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Heuristic Estimation of the Vacuum Energy Density of the Universe: Part II—Analysis Based on Frequency Domain Electromagnetic Radiation

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Abstract

In Part I of this paper, an inequality satisfied by the vacuum energy density of the universe was derived using an indirect and heuristic procedure. The derivation is based on a proposed thought experiment, according to which an electron is accelerated to a constant and relativistic speed at a distance L from a perfectly conducting plane. The charge of the electron was represented by a spherical charge distribution located within the Compton wavelength of the electron. Subsequently, the electron is incident on the perfect conductor giving rise to transition radiation. The energy associated with the transition radiation depends on the parameter L. It was shown that an inequality satisfied by the vacuum energy density will emerge when the length L is pushed to cosmological dimensions and the product of the radiated energy, and the time duration of emission is constrained by Heisenberg's uncertainty principle. In this paper, a similar analysis is conducted with a chain of electrons oscillating sinusoidally and located above a conducting plane. In the thought experiment presented in this paper, the behavior of the energy radiated by the chain of oscillating electrons is studied in the frequency domain as a function of the length L of the chain. It is shown that when the length L is pushed to cosmological dimensions and the energy radiated within a single burst of duration of half a period of oscillation is constrained by the fact that electromagnetic energy consists of photons, an inequality satisfied by the vacuum energy density emerges as a result. The derived inequality is given by $\rho_\Lambda \leq 6.3\times10^{-10}\ \text{J/m}^3$ where ρ_Λ is the vacuum energy density. This result is consistent with the measured value of the vacuum energy density, which is 5.38×10^{-10} J/m. The result obtained here is in better agreement with experimental data than the one obtained in Part I of this paper with time domain radiation.

Keywords

Classical Electrodynamics, Electromagnetic Radiation, Action, Radiated Energy, Photon, Heisenberg's Uncertainty Principle, Dark Energy, Vacuum Energy, Cosmological Constant, Hubble Radius

1. Introduction

The observed accelerated expansion of the universe has led scientists to propose vacuum energy as a possible explanation. However, several attempts to theoretically estimate the vacuum energy density based on the uncertainty principle and quantum fluctuations gave rise to values which are about 10^{50} to 10^{120} times larger than the experimentally observed value [1] [2]. This discrepancy, known as the vacuum energy catastrophe, is named as one of the worst predictions in physics.

The vacuum energy density of the universe is an important parameter whose accurate estimation plays a central role in testing some proposed theories of quantum gravity. A direct estimation of the vacuum energy density is based on the analysis of the energy associated with quantum fluctuations in space-time. However, such attempts have led to extremely large values which are many orders of magnitude larger than the experimental data obtained by analyzing the expansion of the universe. Thus, there is a need to look also for other avenues where at least an indirect estimation of this parameter can be made.

In Part I of this paper, we have studied the electromagnetic radiation fields generated by transition radiation where an electron accelerated to a relativistic speed at a distance L from a perfectly conducting boundary is absorbed into a perfectly conducting plane [3]. In that paper, it was shown that when the length L is pushed to the upper limit that can ever be realized in the current universe, the resulting field equations when restricted by the Heisenberg's uncertainty principle give rise to an inequality satisfied by the vacuum energy density. It was shown that the limit obtained is in agreement with the observed value of the vacuum energy density.

In this paper, we will conduct a similar thought experiment. However, instead of using a transient radiating system as in Part I of this paper, we will analyze the frequency domain radiation fields generated by a chain of electrons oscillating in such a manner to create an oscillating current similar to that of a long dipole located above a perfectly conducting plane. In [4] it was shown that when the length of the chain of electrons is increased to the maximum length allowed by nature, classical electrodynamics will reveal hints of the photonic nature of the electromagnetic radiation. Here, we will utilize the same field equations and obtain an inequality satisfied by the vacuum energy density which is in better agreement with the experimental data than the inequality obtained in Part I of this paper.

The first study that pointed out the possibility of estimating the magnitude of the vacuum energy density by extending the length of oscillating dipoles to cosmological distances was conducted by Cooray and Cooray [5] [6]. However, in that analysis, the radiating system used was a physical antenna and the antenna diameter was used to obtain the final result. In that study, results similar to the ones to be presented here were obtained when it was assumed that the smallest radius of the antenna ever possible in nature is the Bohr radius. In contrast, we assume here that the radiation is generated directly by a chain of oscillating electrons.

As pointed out in Part I of this paper, it is important to understand that the whole exercise presented in the current paper is a hypothetical and theoretical experiment. In the presented analysis, we have studied how the equations of classical electrodynamics will behave if the dimensions of a radiating system are pushed to their natural limits. It is important to stress that the model utilized in this paper cannot be realized experimentally. The maximum length of the radiating system used in the analysis should not be interpreted as that of a real system but that of a hypothetical "gedanken" experiment, the purpose of which is to study the behavior of classical electromagnetism when pushed to extreme limits.

As in Part I of this paper, we assume that vacuum energy is responsible for the expansion of the universe and treat it as an intrinsic and fundamental constant of nature which has the same value throughout the whole universe. Thus, it is directly related to the cosmological constant.

2. The Current and the Maximum Power Dissipated by the Electromagnetic Radiating System

Consider a chain of electrical charges oscillating sinusoidally. The length of the chain is L and it is located over a perfectly conducting plane. The individual oscillators are coupled so that the electric current at any given location along the string, say z, is given by

$$I_0 \sin\left\{(2\pi/\lambda)[L-z]\right\} e^{j\omega t} \quad 0 \le z \le L \tag{1}$$

$$I_0 \sin\left\{(2\pi/\lambda)[L+z]\right\} e^{j\omega t} \quad -L \le z \le 0 \tag{2}$$

In the above equations, I_0 is a constant and ω is the angular frequency of oscillation. The current associated with the oscillating charges is similar to the one originated in a long electric dipole in space [7]. The calculation of the median power, P_{med}, radiated by such a system of charges is a simple procedure and it suffices here to give directly the expression for it. It is given by [4] [7]

$$P_{med} = \frac{q^2 \pi \nu^2}{2\varepsilon_0 c}\left\{\gamma + \ln(4\pi L/\lambda)\right\} \tag{3}$$

In Equation (3), γ is the Euler's constant, ν is the frequency and λ is the wavelength of the electromagnetic radiation.

Note that the median power as given by Equation (3) increases as the value of L increases and as the value of λ decreases. Now, we will push the values of L and λ to their limits allowed by nature. This will give us the maximum power that can be generated by a given charge q. In Part I of this paper and in [4] we have pointed out that the smallest wavelength associated with the oscillating electron is equal to its Compton wavelength and the largest length ever possible for the radiating system is the constant value of the Hubble radius that will occur in a future epoch if the vacuum energy density remains constant with time as assumed in this paper. The justification for these assumptions as given in Part I of this paper and in [4] are detailed below.

2.1. The Lower Limit of the Wavelength

From a classical point of view, the only restriction on the minimum value of the wavelength is that it has to be much larger than the dimension of the oscillating charge. Equation (3) is actually based on the assumption that the wavelength is much larger than the dimension of the oscillating charge. If the wavelength is comparable or smaller than the dimension of the charge, destructive interference will lead to the reduction of the radiated energy in comparison to the expression given in 3. Here we assume that the effective radius of the electron taking part in the emission of electromagnetic radiation is equal to the Compton wavelength, λ_c, of the electron. In the case of electrons, the actual radius of the particle has not been determined and scientists often treat the electron as a point particle [8]. But, as far as the emission of radiation is concerned, one can treat the electron as an extended particle of dimension comparable to its Compton wavelength [9] [10]. It is also of interest to note that Schrodinger's zitterbewegung (jittery) theory suggests that an electron oscillates rapidly [11]. The amplitude of this spatial oscillation was shown to be of the order of the Compton wavelength. In quantum electrodynamics, the zitterbewegung is understood as the result of interaction of the electron with spontaneously forming and annihilating electron-positron pairs. Based on these considerations, we assume that the effective radius of the electron taking part in the emission of radiation is equal to the Compton wavelength. Thus, the minimum value of the wavelength, $\lambda_{\min}$, that should be plugged into the above equation should satisfy the condition $\lambda_{\min} \gg \lambda_c$. If this is not the case, the differences in the phase of the electromagnetic fields and the resulting interference will lead to a reduction in the strength of the electromagnetic field and hence in the energy radiated. It is also of interest to note that if the wavelength of the radiation is comparable to or smaller than λ_c, it can lead to pair production invalidating the classical expression given by Equation (3).

In our previous analysis reported in [4] we have assumed without any analysis that the condition $\lambda_{\min} \gg \lambda_c$ is satisfied if $\lambda_{\min} = 10\lambda_c$. However, here we will consider this point in details. Recall that our goal is to maximize the peak of the electromagnetic field so that we will get the maximum energy output for a given time interval. If we neglect the interference effects, the peak value of the electromagnetic field decreases as the wavelength increases. One can see this effect in the expression for the power given in Equation (3). Now, consider a wavelength comparable to λ_c. Due to interference effects, the peak value of the corresponding electromagnetic field for a given oscillating charge will be less than the one that would be present without the interference effect. This means, as the wavelength increases beyond λ_c two opposing effects will come into play. As the wavelength increases beyond λ_c, the reduction in the interference effects leads to an increase in the peak electric field while the electric field peak that would be present without interference will start to decrease (according to Equation (3)). We have selected the value of λ so that the decrease in the peak value of the electromagnetic field due to further increase of λ will balance the increase in the peak electric field caused by the reduction in interference. Our numerical simulation shows that the two opposing effects will balance when $\lambda_{\min} \approx 18\lambda_c$. Observe that this also satisfies the condition $\lambda_{\min} \gg \lambda_c$ by a good margin.

2.2. Upper Limit of the Length of the Chain of Electrons

Let us now consider the maximum length ever possible in the current universe. Friedmann [12] equations provide the solutions of the equations of general relativity for the evolution of the universe. The growth of the universe depends on the density of the radiation, matter and the vacuum energy (positive cosmological constant) of the universe. In the case where the vacuum energy density dominates both that of radiation and matter, the universe expands exponentially and it will evolve similar to a de Sitter universe [13]. According to measurements, the current universe is almost (asymptotically) equal to a de Sitter space with a positive vacuum energy density and negative pressure. It will evolve like a de Sitter universe in the future when vacuum energy dominates over matter density. In that epoch, the Hubble radius (which is increasing at present) will become constant and it will define the maximum length scale over which events can be in causal contact. In our analysis, we have assumed that the maximum length scale ever possible in the current universe is equal to this steady state value of the Hubble radius. In the paper, this limiting length is used as an input to find out the behavior of classical electrodynamics when the dimension of the radiating system is pushed to these extreme limits. Based on general relativity, the steady state value of the Hubble radius is given by $R_\infty = c^2\sqrt{3/8\pi G\rho_\Lambda}$, where G is the gravitational constant and ρ_Λ is the vacuum energy density [14].

2.3. Maximum Median Power Radiated by the Radiating System

Substituting the minimum value of the wavelength and the maximum value of the length pertinent to the oscillating dipole into Equation (3), the maximum value of the median power is given by (after replacing λ by $\lambda_{\min}$)

$$\langle P_{med}\rangle_{\max} = \frac{e^2\pi\nu^2}{2\varepsilon_0 c}\left\{\gamma + \ln\left(2\pi R_\infty/9\lambda_c\right)\right\} \tag{4}$$

3. Restrictions Due to the Photonic Nature of Electromagnetic Radiation

First observe that the electromagnetic radiation generated by the dipole oscillates in time as $\sin(2\pi\nu t)$. Therefore, the power generated by the dipole consists of continuously repeating bursts, each with a duration of $T/2$, where T is the period of oscillation. Let us consider one of these bursts. The energy dissipated in one of these bursts is given by

$$U_{\max} = \frac{e^2\pi\nu}{4\varepsilon_0 c}\left\{\gamma + \ln\left(2\pi R_\infty/9\lambda_c\right)\right\} \tag{5}$$

Now, consider a truncated sinusoid of duration Δt. The full width of the main fringe of the spectrum of this sinusoid $\Delta\omega$ satisfies the condition $\Delta t\Delta\omega \geq \pi$. In other words, $\Delta t\Delta\nu \geq 1/2$ [15] [16]. This leads to the inequality $\Delta U \geq h/2\Delta t$, where ΔU is the uncertainty in energy. When the duration of the truncated sinusoid is $T/2$ one obtains $\Delta U = h\nu$. Since $U_{\max} \geq \Delta U$ following mathematical statement is valid:

$$\frac{e^2\pi\nu}{4\varepsilon_0 c}\left\{\gamma + \ln\left(2\pi c^2\sqrt{3/8\pi G\rho_\Lambda}/9\lambda_c\right)\right\} \geq h\nu \tag{6}$$

Observe that we have replaced R_∞ by $c^2\sqrt{3/8\pi G\rho_\Lambda}$ when moving from Equations (5) to (6). In Equation (6), $h\nu$ is the energy of a photon corresponding to frequency ν. This restriction of energy using the concept of photons but applied to the electromagnetic radiation calculated using classical electrodynamics is justified due to the following fact. The field equations associated with the classical electromagnetic radiator is identical with the quantum electromagnetic radiator with two exceptions: 1) The radiated energy of a quantum oscillator does not increase continuously as in classical radiator but only in units of $h\nu$. 2) There is no restriction on the energy in the classical radiator that is necessary to generate electromagnetic radiation while the quantum radiator does not radiate in the ground state or the lowest energy state.

4. Results and Discussion

Equation (6) provides an inequality in ρ_Λ as a function of other fundamental constants. In fact, Equation (6) can be written as

$$\rho_\Lambda \leq \frac{\pi\left[m_e c^2\right]^2 c^2}{54Gh^2}\mathrm{e}^{-\left\{\frac{4}{\pi\alpha}-2\gamma\right\}} \approx \frac{m_e c^2}{\left(\pi l_p^2\right)\left(2\pi\lambda_c\right)}\mathrm{e}^{-\left\{\frac{4}{\pi\alpha}-2\gamma\right\}} \tag{7}$$

In the above equation, $\lambda_c = h/m_e c$, m_e is the rest mass of the electron, l_p is the Planck length given by $\sqrt{\frac{hG}{c^3}}$ and α is the fine structure constant which is equal to $e^2/2\varepsilon_0 ch$. Once we substitute the values of the standard constants in the above equation, we obtain our final result, which is

$$\rho_\Lambda \leq 0.63\times 10^{-9}\ \mathrm{J/m^3} \tag{8}$$

The above result shows that the vacuum energy density has to be either equal to 0.63 × 10^{-9} J/m^3 or less than this value. This result is in good agreement with the measured value of the vacuum energy density based on the accelerated expansion of the universe, which is equal to 0.535 × 10^{-9} J/m^3 [17] [18] [19].

Note that an almost identical relationship for the vacuum energy density (*i.e.*, similar to Equation (7)) was obtained in [6]. In that paper, the analysis was based on the frequency domain radiation generated by a long antenna located above a perfectly conducting plane. In that analysis, the minimum possible wavelength was assumed to be the Bohr radius, the smallest radius that can be assigned to a physical antenna. Since the Bohr radius is approximately equal to $20\lambda_c$, the similarity of the results is not surprising.

Observe that the vacuum energy, assumed here to be the intrinsic energy of space, is related to the cosmological constant Λ through the relationship $\Lambda = 8\pi G\rho_\Lambda/c^4$ [14]. Thus, Equation (7) will indeed provide the possible range of values for the cosmological constant. That is

$$\Lambda \leq \left[\frac{2\pi}{3\sqrt{3}\lambda_c}\right]^2 \mathrm{e}^{-\left\{\frac{4}{\pi\alpha}-2\gamma\right\}} \tag{9}$$

Equations (7) and (9) show that the vacuum energy density and the cosmological constant depend very strongly on the charge of the electron or the fine structure constant. A slight change in the charge of the electron will cause a significant change in the vacuum energy density or the cosmological constant. However, due to the logarithmic term the electronic charge or the fine structure constant depends only weakly on the vacuum energy density. Moreover, according to the expression the vacuum energy density decreases with decreasing electronic charge and electronic mass. In Equation (7) we have given also an alternative but approximate expression for the dark energy density. The term outside the exponential corresponds to the mass energy density when the mass of the electron is confined in a ring like structure with cross sectional radius equal to the Planck length and the radius of the ring equal to the Compton wavelength. This corresponds to an energy density of about 10^{49} J/m^3. This energy density is drastically reduced by the exponential term due to the small value of the fine structure constant.

Unfortunately, based on our work, it is not possible to determine whether the vacuum energy density is controlled by the electronic charge or vice versa. Though a deeper analysis on why the physical parameters of the electron and the vacuum energy density should be connected to each other is beyond the scope of this paper, several publications postulate that the stability of the electron is controlled by the vacuum energy density (see [20] and references therein). If this hypothesis is correct, in order to maintain the stability of the electron, the elementary charge should decrease with decreasing vacuum energy density.

Observe that the Equation (6) can also be written as

$$\alpha \geq \frac{2}{\pi\left\{\gamma + \ln\left[\frac{c\nu_c}{3}\sqrt{\frac{\pi}{6G\rho_\Lambda}}\right]\right\}} \tag{10}$$

If we assume the measured value of 0.535×10^{-9} J/m^3 for the vacuum energy density, we obtain

$$\alpha \geq \frac{1}{137.17} \tag{11}$$

The Equation (11) is a reasonably good approximation for the fine structure constant.

Finally, observe that we obtained the inequality pertinent to the vacuum energy density (or the cosmological constant) indirectly and heuristically by pushing the equations of classical electrodynamics to their limits and restricting the energy associated with radiation by appealing to the photonic nature of the electromagnetic radiation. Thus, the value of the vacuum energy density or the cosmological constant obtained here is the one that is necessary to satisfy the field equations of classical electrodynamics once the photonic nature of the radiation is taken into account.

5. Conclusion

In this paper, we have derived an inequality that is satisfied by the vacuum energy density of the universe using an indirect and heuristic procedure. In the proposed thought experiment, the length of a frequency domain radiating system was pushed to cosmological dimensions, and the restrictions resulting from the photonic nature of the radiation were applied to the resulting radiation. The inequality we obtained for the vacuum energy density, namely $\rho_\Lambda \leq 0.63 \times 10^{-9}\ \mathrm{J/m^3}$, agrees well with the experimentally obtained value.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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