)$ is correctly specified. Alternatively, we may estimate this model as a *nonlinear logistic growth* specification by further assuming the correct error distribution for $\upsilon\sim g(\bullet)$ jointly with an orthogonality condition $E(\upsilon|\boldsymbol{x},t)=0$. If TTR is correlated with LGD but omitted from the above model specification in (3), the LGD model developed on the historical defaulted loans will be biased when applied to non-defaulted obligations (Greene, 2012). Furthermore, we may enforce the positivity of the TTR and $E(t|\boldsymbol{z},\boldsymbol{\gamma})>0$ by modeling the TTR as a logarithmic transformation

$$\mathrm{Log}(t)=\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z}+u, \tag{4}$$

where $\boldsymbol{z}$ is a vector of explanatory variables as of the default, $\boldsymbol{\gamma}$ is a vector of coefficients and u is a random error term following a probability density function $u\sim f(\bullet)$. Commonly used parametric distributions for the TTR error term include gamma, lognormal, log-logistic, exponential and Weibull (Allison, 2007), all of which we consider as part of the model development process. It is further assumed that with sufficient time the instance of default will be resolved, which may be a reorganization or liquidation for loans or bonds, so that the missing LGDs resulting from the right censoring of TTR will become eventually become observable if the data sample period can be extended beyond the current sample end date.

In order to estimate the selection equation governing the TTR, a right censored log-likelihood function can be maximized using both resolved and unresolved observations, denoted as

$$\mathrm{LogL}^{ttr}(t,\boldsymbol{z},\boldsymbol{\gamma})=\sum_{resolved}\log\left[f(u|t,\boldsymbol{z},\boldsymbol{\gamma})\right]+\sum_{unresolved}\log\left[1-F(\bar{t})\right], \tag{5}$$

where $F(\bar{t})$ is the *cumulative density function* ("CDF") of u evaluated at $\mathrm{Log}(\bar{t})-\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z}$ in the case of unresolved defaults. As there is a relationship of recursion between Equations (3) and (5), coupled with conditioning upon right censoring of the final date in the LGD data sample, the model of LGD in Equation (3) incorporating TTR for resolved default may be estimated by maximizing the following log-likelihood function with explanatory variables $\{\boldsymbol{x},t\}$ and random disturbance term υ:

$$\begin{aligned}\mathrm{LogL}^{\mathrm{lgd}}\left(\mathrm{LGD},\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x},\delta t\right)&=\sum_{resolved}\log\left[g\left(\upsilon|\mathrm{LGD},\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x},\delta t\right)\right]\\&=\sum_{resolved}\log\left[g\left(\mathrm{LGD}-\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta t\right)\right)\right],\\&=\sum_{resolved}\log\left[g\left(\upsilon|\mathrm{LGD}-\frac{1}{\exp\left(-\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}-\delta t\right)\right)}\right)\right],\end{aligned} \tag{6}$$

where we choose the unit interval logistic growth model functional form,

$$\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta t\right)=\frac{1}{\exp\left(-\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}-\delta t\right)\right)}\in[0,1]. \tag{7}$$

As an alternative to the above derivation, we may assume that the LGD is generated from a distribution in the *linear exponential family*, which means that it inherits a log-likelihood function of the Bernoulli type. In this setting, *quasi-maximum likelihood* estimation may be deployed to estimate the FRLM in the following manner (Papke & Wooldridge, 1996):

$$\begin{aligned}&\mathrm{LogL}^{\mathrm{lgd}}\left(\mathrm{LGD},\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x},\delta t\right)\\&=\sum_{resolved}\mathrm{LGD}\times\log\left[\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta t\right)\right]+(1-\mathrm{LGD})\times\log\left[1-\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta t\right)\right].\end{aligned} \tag{8}$$

In either case of the logistic growth model (7) or the fractional response logit model (8), the expected LGD conditional on the risk factors $\{\boldsymbol{x},t\}$ is given by,

$$\mathrm{E}\left[\mathrm{LGD}(\boldsymbol{x},t)\right]=\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta t\right)=\frac{1}{\exp\left(-\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}-\delta t\right)\right)}. \tag{9}$$

Whether the default instance is resolved or unresolved, the complete log-likelihood function $\mathrm{LogL}^{\mathrm{lgd}}\left(\mathrm{LGD},t,\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x},\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z},\delta\right)$ conditional upon the data sample $\{\mathrm{LGD},t,\boldsymbol{x},\boldsymbol{z}\}$ is given by the summation of the marginal log-likelihood function $\mathrm{LogL}^{\mathrm{ttr}}\left(t,\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z}\right)$ from the TTR Equation (5) and $\mathrm{LogL}^{\mathrm{lgd}}\left(\mathrm{LGD},\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x},\delta t\right)$ from the LGD Equations (6) or (8), conditional upon t:

$$\begin{aligned}&\mathrm{LogL}\left(\mathrm{LGD},t,\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x},\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z},\delta\right)\\&=\sum_{resolved}\log\left[g\left(\upsilon\middle|t\right)\right]+\sum_{resolved}\log\left[f\left(t\right)\right]+\sum_{unresolved}\log\left[1-F\left(\bar{t}\right)\right]\\&=\sum_{resolved}\log\left[h(u,\upsilon)\right]+\sum_{unresolved}\log\left[1-F\left(\bar{t}\right)\right],\end{aligned} \tag{10}$$

where the joint PDF is the product of marginal distribution of TTR and conditional distribution for LGD given TTR $h(u,\upsilon)=g\left(\upsilon\middle|u\right)f\left(u\right)$. This recursive relationship between LGD and TTR implies that the complete log-likelihood function (10) may be computed by separately estimating the LGD model (6) or (8) and the TTR model (5). As the TTR is unknown at the time of default, the expected LGD conditional on the observed explanatory variables $\{\boldsymbol{x},\boldsymbol{z}\}$ is obtained through integrating over t:

$$\mathrm{E}\left(\mathrm{LGD}\middle|\boldsymbol{x},\boldsymbol{z}\right)=\mathrm{E}_t\left(\mathrm{E}_{u|t}\left[\mathrm{LGD}\middle|\boldsymbol{x},t\right]\right)=\int_{s=0}^{\infty}\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta s\right)f\left(s\middle|\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z}\right) \tag{11}$$

A consistent estimator of $\mathrm{E}\left(\mathrm{LGD}\middle|\boldsymbol{x},\boldsymbol{z}\right)$ is obtained by substituting consistent parameter estimates $\left\{\hat{\boldsymbol{\beta}},\hat{\delta},\hat{\boldsymbol{\gamma}}\right\}$ into $\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta s\right)$ and $f\left(t\middle|\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z}\right)$. Furthermore, for credit risk modeling applications such as stress testing or economic capital the LGD's sensitivity to marginal changes in explanatory variable $y\in\{\boldsymbol{x},\boldsymbol{z}\}$ is given by:

$$\frac{\partial\,\mathrm{E}\left(\mathrm{LGD}\middle|\boldsymbol{x},\boldsymbol{z}\right)}{\partial y}=\int_{s=0}^{\infty}\left[\frac{\partial\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta s\right)}{\partial y}f\left(s\middle|\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z}\right)+\frac{\partial f\left(s\middle|\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{z}\right)}{\partial y}\Lambda\left(\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{x}+\delta s\right)\right]\mathrm{d}s \tag{12}$$

We may obtain a consistent estimator of (12) through substitution of consistent $\left\{\hat{\beta},\hat{\delta},\hat{\gamma}\right\}$.

4. Data and Summary Statistics

We have built a database of defaulted firms representative of the U.S. large corporate loss experience (bankruptcies and out-of-court settlements) based upon the December 2023 release of the MURD. It contains data on 6094 defaulted instruments (bonds and bank loans) from 1987 to 2023 for 1162 borrowers, for which there is information on all classes of debt in the capital structure at the time of default. In Table 1 we summarize some key characteristics at the borrower level. On average, defaulted borrowers have around 5 instruments, but there are complex cases where as many as 80 instruments exist. Defaulted obligors have on average $1.26 Billion in defaulted debt, where there are relatively few cases where this is below $100 Million or approaching $100 Billion. The earliest year of default date (resolution) is in 1987 (1988) and the latest year of default or resolution 2022, with a mean TTR of just under a year although there are cases over 10 years with a maximum of almost 13 years. The average family (or estate) level LGD is 46.3%, ranging from −20.1% to 100%.

This dataset is largely representative of the U.S. large-corporate default experience over the last 30 years. All instruments are detailed by debt type, seniority, collateral type, position in the capital structure, original and defaulted amount, resolution outcome, and instrument price or value of securities at the resolution of default. Resolution types include emergence from Chapter 11 bankruptcy as an independent entity, acquisition by a third party, Chapter 7 liquidation or out-of-court settlement of a distressed exchange. It includes either the prices of prepetition instruments at the time of emergence from, or prices of new instruments received in settlement of, bankruptcy or other distressed restructuring, respectively. We calculate economic LGD by discounting nominal LGD by the coupon rate on the instrument prevailing just prior to default[6].

Table 2 presents the summary statistics and Pearson correlation coefficients of the key variables used in our models of LGD and TTR at the instrument level. The average LGD (TTR) is 40.1% (1.07 years), ranging from −40.2% (0 years) to 100% (12.9 years). Principal at default averages $240.4 Million, with some cases of nothing outstanding at default, and the maximum amount being $15.1 Billion. The mean proportion of debt above (below) an instrument in the capital structure is 48.0% (23.8%), with respective minimums and maximums of 0.0% and 20.5% (0.0% and 100.0%). The rank in the collateral structure is on average about 2, with a maximum of 8 and minimum of 1. The macroeconomic variable that we found to have the best performance in our models of LGD and TTR, the annual change in the industrial production index, averages (has a median) −0.15% (0.15%) over the cycle and has recessionary declines (expansionary increases) of −15.1% (13.3%). Regarding the Pearson correlation coefficients in Table 2, it can be seen that none of the risk factors have pairwise correlations

[6]We also replicate results with a risk-free term structure, as in Carey & Gordy (2007), as well as using a high yield index, as in Acharya et al. (2007), and results are not materially different.

Table 1. Summary statistics of obligor level characteristics—Moody's ultimate recovery database™ (December 2023 Release).

	Number of Instruments	Principal at Default ($MM)	Date of Default	Date of Resolution	Family Loss Given Default	Time to Resolution (Years)
Count			1162			
Minimum	1	0.00	4/17/1987	5/1/1988	−0.2013	0.00
25th Percentile	2	165.36	3/5/1999	7/12/2000	0.2596	0.20
Median	4	366.32	9/26/2002	2/7/2004	0.4679	0.59
Average	5.24	1260.66	1/9/2004	4/12/2005	0.4634	0.92
75th Percentile	6	978.12	6/8/2009	3/18/2010	0.6802	1.30
Maximum	80	77739.20	8/23/2022	12/16/2022	1.0000	12.85
Standard Deviation	5.96	3795.86	N/A	N/A	0.2728	1.08

Table 2. Summary statistics of instrument key variables—Moody's ultimate recovery database™ (December 2023 Release).

		Loss Given Default	Time to Resolution (Years)	Principle at Default (Millions)	Percent Debt Above	Percent Debt Below	Collateral Rank	Industrial Production (Annual Change)
Count					6094			
Minimum		−0.4021	0.0000	0.00	0.0000	0.0000	1.0000	−0.1507
25th Percentile		0.0000	0.1890	24.92	0.0000	0.0000	1.0000	−0.0283
Median		0.3615	0.6822	100.00	0.0644	0.1012	2.0000	0.0155
Average		0.4106	1.0749	240.38	0.2384	0.2571	1.8408	−0.0015
75th Percentile		0.7960	1.5205	262.54	0.4552	0.4801	2.0000	0.0295
Maximum		1.0000	12.8493	15977.00	1.0000	1.0000	8.0000	0.1328
Standard Deviation		0.3906	1.2435	581.23	0.3009	0.3046	1.0285	0.0504
Correlations	Loss Given Default		0.0837	−0.0186	0.4797	−0.5339	0.4319	−0.0494
	Time to Resolution			−0.0450	0.0007	−0.0658	−0.0424	0.0749
	Principle at Default (Millions)				−0.0209	−0.0348	0.0567	−0.1358
	Percent Debt Above					−0.5065	0.7649	−0.0740
	Percent Debt Below						−0.4602	0.0259
	Collateral Rank							−0.0797

that would raise an alarm about potential multicollinearity, and furthermore the signs of the correlations are all intuitive and the correlations (even when modest in magnitude) are all statistically significant. Focusing in on LGD and TTR, we see that the strongest drives are the percentage debt above (0.48 and 0.001), percentage debt below (−0.53 and −0.07) and the collateral rank (0.43 and −0.04); where lower means a higher rank) relative to the instrument, respectively. Critical to the theme of this study, we observe a Pearson correlation of TTR and LGD of 8.37% (which happens to be statistically significant), corroborated by the regression results to follow that show that this relationship is robust and in line with our core thesis. The macroeconomic variable that performs best in our model development testing, the annual change in industrial production, has a negative (positive) and significant correlation of −0.05 (0.07) with LGD (TTR), consistent with our expectation that loss severities (resolution periods) are worse (longer) during downturn economic periods. Finally, we find a direct (inverse) relationship between the amount of principal due at default and LGD (TTR) as evidenced by a correlation coefficient of −0.02 (0.05).

Before moving on to the results of the estimation of our econometric model for LGD, we will make note of some interesting descriptive statistics that have bearing upon the implications of this exercise. In Table 3 we show the rank of each defaulted instrument in the default resolution process, as determined by the bankruptcy court or main coalition of creditors. It can be seen that there is a preponderance of more highly rated instruments, with the model classes being at the top or penultimate slots. In general, both the mean LGD and TTR increase with a lower ranking of the instrument, with no clear pattern in the standard deviation.

In Table 4 we show the statistics of LGD and TTR according to the type of collateral. As expected, in terms of mean LGD more tangible collateral types such as cash (2.78%), accounts receivable (7.47%), inventory and accounts receivable (1.6%) have the best recoveries; and less tangible collateral types such non-current assets (358%), real estate (21.2%) and plant/property/equipment (34.9%) experience elevated loss severities; and unsecured credit the most inferior severity experience (50.0%). The pattern in average TTR is broadly consistent with that in LGD along this dimension, with real estate obligations having the most drawn-out resolution episodes (2.29 years) and cash secure instruments the speediest resolutions (0.80 years).

In Table 5 we summarize the average and volatility of LGD and TTR by industry group, as measured by the Moody's broad industry classification. It can be observed that non-bank financials (50.1%) and real estate finance companies (64.7%) have the highest LGDs, while public utilities (5.35%) and securities firms (7.8%) have the best recoveries on average. The patterns in TTR are broadly consistent with those seen in LGD, as real estate companies have the lengthiest resolution periods (2.65 years), while industrial firms are below average (0.99 years).

Table 3. Summary statistics of instrument key variables by collateral rank—Moody's ultimate recovery database™ (December 2023 Release).

		Loss Given Default		Time-to-Resolution	
Collateral Rank	Count	Average	Standard Deviation	Average	Standard Deviation
1	2842	0.2153	0.3061	1.1341	1.3138
2	2018	0.5214	0.3749	1.0597	1.2413
3	808	0.6597	0.3661	0.8934	0.9769
4	281	0.7534	0.3226	1.0697	1.0490
5	97	0.6626	0.3229	1.4252	1.5508
6	36	0.7132	0.3800	0.7375	1.0097
7	5	0.5074	0.4609	0.2608	0.3800
8	7	0.1466	0.1829	0.0000	0.0000
Total	6094	0.4106	0.3906	1.0749	1.2435

Table 4. Summary statistics of instrument key variables by collateral type—Moody's ultimate recovery database™ (December 2023 Release).

Collateral Type	Count	Loss Given Default		Time-to-Resolution	
		Average	Standard Deviation	Average	Standard Deviation
Accounts Receivable	46	0.0747	0.1919	1.3518	1.2853
All Assets	1712	0.1786	0.2780	0.8574	1.0300
All Current Assets	94	0.0140	0.0996	0.5233	0.4515
All Non-current Assets	154	0.3575	0.3105	0.6794	0.7428
Capital Stock	209	0.3322	0.3101	1.7813	1.7533
Cash	17	0.0278	0.1147	0.7963	0.8123
Equipment	130	0.5138	0.3641	1.0170	0.5701
Guarantees	19	−0.0180	0.0470	1.3583	0.9389
Intellectual Property	8	0.4648	0.3110	0.7346	0.4874
Intercompany Debt	1	0.8735	N/A	0.4822	N/A
Inventory	18	0.0690	0.1344	1.4563	1.1113
Inventory and Accounts Receivables	132	0.0160	0.0822	0.9644	1.1347
Most Assets	42	0.1852	0.2345	1.2238	0.8767
Oil and Gas Property	16	0.0084	0.0337	0.6101	0.6947
Other	2	0.5459	0.3519	1.3863	1.4375
PP&E	172	0.3493	0.3072	1.9496	1.2883
Real Estate	25	0.2122	0.3390	2.4563	2.2914
Second Lien	343	0.5111	0.3899	0.6788	0.7295
Third Lien	53	0.6146	0.3917	1.1062	1.1901
Unsecured	2901	0.5895	0.3762	1.1756	1.3526
Grand Total	6094	0.4106	0.3906	1.0749	1.2435

Table 5. Summary statistics of instrument key variables by industry classification—Moody's ultimate recovery database™ (December 2023 Release).

Instrument Type	Loss Given Default			Time-to-Resolution	
	Count	Average	Standard Deviation	Average	Standard Deviation
FINANCE	30	0.2105	0.0714	1.1408	0.2040
INDUSTRIAL	5091	0.4255	0.3925	0.9926	1.2321
OTHER NON-BANK	16	0.5099	0.2044	1.7122	0.8785
PUBLIC UTILITY	212	0.0525	0.1690	2.4101	1.3296
REAL ESTATE FINANCE	25	0.6470	0.2761	3.1369	2.6522
SECURITIES	3	0.0777	0.1346	1.1918	0.0000
STRUCTURED FINANCE	7	0.5247	0.3726	0.3190	0.2874
THRIFTS	2	0.0000	0.0000	1.0055	0.0000
TRANSPORTATION	369	0.4315	0.3682	1.4173	0.9717
UNASSIGNED	336	0.3847	0.3985	0.9353	0.7949
Unknown	3	0.5891	0.2377	0.3251	0.0364
Grand Total	6094	0.4106	0.3906	1.0749	1.2435

In Table 6 we present an interesting analysis that is only tangentially related to the subject matter of this study, as we do not have proxy variables for these characteristics that would be known ex ante, the type of default event and resolution type. First addressing default type, we see that on average bankruptcies (cures and distressed exchanges) have the highest (lowest) mean LGD of 45.9% (3.11% and 29.3%, respectively); and this rank ordering is preserved for mean TTR of 1.31 years (0.14 and 0.01 years, respectively). In terms of resolution type, mean LGD is highest (lowest) in the case of liquidations (acquisitions or emergences) at 54.3% (45.3% or 39%, respectively); and with default type the rank ordering is preserved for TTR, a highest (lowest) mean of 1.65 years (1.54 or 0.96 years, respectively).

We conclude this section by presenting some graphical analyses of LGD and TTR. In Figure 1 we plot the averages of LGD and TTR by quarter, where we observe that while the data is noisy, an elevation in LGD and TTR during economic downturns is visible. In Figure 2 we plot the average and count of LGD by TTR quarter, where we can see that the mean LGD does increase with TTR albeit noisily, and the number of LGD observations clearly declines with TTR. In Figure 3 we show a scatter plot of LGD and TTR along with a linear trend line, where we see that by the relationship is positive (with LGD on average increasing by 2.6% per year TTR), confirming the Pearson correlation result of 0.084 in Table 2.

5. Empirical Results

In this section we discuss the estimation results and empirical findings for the FRLM and survival models as applied to LGD and TTR, respectively. We first

Table 6. Summary statistics of instrument key variables by default and resolution types—Moody's ultimate recovery database™ (December 2023 Release).

Resolution Type		Acquired		Emerged		Liquidated		Total		Cnt.
	Default Type	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.	
LGD	Bankruptcy	0.4674	0.3994	0.4442	0.3882	0.5427	0.4112	0.4590	0.3935	5004
	Default and Cure	0.0000	N/A	0.0046	0.0311	N/A	N/A	0.0046	0.0311	57
	Distressed Exchange	0.2105	0.3129	0.2010	0.2927	N/A	N/A	0.2012	0.2929	998
	Other Restructuring	0.2388	0.3618	0.0978	0.1362	N/A	N/A	0.1219	0.1935	35
	Total	0.4526	0.3990	0.3900	0.3834	0.5427	0.4112	0.4106	0.3906	6094
TTR	Bankruptcy	1.6383	1.5149	1.2192	1.2438	1.6455	1.1079	1.3056	1.2588	5004
	Default and Cure	0.0000	N/A	0.0786	0.1442	0.0000	N/A	0.0786	0.1442	57
	Distressed Exchange	0.0000	N/A	0.0119	0.0518	0.0000	N/A	0.0117	0.0514	998
	Other Restructuring	0.1096	0.0179	0.0080	0.0179	0.0000	N/A	0.0254	0.0624	35
	Total	1.5430	1.5177	0.9647	1.2093	1.6455	1.1079	1.0749	1.2435	6094
Count		371		5052		671				

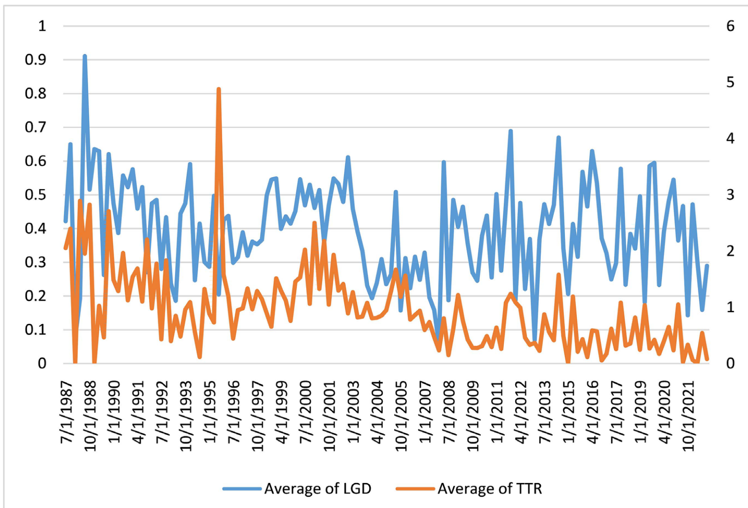

Figure 1. Average loss-given-default and time-to-resolution by quarter—Moody's ultimate recovery database™ (December 2023 Release).

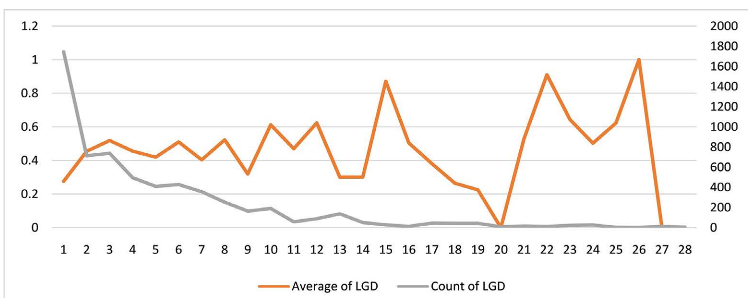

Figure 2. Average loss-given-default and time-to-resolution by quarter—Moody's ultimate recovery database™ (December 2023 Release).

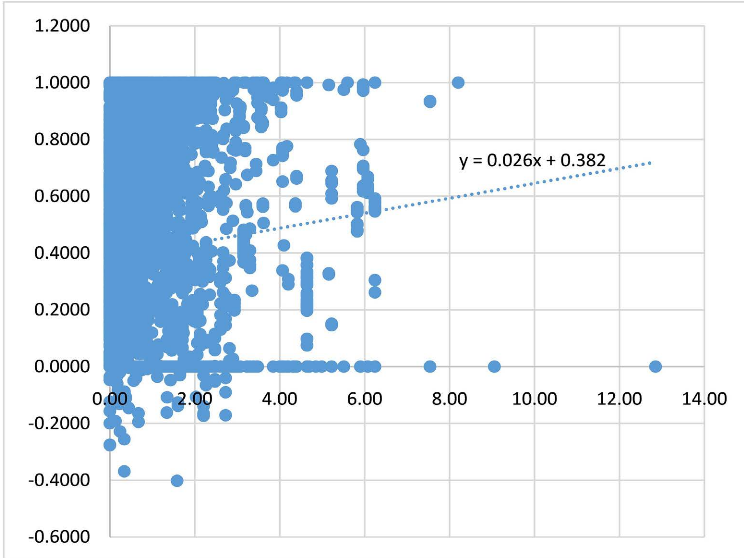

Figure 3. Scatter of loss-given-default and time-to-resolution—Moody's ultimate recovery database™ (December 2023 Release).

study the distributional properties of the target variables, benchmark to an OLS estimation for LGD both including and excluding TTR and describe the empirical strategy for testing the prediction out-of-sample with the MURD dataset. In **Figure 4** we show the histogram of LGD, where it is observed as seen in the summary statistics there are instances of negative LGDs. In the estimation of the FRLM negative LGDs are not permitted, so a modeling option is to floor the LGD observations at zero, the histogram of such observations shown in **Figure 5**. It is observed that the differences in the distributions are minimal, and indeed in an OLS model we find negligible performance differences in modeling floored or raw LGD, which we omit for purposes of brevity. The next consideration is how we can test LGD predictions out-of-sample, in the sense of having observations which are not resolved and TTR is known, so that we can study how incorporating our joint model of LGD and TTR can improve the prediction. As the MURD dataset only has resolved defaults, in order to accomplish this, we partition the dataset into resolved defaults prior to 2018 as an in-sample period, and the remainder of the population (either unresolved defaults as of the end of 2017, else defaulted and resolved defaults after 1997) as an out-of-sample period. In **Figure 6** and **Figure 7** we show the data histograms of LGD for the synthetic in- and out-of-sample periods, respectively, where we can observe that they are rather similar.

Finally for the descriptive part of this section, in **Figure 8** we show the average and count of resolved LGDs by TTR for the in-sample period, where we observe a similar pattern of declining counts and higher mean (as well as more volatile) LGD; and in **Figure 9** below we show the counts of unresolved LGDs by TTR in the out-of-sample period, where the counts are declining and as expected the average TTR is shorter at 2.61 years as compared to 5.09 years for the resolved LGDs.

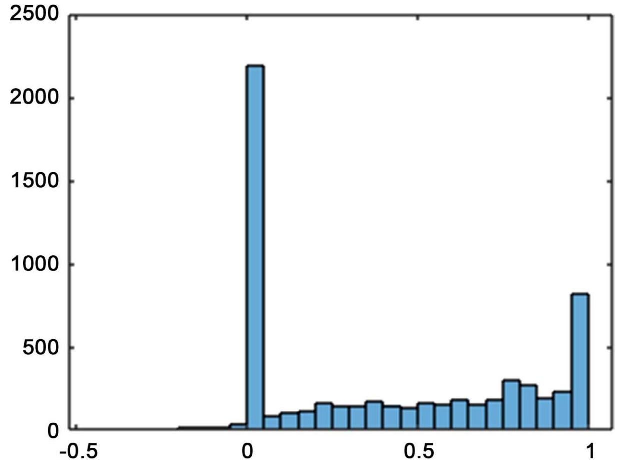

Figure 4. Histogram of raw loss-given-default—Moody's ultimate recovery database™ (December 2023 Release).

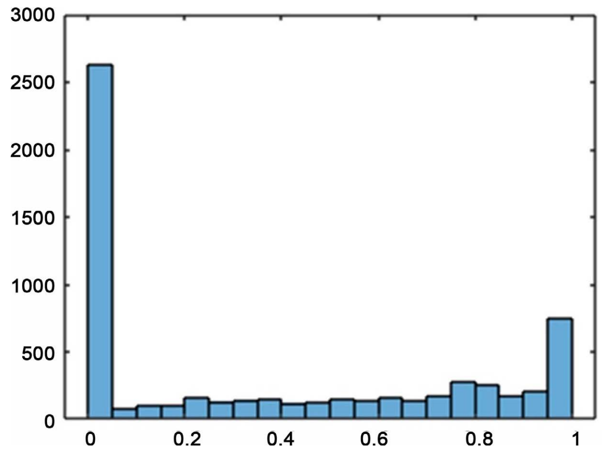

Figure 5. Histogram of floored loss-given-default—Moody's ultimate recovery database™ (December 2023 Release).

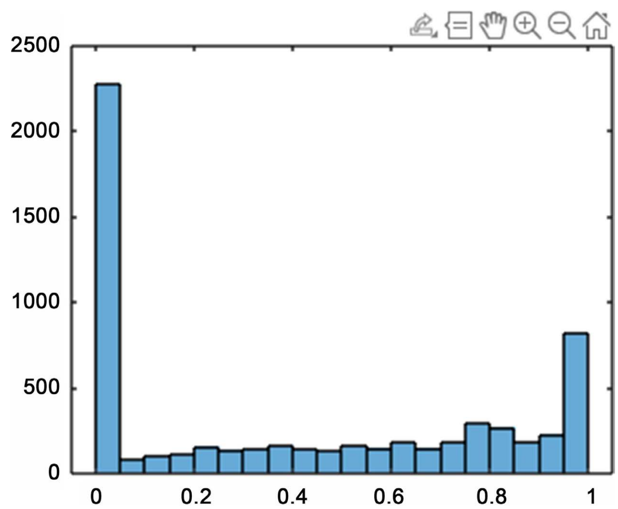

Figure 6. Histogram of floored loss-given-default for defaults prior to 1998—Moody's ultimate recovery database™ (December 2023 Release).

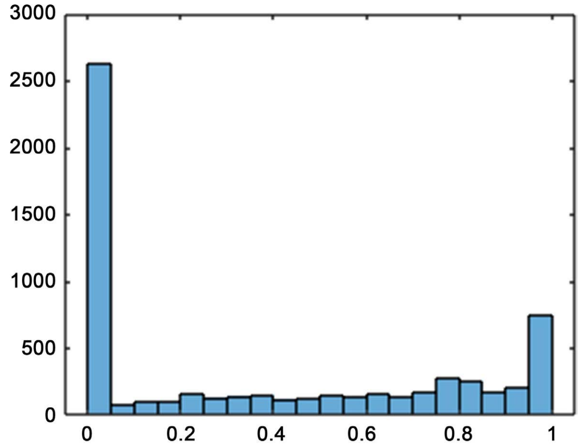

Figure 7. Histogram of floored loss-given-default for defaults subsequent to 1997—Moody's ultimate recovery database™ (December 2023 Release).

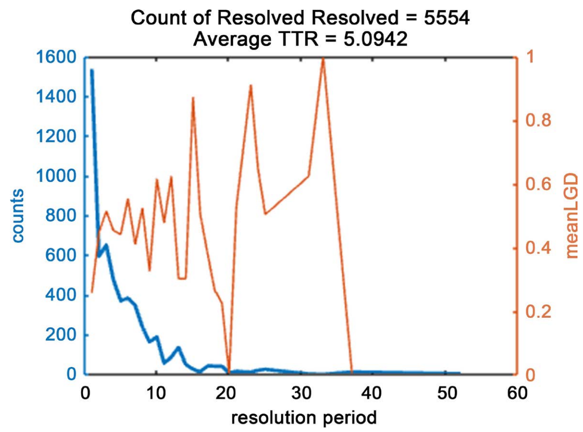

Figure 8. Average and count of loss-given-default observation by time-to-resolution for resolved defaults prior to 1998—Moody's ultimate recovery database™ (December 2023 Release).

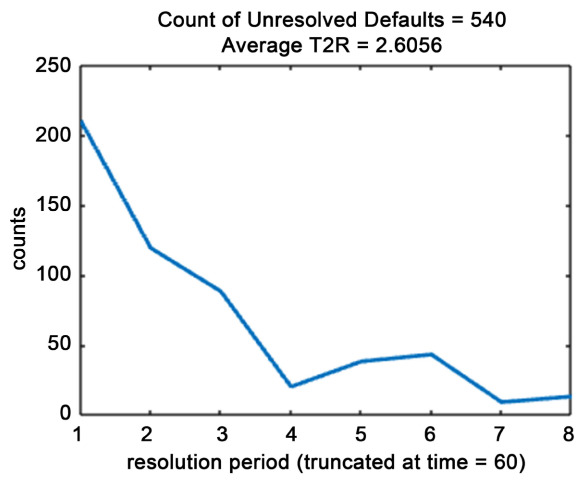

Figure 9. Count of loss-given-default observation by time-to-resolution for unresolved defaults subsequent to 1997—Moody's ultimate recovery database™ (December 2023 Release).

In Table 7 we show the OLS regression estimation results for models of LGD with and without the TTR. All signs on coefficient estimates are as expected based on economic intuition and all are highly statistically significant. Note that we added an additional risk factor, an indicator for highly tangible collateral (accounts receivables, cash, inventories and oil & gas properties), that we found in our model selection process to add considerable predictive power. When including TTR in the model, there is a slight increase in predictive power of 0.3320 to 0.3360 (0.6255 to 0.6259) according to adjusted r-squared (*loss percentage coverage ratio*—"LPCR"). The LPCR is an analogue to an accuracy ratio as used in the PD literature that is used in the LGD literature (Chen, 2018), the associated plots for which are shown in Figure 10 and Figure 11 for the models including and excluding TTR, respectively. In subsequent analysis we will extend this methodology in a rolling out-of-time and sample resampling cross-validation exercise.

The estimation results for the FLRM are shown in Table 8. All signs on coefficient estimates are economically intuitive and statistically significance. However, the magnitudes of the coefficient estimates are materially changed, notably with larger coefficient estimates on most of the risk factors (the balance at default, macroeconomic factor annual change in *industrial production* ("IP"), TTR, collateral rank and tangibility indicator, as well as percent debt above), and lower magnitudes only on the percent debt below, so this specification has greater sensitivity to inputs. As seen in Figure 12, the LPCR of 0.8264 is higher in the FRLM than in the linear models.

In Table 9 we show the estimation results for the survival model of TTR. The four risk factors that we found to be significant—the macroeconomic factor the annual change in IP, collateral rank, percent debt above and the defaulted balance—all have signs as expected (in line with that in the LGD equations) and are all statistically significant. In terms of absolute performance of the TTR model, we show the LPCR curve in Figure 13, where the value is 0.88104. Finally, for the TTR model, in Figure 14 and Figure 15 we show the histograms of predicted vs. observed TTRs for resolved and unresolved LGDS, respectively.

It could be argued that the differences in the performance metrics between the champion and challenger models may not be material, for example the LPCR difference of just under 1%. We test this by performing a *walk forward* 5-*fold cross-validation bootstrap* ("WF5FCVB") with respect to the LPCR measure, the mechanics of which are as follows. We partition between 1986-1999 and 2000-2023 to roughly divide the dataset in halves temporally. We resample with replacement from the in- and out-of-sample datasets repeatedly (1000 bootstraps), where in each iteration we train the model on the former and test the model on the latter partition. Then a year is added to the in-sample partition, subtracted from the out-of-sample partition, and the training and testing iterations are repeated. We proceed until there is the single holdout year of 2023, cumulating to 14K iterations in all, where we collect the LPCR measurement in each iteration for its distribution to be studied.

Table 7. OLS regressions of loss-given-default—Moody's ultimate recovery database™ (December 2023 Release).

	Risk Factor	Coefficient Estimate	Standard Error	Probability Value	Root Mean Squared Error	Adjusted R-Squared
Regression Including Time-to-Resolution	Intercept	0.3886	0.0133	4.71E−174	0.3190	0.3360
	Industrial Production (Annual Change)	−0.1463	0.0867	9.15E−02		
	Time-to-Resolution	0.0049	0.0009	9.86E−09		
	Collateral Rank	0.0384	0.0068	1.96E−08		
	Collateral Tangibility Indicator	−0.1261	0.0226	2.52E−08		
	Percent Debt Below	0.2425	0.0235	8.78E−25		
	Percent Debt Above	−0.4727	0.0169	6.50E−162		
	Balance at Default	−0.0239	0.0074	1.34E−03		
Regression Excluding Time-to-Resolution	Intercept	0.4195	0.0122	1.80E−234	0.3200	0.3320
	Industrial Production (Annual Change)	−0.1192	0.0868	1.70E−01		
	Collateral Rank	0.0359	0.0068	1.46E−07		
	Collateral Tangibility Indicator	−0.1259	0.0227	2.94E−08		
	Percent Debt Below	0.2450	0.0235	4.07E−25		
	Percent Debt Above	−0.4810	0.0168	2.08E−167		
	Balance at Default	−0.0248	0.0075	8.64E−04		

Table 8. Fractional Response logit model estimation of loss-given-default including time-to-resolution—Moody's ultimate recovery database™ (December 2023 Release).

Risk Factor	Coefficient Estimate	Standard Error	Probability Value	Negative Log-Likelihood	Bayesian Information Criterion
Intercept	−0.4205	0.0931	0.00E+00	6166.40	6235.38
Industrial Production (Annual Change)	−2.7533	0.1431	1.41E−12		
Time-to-Resolution	0.8909	0.1594	1.77E−10		
Collateral Rank	0.1912	0.0485	0.00E+00		
Collateral Tangibility Indicator	−1.9941	0.3836	4.57E−03		
Percent Debt Below	0.0254	0.0061	0.00E+00		
Percent Debt Above	−0.8446	0.6319	5.68E−03		
Balance at Default	−0.1414	0.0713	0.00E+00		

Table 9. Survival model estimation of time-to-resolution—Moody's ultimate recovery database™ (December 2023 Release).

Risk Factor	Coefficient Estimate	Standard Error	Probability Value	Negative Log-Likelihood	Bayesian Information Criterion
Intercept	−0.4205	0.1897	5.68E−221	167.14	210.25
Industrial Production (Annual Change)	−2.7533	1.3632	6.61E−05		
Collateral Rank	0.8909	0.0758	2.28E−07		
Percent Debt Above	0.1912	0.2509	1.64E−12		
Balance at Default	−1.9941	0.1166	5.99E−03		

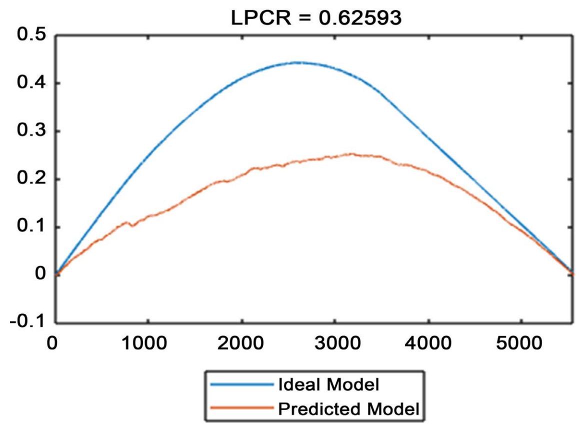

Figure 10. Loss percentage coverage ratio curve for ols model of loss-given-default including time-to-resolution—Moody's ultimate recovery database™ (December 2023 Release).

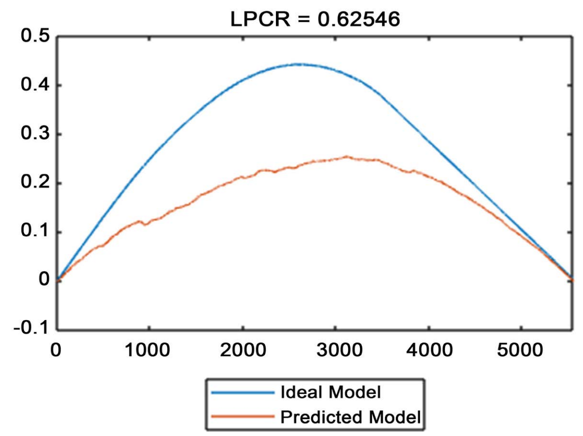

Figure 11. Loss percentage coverage ratio curve for ols model of loss-given-default excluding time-to-resolution—Moody's ultimate recovery database™ (December 2023 Release).

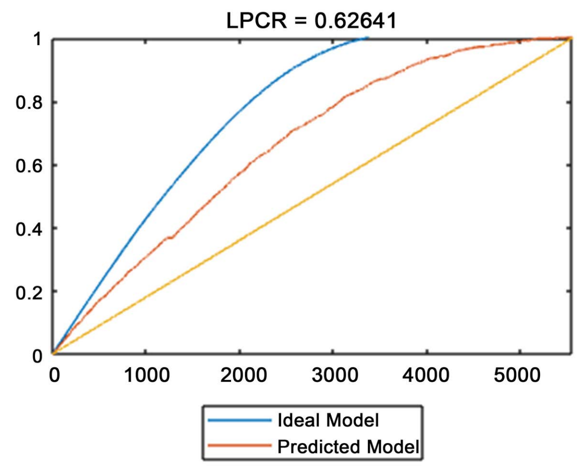

Figure 12. Loss percentage coverage ratio curve for fractional response logit model of loss-given-default including time-to-resolution—Moody's ultimate recovery database™ (December 2023 Release).

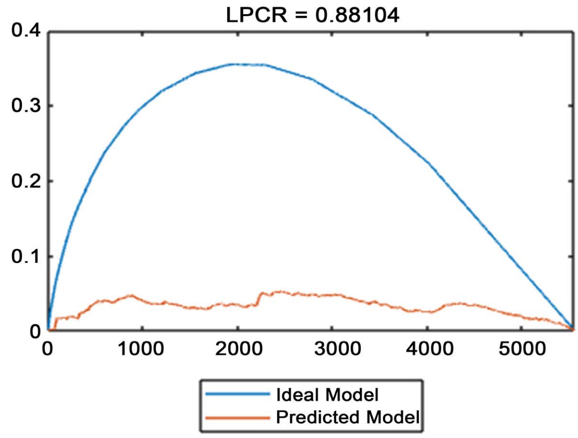

Figure 13. Loss percentage coverage ratio curve for survival model of time-to-resolution—moody's ultimate recovery database™ (December 2023 Release).

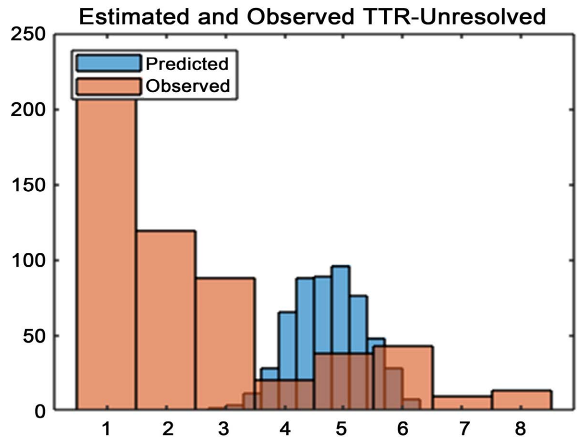

Figure 14. Histograms of observed and predicted values for survival model of resolved time-to-resolution—moody's ultimate recovery database™ (December 2023 Release).

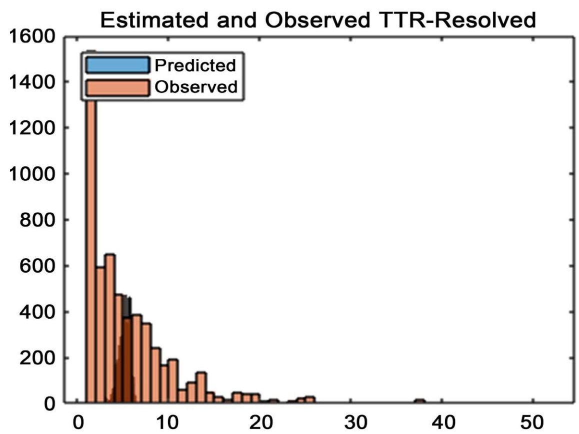

Figure 15. Histograms of observed and predicted values for survival model of unresolved time-to-resolution—Moody's ultimate recovery database™ (December 2023 Release).

In **Table 10** we summarize performance statistics for predicting unresolved LGDs out-of-sample. We compare the champion FRLM and survival models for LGD and TTR, respectively, versus a challenger linear model that is a 3 SLSLRM (Zellner & Theil, 1992). We tabulate the LPCR and three types of correlations measures (Pearson Linear, Kendall tau and Spearmen rank order). It is observed that the champion model outperforms the linear model across each measure of performance. In **Figures 16-19** we show the respective LPCR curves (prediction histograms) for the champion and challenger models.

The results of the WF5FCVB testing analysis are shown in **Figure 20** and **Table 11**, the data histograms and summary statistics of the LPCR measures for the champion and challenger models, respectively. Examining the figure and statistics, while in terms of mean we may reject that the LPCRs are different, the distributions tell a more nuanced story. The challenger model has many more prejudicial outcomes than does the champion model, with approximately 30% of the outcomes in the range of around 60% - 62%, while about half of the challenger's values lying around 63% - 65%, which is a meaningful bifurcation in performance.

6. Discussion and Conclusion

In this study, we have empirically investigated LGD, a critical component of risk management, pricing, and portfolio models of credit along the neglected dimension of the TTR. We have argued that especially when considering loans that may not be traded, taking into consideration when cash was received as well as other losses incurred in the collection process, ultimate LGD is the relevant measure for an input into a regulatory or economic credit capital model. As our primary objective is to provide results of use to agents invested in defaulted securities having time horizons that span the resolution period, who wish to assess expected value upon emergence relative to some benchmark available at default, we have focused on the TTR aspect. Agents who would benefit include bank workout specialists, risk managers or vulture hedge fund investors. Furthermore, our results would be relevant for financial institutions attempting to quantify economic LGD for purposes of the Basel AIRB approach to regulatory capital, which requires estimation of the ultimate LGD.

We have highlighted that the length of the data sample used for estimating LGD may be relatively short, while the TTR may be relatively long, resulting in a right censoring of recovery cash flows due to the end date of the sample. Arguably, it is likely that censored observations may have different characteristics from those of non-censored or closed counterparts, and furthermore as we have shown if the TTR is correlated with LGD but omitted from the specification of the LGD model, then such model will suffer from a specification error due to both an omitted variable bias as well as a sample selection bias as demonstrated by our empirical results.

We have also emphasized that another important aspect of considering TTR in the context of LGD modeling is the implications for *stress testing*—where

Table 10. Out-of-Sample unresolved lgd performance measures for champion fractional response logit model for loss-given-default and survival model for time-to-resolution vs. challenger 3 stage least squares simultaneous equation linear model for lgd & ttr—moody's ultimate recovery database™ (December 2023 Release).

Performance Measure	Champion Fractional Response Logit Model for LGD & Survival Model for TTR	Challenger Linear 3 Stage Least Squares Simultaneous Equation Model
Loss Percent Coverage Ratio	0.6211	0.6131
Pearson Correlation	0.4710	0.4319
Kendall Correlation	0.6185	0.5974
Spearman Correlation	0.6211	0.6131

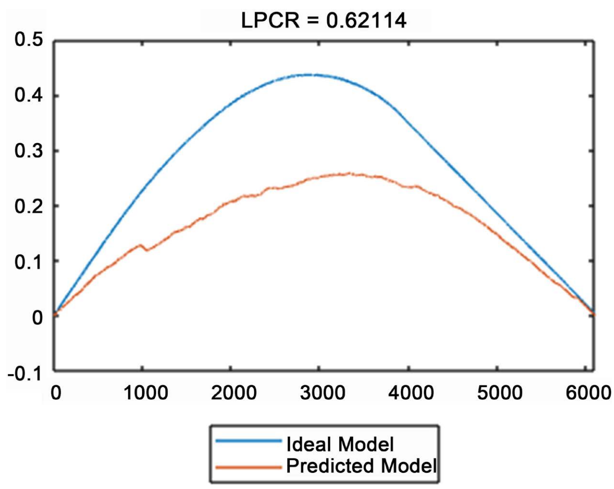

Figure 16. Loss percentage coverage ratio curve for champion fractional response logit model for loss-given-default survival model for time-to-resolution in predicting out-of-sample unresolved lgds—moody's ultimate recovery database™ (December 2023 Release).

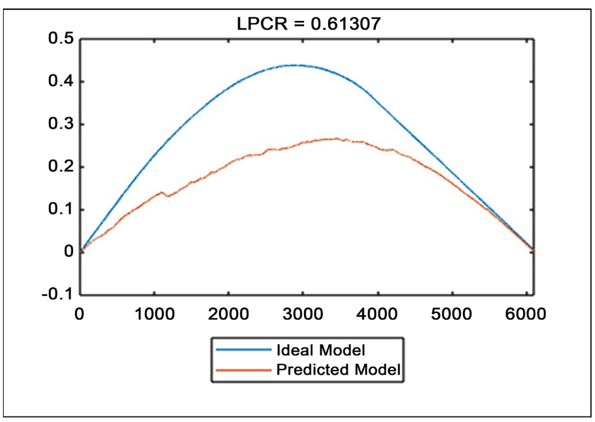

Figure 17. Loss percentage coverage ratio curve for challenger linear 3-stage least squares linear simultaneous equation model for loss-given-default and time-to-resolution in predicting out-of-sample unresolved lgds—moody's ultimate recovery database™ (December 2023 Release).

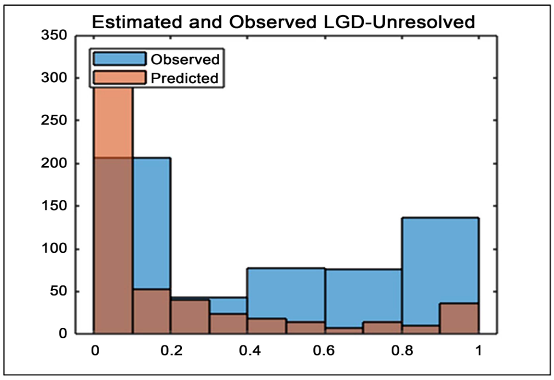

Figure 18. Prediction histograms for champion fractional response logit model for loss-given-default survival model for time-to-resolution in predicting out-of-sample unresolved lgds—moody's ultimate recovery database™ (December 2023 Release).

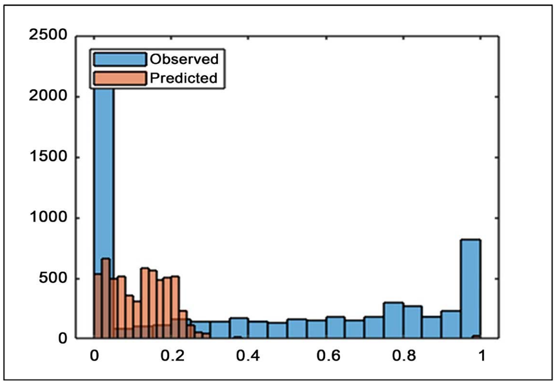

Figure 19. Prediction histograms for challenger linear 3-stage least squares linear simultaneous equation model for loss-given-default and time-to-resolution in predicting out-of-sample unresolved lgds—moody's ultimate recovery database™ (December 2023 Release).

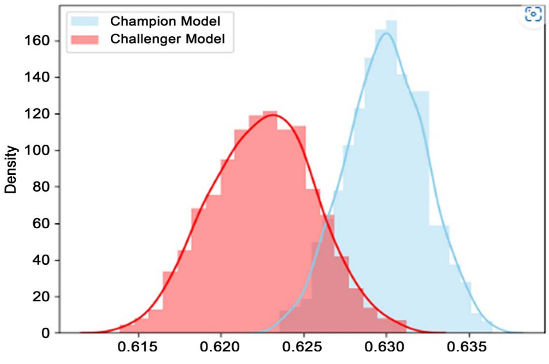

Figure 20. Data histograms for walk forward 5-fold cross-validation bootstraps of the loss percentage coverage ratio of the champion fractional response logit and the challenger linear 3-stage least squares linear simultaneous equation model for loss-given-default and time-to-resolution in predicting out-of-sample unresolved lgds—moody's ultimate recovery database™ (December 2023 Release).

Table 11. Summary statistics for walk forward 5-fold cross-validation bootstraps of the loss percentage coverage ratio of the champion fractional response logit and the challenger linear 3-stage least squares linear simultaneous equation model for loss-given-default and time-to-resolution in predicting out-of-sample unresolved lgds—moody's ultimate recovery database™ (December 2023 Release).

	Champion Model	Challenger Model
Count	14,000	
Minimum	0.6227	0.6138
5th Percentile	0.6260	0.6176
25th Percentile	0.6283	0.6204
Median	0.6200	0.6224
Average	0.6300	0.6225
75th Percentile	0.6317	0.6247
95th Percentile	0.6338	0.6276
Maximum	0.6367	0.6309
Standard Deviation	0.0024	0.0031
Skewness	0.0182	0.0183
Kurtosis	−0.4295	−0.2215

currently these exercises are accepted as the primary means of supporting capital planning, business strategy and portfolio management decision making—as a model for LGD that explicitly accounts for the time dimension is naturally suited to the development of forward looking forecast scenarios in stress testing exercises.

We have shown the modeling utility of considering the joint distributional properties of LGD and TTR. While the available theory and empirical evidence suggest both LGD and TTR to be stochastic and predictable with respect to other variables, in most extant credit models LGD has been treated as either deterministic or as an exogenous stochastic process, and TTR has been completely ignored. The quest for tractability gives rise to such assumptions, but in practical applications this results in understated capital, mispricing, and unrealistic dynamics of model outputs. Our research helps to resolve such deficiencies by modeling ex ante the distributions of LGD and TTR as functions of empirical determinants such as contractual features, firm capital structure, borrower characteristics and systematic factors. In order to empirically investigate the determinants of LGD and TTR we have built predictive econometric models using the Moody's MURD, a dataset containing the complete capital structures of each obligor, and a sample that is highly representative of the U.S. large corporate loss experience over the last two decades.

This study has contributed to the research on LGD in several respects. The methodology has constructed an internally consistent model of LGD and TTR in line with prior empirical findings and theoretical expectations, which has favorable features in line with industry and supervisory expectations regarding model validation (Board of Governors of the Federal Reserve System, 2011). Furthermore, we have extended the prior literature on considering LGD and TTR through using an extensive sample of corporate bond and loan defaults with the consideration of both obligor complete capital structure characteristics and macroeconomic factors. In the process, we have not only addressed several academic aspects of LGD, but also have offered an actionable approach that may be applied by bank portfolio and risk managers, as well as supervisors and market participants in this space.

In the stress testing domain, in satisfaction of the CECL standards and the DFAST program, we observed that the predominant types of models used in the industry differ slightly from the earlier context of the financial crisis, as the applications must meet particular capital adequacy and accounting requirements not previously a consideration, which speaks to the importance of loan level modeling which is naturally the level at which LGD analytics is performed. Furthermore, our model that considers the time dimension of LGD modeling through consideration of the TTR, is a framework is particularly suited to this use case.

We have found that in the MURD database sample that LGD and TTR are positively correlated, and covary significantly and intuitively with a set of standard risk factors where some are common to each. We compared a linear regression model for both LGD and TTR to a champion approach, the FRLM for LGD and a survival regression model for TTR, showing that the chosen approach has better qualitative aspects (including increased sensitivity to risk factors) and superior predictive performance. We then considered a challenger simultaneous linear 3SLSLRM approach, and demonstrated that the favored approach outperforms on an out-of-sample basis in predicting unresolved LGDs.

Finally, we make note of the contributions that have been made in this study to the science and practice of model validation. We have highlighted challenges around demonstrating the conceptual soundness and adequacy of performance for LGD models, as doing so is critical in the model validation process that lies at the heart of programs to measure and manage model risk. We have also pointed out a well-known observation in the industry that models for LGD suffer from a depth and breadth of data for model development, contributing to a proliferation of model validation issues handed down from independent validation groups and other 3rd party reviewers. In order to help address such challenges, we have contributed to their resolution by studying the TTR, which allows institution to leverage the information on recoveries embedded in non-resolved instances of default, which may lead to more robust models and greater accuracy in computing portfolio credit risk measures. Furthermore, aside from the possibility of producing more accurate models of LGD, we also addressed a conceptual gap, as it is recognized that resolution uncertainty is the 2nd main source of uncertainty in recoveries on defaulted debt, yet most models of LGD seen in the industry do not account for this aspect. In addressing the violation of the assumption that TTR does not influence LGD, this study demonstrated the measurement of model risk more broadly, as opposed to the narrower concept of parameter uncertainty in a statistical model of LGD.

There are several dimensions along which we may extend this research, including but not limited to:

- Alternative data sources, for example, bank loan data;
- Alternative novel estimation techniques, such as machine learning algorithms; and,
- An application to CECL or CCAR stress testing.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Acharya, V. V., Bharath, S. T., & Srinivasan, A. (2007). Does Industry-Wide Distress Affect Defaulted Firms? Evidence from Creditor Recoveries. *Journal of Financial Economics, 85,* 787-821. https://doi.org/10.1016/j.jfineco.2006.05.011

Allison, P. (2007). *Survival Analysis Using SAS: A Practical Guide.* School of Advanced Studies.

Altman, E. I. (1968). Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. *The Journal of Finance, 23,* 589-609. https://doi.org/10.1111/j.1540-6261.1968.tb00843.x

Altman, E. I. (2006). *Default Recovery Rates and LGD in Credit Risk Modeling and Practice: An Updated Review of the Literature and Empirical Evidence.* Working Paper, NYU Salomon Center.

Altman, E. I., & Eberhart, A. C. (1994). Do Seniority Provisions Protect Bondholders' Investments? *The Journal of Portfolio Management, 20,* 67-75. https://doi.org/10.3905/jpm.1994.67

Altman, E. I., & Fanjul, G. (2004). *Defaults and Returns on High Yield Bonds: First Quarter 2004 (April).* NYU Law and Economics Research Paper No. 04-008. https://doi.org/10.2139/ssrn.550482

Altman, E. I., & Kishore, V. M. (1996). Almost Everything You Wanted to Know about Recoveries on Defaulted Bonds. *Financial Analysts Journal, 52,* 57-64. https://doi.org/10.2469/faj.v52.n6.2040

Altman, E. I., & Ramayanam, S. (2006). *The High Yield Bond Default and Return Report: 3rd Quarter 2006 Review.* Special Report, New York University Salomon Center.

Altman, E. I., Brady, B., Resti, A., & Sironi, A. (2005). The Link between Default and Recovery Rates: Theory, Empirical Evidence, and Implications. *The Journal of Business, 78,* 2203-2228. https://doi.org/10.1086/497044

Altman, E. I., Haldeman, R. G., & Narayanan, P. (1977). ZETATM Analysis a New Model to Identify Bankruptcy Risk of Corporations. *Journal of Banking & Finance, 1,* 29-54. https://doi.org/10.1016/0378-4266(77)90017-6

Altman, E. I., Resti, A., & Sironi, A. (2001). *Analyzing and Explaining Default Recovery*

Rates. ISDA Research Report.

Araten, M., Jacobs, Jr., M., & Varshney, P. (2004). Measuring Loss Given Default on Commercial Loans for the JP Morgan Chase Wholesale Bank: An 18 Year Internal Study. *RMA Journal, 86,* 96-103.

Bakshi, G., Madan, D. B., & Zhang, F. X. (2001). *Understanding the Role of Recovery in Default Risk Models: Empirical Comarisons and Implied Recovery Rates.* Finance and Economics Discussion Series. https://doi.org/10.17016/feds.2001.37

Basel Committee on Banking Supervision (2003). *The New Basel Accord. Consultative Document.* Bank for International Settlements.

Basel Committee on Banking Supervision (2004). *International Convergence on Capital Measurement and Capital Standards.* Bank for International Settlements.

Basel Committee on Banking Supervision (2005). *Guidance on Paragraph 468 of the Framework Document.* Bank for International Settlements.

Basel Committee on Banking Supervision (2017). *Basel III: Finalizing Post-Crisis Reforms.* Bank for International Settlements.

Bastos, J. A. (2010). Forecasting Bank Loans Loss-Given-Default. *Journal of Banking & Finance, 34,* 2510-2517. https://doi.org/10.1016/j.jbankfin.2010.04.011

Board of Governors of the Federal Reserve System (2009). *The Supervisory Capital Assessment Program: Overview of Results (White Paper), Washington DC.*

Board of Governors of the Federal Reserve System (2011). *SR 11-7: Guidance on Model Risk Management, Washington DC.*

Board of Governors of the Federal Reserve System (2016). *Dodd-Frank Act Stress Test 2016: Supervisory Stress Test Methodology and Results, Washington DC.*

Cantor, R., & Varma, P. R. (2004). *Determinants of Recovery Rates on Defaulted Bonds and Loans for North American Corporate Issuers: 1983-2003.* Moody's Investor Service.

Cantor, R., Hamilton, D., & Varma, P. R. (2003). *Recovery Rates on Defaulted Corporate Bonds and Preferred Stocks.* Moody's Investor Service.

Carey, M., & Gordy, M. (2007). *Systematic Risk in Recoveries on Defaulted Debt.* Working Paper, Federal Reserve Board.

Carey, M., & Gordy, M. B. (2016). *The Bank as Grim Reaper: Debt Composition and Bankruptcy Thresholds.* Finance and Economics Discussion Series. https://doi.org/10.17016/feds.2016.069

Chen, H. Z. (2018). A New Model for Bank Loan Loss Given Default by Leveraging Time to Recovery. *The Journal of Credit Risk, 14,* 1-29. https://doi.org/10.21314/jcr.2017.237

Dermine, J., & de Carvalho, C. N. (2006). Bank Loan Losses-Given-Default: A Case Study. *Journal of Banking & Finance, 30,* 1219-1243. https://doi.org/10.1016/j.jbankfin.2005.05.005

Dwyer, D., Rathore, S., & Russell, H. (2014). *Stressed LGD Model.* Moody's Analytics Methodology Paper.

Eberhart, A. C., Moore, W. T., & Roenfeldt, R. L. (1990). Security Pricing and Deviations from the Absolute Priority Rule in Bankruptcy Proceedings. *The Journal of Finance, 45,* 1457-1469. https://doi.org/10.1111/j.1540-6261.1990.tb03723.x

Emery, K., Cantor, R., Keisman, D., & Ou, S. (2007). *Moody's Ultimate Recovery Database: Special Comment.* Moody's Investor Service.

Financial Accounting Standards Board (2016). *Accounting Standards Update No. 2016-13, FinancialInstruments-Credit Losses (Topic 326): Measurement of Credit Losses on Fi-*

nancial Instruments.

Financial Services Authority UK (2008). *Stress Testing and Scenario Testing (Consultation Paper 8/24).*

Fridson, M. S., Garman, C. M., & Okashima, K. (2000). *Recovery Rates: The Search for Meaning.* Merrill Lynch & Company.

Frye, J. (2000a). Collateral Damage. *Risk,* 91-94.

Frye, J. (2000b). *Collateral Damage Detected.* Federal Reserve Bank of Chicago, Emerging Issues Series.

Frye, J. (2000c). Depressing Recoveries. *Risk,* 108-111.

Global Credit Data (2019). Current Expected Credit Losses (CECL) Benchmarking Survey.

Greene, W. H. (2012). *Econometric Analysis* (7th ed.). Prentice Hall.

Grunert, J., & Weber, M. (2009). Recovery Rates of Commercial Lending: Empirical Evidence for German Companies. *Journal of Banking & Finance, 33,* 505-513. https://doi.org/10.1016/j.jbankfin.2008.09.002

Gürtler, M., & Hibbeln, M. (2011). *Pitfalls in Modeling Loss Given Default of Bank Loans.* Working Paper IF35V1, InstitutfürFinanzwirtschaft, Technische Universität Braunschweig.

Hamilton, D. T. (2001). Default and Recovery Rates of Corporate Bond Issuers: 2000. *SSRN Electronic Journal.* https://doi.org/10.2139/ssrn.277999

Hotchkiss, E. S. (1993). The Liquidation/Reorganization Choice of Firms Entering Chapter 11. New York University.

Hu, Y. T., & Perraudin, W. (2002). *The Dependence of Recovery Rates and Defaults.* Working Paper, Birkbeck College.

Jacobs, Jr., M. (2022). Quantification of Model Risk with an Application to Probability of Default Estimation and Stress Testing for a Large Corporate Portfolio. *Journal of Risk Model Validation, 16,* 73-111. https://doi.org/10.21314/jrmv.2022.023

Jacobs, M., & Karagozoglu, A. K. (2011). Modeling Ultimate Loss Given Default on Corporate Debt. *The Journal of Fixed Income, 21,* 6-20. https://doi.org/10.3905/jfi.2011.21.1.006

Jarrow, R. (2001). Default Parameter Estimation Using Market Prices. *Financial Analysts Journal, 57, 7*5-92. https://doi.org/10.2469/faj.v57.n5.2483

Jokivuolle, E., & Peura, S. (2003). Incorporating Collateral Value Uncertainty in Loss Given Default Estimates and Loan-to-Value Ratios. *European Financial Management, 9,* 299-314. https://doi.org/10.1111/1468-036x.00222

Keisman, D., & van de Castle, K. (2000). *Suddenly Structure Mattered: Insights into Recoveries of Defaulted Debt.* Corporate Ratings, Commentary, Standard and Poors.

Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance, 29,* 449-470. https://doi.org/10.1111/j.1540-6261.1974.tb03058.x

Miu, P., &Ozdemir, B. (2006). Basel Requirements of Downturn Loss Given Default: Modeling and Estimating Probability of Default and Loss Given Default Correlations. *The Journal of Credit Risk, 2,* 43-68. https://doi.org/10.21314/jcr.2006.037

Papke, L. E., & Wooldridge, J. M. (1996). Econometric Methods for Fractional Response Variables with an Application to 401(k) Plan Participation Rates. *Journal of Applied Econometrics, 11,* 619-632. https://doi.org/10.1002/(sici)1099-1255(199611)11:6<619::aid-jae418>3.0.co;2-1

Parnes, D. (2009). Modeling Bankruptcy Proceedings for High-Yield Debt Portfolios. *The Journal of Fixed Income, 19,* 23-33. https://doi.org/10.3905/jfi.2009.19.2.023

Pineau, E. (2023). Three PD-LGD Models for a Stress Test Exercise. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4624768

Qi, M., & Zhao, X. (2011). Comparison of Modeling Methods for Loss Given Default. *Journal of Banking & Finance, 35,* 2842-2855. https://doi.org/10.1016/j.jbankfin.2011.03.011

Shleifer, A., & Vishny, R. W. (1992). Liquidation Values and Debt Capacity: A Market Equilibrium Approach. *The Journal of Finance, 47,* 1343-1366. https://doi.org/10.1111/j.1540-6261.1992.tb04661.x

Weiss, L. A. (1990). Bankruptcy Resolution. *Journal of Financial Economics, 27,* 285-314. https://doi.org/10.1016/0304-405x(90)90058-8

Yashkir, O., & Yashkir, Y. (2013). Loss Given Default Modeling: A Comparative Analysis. *The Journal of Risk Model Validation, 7,* 25-59. https://doi.org/10.21314/jrmv.2013.101

Zellner, A., & Theil, H. (1992). Three-Stage Least Squares: Simultaneous Estimation of Simultaneous Equations. In B. Raj, & J, Koerts (Eds.), *Henri Theil's Contributions to Economics and Econometrics: Econometric Theory and Methodology* (pp. 147-178). Springer.