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Hints of the Photonic Nature of the Electromagnetic Fields in Classical Electrodynamics

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Abstract

Several recent publications show that the electromagnetic radiation generated by transmitting antennas satisfy the following universal conditions: The time domain radiation fields satisfy the condition $A \geq h/4\pi \Rightarrow q \geq e$ where A is the action of the radiation field, which is defined as the product of the radiated energy and the duration of the radiation, h is the Planck constant, e is the electronic charge and q is the charge associated with the radiating system. The frequency domain radiation fields satisfy the condition $U \geq h\nu \Rightarrow q \geq e$ where U is the energy radiated in a single burst of radiation of duration $T/2$ and ν is the frequency of oscillation. The goal of this paper is to show that these conditions, which indeed are expressions of the photonic nature of the electromagnetic fields, are satisfied not only by the radiation fields generated by physical antennas but also by the radiation fields generated by accelerating or decelerating electric charges. The results presented here together with the results obtained in previous studies show that hints of the photonic nature of the electromagnetic radiation remain hidden in the field equations of classical electrodynamics, and they become apparent when the dimension of the radiating system is pushed to the extreme limits as allowed by nature.

Keywords

Classical Electrodynamics, Electromagnetic Radiation, Action, Radiated Energy, Photon, Heisenberg's Uncertainty Principle, Dark Energy, Vacuum Energy, Hubble Radius

1. Introduction

Classical electrodynamics is an old subject that has its mathematical origins in the 19th century with the pioneering work due to James Clerk Maxwell. The topic has been developed and expanded extensively over the years and as it stands today, it is a subject which is fully matured and thoroughly explored. However, recent studies show that there are very interesting features hidden within classical electrodynamics that went unnoticed for nearly 160 years.

In analyzing the electromagnetic fields generated by accelerating charged particles and long transmitting antennas, scientists and engineers utilize electromagnetic field equations pertinent to various charge and current distributions both in the time and the frequency domains. These field equations, the subject matter of this paper, are familiar to those working in the fields of antenna theory and electromagnetic radiation. The goal of this paper is to show that the energy radiated by electromagnetic radiation fields and the charge of an electron are connected by two universal conditions which are satisfied by all radiating systems. Interestingly, these conditions remained hidden and went unnoticed until recently [1]-[7].

The first study that pointed out the presence of a hidden connection between the electronic charge and the energy radiated as electromagnetic radiation was conducted by Cooray and Cooray [1]. In that paper, radiation fields in both the time domain and the frequency domain were considered. The time domain analysis was conducted using a transient current waveform propagating along a long antenna and the frequency domain analysis was conducted by considering a long antenna containing an oscillating current. The time domain analysis was further improved by Cooray and Cooray [2] [3] and the frequency domain analysis was further extended by Cooray and Cooray [4]. Cooray *et al.* [5] showed that the time domain results obtained in [1] [2] and [4] are valid for any arbitrary current waveform. A review of the work, both in the time and the frequency domain, was given by Cooray and Cooray [6]. Cooray and Cooray [7] showed that the energy radiated by a decelerating electron as transition radiation also satisfies a condition similar to that obtained for time domain radiators in [1] [2] [3]. Even though the conclusions to be reached in the present study are identical to those obtained in the above-mentioned studies, the radiating system used in obtaining these conclusions in the present study is different. In the frequency domain analysis presented in the present paper, instead of a real transmission antenna, a chain of oscillating electrons that simulated the current in the transmission antenna is used as the frequency domain radiating system. In this case, the physical radius of the transmission antenna does not enter into the field expressions. Moreover, in the study conducted by Cooray and Cooray [7], the non-relativistic Heisenberg's uncertainty principle was used in a context which is highly relativistic and this led to a limitation on the speed of the charged particle. This deficiency is corrected in the current paper.

The analysis to be presented here is carried out within classical electrodynamics and it is conducted without utilizing any quantum mechanical concepts. However, in this work we will use as input the smallest dimension allowed by nature for the charged particles. Since the smallest charged radiators that exist in nature are the charged elementary particles and, being quantum mechanical in nature, their size as defined in the literature is based on quantum mechanics. Except for the choice of the value of this input parameter, the analysis is based purely on classical electrodynamics. Our goal here is to show that information pertinent to the photonic nature of electromagnetic radiation or at least hints as to that fact exist hidden in the field equations of classical electrodynamics. These hidden features become apparent when the dimension of the radiating system is pushed to its natural upper limit. As we will see later, the longest length ever possible for a radiating system is given by general relativity and the smallest dimension of the radiating system, in our case radiating charged particles, is given by quantum mechanics.

Our analysis will be separated into two parts: the first part where electromagnetic radiation is analyzed in the time domain and the second part where the electromagnetic radiation is analyzed in the frequency domain. The radiation fields we will analyze in the time domain are the ones generated by transition radiation when moving charged particles are accelerated or decelerated at dielectric or conducting boundaries, while in the frequency domain the source of the radiation is a long chain of oscillating charges giving rise to an oscillating current.

2. Definition of Parameters

Action associated with the time domain radiation field—A; The period of oscillation of the frequency domain oscillator—T; The energy radiated within time $T/2$ in the frequency domain radiation fields—U; The energy radiated by the decelerating electron—ΔU; Planck constant—h; The charge associated with the decelerating charged particle in the time domain and the peak value of the charge in the oscillator in the frequency domain—q; The frequency of oscillation of the frequency domain radiator—ν; The wavelength of the frequency domain radiation field—λ; the angular frequency of the frequency domain radiation field—ω; The wave number—k; The speed of propagation of the charged particle—v; The ratio of the speed of the charged particle to the speed of light—β; The distance between the initiation of the motion of the charged particle and its point of incidence on the perfectly conducting ground plane in the case of the time domain radiating system and the length of the antenna above the conducting plane in the frequency domain—$L/2$; The total duration of the time domain current waveform—Γ; The effective duration of the time domain current waveform—τ; The Compton wavelength of the electron—λ_c. The average power radiated over one period by the frequency domain radiator—P; The sine integral—S_i; The cosine integral—C_i; Euler's constant—γ; The median value of the power radiated by the frequency domain radiator—P_{med}; The median energy radiated within a burst of *T*/2 duration by the frequency domain radiator—U_{med}; Maximum value of the median energy radiated by the frequency domain radiator—$\langle U_{med} \rangle_{\max}$; The dark energy density—ρ_v; The steady state value of the Hubble radius—R_∞.

3. Time Domain Electromagnetic Fields—Transition Radiation

The transition radiation is generated when a charged particle moving with constant speed is ejected or incident either on a conducting or dielectric boundary. Here, we study the transition radiation generated when a moving charged particle is incident on a perfectly conducting boundary. This is the case where, for a given charge and speed, one can obtain the highest energy in the transition radiation. Observe that by definition the transition radiation is the radiation generated during the acceleration or deceleration of charge and it is not the radiation, if any, generated during the deceleration of material particles. In the literature, the analysis of transition radiation is usually carried out in the frequency domain but here we will confine our analysis to the time domain. Moreover, in analyzing the transition radiation, the charge is usually considered to be concentrated onto a point, *i.e.*, a point charge, but in the present study we represent the charge as having a spatial distribution. In this case, the movement of the charge can be represented by a current pulse of finite duration. Of course, when the duration of the current pulse approaches zero (*i.e.*, when it can be expressed as a Dirac delta function), it represents the movement of a point charge. The temporal variation of the current pulse associated with the moving charge is related to the way in which the charge is distributed across the charged particle. The incidence or emission of a charged particle from a perfectly conducting plane can then be represented by a current pulse that moves into or out of the perfect conductor. Once the speed of the charged particle and the current pulse that represents the movement of the charged particle are defined, the electromagnetic fields associated with the transition radiation can be obtained using standard techniques.

The geometry relevant to the analysis is shown in **Figure 1**. A negatively charged particle with a charge of magnitude q is accelerated to speed v at a distance *L*/2 from the perfectly conducting plane. This charge is incident on the perfectly conducting plane giving rise to the transition radiation. If the movement of the charged particle can be represented by a current pulse $i(t)$, the radiation field generated during the deceleration of the charge during the encounter with the perfect conductor is given by [8]

$$\boldsymbol{E}_{tran}(t) = \frac{i(t - L/2v - r/c)v\sin\theta}{4\pi\varepsilon_0 c^2 r}\left\{\frac{1}{1-\beta\cos\theta} + \frac{1}{1+\beta\cos\theta}\right\}\boldsymbol{a}_\theta \tag{1}$$

where $\beta = v/c$. Note that $\boldsymbol{a}_r$ and $\boldsymbol{a}_\theta$ are unit vectors in the direction of increasing r and θ (see **Figure 1**). The first term inside the brackets is generated by the absorption of the current pulse by the conducting plane and the second term is due to its image in the perfectly conducting plane.

Now, in addition to the transition radiation, two more components of electromagnetic radiation exist in space. The first is the radiation field generated

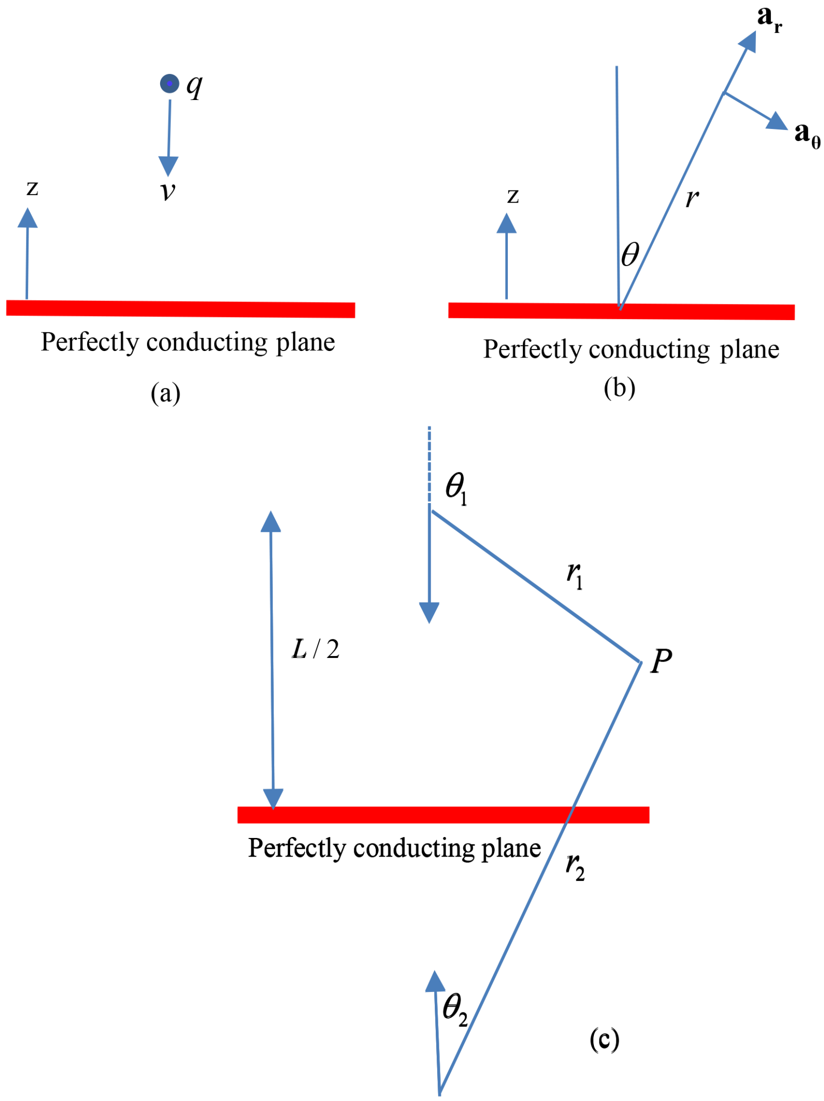


Figure 1. (a) A charged particle moves along the z-axis towards the perfectly conducting plane. (b) The definition of parameters pertinent to the expression derived for the electric radiation field. In the diagram, $\boldsymbol{a}_r$ and $\boldsymbol{a}_\theta$ are unit vectors in the increasing radial r and θ directions. (c) The geometry relevant to the study of overlapping of the signals.

during the initial acceleration of the charge and the second is its reflection from the perfectly conducting plane (represented by a wave generated by an image source located at a depth $L/2$ below the conducting plane). These two components are given respectively by (see **Figure 1(c)**)

$$\boldsymbol{E}_i\left(\theta_1,t\right)=-\frac{i\left(t-r_1/c\right)v\sin\theta_1}{4\pi\varepsilon_0c^2r_1}\left\{\frac{1}{1+\frac{v}{c}\cos\theta_1}\right\}\boldsymbol{a}_{\theta_1} \tag{2}$$

$$\boldsymbol{E}_{i,r}\left(\theta_2,t\right)=-\frac{i\left(t-r_2/c\right)v\sin\theta_2}{4\pi\varepsilon_0c^2r_2}\left\{\frac{1}{1-\frac{v}{c}\cos\theta_2}\right\}\boldsymbol{a}_{\theta_2} \tag{3}$$

Before analyzing the energy radiated by the transition radiation, let us define the current waveform associated with the moving charge.

4. The Current Pulse Associated with the Moving Charge

The current distribution associated with a moving charged particle with charge q depends on how the electric charge is distributed on the particle. As a first step we will assume that the charge of the particle is distributed uniformly within the particle. In this case the current associated with the moving charge can be described by (after normalizing the peak amplitude to i_0)

$$i(t) = 4i_0 \frac{t}{\Gamma}\left(1 - \frac{t}{\Gamma}\right) \quad 0 \le t \le \Gamma \tag{4}$$

The expression for the current waveform can also be given in terms of the magnitude of the moving charge by

$$i(t) = \frac{6q}{\Gamma^2} t \left(1 - \frac{t}{\Gamma}\right) \tag{5}$$

Observe that when $\Gamma \to 0$ the current pulse reduces to that of a point charged particle. The effective duration of the current pulse over which radiation is emitted, τ, can be defined as $\tau i_0 = q$ which gives $\tau = 4\Gamma/6$.

5. Energy Released by the Transition Radiation

As described in Section 3, the system under consideration contains two radiation bursts in addition to the transition radiation. Now, one can see directly that the transition radiation overlaps with the reflected wave (*i.e.* the wave coming out from the image location) at very small angles of θ. Let us denote this angle by θ_0. The separation between the electric field pulses when they begin to overlap is Γ. With this parameter, it is a simple matter to show that $\cos\theta_0 = (c/v)(1 - 2v\Gamma/L)$. This can also be written as $\cos\theta_0 = (c/v)(1 - 2d/L)$ where d is the effective diameter of the charged particle. Observe that if $(L/2v) - (L\cos\theta/2c) > \Gamma$ fields do not overlap. Note also that in making this evaluation we are assuming that the point of observation is such that $r_1 \approx r_2 \approx r$ and $\theta_1 \approx \theta_2 \approx \theta$. In our analysis, we will consider current pulse durations which are much shorter than the time $L/2v$. Moreover, in this case, $\cos\theta_0$ becomes almost equal to unity and the value of θ_0 reduces to $\theta_0 = \sqrt{2(1 - c/v + 2dc/vL)}$. In the region where $\theta < \theta_0$ (*i.e.*, in the region where the pulses overlap) the resultant radiation field is

$$\mathbf{E}_{overlap}(t) = \frac{v\sin\theta}{4\pi\varepsilon_0 c^2 r(1 - \beta\cos\theta)}\left\{ i(t)\left[1 + \frac{1 - \beta\cos\theta}{1 + \beta\cos\theta}\right] - \left[i(t - \Delta t)\right]\right\}\mathbf{a_0} \tag{6}$$

In the above equation $\Delta t = (L/2)(1/v - \cos\theta/c)$. The Poynting vector associated with the transition radiation in the non-overlapping region is given by

$$\boldsymbol{S}(t) = \frac{i(t - r/c)^2 \beta^2 \sin^2\theta}{4\pi^2\varepsilon_0 c r^2} \frac{1}{\left(1 - \beta^2\cos\theta^2\right)^2}\boldsymbol{a}_r \tag{7}$$

The Poynting vector in the overlapping region is given by

$$\boldsymbol{S}_{overlap}(t)=\frac{\beta^2\sin^2\theta}{16\pi^2\varepsilon_0 cr^2(1-\beta\cos\theta)^2}\left\{i(t)\left[1+\frac{1-\beta\cos\theta}{1+\beta\cos\theta}\right]-\left[i(t-\Delta t)\right]\right\}^2\boldsymbol{a}_r \quad (8)$$

The power radiated by the transition radiation in the region where the pulses do not overlap is

$$P(t)=\frac{i(t-r/c)^2\beta^2}{2\pi\varepsilon_0 c}\int_{\theta_0}^{\pi/2}\frac{\sin^3\theta}{\left(1-\beta^2\cos^2\theta\right)^2}\mathrm{d}\theta \quad (9)$$

The power generated by the pulses in the overlapping region is

$$P_{overlap}(t)=\frac{\beta^2}{8\pi\varepsilon_0 c}\int_0^{\theta_0}\frac{\sin^3\theta}{(1-\beta\cos\theta)^2}\left\{i(t)\left[1+\frac{1-\beta\cos\theta}{1+\beta\cos\theta}\right]-\left[i(t-\Delta t)\right]\right\}^2\mathrm{d}\theta \quad (10)$$

Now, the energy radiated depends on the shape of the current waveform. However, our calculations demonstrate that irrespective of the shape of the current waveform, the energy radiated in the overlapping region can be neglected in comparison to the energy radiated in the non-overlapping region. Thus, the energy radiated by the transition radiation is completely controlled by Equation (9).

One can see from Equation (9) that the power radiated by the transition radiation increases with increasing speed of the charged particle. Here, we assume that the speed is highly relativistic and therefore $\beta\approx 1$. In this case, the value of θ_0 becomes equal to $\sqrt{4d/L}$. The integral in Equation (9) can be performed easily resulting in the following expression for the energy radiated by the transition radiation (note that this energy is equal to the change in the energy of the charged particle due to the emission of transition radiation):

$$\Delta U=\frac{1}{4\pi\varepsilon_0 c}\left[\log\frac{L}{d}\right]\int_0^{\Gamma}i(t)^2\,\mathrm{d}t \quad (11)$$

Performing the time integral associated with Equation (11), we obtain

$$\Delta U=\frac{q^2}{4\pi\varepsilon_0 c}\frac{6}{5\Gamma}\left[\log\frac{L}{d}\right] \quad (12)$$

6. The Action Associated with the Transition Radiation and Its Maximum Value

The total energy radiated by the transition radiation is given by Equation (12). The action associated with the transition radiation, which is defined as the product of the radiated energy and the effective duration of the time over which the charged particle had been decelerated during the emission of radiation, *i.e.*, τ as defined in Section 4, is given by

$$A=\tau\Delta U=\frac{q^2}{5\pi\varepsilon_0 c}\left[\log\frac{L}{d}\right] \quad (13)$$

Let us estimate the absolute theoretical maximum value of the action ever possible associated with the radiation field. For a given charge, the action increases with increasing *L*/*d*. Let us consider the natural limits imposed on this ratio by nature. Classical electrodynamics does not impose any limit on these parameters and in principle the maximum length of the radiating system can reach infinity and the smallest dimension of the radiating system can reach values close to zero. However, general relativity and quantum mechanics provide limits related to the maximum and minimum dimensions of the radiating system, respectively. Let us first consider the smallest value of *d* ever allowed by nature. The smallest charged particle that exists in nature is the electron. Currently, there are no observations that determine the size of the electron. In our analysis, we assume that the radius of the electron that takes part in the emission of radiation is equal to its Compton wavelength. Let us now consider *L*. The largest possible value of the spatial length (in our case the distance between the point of origin of the charged particle and its image in the perfectly conducting plane) that one can have in nature is equal to the radius of the universe where events can be in causal contact. This radius is defined as the Hubble radius. At present the Hubble radius increases with time. However, according to the current understanding, the Hubble radius of the universe becomes constant at some future epoch and this value is given by the steady state value of the Hubble radius [9] [10] [11]. Let us denote this radius by R_∞. We assume this to be the longest length scale ever possible in the current universe. The validity of the assumptions made above will be discussed in Section 8. Substituting these parameters into our expression for the action associated with the transition radiation given by (13), we obtain

$$\langle A \rangle_{\max} = \tau \Delta U = \frac{q^2}{5\pi\varepsilon_0 c}\left[\log\frac{R_\infty}{2\lambda_c}\right] \tag{14}$$

In Equation (14), $\langle A \rangle_{\max}$ is the absolute maximum value of the action that can be achieved by a given charge. Based on general relativity, the steady state value of the Hubble radius is given by $R_\infty = c^2\sqrt{3/8\pi G\rho_\Lambda}$, where G is the gravitational constant and ρ_Λ is the dark energy density [12]. Substituting this expression into Equation (14) yields

$$\langle A \rangle_{\max} = \tau \Delta U = \frac{q^2}{5\pi\varepsilon_0 c}\left[\log\frac{c^2\sqrt{3/8\pi G\rho_\Lambda}}{2\lambda_c}\right] \tag{15}$$

Figure 2 shows a plot of $\langle A \rangle_{\max}$ as a function of the magnitude of the charge. In the calculation, the dark energy density was taken to be 5.38×10^{-10} J/m^3 [13]. We have represented the action in units of $h/4\pi$, which is the atomic unit of the angular momentum with h representing the Planck constant. According to the results presented in **Figure 2**, Equation (15) predicts that when $q = 1.6\times10^{-19}$ C, $\langle A \rangle_{\max} = h/4\pi$. Note that this value of q is accurate to within one percent of the charge of an electron. Thus, Equation (15) can be written as

$$\frac{e^2}{5\pi\varepsilon_0 c}\left[\log\frac{c^2\sqrt{3/8\pi G\rho_\Lambda}}{2\lambda_c}\right] \approx \frac{h}{4\pi} \tag{16}$$

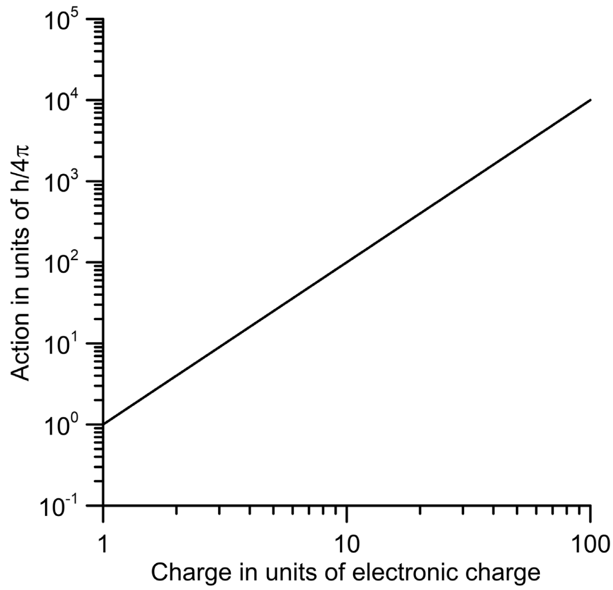


Figure 2. The absolute maximum value of the action $\langle A \rangle_{\max}$ as a function of the charge.

Observe that $\langle A \rangle_{\max}$ is the absolute maximum action that can be realized by any given charge. Thus, the results can be summarized by the mathematical statement

$$\tau \Delta U \geq h/4\pi \Rightarrow q \geq e \quad (17)$$

The meaning of this mathematical statement is the following: *If* $\tau \Delta U \geq h/4\pi$ *then* $q \geq e$. A statement identical to that given by Equation (17) was derived previously in [1] [2] [4] by analyzing transient currents in long antennas. However, since the effective value of the radius of the electron is not defined exactly, one may treat this relationship as an order of magnitude relationship. It is important to point out here that the reverse of this mathematical statement, *i.e.*, $q \geq e \Rightarrow A \geq h/4\pi$ is not valid. Indeed, even in the case where the charge is larger than the electronic charge, by decreasing the speed of propagation or by decreasing the length between the point of initiation of the movement of the charged particle and its image in the conducting plane (*i.e.*, length L), one can satisfy the condition $A < h/4\pi$.

In order to test whether the condition given by Equation (17) is also valid for a proton undergoing transition radiation, we have repeated the calculation for a proton by taking into account the measured size of the proton and the theoretically estimated charge distribution [14] [15]. The results of this exercise show that the mathematical statement given above is also valid for radiation fields generated by decelerating (or accelerating) protons.

7. Frequency Domain Electromagnetic Fields

Consider a chain of electrical charges oscillating sinusoidally. The length of the chain is $L/2$ and it is located over a perfectly conducting plane. The individual oscillators are coupled so that the electric current at any given location along the string, say z, is given by

$$I_0 \sin\left\{(2\pi/\lambda)\left[(L/2)-z\right]\right\} e^{j\omega t} \quad 0 \le z \le L/2 \tag{18}$$

$$I_0 \sin\left\{(2\pi/\lambda)\left[(L/2)+z\right]\right\} e^{j\omega t} \quad -L/2 \le z \le 0 \tag{19}$$

In the above equations, I_0 is a constant and ω is the angular frequency of oscillation. The current associated with the oscillating charges is similar to the one originated in a long electric dipole in space [16]. The calculation of the average energy radiated by such a system of charges is a simple procedure and it suffices here to give directly the expression for it. It is given by [16]

$$\begin{aligned} P = \frac{I_0^2}{8\pi\varepsilon_0 c}\Big\{ & \gamma + \ln(kL) - C_i(kL) + \frac{1}{2}\sin(kL)\left[S_i(2kL) - 2S_i(kL)\right] \\ & + \frac{1}{2}\cos(kL)\left[\gamma + \ln(kL/2) + C_i(2kL) - 2C_i(kL)\right]\Big\} \end{aligned} \tag{20}$$

where $k = \omega/c$, C_i is the cosine integral, S_i is the sine integral, and γ is Euler's constant. In terms of the magnitude of the oscillating charge in any given element, q, the average power is given by

$$\begin{aligned} P = \frac{q^2(2\pi\nu)^2}{8\pi\varepsilon_0 c}\Big\{ & \gamma + \ln(kL) - C_i(kL) + \frac{1}{2}\sin(kL)\left[S_i(2kL) - 2S_i(kL)\right] \\ & + \frac{1}{2}\cos(kL)\left[\gamma + \ln(kL/2) + C_i(2kL) - 2C_i(kL)\right]\Big\} \end{aligned} \tag{21}$$

One can observe from this equation that the value of P oscillates rapidly with kL for large values of kL. The upper and lower bounds of P occur when $kL = n\pi$ and $kL = m\pi$, where n and m are even and odd integers (*i.e.*, when $\cos(kL) = 1$ or $\cos(kL) = -1$). The median value of P is given by

$$P_{med} = \frac{q^2\pi\nu^2}{2\varepsilon_0 c}\left\{\gamma + \ln(kL) - C_i(kL)\right\} \tag{22}$$

Note that for large values of kL, the cosine integral varies as $\cos(2kL)/(2kL)^2$ and it can be neglected with respect to other terms. Thus, for large values of kL, the expression for the median power reduces to

$$P_{med} = \frac{q^2\pi\nu^2}{2\varepsilon_0 c}\left\{\gamma + \ln(kL)\right\} \tag{23}$$

Observe that due to the sinusoidal nature of the oscillation, the power generated by the oscillatory charges consists of bursts of energy of duration $T/2$, where T is the period of oscillation. The median energy dissipated by a single burst of energy U_{med} of duration $T/2$ is given by

$$U_{med} = \frac{q^2\pi\nu}{4\varepsilon_0 c}\left[\gamma + \ln(kL)\right] \tag{24}$$

Note that the radiated energy as given by Equation (24) increases with increasing value of L and with decreasing value of λ (observe that $k = 2\pi/\lambda$). As mentioned earlier, in order to extract hidden information from nature one has to push the parameters of interest to their extreme values. In our case, we have to push L and λ to their limits.

From a classical point of view, the only restriction on the minimum value of the wavelength is that it has to be much larger than the dimension of the oscillating charge. If this is not the case, destructive interference will lead to the reduction of the radiated energy. Accordingly, the minimum value of λ has to be much larger than the Compton wavelength of the electron, *i.e.*, the assumed radius of the electron. Based on these considerations, the minimum value of the wavelength that should be plugged into the above equation is about $10\lambda_c$, where λ_c is the Compton wavelength of the electrons (see section 8).

As in the time domain case, the largest value of L ever possible in the current universe is the steady state value of the Hubble radius. Combining this with the information given in the previous paragraph, the expression for the maximum energy that the chain of charges can generate is given by

$$\langle U_{med} \rangle_{\max} = \frac{q^2 \pi \nu}{4\varepsilon_0 c}\left[\gamma + \ln\left(2\pi c^2 \sqrt{3/8\pi G\rho_\Lambda}/10\lambda_c\right)\right] \tag{25}$$

Figure 3 shows a plot of $\langle U_{med} \rangle_{\max}$ as a function of the magnitude of the oscillating charge. According to the results presented in **Figure 3**, Equation (25) predicts that $q = 1.6023\times10^{-19}$ when $\langle U_{med} \rangle_{\max} = h\nu$. Note again that this value of q is accurate to within one part in 10^3 of the charge of an electron. It is of interest to point out that had we used the value 4.96×10^{-10} J/m^3 for the dark energy density, we would have obtained an exact match for e when $\langle U_{med} \rangle_{\max} = h\nu$. In this case the equation reduces to

$$\frac{e^2\pi}{4\varepsilon_0 c}\left[\gamma + \ln\left(2\pi c^2 \sqrt{3/8\pi G\rho_\Lambda}/10\lambda_c\right)\right] = h \tag{26}$$

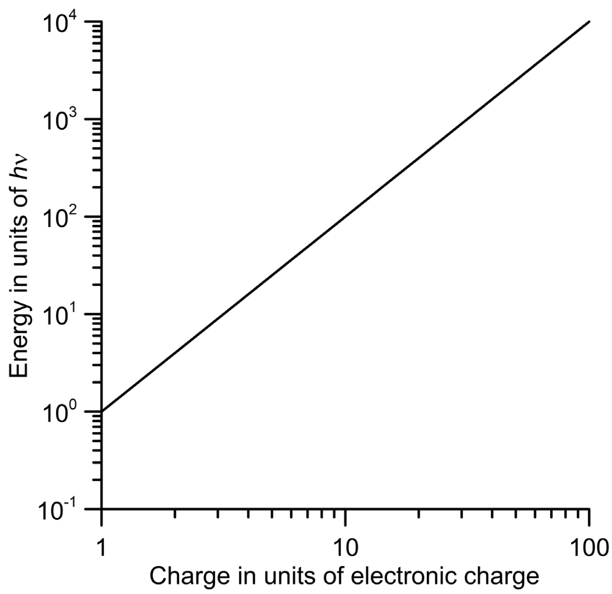


Figure 3. The absolute value of the median energy transported by a single burst of radiation, $\langle U_{med} \rangle_{\max}$, as a function of the charge q as given by Equation (25).

Note that, since $\langle U_{med} \rangle_{\max}$ is the maximum energy that can be radiated within a single burst of power for any given charge, the results can be summarized by the mathematical statement

$$U \geq h\nu \Rightarrow q \geq e \tag{27}$$

where U is the energy radiated in a single power burst of duration $T/2$, h is the Planck constant, ν is the frequency of oscillation, and q is the peak value of the oscillating charge. Note that the mathematical statement given by Equation (27) is universal and it is satisfied by any radiating system created by oscillating charges and currents. Of course, since the value of the effective radius of the electrons cannot be defined exactly, one may treat this relationship also as an order of magnitude relationship. A result identical to that given in Equation (27) was derived previously in [1] and [4] by analyzing electromagnetic radiation from a long antenna containing an oscillating current. It is important to point out that, as in the case of the time domain radiator, the reverse of this mathematical statement, *i.e.*, $q \geq e \Rightarrow U \geq h\nu$ is not necessarily true. For example, even if the oscillating charge is larger than the electronic charge, by decreasing the length of the chain of oscillators one can make the energy be sufficiently low for the relation $U < h\nu$ to be satisfied. Furthermore, as in the time domain example, the derivation is purely based on classical electrodynamics and the appearance of the Planck constant in the expression is due to the use of atomic units to describe the energy.

8. A Discussion on the Assumptions Made in the Analysis

In what follows, we discuss the assumptions made in the presented analysis.

1) It is assumed that the dark energy density of the universe is finite and positive. It is also assumed that it is the same throughout the Hubble sphere and that the dark energy density does not change as the universe age. The first assumption is supported by the current experimental observations [13]. However, at present scientists do not have information on how the dark energy density will change in the future.

2) Classical electrodynamics does not have any restrictions on the dimensions of the radiating system. However, in the analysis it is assumed that due to the physical nature of the universe, there are upper and lower bounds for the radiating systems which result from both general relativity and quantum theory. Friedmann [17] equations provide the solutions of the equations of general relativity for the evolution of the universe. The growth of the universe depends on the density of the radiation, matter and the dark energy (positive cosmological constant) of the universe. In the case where the dark energy density dominates both that of radiation and matter the universe expands exponentially and it will evolve similar to a de Sitter universe [18]. According to the measurements, the current universe is almost (asymptotically) equal to de Sitter space with a positive dark energy density and negative pressure. It will evolve like a de Sitter universe in the future when dark energy dominates over matter density. In that

epoch the Hubble radius (which is increasing at present) becomes a constant and it will define the maximum length scale over which events can be in causal contact. In our analysis we have assumed that the maximum length scale ever possible in the current universe is equal to this steady state value of the Hubble radius. In the paper, this limit is used as an input to find out the behavior of classical electrodynamics when the dimension of the radiating system is pushed to these extreme limits.

3) In the analysis, we have assumed that the radius of the electron is equal to its Compton wavelength. Actually, the current understanding is that electrons do not have a structure and can be treated as point particles. However, both assumptions, *i.e.*, point particle or a particle with a finite size, lead to inconsistencies. For example, the assumption of a finite size for a fundamental particle leads to inconsistencies in relativity and the assumption of point particle leads to infinities when internal energy is concerned [19]. However, due to quantum effects, as far as the emission and absorption of radiation are concerned, one may treat the electron as a fuzzy spherical region with a radius roughly in the order of the Compton wavelength [20]. Moreover, the work of Moniz and Sharp [21] [22] indicates that an electron behaves as an extended particle with the size of the Compton wavelength. It is also of interest to note that Schrodinger's zitterbewegung (jittery) theory suggests that an electron oscillates rapidly [23]. The amplitude of this spatial oscillation was shown to be of the order of the Compton wavelength. This provides support to our assumption to treat the electron as a fuzzy spherical charge distribution with dimensions in the order of the Compton wavelength. Furthermore, observe that the size of the electron, or more strictly the size of the region where the charge is distributed, comes into the equations as a result of the time over which the electron decelerates when crossing the boundary. Since the electron cannot be located to a better accuracy than its Compton wavelength, this time cannot be assigned to an accuracy better than λ_c/v. This justifies the assumption that the duration of the radiation field is about λ_c/v. Furthermore, even if the actual size of the electron can be approximated by a point, pair production in the vicinity of the electron polarizes the space in the vicinity of the electron making the charge of the electron to disperse over region larger than the actual size of the electron. Having explained the reason for our decision to consider the electron as a fuzzy charged sphere with dimensions comparable to Compton wavelength, let us consider the consequences of treating the electron as a point particle. There is general consensus today that the smallest material particle cannot be smaller than the Planck length. If we use the Planck length as the size of the electron, the results would differ only by about 20% from the one we obtained by assuming Compton wavelength to be the size of the effective electron radius. The reason for this low sensitivity of the results to the size of the electron is due to the fact that this parameter appears in the equations inside a logarithmic term. For this reason, our conclusions remain valid even for the size of electron much smaller than the Compton wavelength.

4) In the time domain analysis, we have assumed that the charge of the electron is distributed uniformly over a finite region. Our calculations show, however, that the assumption of how the charge of the particle is distributed in space is not that critical to the final result. We have repeated the calculations by assuming a) the charge of the electron is spread over a thin shell around the particle (note that there are models where the charge of the electron is assumed to be distributed over a thin shell [24]); b) the charge density decreases linearly towards the center; c) the charge density increases linearly towards the center and d) the charge density increases exponentially towards the center. These calculations show that the results would not change significantly if we assume other charge distributions. For example, the value of the charge that makes the action equal to $h/4\pi$ does not change more than about 15% when different charge distributions are assumed. This result is also in agreement with the results obtained in previous studies by analyzing transient currents propagating along long antennas [6].

5) In the calculation of the frequency domain radiation, we had to select a wavelength which is much larger than the Compton wavelength of the electron. We have assumed that the smallest wavelength that satisfies this condition (*i.e.* $\lambda \gg \lambda_c$) is $10\lambda_c$. Observe that had we assumed $5\lambda_c$ or even λ_c the derived condition would not have changed significantly. Of course, observe that Equation (28) remains valid also for *any* wavelength larger than $10\lambda_c$.

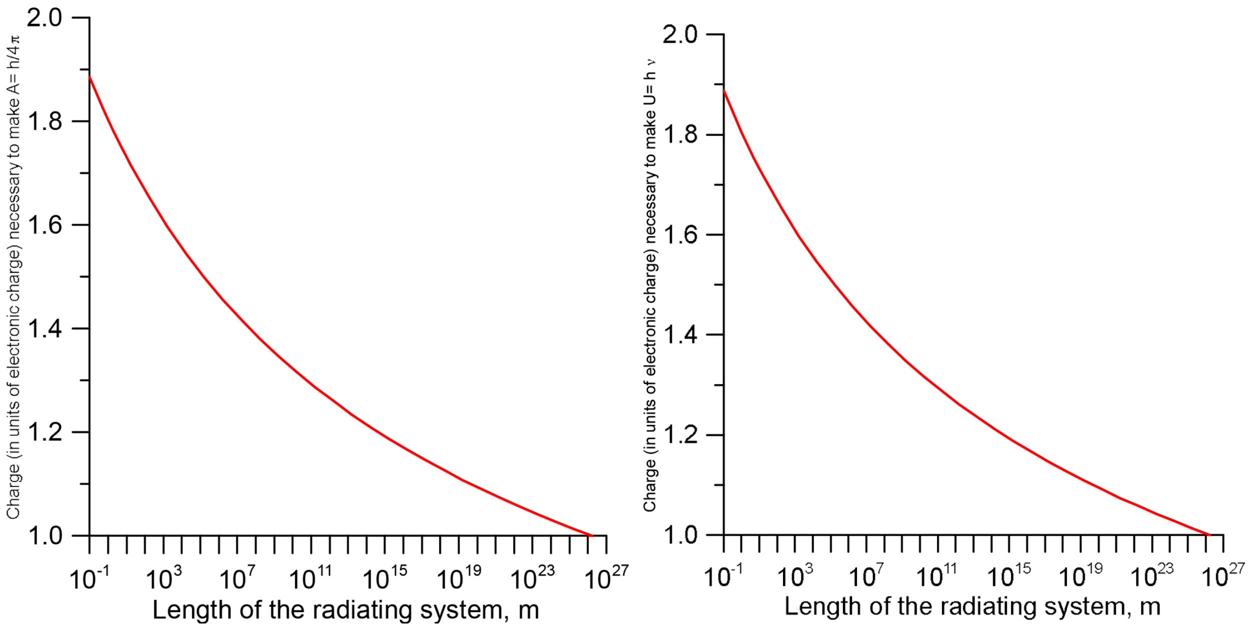
6) Observe that in the analysis we have assumed that the maximum length of the radiating system located above the perfectly conducting plane to be $R_\infty/2$ instead of R_∞ (*i.e.* $L = R_\infty$). The reason for this choice is that this value makes it possible for the perfectly conducting plane to have a diameter larger than the maximum possible distance between the oscillating charges and their image in the perfectly conducting plane. The latter is a requirement for the validity of the field equations used in the analysis. However, had we assumed $L = 2R_\infty$, the results would have remained almost identical to the ones presented here.

7) It is important to understand that the whole exercise presented in the current paper is a hypothetical theoretical experiment or a thought experiment. In the presented analysis, we have studied how the equations of classical electrodynamics will behave if the dimensions of a radiating system are pushed to their natural limits. However, it is important to stress that the model utilized in this paper cannot be realized experimentally. The maximum length of the radiating system used in the analysis should not be interpreted as that of a real system but that of a hypothetical "gedanken" experiment the purpose of which is to study the behavior of the classical electromagnetism when pushed to extreme limits. Thus, the hypothetical experiment should not be interpreted as a real one. The results obtained has to be understood only as due to an effect of scaling on the electromagnetic field equations of classical electrodynamics.

9. Significance of the Results

The derivation presented in this paper is based purely on the field equations of classical electrodynamics. The maximum possible length scale ever possible in the current universe is obtained from general relativity and the effective size of the electrons and the minimum wavelength that should be plugged into the frequency domain analysis are obtained from quantum mechanics. In the two inequalities derived here, the right-hand portion of the mathematical statement is $q \geq e$. That is, the radiating charge has to be larger than or equal to the charge of an electron. This is actually not a consequence of classical electrodynamics because the theory does not specify any limits on the magnitude of the charge. However, from experimental observations, we know today that the minimum charge that exists in nature is equal to the electronic charge or the elementary charge. This shows that the two inequalities $U \geq h\nu$ and $\Delta U \tau \geq h/4\pi$ appearing respectively in (17) and (27) make sense. Otherwise we cannot satisfy the condition $q \geq e$. The inequality $U \geq h\nu$ states that the energy associated with a single burst of radiation has to be larger than $h\nu$, which with hindsight we recognize as the energy of a photon, a result emanating from the photonic nature of electromagnetic radiation. Similarly, the inequality $\Delta U \tau \geq h/4\pi$ can be recognized as a result coming out from the time-energy uncertainty principle as applicable for the energy loss from the charged particle during its deceleration at the boundary of the conductor. This is also related to the photonic nature of the electromagnetic fields. These results show that when classical electrodynamics is pushed to its extremes it can either reveal or at least provide hints as to the true nature of the electromagnetic radiation.

It is important to point out that if the length of the radiating system is less than or equal to the steady state value of the Hubble radius, the two mathematical statements given by Equations (17) and (27) remain strictly valid. Furthermore, it is of interest to observe that over a very large span of the lengths of the radiating system, from Hubble radius to lengths less than a meter, the charge necessary to make $\tau \Delta U = h/4\pi$ and $U = h\nu$ remains still within the order of magnitude of the electronic charge. For example, **Figure 4(a)** and **Figure 4(b)** depict the value of the charge (in units of electronic charge) that is necessary to make $\tau \Delta U = h/4\pi$ in the transition radiation and $U = h\nu$ in the frequency domain. First observe that the charge for a given length is almost same for both the transition radiation and the frequency domain radiation systems. Second, note that the value of the charge remains within the same order of magnitude of the electronic charge even for laboratory scale lengths of the radiating system. This indeed makes it possible to test the validity of our hypothesis using actual laboratory scale experiments. For example, the mathematical statement derived here, namely $U \geq h\nu \Rightarrow q > e$ should be valid for Hertzian, half-wave and full-wave dipole radiation and this could be tested easily in the laboratory. Moreover, the mathematical statement $\tau \Delta U \geq h/4\pi \Rightarrow q > e$, which was also derived in [2] and [3] based on propagation of current pulses in long antennas, should be valid for the radiation from accelerating charged particles as described by Larmor formula and this also could be tested using laboratory experiments. This is the first time that this information is presented in the literature except in our preliminary work referenced previously.



(a) (b)

Figure 4. (a) The charge (as a fraction of the electronic charge) necessary to make the action equal to $h/4\pi$ in the transition radiation as a function of the length of the radiating system. (b) The charge (as a fraction of the electronic charge) necessary to make the energy equal to $h\nu$ in the frequency domain radiating system as a function of the length of the radiating system.

In this paper, we have kept our analysis strictly within classical electrodynamics. However, had we utilized the photonic nature of the electromagnetic fields as an input in the analysis, we could have derived indirectly an expression for the dark energy density in terms of the other natural constants. Such an analysis was presented in [1] [2] and [4].

10. Conclusion

The results presented in this paper show that classical electrodynamics, when combined with the fact that the maximum dimension of any radiating system cannot be larger than the steady state value of the Hubble radius (*i.e.*, the ultimate upper limit of the size of the universe where events are in causal contact) and that the smallest free charge that exists in nature is the electronic charge, leads to two inequalities which with hindsight can be interpreted as a result of the photonic nature of the electromagnetic radiation. From this, one can conclude that hints of the photonic nature of the electromagnetic radiation remain hidden in the field equations of classical electrodynamics and they become apparent when the dimension of the radiating system is pushed to the extreme limits as allowed by nature.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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