A Theoretical Study on Energy of a Gaseous System Vis-a-Vis Mass and Temperature

Santosh K. Karn[1]*, Necati Demiroglu[2]

[1]IGTMS University, Arunachal Pradesh, India
[2]DSI (State Hydraulic Works), 9th Regional Directorate 12th Branch Directorate, Elazig, Turkey
Email: *skarn03@yahoo.com, nct_dmr@hotmail.com

Abstract

To study various properties of a gas has been a subject of rational curiosity in pneumatic sciences. A gaseous system, in general, is studied by using four measurable parameters namely, the pressure, volume, number of moles and temperature. In the present work, an attempt is made to study the variation of energy of an ideal gas with the two measurable parameters, the mass and temperature of the gas. Using the well known ideal gas equation, $PV = nRT$ where symbols have their usual meanings and some simple mathematical operations widely used in physics, chemistry and mathematics in a transparent manner, an equation of state relating the three variables, the energy, mass and temperature of an ideal gas is obtained. It is found that energy of an ideal gas is equal to the product of mass and temperature of the gas. This gives a direct relationship between the energy, mass and temperature of the gas. Out of the three variables, the energy, mass and temperature of an ideal gas, if one of the parameters is held constant, the other two variables can be measured. At a constant temperature, when the power or energy is stabilized, the increase in the mass of the gas may affect the new works and an engine can therefore be prevented from overheating.

Keywords

Hydrodynamics, Low Temperature Fluid Flow, Ideal Gas, Equation of State, Energy, Mass, Temperature and Their Relation

1. Introduction and a Brief Review of the Field and Related Areas

In this section, we present the introduction and review briefly the works done in

*Corresponding author; associated with Lifelong Founder Chancellor in New Delhi.

the field and related areas. We know, heat and thermodynamics plays an important role in understanding the nature and natural phenomena. To understand the nature and natural phenomena regularly occurring in the Universe, gaseous state is a very significant fundamental state of matter. The study of changes occurring in a gaseous system with varying conditions has therefore been and is a subject of interest in human curiosity. In general, a gaseous system is studied by using the four measurable parameters namely, pressure (P), volume (V), number of moles (n), and temperature (T). Two types of gases namely, ideal and real gases exist. An ideal gas is governed by a law called ideal or general gas law. This law is expressed by a mathematical equation that relates all of these four parameters. The ideal gas law is a generalization of different gas laws that describe the behaviour of gases, namely Boyle's law, Charles's law, Gay-Lussac law and Avogadro law. Boyle's law states that the pressure of a gas is inversely proportional to its volume ($P \, \alpha \, (1/V)$) at constant temperature and number of moles of the gas. Charles law states that volume of a gas is directly proportional to temperature ($V \, \alpha \, T$) at constant pressure and number of moles of the gas. Gay-Lussac law [1] states that the pressure of an enclosed gas is directly proportional to its temperature ($P \, \alpha \, T$) at a constant volume and number of moles. Avogadro law states that the volume of a gas is directly proportional to number of moles ($V \, \alpha \, n$) at constant pressure and temperature of the gas. Clapeyron [2] in 1834 combined all of these gas laws together and is called ideal gas law. The ideal gas law relates all the four variables together, namely pressure (P), volume (V), moles of gas (n), and temperature (T) and is expressed by $PV = nRT$ where R is a proportionality constant, called the ideal gas constant or Universal gas constant and is equal to 8.314 Joule per mole per Kelvin. The ideal gas equation, $PV = nRT$ is directly and implicitly contributing a lot in the direction. This equation deals with measurable four significant parameters, the pressure, volume, number of moles and temperature. This equation is based on basic assumptions that is, ideal gas consists of a large number of negligibly small sized molecules compared to the volume occupied by the gas; molecules are in frictionless random motion obeying Newton's laws of motion and are colliding elastically amongst each other and also with the walls containing the gas for a negligible duration; and the size of a molecule is significantly smaller than the distance covered by the molecule between two consecutive collisions. The law can be derived from kinetic theory of gases.

The ideal gas law is applicable for various gases under nearly ideal conditions of high temperatures and low pressures. Ideal gas under the basic assumptions does not become liquid at room temperature. It is found that many gases deviate from the ideal behaviour and become liquid at room temperature. It is to mention here that a real gas is described on the basis of considerations that gas molecules have significant volume and the molecules interact among themselves and also with the walls of the container. In 1873, Van der Waals [3] [4] gave an equation of state for the real gas by considering finite size of molecules and in-

teractions between the molecules (intermolecular interactions) of the gas as $(P + a(n^2/V^2))\cdot(V - nb) = nRT$ where "a" and "b" are constants for a gas, also called Van der Waals constants, and are determined experimentally. The parameter "a" represents intermolecular interactions whereas "b" represents a fraction of volume occupied by gas molecules (due to finite molecular size) and therefore "nb" is subtracted from the actual volume of the gas V in the equation. At constant temperature, an increase in pressure decreases the volume of a gas. Therefore, for gases with strong molecular forces, "a" has larger value whereas for gases with weak intermolecular forces (like, in inert gases) "a" has smaller value. That is, larger the value of "a", the larger the pressure (force) and smaller the volume, and for smaller value of "a", the pressure (force) decreases and volume increases. The parameter "b" represents volume occupied by one mole of the gas and it affects (adjusts) the net volume of the gas. Volume of a gas is large compared to the volume occupied by molecules of the gas. Therefore, for larger value of "b", volume of the gas decreases and pressure (force) increases, and for a smaller value of "b", the volume increases and pressure decreases. Pressure of a gas depends on density and temperature of the gas molecules. Therefore, in the equation, when pressure becomes low ($P \to 0$); $nb < V$ and "$a(n^2/V^2)$" becomes negligible, and the real gas equation reduces to the ideal gas equation. Thus, a real gas behaves as an ideal gas at extremely low density (high temperature). When pressure becomes very high ($P \to$ infinity); V becomes very small (high density), and approaches the value "nb". At high temperatures and low pressures both the constants "a" and "b" become zero (negligibly small), the Van der Waals equation reduces to ideal gas equation and real gases behave as an ideal gas. It is to mention here that plasma scientists study gases at very high temperatures whereas particle physicists study particles at extremely high temperatures and/or density. Behaviour of matter at extremely high densities and/or temperatures, being explained also on hydrodynamical model, is an exciting field of research, has wide implications and is beyond the scope of the paper. As the study of changes occurring in a gaseous system with varying conditions is a subject of interest, various attempts in this context are being made these days. J. Lungu *et al.* [5] have studied a correlation between performance, engine speed, load and exhaust gas temperature of a spark ignition gasoline engine by performing experiments and by numerical simulations using an advanced computational fluid dynamics modeling code, Kiva4. It is shown that how the performance of an engine is affected by the exhaust gas temperature. The engine is fairly maintained at a constant working temperature by a cooling system but the actual engine temperature varies due to a number of reasons. In the experiment, engine torque is measured using an eddy current dynamometer. Torque of a motor is directly proportional to the energy or power. Torque varies with mass and temperature. When an engine works continuously for a specific time a stage would reach where the temperature reaches the maximum value and if the temperature exceeds the value the engine may get damaged. It is to mention that the model has shown significant predictability for combustion characteristics. An appreciable

agreement with a percentage error of 2% to 3% is obtained between the experimental results and the results obtained from the simulations using the kiva4. Whereas L. Titelman has considered [6] [7] energy as a generalized product of forces (gas pressures and mechanical strengths) and displacements (textural properties of adsorbent, the surface area, volume and size of pores) and obtained the energies of a gas and sorbent as generalized variables. The interrelationships between them and the storage capacity for metal-organic frameworks, polar organic polymers and activated carbons have been studied. It is found that the generalized variables of adsorbents for gas correlates well with sorbent capacity and gives an improvement in the relationship between energy and capacity. It is to mention here that in the work, for calculating the forms of energy in porous materials in the equation $PV = $ constant at a constant temperature T, (PV) is considered [8] as energy (work) with P as force and V as displacement in a broad sense. Kelvin M. Tenny and Jeffrey S. Cooper [9] have studied in detail various aspects of ideal gas behaviour and its implications in related areas especially in medical care even in emergencies like operations, anaesthesia etc. and for details on it we refer the references therein. It is pertinent to note here in the light of the above discussion that energy of a gas depends on its temperature. We also find in these works that a direct relation among the three variables of a gas, namely energy, mass and temperature is missing although we know that the number of moles is related to the mass of the gas. Therefore, in the present work, we have made an attempt to understand the variation of energy of an ideal gas with mass and temperature of the gas. For this, we have computed the variation of energy of an ideal gas in terms of mass and temperature of the gas by using well known ideal gas equation and some mathematical operations in the next Section. A list of all symbols used in the work is given in Section 3 whereas summary and results of the work are presented in Section 4.

2. Present Model

Consider an ideal gas having mass m is kept inside a container having surface area S, height h and volume V. The gas is at pressure P and temperature T. We know the ideal gas equation is

$$PV = nRT \tag{1}$$

where P is the pressure of an ideal gas, V the volume of the ideal gas, n the number of moles (total amount of the ideal gas is measured in terms of moles), R the Universal gas constant and T the temperature of the gas. Taking derivative on both sides of Equation (1), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(PV) = \frac{\mathrm{d}}{\mathrm{d}t}(nRT)$$

The L.H.S. of the equation is equal to

$$V\frac{\mathrm{d}P}{\mathrm{d}t} + P\frac{\mathrm{d}V}{\mathrm{d}t} = V\frac{\mathrm{d}\left(\dfrac{F}{S}\right)}{\mathrm{d}t} + P\frac{\mathrm{d}(Sh)}{\mathrm{d}t}$$

where $P = (F/S)$, $V = Sh$ with F is the force, S the surface area and h the height of the gas container. Therefore, we have

$$V \frac{\mathrm{d}(F/S)}{\mathrm{d}t} + P \frac{\mathrm{d}(Sh)}{\mathrm{d}t} = R \left[T \frac{\mathrm{d}n}{\mathrm{d}t} + n \frac{\mathrm{d}T}{\mathrm{d}t} \right] \tag{2}$$

We know the number of moles, $n = m/M$ where m is the mass of the gas and M the molar mass and $V = Sh$. Substituting these and $P = (F/S)$, $PS = F$ and $Ph = (F/S)h$, in Equation (2), we have

$$h \frac{\mathrm{d}F}{\mathrm{d}t} - \frac{Fh}{S} \frac{\mathrm{d}S}{\mathrm{d}t} + \frac{Fh}{S} \frac{\mathrm{d}S}{\mathrm{d}t} + F \frac{\mathrm{d}h}{\mathrm{d}t} = R \left[T \frac{M \frac{\mathrm{d}m}{\mathrm{d}t} - m \frac{\mathrm{d}M}{\mathrm{d}t}}{M^2} + n \frac{\mathrm{d}T}{\mathrm{d}t} \right] \tag{3}$$

As M is constant, $\frac{\mathrm{d}M}{\mathrm{d}t} \cong 0$ and therefore from Equation (3), we get

$$h \frac{\mathrm{d}F}{\mathrm{d}t} + \frac{F\mathrm{d}h}{\mathrm{d}t} = \frac{RT}{M} \frac{\mathrm{d}m}{\mathrm{d}t} + Rn \frac{\mathrm{d}T}{\mathrm{d}t}$$

$$\Rightarrow h\mathrm{d}F + F\mathrm{d}h = \frac{RT}{M} \mathrm{d}m + Rn\mathrm{d}T \tag{4}$$

We know $F = ma$, where a is the acceleration. Taking derivative on both sides of this equation, we get $\mathrm{d}F = a\mathrm{d}m + m\mathrm{d}a$. Substituting this in Equation (4), we have

$$\left(ha\mathrm{d}m + hm\mathrm{d}a \right) + ma\mathrm{d}h = \frac{RT}{M} \mathrm{d}m + Rn\mathrm{d}T \tag{5}$$

Dividing both sides of the Equation (5) by ($mahT$), we get

$$\frac{1}{T} \frac{\mathrm{d}m}{m} + \frac{1}{T} \frac{\mathrm{d}a}{a} + \frac{1}{T} \frac{\mathrm{d}h}{h} = \frac{R}{Mah} \frac{\mathrm{d}m}{m} + \frac{Rn}{mah} \frac{\mathrm{d}T}{T} \tag{6}$$

Integrating both sides, we get

$$\frac{1}{T} \ln(mah) = \frac{R}{Mah} \left(\ln(m) + \ln(T) \right) = \frac{R}{Mah} \ln(mT)$$

As $F = ma$, $\Rightarrow mah = Fh = E$, the energy and $n = m/M \Rightarrow M = m/n$, we get

$$\ln(E) = \frac{RT}{Mah} \ln(mT) = \frac{RT}{ah} \frac{n}{m} \ln(mT) = \frac{nRT}{E} \ln(mT)$$

$$\Rightarrow E \ln(E) = nRT \ln(mT) \Rightarrow E^E = (mT)^{nRT} \tag{7}$$

Now, $PV = nRT \Rightarrow (F/S)(Sh) = nRT \Rightarrow Fh = E = nRT$. Substituting $nRT = E$ in Equation (7), we have

$$E = mT \tag{8}$$

where $m = nM \Rightarrow E = mT = nMT$, m is mass of an ideal gas, n number of moles, M molar mass, E energy and T temperature of a gas in a container having surface area S, height h and volume V. Equation (8) implies that

$$\frac{E_1}{m_1 T_1} = \frac{E_2}{m_2 T_2} = \frac{E_3}{m_3 T_3} = \cdots \tag{9}$$

Thus, from Equation (8) we find that the energy E of an ideal gas is equal to

the product of the mass m and temperature T of the gas. Equation (8) shows that the energy of a gas is proportional to the temperature of the gas when mass of the gas is kept constant. For example, for a fixed (constant) value of a 10 kg of an ideal gas at different temperatures, say at T in ^{0}C = −200, −150, −100, −50, 0, 50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700 (or, T in K = 73, 123, 173, 223, 273, 323, 373, 423, 473, 523, 573, 623, 673, 723, 773, 823, 873, 923, 973, as T in K = T in ˚C + 273); the value of energy in kilogram·Kelvin (kg·K), from Equation (8), will respectively be 730, 1230, 1730, 2230, 2730, 3230, 3730, 4230, 4730, 5230, 5730, 6230, 6730, 7230, 7730, 8230, 8730, 9230, 9730. Likewise, when the temperature is kept constant, the energy of a gas changes with the amount of mass of the gas. This gives a direct relationship between the amount of matter (mass) and energy. It is to note here that an increase in the amount (mass) of a substance when the power or energy of an engine is stabilized, may affect new works and therefore prevent the engine from overheating. Moreover, out of the three variables, the energy, mass and temperature of an ideal gas, if one of the parameters is held constant, the other two variables can be measured.

3. List of all Symbols Used in the Work

P - Pressure, V - Volume, n - Number of moles, R - Universal gas constant, T - Temperature, F - Force, S - Surface area of a gas container, h - Height of the gas container, m - Mass of an ideal gas, M - Molar mass, a - Acceleration, and E - Energy of the gas.

4. Summary and Results

Study of material substances (having mass), energy associated with and their interconversions form basis of all natural phenomena occurring in nature in the observable universe and probably beyond too. Study of properties of a gas to understand the nature and natural phenomena is of paramount importance. Therefore, various properties of gases under ordinary and specific conditions as per requirement are studied by physicist, chemical scientist, engineer, technocrat, and others. We know that an ideal gas differs from a real gas and four fundamental parameters namely, pressure P, volume V, number of moles n and temperature T are in general required to deal with the behaviour of a gas. An ideal gas is described by the equation $PV = nRT$, where symbols have their usual meanings and R is the universal gas constant while a real gas is described by $(P + a(n^2/V^2))\cdot(V - nb) = nRT$, where the parameter "$a$" represents intermolecular interactions and the parameter "b" represents a fraction of volume occupied by gas molecules and nb is subtracted from the actual volume of the gas V. In general, we thus find that a gaseous system is studied by using all the four measurable parameters namely, pressure (P), volume (V), number of moles (n), and temperature (T). It is observed that a direct relation among the three variables of a gas, namely the energy, mass and temperature is missing although we know that the number of moles is related to the mass of the gas.

Therefore, in the present work, we have made an attempt to study properties of an ideal gas in terms of the three variables, the energy, mass and temperature by considering the ideal gas equation and some mathematical operations widely used in physics, chemistry and mathematics in a transparent manner. For this, we have computed the energy of an ideal gas in terms of mass and temperature of the gas. An equation of state relating the energy, mass and temperature of an ideal gas is found. We find that the energy of an ideal gas is equal to the product of mass and temperature of the gas. This shows a linear variation of energy of an ideal gas with mass and temperature of the gas. The main outcome of the study is the direct relationship between the amount (mass) of the gas and the energy at a constant temperature. It is pertinent to mention here that when the power or energy is stabilized, the increase in the mass of the gas may affect the new works and can therefore prevent an engine from overheating. Thus, out of the three variables, the energy, mass and temperature of an ideal gas, if one of the parameters is held constant, the other two variables can be measured (quantified) and a table containing energy, mass and temperature of a gas can be made. Thus, in a closed system the mass of the gas can be kept constant and the energy of the gas changes with its temperature. This methodology can also be applied to real gases, mixture of different gases in varying proportions, automobiles, the work in this direction is in progress, and we hope to address it in coming days.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Gay-Lussac, J.L. (1809) Memoir on the Combination of Gaseous Substances with Each Other. *Mémoires de la Société d'Arcueil*, **2**, 207.

[2] Clapeyron, E. (1834) Memoire sur la puissance motrice de la chaleur. *Journal de l' Ecole Polytechnique*, **23**, 153190. (In French)

[3] Van der Waals, J.D. (1873) Over de Continuiteit van den Gas-en Vloeistoftoestand [About the Continuity of the Gas and Fluid States]. University of Leiden, Leiden. (In Dutch)

[4] Van der Waals, J.D. (1873) The Equation of State for Gases and Liquids. Nobel Lectures, Physics 1910, 254-265.

[5] Lungu, J., Siwale, L., Kashinga, R., Chama, S. and Bereczky, A. (2021) Correlation of

Performance, Exhaust Gas Temperature and Speed of a Spark Ignition Engine Using Kiva4. *Journal of Power and Energy Engineering*, **9**, 53-78.
https://doi.org/10.4236/jpee.2021.98004

[6] Titelman, L. (2021) Generalized Parameters of Porous Materials as Similarity Numbers. *Advances in Materials Physics and Chemistry*, **11**, 177-201.
https://doi.org/10.4236/ampc.2021.1111017

[7] Titelman, L. (2022) Adsorbents for Gas Storage: Gas Energy, Sorbent Energy and Their Relationship to Capacity. *Advances in Materials Physics and Chemistry*, **12**, 221-239.
https://doi.org/10.4236/ampc.2022.1210016

[8] Titelman, L. (2012) Generalized Processing - Structural Functions for Porous Materials. *Journal of Porous Materials*, **19**, 1-13.
https://doi.org/10.1007/s10934-010-9440-y

[9] Tenny, K.M. and Cooper, J.S. (2021) Ideal Gas Behavior. StatPearls (Internet) Publishing LLC, Treasure Island, FL.