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A Spectrum Occupancy Model for Primary Users in Cognitive Radio Systems

Yingying Cao, Yiqian Zhao, Haihua Li*

School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, China
Email: *haihli@163.com

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Abstract

There are abundant research results related to cognitive radio systems (CR systems), but using queueing models to portray CR systems is a new research trend. In this paper, a single-server retrial cognitive radio system with a linear retrial rate has been considered. The system has two types of users: primary users and secondary users. Secondary users have no effect on primary users because primary users have preemptive precedence. As a result, our purpose is to examine some performance indicators such as the expected queue length for primary users, the probability of the system being idle or occupied by a secondary user, and the probability of the system being busy. This paper begins by deriving the expressions for the generating functions based on the balance equations, so that we can calculate our goal conveniently.

Keywords

Cognitive Radio System, Retrial Queue, Reneging, Stable-State Distribution

1. Introduction

Cognitive radio originated from Joseph Mitola's groundbreaking work, and its main technological innovation compared to traditional radio systems is that cognitive radio has the ability to learn, perceive, and utilize the available spectrum in that space, and reduce the occurrence of matching occupation conflicts. The FCC in the United States has provided a relatively broad definition, which suggests in FCC-03322 that any radio with adaptive spectrum awareness should be referred to as a cognitive radio system. In cognitive radio systems, unauthorized users (secondary users (SU)) are able to perceive the spectrum environment, detect the presence of legitimate authorized users (referred to as primary users (PU)), adaptively occupy the immediately available local spectrum, and do not cause harmful interference to the primary user throughout the entire communication process. In cognitive radio, secondary users dynamically search for spectrum holes for communication, and this technology is called dynamic spectrum access. When the primary user occupies an authorized frequency band, the secondary user must exit from that frequency band and search for other idle frequency bands to complete their communication.

Currently, there are abundant research results related to cognitive radio systems (CR systems). For example, Yuan *et al.* [1] considered introducing multiple intelligent reflecting surface (IRS) into a downlink multiple-input single-output (MISO) CR system with the goal of optimizing the beam forming at SU-TX and the reflecting coefficients at each IRS. The Self-Learning Salp Swarm Algorithm (SLSSA) is a newly proposed meta-heuristic algorithm that can design efficient CR systems. Mittal *et al.* [2] used SLSSA to adapt the parameters of the CR system and got some results. Mahendru [3] considered a spectrum sensing technique at low SNR for cognitive radio systems. Mathematical modeling and critical measurement of detection probability for low SNR spectrum sensing based on energy detection in an uncertain noisy environment were carried out. A new parameter ``Threshold Wall" has been proposed for optimum threshold selection to overcome sensing failure. In order to avoid disturbances, further improve the output, and remove the noise in the received signal, an adaptive learning method was proposed. Surekha *et al.* [4] proposed a variable-step adaptive learning algorithm (VSALA) based on regularization, analyzed the proposed learning methodology for estimating the direction of arrival (DOA) and gave the corresponding beam forming patterns. Lee *et al.* [5] proposed a multi-channel underlay cognitive radio (CR) systems resource allocation strategy based on an integrated deep learning framework and demonstrated that the proposed strategy can be implemented at a rate of less than 1.5 ms.

However, using queueing models to portray cognitive radio systems is a new research trend. Zhu *et al.* [6] proposed a Markov chain analysis method for spectrum access in licensed cognitive radio bands and derived the forced termination probability, blocking probability, and traffic throughput in licensed cognitive radio bands. A channel reservation scheme for cognitive radio spectrum handoff is also proposed. Zhang and Wang [7] modeled the inherent hierarchical structure of cognitive radio networks as a priority queuing system. In an $M/G/1$ system with one primary user and multiple secondary users, the analytical expressions of delay and throughput for different users are obtained with the function of traffic and channel conditions. Chen and Wyglinski [8] presented a feasibility analysis of implementing a vehicle dynamic spectrum access method for idle TV channels via queuing theory. In this paper, they use $M/M/m$ model and $M/G/m$ model to estimate the probability that vehicles will find all lanes busy, and derive the expected waiting time of vehicles. Also considered are cases where there are multiple service request priority categories, such as channel requests for first responder vehicles. Zahed *et al.* [9] proposed prioritized proactive spectrum handoff decision-making schemes using the preemptive resume priority (PRP) $M/G/1$ queue model to reduce the handoff delay and the total service time. The performance of the proposed handoff scheme was evaluated and compared with the existing spectrum handoff schemes. Balapuwaduge *et al.* [10] proposed introducing queues for secondary users and possibly providing services later. Based on the delay tolerance of interrupted elastic services, two queuing schemes were proposed. On this basis, continuous-time Markov chain models are established, and the correctness and the preciseness of the theoretical models are verified. Tadayon and Aissa [11] developed a multi-channel cognitive radio network model using the priority queue theory. Performance analysis was performed by deriving the probability mass function (PMF) of queue length. Finally, the conditions for the existence of an optimal trade-off between the primary users (PUs) interference and the quality-of-service of the secondary users (SUs) are given, and the optimal mixed strategy is obtained when these conditions are satisfied. Dudin *et al.* [12] analyzed a novel priority retry queuing model with multi-type customers and servers reservation for access optimization. An effective approach for the analysis of multi-server queue-to-service process with multi-type customers and heterogeneous requirements has been provided and applied. Vasquez-Toledo *et al.* [13] analyzed highly scalable cognitive radio systems and proposed a new mathematical analysis method combining queuing theory with game theory to reduce the blocking probability of cognitive radio systems. Finally, it is found that the proposed strategies improve general performance and reduce the blocking probability for a cognitive system.

In the above literature, if the authorized spectrum is occupied by a service request from the primary user, a newly PU service request is not allowed to retry, which is not in line with objective reality. In this paper, assuming that the primary user who fails to immediately obtain spectrum resources upon arrival will join a retrial space and retry the server at a certain rate. In addition, the primary user is impatient and will exit the retrial space with a certain probability. This paper only considers the spectrum access model of the primary user. The rest of the paper consists of three parts. Section 2 gives the model description. In section 3, we consider the stable-state distribution of our proposed model and obtain the expression of generating functions and the expected queue length and so on. Finally, section 4 offers conclusions and future work.

2. Model Description

This paper considered a single-server retrial cognitive radio system with a linear retrial rate. There are two kinds of users in the system: primary users and secondary users. If a primary user arrives while a secondary user is being served, the server will immediately stop serving the secondary user and serve the latter instead. Since secondary users have no influence on primary users, this paper will mainly discuss the spectrum access for primary users. Potential primary users arrive according to the Poisson process with rate λ and there is no waiting space in front of the spectrum. Upon arrival, an arriving primary user will immediately occupy the spectrum and get service if the server is in an idle state or occupied by a secondary user. Otherwise, if the spectrum is in a busy state, the user will enter a retrial orbit with infinite capacity. j users in the orbit retry linearly with probability q or leave with probability $1-q$. Therefore, the retrial rate is $j\xi q$ and the leaving rate is $j\xi(1-q)$. The corresponding flow chart is shown in Figure 1.

Let $N(t)$ be the number of PU service requests in the orbit at time t and denote by $I(t)$ the state of the server at time t. $I(t)=0$ means that the server is in an idle state or occupied by a secondary user, and $I(t)=1$ means that the server is in a busy state. Namely, $I(t)$ can be written as

$$I(t)=\begin{cases}0 & \text{idle},\\ 1 & \text{busy}.\end{cases}$$

It is clear that the process $\{(I(t),N(t)),t\ge 0\}$ is a two-dimensional continuous-time Markovian chain with a state space $\Omega=\{(0,i),i\ge 0;(1,j),j\ge 0\}$. The corresponding transition rate diagram of the system is shown in Figure 2.

3. Stable-State Distribution

From Figure 1, the following balance equations are obtained:

$$\begin{cases}(\lambda+j\xi)\pi_{0,j}=\mu\pi_{1,j}+(j+1)\xi(1-q)\pi_{0,j+1},\\ (\lambda+\mu+j\xi(1-q))\pi_{1,j}=\lambda p\pi_{1,j-1}+\lambda\pi_{0,j}+(j+1)\xi q\pi_{0,j+1}+(j+1)\xi(1-q)\pi_{1,j+1},\end{cases} \tag{3.1}$$

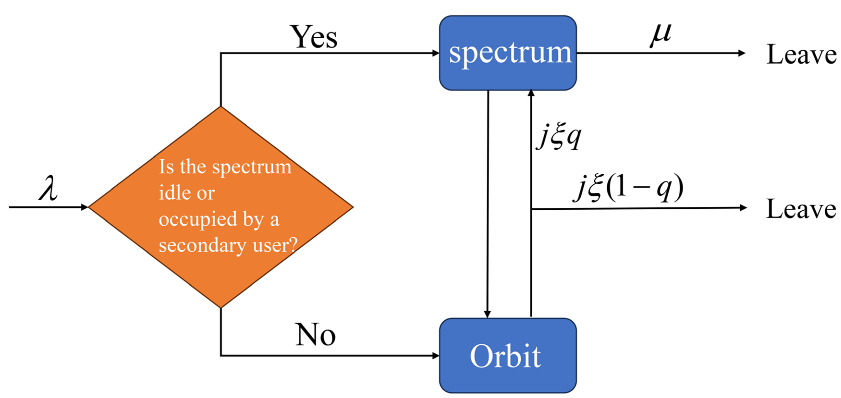

Figure 1. Flow chart of PU service requests.

where $\pi_{1,-1}=0$ and $j=0,1,2,\cdots$.

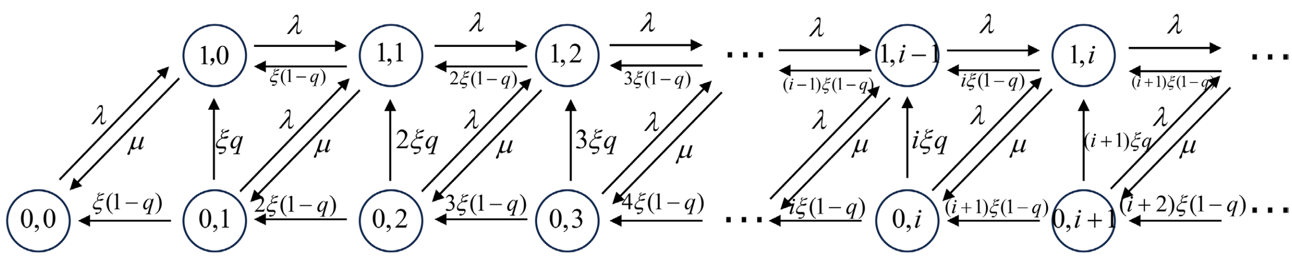

Figure 2. Transition rate diagram of the system.

Let $\Phi_0(x)$ and $\Phi_1(x)$ are the generating functions, defined by

$$\Phi_0(x) = \sum_{j=0}^{\infty} \pi_{0,j} x^j, \tag{3.2}$$

$$\Phi_1(x) = \sum_{j=0}^{\infty} \pi_{1,j} x^j. \tag{3.3}$$

Generating function is an important theory and tool in mathematics. It's idea is to correspond the discrete sequence with the power series one by one, and to correspond the relation of the combination of the discrete sequence to the operation relation of the power series, and the structure of discrete sequence is determined by the form of power series in the end. In this paper, we can derive some probabilities and the expected queue length in the system by using the generating functions.

Then the following theorem can be obtained.

Theorem 3.1. *For the retrial cognitive radio system with a linear retrial rate, the generating functions* $\Phi_0(x)$ *and* $\Phi_1(x)$ *are computed by*

$$\Phi_0(x) = c_2 e^{\frac{\lambda x}{\xi(1-q)}} \int \exp\left(\kappa(x) - \frac{2\lambda x}{\xi(1-q)}\right) dx + c_1 e^{\frac{\lambda x}{\xi(1-q)}},$$

and

$$\begin{aligned}\Phi_1(x) = {} & \frac{c_2\left(\xi x - \xi(1-q)\right)}{\mu} \exp\left(\kappa(x) - \frac{\lambda x}{\xi(1-q)}\right) \\ & + \frac{\xi x - \xi(1-q)}{\mu}\left(\frac{c_2 \lambda e^{\frac{\lambda x}{\xi(1-q)}}\left(\int \exp\left(\kappa(x) - \frac{2\lambda x}{\xi(1-q)}\right) dx\right)}{\xi(1-q)} + \frac{c_1 \lambda e^{\frac{\lambda x}{\xi(1-q)}}}{\xi(1-q)}\right) \\ & + \frac{\lambda}{\mu}\left(c_2 e^{\frac{\lambda x}{\xi(1-q)}}\left(\int \exp\left(\kappa(x) - \frac{2\lambda p x}{\xi(1-q)}\right) dx\right) + c_1 e^{\frac{\lambda x}{\xi(1-q)}}\right).\end{aligned}$$

where c_1 and c_2 can be obtained from the following equations:

$$\Phi_0(1) + \Phi_1(1) = 1, \tag{3.4}$$

$$\lim_{x \to 0+} \lambda p \Phi_0(x) + \left[x\xi - \xi(1-q)\right]\Phi_0'(x) = 0. \tag{3.5}$$

Proof. Multiplying Equation (3.1) by x^j and summing up over *j*, then we get

$$\lambda\Phi_0(x) + x\xi\Phi_0'(x) = \mu\Phi_1(x) + \xi(1-q)\Phi_0'(x),$$

$$\begin{aligned}&(\lambda+\mu)\Phi_1(x) + \xi(1-q)x\Phi_1'(x) \\ &= \lambda x\Phi_1(x) + \lambda\Phi_0(x) + \xi q\Phi_0'(x) + \xi(1-q)\Phi_1'(x).\end{aligned}$$

The following equations can be derived after some algebraic manipulations, namely

$$\lambda\Phi_0(x) + \left[x\xi - \xi(1-q)\right]\Phi_0'(x) = \mu\Phi_1(x), \tag{3.6}$$

$$(\lambda - \lambda x + \mu)\Phi_1(x) + \left[\xi(1-q)x - \xi(1-q)\right]\Phi_1'(x) = \lambda\Phi_0(x) + \xi q\Phi_0'(x). \tag{3.7}$$

From (3.6), we easily get

$$\Phi_1(x)=\frac{\lambda}{\mu}\Phi_0(x)+\frac{x\xi-\xi(1-q)}{\mu}\Phi_0'(x), \tag{3.8}$$

$$\Phi_1'(x)=\frac{\lambda}{\mu}\Phi_0'(x)+\frac{\xi}{\mu}\Phi_0'(x)+\frac{x\xi-\xi(1-q)}{\mu}\Phi_0''(x). \tag{3.9}$$

Substituting Equations (3.8)-(3.9) into (3.7), we have

$$\begin{aligned}&\frac{\lambda-\lambda x+\mu}{\mu}\left(\lambda\Phi_0(x)+\left[x\xi-\xi(1-q)\right]\Phi_0'(x)\right)\\&+\frac{\xi(1-q)x-\xi(1-q)}{\mu}\left(\lambda\Phi_0'(x)+\xi\Phi_0'(x)+\left(x\xi-\xi(1-q)\right)\Phi_0''(x)\right)\\&=\lambda\Phi_0'(x)+\xi q\Phi_0''(x).\end{aligned} \tag{3.10}$$

For convenience, we rewrite (3.10) as

$$a(x)\Phi_0(x)+b(x)\Phi_0'(x)+c(x)\Phi_0''(x)=0, \tag{3.11}$$

where

$$a(x)=(\lambda-\lambda x)\lambda,$$

$$b(x)=(\lambda-\lambda x+\mu)\left(x\xi-\xi(1-q)\right)+\left(\xi(1-q)x-\xi(1-q)\right)(\lambda+\xi)-\xi q\mu,$$

$$c(x)=\left(\xi(1-q)x-\xi(1-q)\right)\left(x\xi-\xi(1-q)\right).$$

The corresponding characteristic equation of the homogeneous linear difference Equation (3.11) is

$$a(x)+b(x)r+c(x)r^2=0. \tag{3.12}$$

The solution to the characteristic Equation (3.12) is

$$r=\frac{\lambda}{(1-q)\xi}.$$

According to Liouville Formula, we have $\Phi_0(x)=c_1y_1+c_2y_2$, where c_1 and c_2 are two constants, and

$$y_1=\mathrm{e}^{\frac{\lambda x}{\xi(1-q)}},$$

$$y_2=\mathrm{e}^{\frac{\lambda x}{\xi(1-q)}}\int\exp\left(\kappa(x)-\frac{2\lambda x}{\xi(1-q)}\right)\mathrm{d}x,$$

where $\kappa(x)=-\frac{1}{6}\xi x\left(\lambda\left(6q(x-2)+2x^2-9x+12\right)-3(x-2)(\mu+\xi-\xi q)\right)$.

Hence, $\Phi_0(x)$ is

$$\Phi_0(x)=c_2\mathrm{e}^{\frac{\lambda x}{\xi(1-q)}}\int\exp\left(\kappa(x)-\frac{2\lambda px}{\xi(1-q)}\right)\mathrm{d}x+c_1\mathrm{e}^{\frac{\lambda x}{\xi(1-q)}}. \tag{3.13}$$

Then from (3.8), $\Phi_1(x)$ can be computed from

$$\Phi_1(x)=\frac{c_2(\xi x-\xi(1-q))}{\mu}\exp\left(\kappa(x)-\frac{\lambda px}{\xi(1-q)}\right)$$
$$+\frac{\xi x-\xi(1-q)}{\mu}\left(\frac{c_2\lambda p\mathrm{e}^{\frac{\lambda px}{\xi(1-q)}}\left(\int\exp\left(\kappa(x)-\frac{2\lambda px}{\xi(1-q)}\right)\mathrm{d}x\right)}{\xi(1-q)}+\frac{c_1\lambda p\mathrm{e}^{\frac{\lambda px}{\xi(1-q)}}}{\xi(1-q)}\right)$$
$$+\frac{\lambda p}{\mu}\left(c_2\mathrm{e}^{\frac{\lambda px}{\xi(1-q)}}\left(\int\exp\left(\kappa(x)-\frac{2\lambda px}{\xi(1-q)}\right)\mathrm{d}x\right)+c_1\mathrm{e}^{\frac{\lambda px}{\xi(1-q)}}\right). \tag{3.14}$$

Obviously, c_1 and c_2 can be obtained from (3.4)-(3.5). This completes the proof.

Corollary 3.2. *Denote by P_0 and M_0 the probability and the expected queue length that the system is idle or occupied by a secondary user, respectively. Let P_1 and M_1 represent the probability and the expected queue length that the system is in the busy state, respectively. For the retrial cognitive radio system with a linear retrial rate, we have*

$$P_0=\Phi_0(1), P_1=\Phi_1(1); \tag{3.15}$$

$$M_0=\Phi_0'(1), M_1=\Phi_1'(1). \tag{3.16}$$

Proof. According to (3.2) and (3.3), we easily get the following equations:

$$\Phi_0(1)=\sum_{j=0}^{\infty}\pi_{0,j}, \Phi_1(1)=\sum_{j=0}^{\infty}\pi_{1,j};$$

$$\Phi_0'(1)=\sum_{j=0}^{\infty}j\pi_{0,j}, \Phi_1'(1)=\sum_{j=0}^{\infty}j\pi_{1,j}.$$

Obviously, $\Phi_0(1)$ and $\Phi_0'(1)$ are the probability and the expected queue length that the system is in the idle state or occupied by a secondary user, respectively; $\Phi_1(1)$ and $\Phi_1'(1)$ are the probability and the expected queue length that the system is in the busy state, respectively. This completes the proof.

4. Conclusions and Future Work

This paper studied a single-server retrial cognitive radio system with a linear retrial rate. There are two types of users in this system: primary users and secondary users. A primary user who fails to immediately obtain spectrum resources upon arrival will join a retrial space and retry the server at a certain rate. In addition, the primary user is impatient and will exit the trial space with a certain probability. Based on the above background, the expressions for the generating functions $\Phi_0(x)$ and $\Phi_1(x)$ are obtained. Some performance measures were given, such as the expected queue length for primary users, the probability that the system is idle or occupied by secondary users, and the probability that the system is busy, respectively. This information can help users to determine their waiting time, and provide help for subsequent analysis of equilibrium strategies and social welfare, and so on.

This paper has only considered the performance measures associated with the primary users, not issues such as queue length and waiting time for secondary users. There are also games between the primary and secondary users, but their behavioral strategies are not analyzed. These are our future work.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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