

Point Charge between Two Grounded and Conducting Planes Forming a 60° Angle and for Yukawa's Potential and Coulomb's Potential

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Abstract

For a point charge between two grounded conductor planes forming a 60° angle, the potential and electric field generated by point charge for Yukawa's potential ($e^{-\mu r}/r$) and Coulomb's potential ($1/r$) are modeled and simulated. The expression for the potential that generalizes the cases discussed in López-Mariño, M. and Trujillo Caballero, J. (2017) Point Charges and Conducting Planes for Yukawa's Potential and Coulomb's Potential. *Journal of Electromagnetic Analysis and Applications*, 9, 135-146.

<https://doi.org/10.4236/jemaa.2017.910012> is presented. Graphs for the potential and electric field for both cases are showed using Maple, that of Coulomb and that of Yukawa for different values of μ . The purpose of this work is to offer students a practical guide for problem analysis of electrostatics using Maple's capabilities as a computational tool.

Keywords

Yukawa's Potential, Coulomb's Potential, Method of Images

1. Introduction

In a recent work [1], and using the Image Method (IM) [2] [3] [4] [5], we modeled and simulated the field and the electric potential produced by electrical charges in the region of grounded conducting planes using Coulomb and Yukawa potential. Our interest had its origin in the interesting works of Griffiths and Uvanovic [6], and Sallabi *et al.* [7], who studied the load distribution in conductors using Yukawa's potential ($e^{-\mu r}/r$).

In [1], we presented the expressions for the electric potential, as well as the graphs of the electric potential and the electric field for the case of a localized point charge in the region of a grounded conducting plane and for the case of two grounded conducting planes, perpendicular to each other and a point charge placed in a quadrant between the planes.

Our goal in this work is to analyze the case of a point charge between two grounded and conducting planes forming a 60° angle for Yukawa's potential and Coulomb's potential and to discuss the relationship between the expression of the electric potential and the cases discussed in [1]. Also, we want our work to motivate students to use computational tools in their learning of basic science subjects such as electromagnetism. Therefore, we include an **Appendix** with the necessary Maple's instructions to obtain the expressions of the potential and the electric field, as well as their graphs.

The motivation for this work coincides with the results of the study carried out by López *et al.* [8] on the modeling and simulation of phenomena of physics, chemistry and biology in articles published and existing in various databases. It was found that publications on simulating greatly outnumber those on modeling. The explanation they presented is that for modeling it is necessary, in addition to mastering the subject, the knowledge of a programming language. For this reason, we offer our students, in a first stage of the course, the solution of the problem and some lines of code of the computational tool [1] [9]. With this, students will be able to carry out simulations that will allow them to analyze the study system.

The parts of this work are the following: Section 2, where we discuss the case of a point charge located between two grounded conductor planes forming a 60° angle and the distribution of the real charge and image ones; Section 3, where we obtain the function of the electric potential and show both the potential and electric field for different values of μ ; Section 4, where we offer our final comments.

2. Point Charge between Two Grounded Conducting Planes Forming a 60° Angle

Our goal for this section is to establish the distribution of the actual, Q , charge and the image charges in the region of the conducting planes, as shown in **Figure 1** [10] [11].

To find the function of potential between the conducting planes, we must first make sure that each one of them has a potential $V = 0$. To analyze this aspect, we refer to **Figure 2**, which shows the geometry of the problem in a general way. It can also be seen that the origin of the planes was displaced to point $C = (h, t)$.

We can observe that one of the conductors is placed on the horizontal while the second plane is located at an α from the aforementioned one. The charge is placed at a θ angle from the horizontal plane and at a distance r from C . Note that $\theta < \alpha$, and, in our case, $\alpha = 2\theta$. In addition, we have the restriction $\frac{360^\circ}{\alpha} \in \mathbb{Z}$.

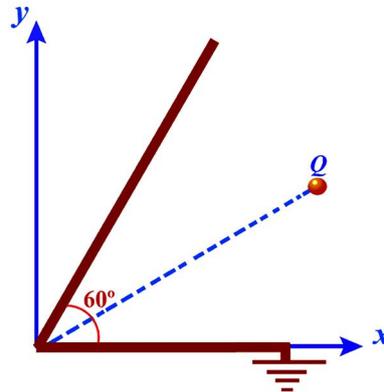


Figure 1. Q charge between planes separated by a 60° angle.

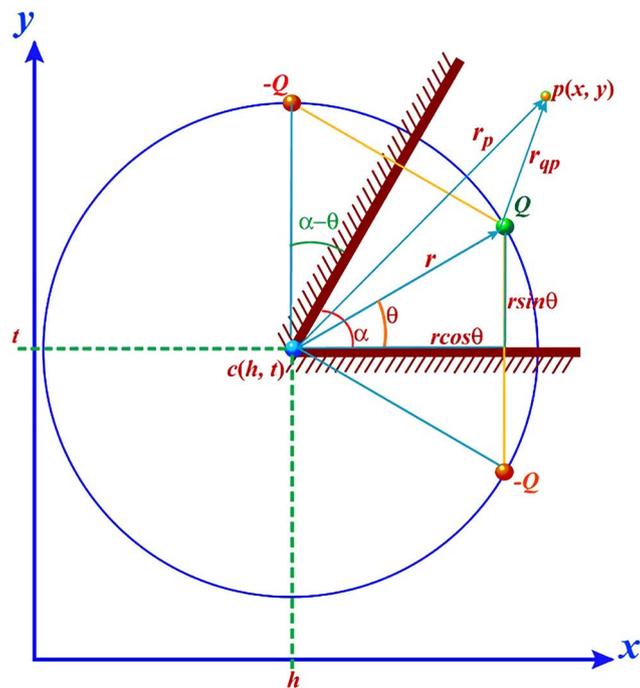


Figure 2. Configuration of Q charge between planes separated by a 60° angle.

In order to make the potential of the plane located at an α angle equal to zero, it is necessary to place an image charge $-Q$ at a 3θ angle from the horizontal plane. Since this does not guarantee a potential on the horizontal plane equal to zero, an image charge $-Q$ is placed at a $-\theta$ angle. As a result, potential in the inclined plane is no longer zero, making it necessary to place a Q charge at 5θ angle. But now, the horizontal plane does not have a potential equal to zero, so a Q charge at a -3θ angle is placed. Finally, it is guaranteed that the potential of both planes becomes zero by placing a charge $-Q$ at a $\theta + \pi$ angle. All charges are at a distance r from the origin of the plane.

The configuration of the planes, the actual Q charge and their images are shown in Figure 3. As we can see, the position of each charge (numbered anti clock-wise) is as follows:

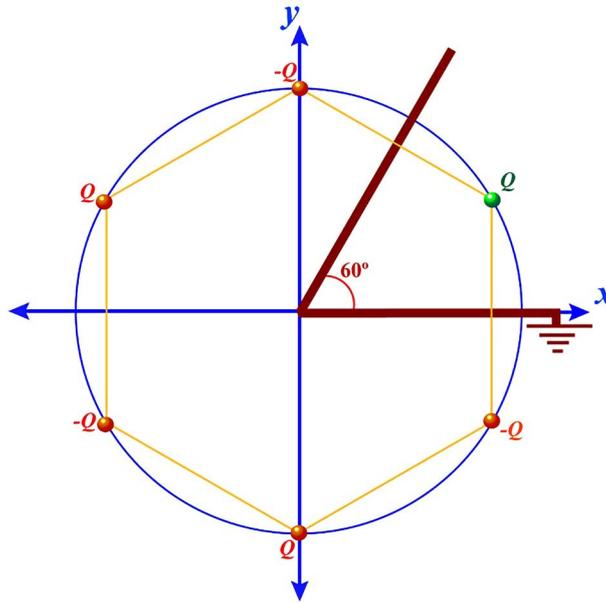


Figure 3. *Q* charge and image charges.

$$q_0 = Q, \text{ position } \theta \tag{1}$$

$$q_1 = -Q, \text{ position } 3\theta = 2(\alpha - \theta) + \theta \tag{2}$$

$$q_2 = Q, \text{ position } 5\theta = 2(\alpha - \theta) + 3\theta \tag{3}$$

$$q_3 = -Q, \text{ position } 7\theta = 4(\alpha - \theta) + 3\theta \tag{4}$$

$$q_4 = Q, \text{ position } 9\theta = 4(\alpha - \theta) + 5\theta \tag{5}$$

$$q_5 = -Q, \text{ position } 11\theta = 6(\alpha - \theta) + 5\theta \tag{6}$$

In this section, we have determined the location of the *Q* charge and the image charges. In the next section, we will present the expression of the electrical potential and we will show the graphs of the potential and the electric field generated by the set of charges.

3. Electric Potential Function and Graphics for Some Values of μ

To calculate the total potential produced by the charges in the space between the planes, we find the actual *Q* charge in $(r \cos \theta, r \sin \theta)$ and a point $P(x, y)$, as shown in **Figure 2**.

By the problem's geometry, we have magnitude of r_{qp} given by

$$|r_{qp}| = \sqrt{(x - h - r \cos \theta)^2 + (y - t - r \sin \theta)^2} \tag{7}$$

and if we consider $a = x - h$ and $b = y - t$, and keep in mind that $\alpha = 2\theta$, it is possible to rewrite (7) as

$$|r_{qp}| = \sqrt{(a - r \cos \theta)^2 + (b - r \sin \theta)^2}. \tag{8}$$

Taking in to account the superposition principle and the previous expressions,

we can express the total electric potential produced by the charges (real and image ones) as

$$V = \sum_{i=0}^5 \frac{(-1)^i Q}{4\pi\epsilon_0 \sqrt{(a-r\cos\varphi_i)^2 + (b-r\sin\varphi_i)^2}}, \tag{9}$$

where

$$\varphi_i = [i+(i\%2)](\alpha-\theta) + [i+[(i+1)\%2]]\theta. \tag{10}$$

For our discussed case, with the following values $\alpha = \frac{\pi}{3}$, $\theta = \frac{\pi}{6}$, we can find the positions of the real charge and image charges, Equations (1)-(6).

To graphically illustrate the total potential, the equipotential curves and the electric field in the region where the actual charge is located, the following values are considered: $r = 2$, $q = 4\pi \times 8.85 \times 10^{-12}$ Coul, $(h,t) = (0,0)$, $\alpha = \frac{\pi}{3}$, $\theta = \frac{\pi}{6}$.

First, considering the case of Coulomb, **Figure 4** shows the potential and the electric field

Then, secondly, for the Yukawa's case, **Figure 5** and **Figure 6**, show the potentials and electric fields for $\mu = 1$ and $\mu = 5$, respectively; some field lines are also shown.

In this section, we have presented the expression of the electrical potential for the configuration of charges in **Figure 2**, as well as its graphs. The exponential behavior is notorious for the case of the Yukawa potential. In the next section, we will discuss the results obtained in this section and offer our point of view when using computational tools in the teaching-learning process in subjects such as electromagnetism.

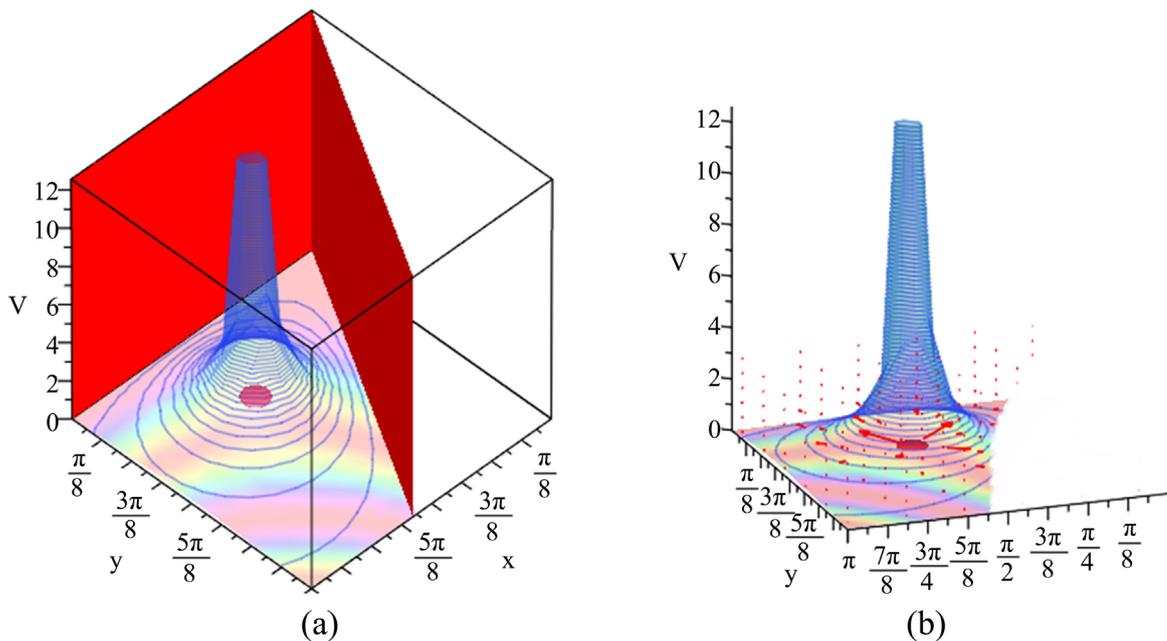


Figure 4. (a) Coulomb's potential and (b) Potential and electric field.

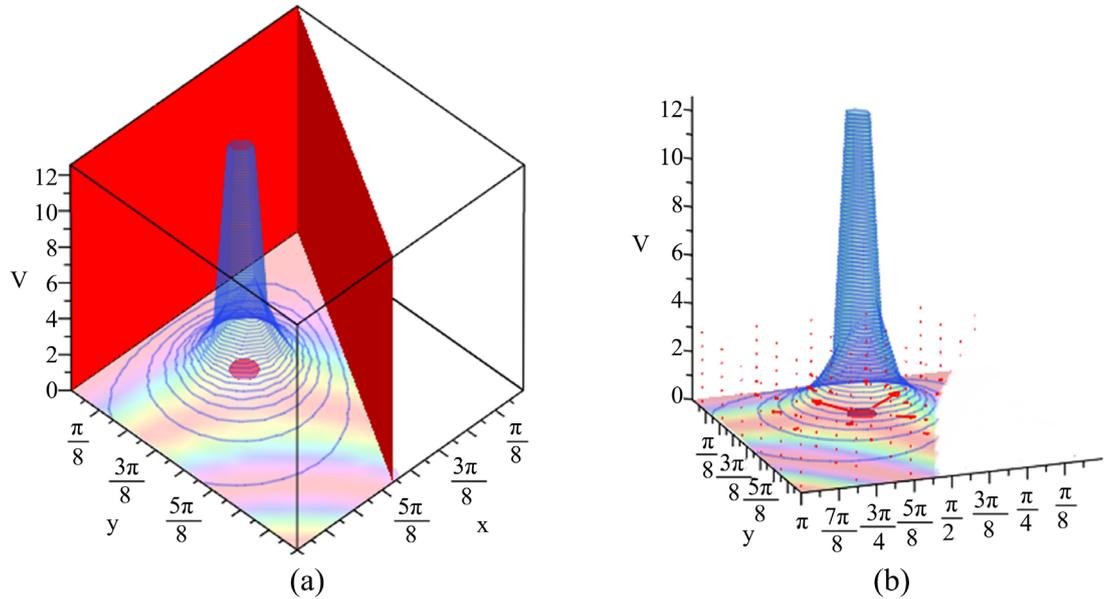


Figure 5. (a) Yukawa's potential for $\mu = 1$ and (b) potential and electric field.

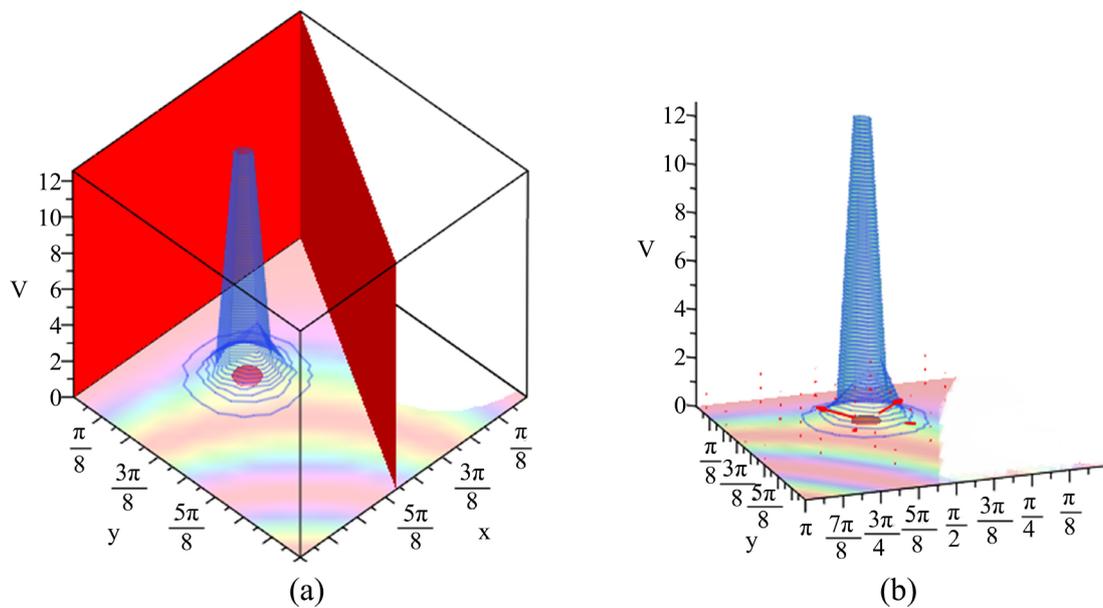


Figure 6. (a) Yukawa's potential for $\mu = 5$ and (b) potential and electric field.

4. Discussion of Results

From the superposition of the potentials of each of the charges, we obtained the expression for the total potential, (9), where φ_i , (10), provides the locations of the image charges from the location of the actual charge and the value for α . For the case we have discussed in this work, we can verify this if we consider $\alpha = 60^\circ$ and the position of the actual charge as $\theta = 30^\circ$, since we find the locations of the image charges of the system: $\varphi_1 = 90^\circ$, $\varphi_2 = 150^\circ$, $\varphi_3 = 210^\circ$, $\varphi_4 = 270^\circ$, $\varphi_5 = 330^\circ$. On the other hand, if we take as the location of the actual charge $\theta = 90^\circ$ and $\alpha = 180^\circ$, we find that the only possible image charge is at

$\varphi_1 = 270^\circ$, which is the case of a charge in the region of a conducting plane. Also, if we choose the position of the actual charge as $\theta = 45^\circ$ and $\alpha = 90^\circ$, we obtain the locations of the three necessary image charges: $\varphi_1 = 135^\circ$, $\varphi_2 = 225^\circ$, $\varphi_3 = 315^\circ$ and this result corresponds to the case of a charge between two perpendicular conducting planes. These last two cases were recently discussed [1].

As physics professors, we are interested in continuing to support student learning through modeling and simulation with the help of computational tools, such as Maple, among others. It is clear that a software capable of performing numerical and symbolic calculations in addition to graphing can be helpful to develop sensitivity analysis in prototypes. All of the above, as additional benefits to one discussed in this paper: thoroughly understanding electromagnetic theory.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

It can be used Yukawa potential ($e^{-\mu r}/r$) or Coulomb potential ($1/r$)

Procedure for the calculation of the potential and electric field of a charge Q between two planes forming an angle of 60°

```

> restart;
> with(plots):
> with(plottools):
> with(linalg):
> with(StringTools):
> poteimgs := proc(cpoint, centerp, angplan, r, q, typec)
> local charget, i, n, chargex, phi, k, epsilon, angplanx, rxy;
> charget := 0;
> n := abs((360)/convert(angplan[1], units, radians, degrees)) - 1;
> epsilon := 8.85*10^(-12);
> k := 1/(4*Pi*epsilon);
> angplanx := `if`(type(r, list) or type(r, vector) or type(r, matrix), in-
vfunc[tan](`if`(traperror(r[2]/r[1]) = "numeric exception: division by zero",
infinity, (r[2]/r[1])), angplan[2]));
> rxy := `if`(type(r, list) or type(r, vector) or type(r, matrix), sqrt(r[1]^2
+ r[2]^2), r);
> chargex := (i) -> (((-1)^i)*k*q)/sqrt((cpoint[1] - centerp[1] -
rxy*cos(phi(i)))^2 + (cpoint[2] - centerp[2] - rxy*sin(phi(i)))^2 +
`if`(vectdim(cpoint) > 2, cpoint[3],0)^2);
> phi := proc(i) (i + modp(i, 2))*(angplan[1] - angplanx) + (i + modp(i +
1, 2))*(angplanx); end proc;
> if (abs(evalf(angplan[1])) >= abs(evalf(angplanx))) then
> for i from 0 to n do charget := charget + chargex(i); end do;
> `if`(UpperCase(convert(typec, string)) = "CAMPO" , -grad(charget, [x, y,
z]), charget);
> else printf("El angulo al que se encuentra la carga no debe ser mayor al
ángulo de abertura de los planos..."); end if;
> end proc;
The stntax with which the potential in calculated is
> v(x, y) := poteimgs([x, y], [coordcx, coordcy], [alphax, thetax], rx, qx,
potencial);
and for the field
> campo(x, y) := poteimgs([x, y], [coordcx, coordcy], [alphax, thetax], rx,
qx, campo);
To obtain the graphs of the potential and the field, it is necessary to execute
the following instructions
> plotCP3d := proc(cpoint, centerp, angplan, r, q, typec)
> local plotsx, plotxyz, ik, elcamp, charc, charge;
> elcamp := seq(implicitplot3d(y - (ik*0.2) = x*tan(angplan[1]), x = -Pi..Pi, y

```

```
= -Pi..Pi, z = -Pi..Pi, thickness = 10, style = wireframe, color = COLOR(RGB, 1, 1, 1)), ik = 1..25):
```

```
> charc := sphere(['if'(type(rx, list) or type(rx, vector) or type(rx, matrix), rx[1], rx*cos(thetax)), 'if'(type(rx, list) or type(rx, vector) or type(rx, matrix), rx[1], rx*sin(thetax)), 0], 0.15):
```

```
> charge := plots[display](charc, scaling=constrained, style = patchngrid, axes=boxed, color = COLOR(RGB, 0.75, 0.15, 0.25)), implicitplot3d(y = x*tan(angplan[1]), x = 0..Pi, y = 0..Pi, z=-Pi..'if'(UpperCase(convert(typex, string)) <> "POTENCIAL", Pi, 4*Pi), axes = boxed, thickness = 4, style = 'if'(UpperCase(convert(typex, string)) <> "POTENCIAL", wireframe, patchngrid), orientation = [-90, 0.0001], color = red), implicitplot3d(y = x*tan(Pi/14400), x = 0..Pi, y = 0..Pi, z=-Pi..'if'(UpperCase(convert(typex, string)) <> "POTENCIAL", Pi, 4*Pi), axes = boxed, thickness = 4, style = 'if'(UpperCase(convert(typex, string)) <> "POTENCIAL", wireframe, patchngrid), orientation = [-90, 0.0001], color = red):
```

```
> plotsx[1] := plot3d(poteimgs([x, y], centerp, angplan, r, q, potencial), x = -Pi..Pi, y = -Pi..Pi, axes = boxed, scaling = unconstrained, orientation = [-90, 0.0001], labels = ["x", "y", "V"], color = cos(x*y), lightmodel = light1, transparency = 0.65, style = patchngrid, numpoints = 1000, shading = XYZ), contourplot3d(poteimgs([x, y], centerp, angplan, r, q, potencial), x = -Pi..Pi, y = -Pi..Pi, axes = boxed, scaling = unconstrained, color = COLOR(RGB, 0.15, 0.45, 0.95), numpoints = 1000, contours = 'if'(UpperCase(convert(typex, string)) = "POTENCIAL", 75, 25)):
```

```
> plotsx[2] := gradplot3d(-poteimgs([x, y, z], centerp, angplan, r, q, potencial), x = -Pi..Pi, y = -Pi..Pi, z = -Pi..Pi, axes = boxed, scaling = unconstrained, arrows = THICK, color = COLOR((RGB, 0.1, 0.35, 0.9)), orientation = [45, 45], labels = ["x", "y", "z"], style = patchngrid, grid = [12, 12, 12]):
```

```
> plotxyz := 'if'(UpperCase(convert(typex, string)) = "POTENCIAL", [plotsx[1], charge], 'if'(UpperCase(convert(typex, string)) = "CAMPO", [plotsx[2], elcamp, charge], [plotsx[1], plotsx[2], elcamp, charge]]):
```

```
> display(plotxyz, axes = boxed, view = [0..Pi, 0..Pi, 0..'if'(UpperCase(convert(typex, string)) <> "POTENCIAL", Pi, 4*Pi)]);
> end proc:
```

To show the potential graph, use

```
> plotCP3d([x, y, z], [coordcx, coordcy], [alphax, thetax], rx, qx, potencial);
```

and for the field, use

```
> plotCP3d([x, y, z], [coordcx, coordcy], [alphax, thetax], rx, qx, campo);
```

To graph the potential and the field together, run

```
> plotCP3d([x, y], [coordcx, coordcy], [alphax, thetax], rx, qx, "");
```