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Heuristic Estimation of the Vacuum Energy Density of the Universe: Part I—Analysis Based on Time Domain Electromagnetic Radiation

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Abstract

In this paper, an inequality satisfied by the vacuum energy density of the universe is derived using an indirect and heuristic procedure. The derivation is based on a proposed thought experiment, according to which an electron is accelerated to a constant and relativistic speed at a distance L from a perfectly conducting plane. The charge of the electron is represented by a spherical charge distribution located within the Compton wavelength of the electron. Subsequently, the electron is incident on the perfect conductor giving rise to transition radiation. The energy associated with the transition radiation depends on the parameter L. It is shown that an inequality satisfied by the vacuum energy density will emerge when the length L is pushed to cosmological dimensions and the product of the radiated energy and the time duration of emission are constrained by Heisenberg's uncertainty principle. The inequality derived is given by $\rho_\Lambda \le 9.9\times10^{-9}\ \mathrm{J/m^3}$ where ρ_Λ is the vacuum energy density. This result is consistent with the measured value of the vacuum energy density, which is 0.538×10^{-9} J/m. Since there is a direct relationship between the vacuum energy density and the Einstein's cosmological constant, the inequality can be converted directly to that of the cosmological constant.

Keywords

Classical Electrodynamics, Electromagnetic Radiation, Action, Radiated Energy, Photon, Heisenberg's Uncertainty Principle, Dark Energy, Vacuum Energy, Cosmological Constant, Hubble Radius

1. Introduction

The observed accelerated expansion of the universe has led scientists to propose an unknown form of energy, called the dark energy, as a possible reason for this expansion. The exact form of the dark energy is unknown. Different forms of energy have been suggested as possible candidates; one of them is the energy of the vacuum, which is directly connected to Einstein's cosmological constant. However, several attempts to theoretically estimate the vacuum energy density based on the uncertainty principle and quantum fluctuations gave rise to values which are about 10^{50} to 10^{120} times larger than the observed value [1] [2]. This discrepancy, known as the vacuum energy catastrophe, is named as one of the worst predictions in physics.

The vacuum energy density of the universe is an important physical parameter, whose accurate estimation plays a central role in testing some proposed theories of quantum gravity. A direct estimation of the vacuum energy density is based on the analysis of the energy associated with quantum fluctuations in space-time. However, as mentioned earlier, such attempts have led to extremely large values which are many orders of magnitude larger than the experimental data obtained by analyzing the expansion of the universe. Thus, there is a need to look also for other avenues where at least an indirect estimation of this parameter can be made. As we will show here, such an estimation can be made when the length scale in classical electrodynamics is pushed to cosmological distances.

Classical electrodynamics accurately predicts the electromagnetic radiation fields generated by accelerating or decelerating charges [3]. The corresponding field equations contain the length of the radiating system as a parameter. For example, field expressions for dipole radiation contain the dipole length as a parameter, and the field equations associated with the radiation generated by a linear antenna contain the antenna length as a parameter. In classical electrodynamics, there is no restriction as to the upper limit of the length of the radiating system. In this paper, we will study the behavior of the field equations of classical electrodynamics when the length of the radiating system is pushed to the maximum value that can ever be realized in the current universe. In several previous studies, it was shown that when the length of the radiating systems is pushed to its limits, classical electromagnetics reveals features that provide hints as to the photonic nature of the electromagnetic fields [4]-[10].

The source of electromagnetic radiation used in the current study is the transition radiation generated by a moving charge incident on a perfectly conducting plane. The field equations pertinent to transition radiation and their features at the extreme length scales have been previously studied [10] [11]. These field equations contain the length over which the movement of the charged particle takes place as a parameter. In the present analysis, we will push this length to the maximum value ever possible in the current universe. When the resulting energy radiated by the charge during its time of deceleration at the conducting boundary is constrained by the Heisenberg's uncertainty principle, the field equations lead to an inequality that has to be satisfied by the vacuum energy density. Based on this analysis, we will estimate the possible range of vacuum energy density that is necessary to satisfy the electromagnetic field equations at the extreme length scale.

It is important to stress that the model utilized in this paper cannot be realized experimentally. The maximum length of the radiating system used in the analysis should of course not be interpreted as that of a real system but that of a hypothetical "gedanken" experiment, the purpose of which is to study the behavior of classical electromagnetism when pushed to extreme limits. The results obtained have to be understood only as due to an effect of scaling on the electromagnetic field equations of classical electrodynamics.

In our analysis, we assume that vacuum energy is responsible for the expansion of the universe and treat it as an intrinsic and fundamental constant of nature which has the same value throughout the whole universe. Thus, it is directly related to the cosmological constant.

2. The Energy Radiated by the Transition Radiation

The transition radiation is generated when a charged particle moving with constant speed is ejected from or incident on either a conducting or a dielectric boundary. Here, we study the transition radiation generated when a moving charged particle is incident on a perfectly conducting boundary. This is the case where, for a given charge and speed, one can obtain the highest energy in the transition radiation. Consider a charged particle which was accelerated to a speed v at a height L above a perfectly conducting boundary. We assume that the charge distribution across the charged particle is such that its movement can be represented by a propagating current pulse, $i(t)$. The expressions for the electromagnetic fields and the energy radiated as transition radiation during the deceleration of the charge at the perfectly conducting boundary were given previously in reference [10]. The expression for the energy, U, associated with the transition radiation when v is highly relativistic, *i.e.*, $v \approx c$, is given by (for a current symmetric around time zero; see Section 3)

$$U = \frac{1}{4\pi\varepsilon_0 c}\left[\log\frac{2L}{d}\right]\int_{-\Gamma/2}^{\Gamma/2} i(t)^2\,\mathrm{d}t \tag{1}$$

In the above expression, d is the diameter or the spatial extension of the charged region of the charged particle and Γ is the duration of the current pulse. The expressions for $i(t)$ and Γ are given in the next section.

3. The Current Pulse Associated with the Moving Charge

In our analysis, we will consider an electron as our charged particle, *i.e.*, $q = e$. In order to calculate the energy released by the decelerating electron, it is necessary to know the effective diameter of the electron pertinent to the emission of the radiation. Based on the pioneering work of Compton [12], we treat the electron, as far as the emission of radiation is concerned, as a uniformly charged sphere with a radius equal to the Compton wavelength, λ_c. Thus, in our analysis, the effective diameter d of the electron was assumed to be equal to $2\lambda_c$. The actual physical radius of the electron is believed to be much less than this [13].

Observe that we are analyzing the electromagnetic radiation generated by accelerating or decelerating electric charges. The analysis provided in reference [3] shows that the field equations of accelerating charges are valid both in the classical and in the quantum mechanical regimes, provided that the energy of the photons is smaller than the total energy of the radiating charged particle.

The effective current associated with the assumed spherical and uniform charge distribution is given by

$$i(t) = i_0\left(1 - \frac{|t|^2}{(\Gamma/2)^2}\right) \quad -\Gamma/2 \le t \le \Gamma/2 \tag{2}$$

The expression for the current waveform can also be given in terms of the electronic charge e by

$$i(t) = \frac{3e}{2\Gamma}\left(1 - \frac{|t|^2}{(\Gamma/2)^2}\right) \quad -\Gamma/2 \le t \le \Gamma/2 \tag{3}$$

Observe that when $\Gamma \to 0$ the current pulse reduces to that of a point charged particle. The effective duration of the current pulse can be defined as $\tau = \int_{-\infty}^{+\infty} i(t)\mathrm{d}t \Big/ \langle i(t) \rangle_{peak}$ where $\langle i(t) \rangle_{peak}$ is the peak value of the current. For the current given by Equation (3), $\tau = 2\Gamma/3$. It is important to point out that the waveform used in [9] to represent the current caused by a uniform charge distribution was asymmetric in time. However, both expressions lead to identical results.

4. Restrictions due to the Time-Energy Uncertainty Principle

According to the analysis presented above, the effective time over which the radiation occurs is equal to τ. During this time, a large number of photons with different energies are emitted. We cannot predict exactly when a given photon will be emitted except that it will be emitted during a time interval equal to τ. According to Heisenberg's time-energy uncertainty principle, the uncertainty in the energy of the photon is equal to $h/4\pi\tau$. This is true for any given photon irrespective of its energy. This in turn leads to a minimum uncertainty in the total energy of the radiation ΔU given by $\Delta U \approx h/4\pi\tau$. This also means that, if the hypothetical experiment is repeated again and again, the total energy will vary slightly from one case to another and the uncertainty in the energy will be equal to the value given above. This result can also be derived in a different way. Consider an electron moving towards a perfectly conducting plane along a direction perpendicular to the plane (say the *z*-axis). At a certain time, the electron will approach the conducting plane and, just at the moment it interacts with the plane, it gives rise to the transition radiation. Let us attempt to detect the location of the electron in the *z*-direction during the time of emission of the transition radiation. The transition radiation allows the location of the electron along the z-axis only to an accuracy of about $\tau v \approx \tau c$. Observe that τc is in the order of the Compton wavelength of the electron. This is the uncertainty in distance over which the transition radiation is emitted. Note also that τc is the effective wavelength associated with the transition radiation and the location of the electron can be obtained only to an accuracy comparable to this wavelength. During this process, the momentum of the electron along the *z*-axis will change by an amount U/c. This is the longitudinal momentum that is being lost to the transition radiation. Observe that this expression for the change in the *z*-momentum is valid since, due to the relativistic speed of the particle, the radiation and hence the Poynting vector are directed almost along the z-axis. The uncertainty in momentum of the electron is then given by $\Delta U/c$. The product of the uncertainty in position and momentum should satisfy the uncertainty principle, and therefore, $\Delta U \approx h/4\pi\tau$. This result also indicates that the condition $U\tau \geq h/4\pi$ is valid since the smallest value of U cannot be smaller than ΔU. Thus, one can write using Equation (1)

$$\frac{\tau}{4\pi\varepsilon_0 c}\left[\log\frac{2L}{d}\right]\int_{-\Gamma/2}^{\Gamma/2} i(t)^2\,\mathrm{d}t \geq \frac{h}{4\pi} \tag{4}$$

The inequality given by Equation (4) is valid for any value of *L*. Let us consider the largest value of *L* ever possible in the current universe. As we will show in a moment, this choice will make it possible to connect the vacuum energy density of the universe or the cosmological constant to the field equations of classical electrodynamics.

Classical electrodynamics does not impose any limit on the length scale associated with electromagnetic phenomena and in principle the maximum length of the radiating system can reach infinity. However, general relativity provides a limit related to the maximum dimensions of the radiating system. The Friedmann equations [14] provide the solutions of the equations of general relativity for the evolution of the universe. The growth of the universe depends on the density of the radiation, matter and vacuum energy (positive cosmological constant) of the universe. In the case where the vacuum energy density dominates both the densities of radiation and matter, the universe expands exponentially, and it will evolve similar to a de Sitter universe [15]. According to the measurements, the current universe is almost (asymptotically) equal to a de Sitter space with a positive vacuum energy density and a negative pressure. It will evolve like a de Sitter universe in the future when vacuum energy dominates completely over matter density. In that epoch, the Hubble radius (which is increasing at present) will become a constant and it will define the maximum length scale over which events can be in causal contact. In our analysis, we have assumed that the maximum length scale ever possible in the current universe is equal to this steady state value of the Hubble radius. Let us represent the steady state value of the Hubble radius by R_∞. This value is given by [16]

$$R_\infty = c^2 \sqrt{3/8\pi G \rho_\Lambda} \tag{5}$$

In the above equation, ρ_Λ is the vacuum energy density and G is the gravitational constant. The vacuum energy, assumed to be the intrinsic energy of space is related to the cosmological constant Λ through the relationship $\Lambda = 8\pi G \rho_\Lambda / c^4$ [16]. Substituting the expression for the steady state value of the Hubble radius in Equation (4), we obtain

$$\frac{\tau}{4\pi\varepsilon_0 c}\left[\log\frac{2c^2\sqrt{3/8\pi G\rho_\Lambda}}{d}\right]\int_{-\Gamma/2}^{\Gamma/2} i(t)^2\,\mathrm{d}t \ge \frac{h}{4\pi} \tag{6}$$

After solving the integration associated with the current in Equation (6), we obtain

$$\frac{6e^2\tau}{20\pi\varepsilon_0 c\Gamma}\left[\log\frac{2c^2\sqrt{3/8\pi G\rho_\Lambda}}{d}\right] \ge \frac{h}{4\pi} \tag{7}$$

Substituting $d = 2\lambda_c$ and $\tau = 2\Gamma/3$, the above equation reads

$$\frac{e^2}{5\pi\varepsilon_0 c}\left[\log\frac{c^2\sqrt{3/8\pi G\rho_\Lambda}}{\lambda_c}\right] \ge \frac{h}{4\pi} \tag{8}$$

Observe that the inequality derived above is independent of τ and Γ. Note also that the parameter λ_c appears in this equation only inside the logarithmic term. Equation (8) can be written as an inequality in ρ_Λ as

$$\rho_\Lambda \le \frac{3c^4}{8\pi G\lambda_c^2}\mathrm{e}^{-\frac{5}{4\alpha}} \tag{9}$$

In the above equation, α is the fine structure constant. The fine structure constant is given by $e^2 / 2\varepsilon_0 hc$.

5. Results and Discussion

Equation (9) provides an inequality expressing ρ_Λ as a function of other fundamental constants. In fact, it can be rewritten as (with $\rho_{\Lambda w}$ representing the right-hand side of Equation (9))

$$\rho_\Lambda \le \rho_{\Lambda w} \tag{10}$$

For the assumed uniform charge distribution, we obtain $\rho_\Lambda \le 9.9\times10^{-9}\ \mathrm{J/m^3}$. This inequality has to be compared with the measured value of the vacuum energy density of 0.538×10^{-9} J/m^3 which is based on the accelerated expansion of the universe [17] [18] [19]. The agreement between the prediction and the experiment is reasonable. Since ρ_Λ is directly related to Λ, the data presented here provide the possible range of values for the cosmological constant.

In our analysis, we have assumed that the charge density of the electron is distributed uniformly inside the Compton radius. The effect of making the charge density to increase towards the center of the charge distribution is to decrease the value of $\rho_{\Lambda w}$.

Observe that Equation (9) provides a connection between the vacuum energy density and the charge of the electron. Though a deeper analysis on why the electronic charge and the vacuum energy density are connected to each other is beyond the scope of this paper, such a connection has been postulated in several publications (see [20] and references therein).

In our analysis, we have assumed that $L = R_\infty$. The effect of decreasing L is to decrease the value of $\rho_{\Lambda w}$. That is, the maximum value of $\rho_{\Lambda w}$ is obtained for $L = R_\infty$. Therefore, the results that will be obtained for any smaller value of L compared to R_∞ will also be in agreement with the results presented here.

In this paper, we have utilized the time domain radiation fields of a decelerating electron to obtain the vacuum energy density that is necessary to satisfy the field equations of classical electrodynamics and Heisenberg's uncertainty principle. Since the uncertainty principle as applied here can fix the relevant parameters only to an order of magnitude and since there is an uncertainty in the correct charge density distribution that should represent the electron, the results obtained here have to be considered as an order of magnitude estimation. In Part II of this paper, we will analyze the frequency domain electromagnetic fields generated by oscillating charges. Using a technique similar to that applied here and appealing to the photonic nature of the electromagnetic fields, we will extract the magnitude of the vacuum energy density or the cosmological constant. As we will show in that paper, a better estimate for the vacuum energy density that is in good agreement with experimental data can be obtained using frequency domain radiation.

6. Conclusion

In this paper, we have derived an inequality that is satisfied by the vacuum energy density of the universe using an indirect and heuristic procedure. The derivation is based on a proposed thought experiment, according to which an electron is accelerated to a constant and relativistic speed at a distance L from a perfectly conducting plane. Subsequently, the electron is incident on the perfect conductor giving rise to transition radiation. The energy associated with the transition radiation depends on the parameter L. It was shown in the paper that an inequality satisfied by the vacuum energy density will emerge when the length L is pushed to cosmological dimensions and the product of the radiated energy and the time duration of emission were constrained by Heisenberg's uncertainty principle. The resulting inequality is given by $\rho_\Lambda \le 9.9 \times 10^{-9}\ \mathrm{J/m^3}$ where ρ_Λ is the vacuum energy density. This value is consistent with the measured value of the vacuum energy density, which is 0.538×10^{-9} J/m. Since the vacuum energy density is directly connected to the cosmological constant, the inequality given here can be directly converted to that of the latter.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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