

Channel Coding at Finite Blocklength and Its Application in 6G URLLC

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Abstract

To support mission-critical applications, such as factory automation and autonomous driving, the ultra-reliable low latency communications (URLLC) is adopted in the fifth generation (5G) mobile communications network, which requires high level of reliability and low latency. Naturally, URLLC in the future 6G is expected to have a better capability than its 5G version which poses an unprecedented challenge to us. Fortunately, the potential solution can still be found in the well-known classical Shannon information theory. Since the latency constraint can be represented equivalently by blocklength, channel coding at finite blocklength plays an important role in the theoretic analysis of URLLC. Applying these achievements in rapidly development of massive MIMO techniques gives rise to a new theory on space time exchanging. It tells us that channel coding can also be performed in space domain, since it is capable of providing the same coding rate as that in time domain. This space time exchanging theory points out an exciting and feasible direction for us to further reduce latency in 6G URLLC.

Keywords

Channel Coding, Finite Blocklength, MIMO, Space Time Exchanging, URLLC

1. Introduction

In the fifth generation (5G) mobile communications network, the ultra-reliable low latency communications (URLLC) is designed to support mission-critical applications, such as autonomous vehicles and virtual reality, which requires high level of reliability and low latency. According to the third generation partnership project (3GPP), the main objective of URLLC is to minimize the latency

down to 1ms while ensuring packet error rates of less than 99.999% [1]. As we know, in 4G, reliability is obtained by the hybrid automatic repeat request procedure. However, the strict latency constraint in URLLC does not endorse multiple retransmissions. Through extensively studying, the researchers in industry and academia solved this conflicting problem and proposed many advanced techniques from the period of access and connection, such as SR, mini-slot and CORESET [2].

Naturally, uRLLC in 6G is expected to have a better capability than its 5G version. This is driven by the factor that almost all the applications, such as robotics and autonomous systems, hunger for higher reliability and lower latency. On the other hand, some upcoming new applications will blur totally the boundary between eMBB and uRLLC. For example, it is widely accepted that as a full immersive technique, extended reality (XR), will emerge in many commercial services in the future. XR is expected to encompass augmented mixed, and virtual reality (AR/MR/VR) technique and can capture all the perceptual inputs stemming from human senses, cognition, and physiology [3]. However, current 5G cannot support XR due to its inability to deliver huge data in a very low latency. Combining eMBB and uRLLC may generate a new application scenario in 6G, which is characterized by high reliability, low latency and high data rate [4] [5] [6] [7] [8].

The new wideband requirement and ultra-low latency pose an unprecedented challenge to the basic principle of designing 6G. Fortunately, the potential solution can still be found in the well-known classical Shannon information theory. All along, packet error probability ε , packet length (*i.e.* codeword size) n and coding rate R (the number of information bits per complex symbol) are the three fundamental parameters involved in communication systems. They are highly correlated to each other and no one is dispensable, just like the vertexes of a triangle. In 1948, C. E. Shannon founded the mathematical basis of communication theory and revealed that the channel capacity C is the maximal rate at which the packet error probability P_e vanishes as the packet length n tends to infinity [9]. After that, through using error exponent corresponding to a fixed rate $R < C$, R. G. Gallager derived the packet error probability ε for that rate in 1965 [10]. Then, all the remainder (the last vertex of the triangle) is only about the packet length. That is, how does n affect R and ε ? It becomes a long-standing problem confusing researcher many years. Fortunately, during the last few years, significant progress has been made within the information theory community. Building upon Dobrushin's and Strassen's previous asymptotic results, Y. Polyanskiy *et al.* recently provided a unified approach to obtain tight bounds on R as a function of n and ε [11] [12] [13] [14]. Since the latency constraint can be represented equivalently by packet length, this achievement paves a flat road to analyze the system performance of URLLC from an information-theoretic point of view.

The use of multiple antennas for wireless communication systems has gained overwhelming interest during the last decade-both in academia and industry. Multiple antennas can be utilized in order to accomplish a multiplexing gain, a

diversity gain, or an antenna gain, thus enhancing the bit rate, the error performance, or the signal-to-noise-plus-interference ratio of wireless systems, respectively [15] [16] [17]. With an enormous amount of yearly publications, the field of multiple-antenna systems, often called multiple-input multiple-output (MIMO) systems, has evolved rapidly. Massive MIMO was first proposed by Marzetta in 2010 [18]. Compared to traditional MIMO, massive MIMO has hundreds of antennas in BSs and can then acquire more multiplexing gain and diversity gain, which could improve data rate and reliability respectively. Those improvements are due to the increase in spatial degrees of freedom brought by the larger dimension of the arrays along with the larger number of antennas.

Based on the rapidly development of massive MIMO techniques and the theoretical achievement in channel coding at finite block length, a new theory on space time exchanging is proposed in [19] and [20]. Its academic achievements illustrate that channel coding can be performed in the space domain, since it is capable of providing the same coding rate which provides us a feasible way to design 6G URLLC.

2. Channel Coding with Finite Block Length

For engineers, the probability density functions are more straight-forward compared to the Radon-Nikodym derivative of probability measures. And the (generalized) probability density functions should suffice for most situations. For example, consider two probability measures P and Q define on the product measurable space $(\mathbf{X} \times \mathbf{Y}, \mathcal{F}_{\mathbf{X} \times \mathbf{Y}})$ and P, Q are absolutely continuous with respect to the Lebesgue measure, we have

$$\frac{dP}{dQ}(x, y) = \frac{\frac{dP}{dx}(x, y)}{\frac{dQ}{dx}(x, y)} = \frac{p(x, y)}{q(x, y)} \quad (1)$$

where $p(x, y)$ and $q(x, y)$ are the probability density functions of probability measure P and Q , respectively.

Now let us consider under additive white Gaussian noise (AWGN) channel, *i.e.*

$$y = x + w \quad (2)$$

where x, w and y represent the transmit signal, noise, and received signal. From the probability density functions, we may define a function of x and y as below:

$$i(x, y) \triangleq \log \frac{dP_{x,y}}{dP_x \times P_y}(x, y) = \log \frac{p(x, y)}{p(x)p(y)}. \quad (3)$$

Here $i(x, y)$ is named as the information density function, in contrast to the probability function. It should be noticed that its expectation

$$\mathbb{E}_{\mathbf{X} \times \mathbf{Y}} [i(x, y)] = \int_{\mathbf{X} \times \mathbf{Y}} \log \frac{p(x, y)}{p(x)p(y)} p(x, y) dx dy \quad (4)$$

is, in fact, an alternative expression of the well-known Shannon capacity.

2.1. Channel Dispersion

For a block (or time slot) consists of n symbols, the received signal can be written by

$$Y^n = X^n + W^n \quad (5)$$

where X^n and W^n denote the transmit symbols and noise vectors, respectively. Since the AWGN channel is memoryless, we obtain

$$\begin{aligned} i(X^n, Y^n) &= \log \frac{dP_{X^n Y^n}}{dP_{X^n \times Y^n}}(X^n, Y^n) \\ &= \sum_{i=1}^n \log \frac{p(x_i, y_i)}{p(x_i) p(y_i)} \\ &= \sum_{i=1}^n i(x_i, y_i). \end{aligned} \quad (6)$$

Notice that, in the non-strict sense, $i(X^N, Y^N)$ usually obeys Gaussian distribution for $N \geq 20$. So, its variance

$$V = \text{Var}[i(X = x, Y = y)] \quad (7)$$

can be viewed as the channel dispersion which is of great interest in our study. According to equation [2], for a code ensemble \mathcal{C} of codeword length n , the achievable code rate R under codeword-error-rate ε can be closely approximated by the following formula

$$R(n, \varepsilon) \approx C - \sqrt{\frac{V}{n}} Q^{-1}(\varepsilon) \quad (8)$$

where

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \quad (9)$$

denotes the Gaussian Q -function and C represents the Shannon capacity. Meanwhile, ε can be viewed as the approximate probability of codeword outage, *i.e.*, the event that $i(X^n, Y^n)$ is less than $R(n, \varepsilon)$. So, the dispersion V finally determines the rate penalty from capacity, for codeword length (or block length) n . And we should minimize V to reduce the penalty. Unfortunately, it is by no means an easy task. Since

$$i(x, y) = \log \frac{p(x, y)}{p(x) p(y)} = \log p(x) + \log \frac{p(x|y)}{p(y)}, \quad (10)$$

we can apply some new constraint on the codeword ensemble \mathcal{C} to minimize V . Under the constraint that all codewords should have equal transmit power, the minimum dispersion of AWGN channel was achieved in [2] which has the form as

$$V_{\text{AWGN}} = \frac{1}{2} \left(1 - \frac{1}{(1+\rho)^2} \right) \log_2 e \quad (11)$$

where ρ denotes the signal-to-noise ratio (SNR). In contrast, when \mathbf{X} is an independent and identically distributed (i.i.d) Gaussian variable, the variance of information density can be written by

$$V_{\text{Gauss}} = \frac{1}{2} \left(1 - \frac{1}{1+\rho} \right) \log_2^2 e, \quad (12)$$

and it can be verified that $V_{\text{Gauss}} \geq V_{\text{AWGN}}$. Nevertheless, V_{Gauss} was regarded as the optimal dispersion in many previous papers. We have to stress that, to reduce dispersion, the codeword power equality constraint is necessary under AWGN channel, and later MIMO channel.

2.2. Achievable Coding Rate in MIMO

We consider a narrow-band MIMO system with N_t transmitter antennas and N_r receiver antennas operating over a frequency-flat quasi-static fading channel. For a block of n symbols, we have

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W} \quad (13)$$

where \mathbf{H} is the channel matrix of size $N_r \times N_t$, \mathbf{X} is the channel input matrix of size $N_t \times n$ and \mathbf{Y} is the matrix valued received signal of size $N_r \times n$. The average total transmit power is constrained to P . Assume \mathbf{H} is available both at transmitter and receiver which is a typical scenario in existing MIMO systems. For the simplicity of analysis, we further assume that there is no water-filling power allocation. In this case, the channel capacity can be express by

$$C(\mathbf{H}, P) = \log \det \left(\mathbf{I}_r + \frac{P}{N_t} \mathbf{H}\mathbf{H}^H \right). \quad (14)$$

which is achieved when the input signals follow the i.i.d Gaussian distribution, *i.e.* $X(i, j) \sim \mathcal{N}(0, P/N)$, where $X(i, j)$ denotes the element in the i -th row and the j -th column of \mathbf{X} [8].

According to [9], the MIMO channel can be transformed into the eigen-mode channel, which is composed of multiple parallel AWGN links, *i.e.*

$$C_{\text{eig}}(\mathbf{H}, P) = \sum_{i=1}^{N_{\min}} \log \left(\mathbf{I}_r + \frac{P\lambda_i}{N_t} \right) \quad (15)$$

where N_{\min} represents the minimal value of N_r and N_t , λ_i denotes the i -th eigen value. If the spherical constraint is satisfied for each individual subchannel, the minimum total dispersion is therefore achieved, which is the sum of dispersion of subchannels multiplied a scaling factor, *i.e.*

$$\begin{aligned} V_{\text{eig}}(\mathbf{H}, P) &= \sum_{i=1}^{N_{\min}} V_{\text{AWGN}} \left(\frac{P}{N_t} \lambda_i \right) \\ &= \log_2^2 e \left(N_{\min} - \sum_{i=1}^{N_{\min}} \frac{1}{(1 + P\lambda_i/N_t)^2} \right). \end{aligned} \quad (16)$$

Nevertheless, eigen-mode transmission can be resource-demanding for many system settings, if perfect CSIT is not readily available. Therefore we consider

the situation that the receiver performs maximum-likelihood decoding, and the adaptive code-rate R is feedback. The transmitter will send a code-word conforming to the code-rate R . In that case, the spherical code-word constraint is imposed on the whole channel input matrix that

$$\|\mathbf{X}\|_F^2 = \sum_{i=1}^{N_t} \sum_{j=1}^n X_{i,i}^2 = nP, \quad (17)$$

and in this case, the channel dispersion is given by the following expression:

$$V_{\text{CSIR}}(\mathbf{H}, P) = \sum_{i=1}^{N_{\min}} V_{\text{AWGN}}\left(\frac{P}{N_t} \lambda_i\right) + \left(\frac{P}{N_t}\right)^2 \log_2^2 e \left(\sum_{i=1}^{N_{\min}} c^2(\lambda_i)\right) - \left(\frac{P}{N_t}\right)^2 \frac{\log_2^2 e}{N_t} \left(\sum_{i=1}^{N_{\min}} c(\lambda_i)\right)^2 \quad (18)$$

where

$$V_{\text{AWGN}}(P) = \log_2^2 e \left(1 - \frac{1}{(1+P)^2}\right), \quad (19)$$

$$c(\sigma) = \frac{\sigma}{1 + P\sigma/N_t}. \quad (20)$$

3. Application in URLLC

In fact, the rationale behind channel coding with finite block length can be viewed as that when n is finite; the coding rate becomes a random variable composed of channel capacity and dispersion. Channel dispersion is also a random variable introduced as a rate penalty to characterize the impact of n . On the other hand, the probability that the coding rate is smaller than R gives the packet error probability ε . Taking n and R as the latency constraint and bandwidth requirement, we can analyze the performance of reliability for 6G URLLC from an information-theoretic point of view.

3.1. Latency Analysis

In wireless communication systems, the latency from end to end is generally composed of multiple delay sources, that is

$$L = T_q + T_t + T_f + T_p + T_r \quad (21)$$

where T_t and T_r denote the processing delays at transmitter and receiver. T_q and T_f illustrate the queuing delay and packet (or block) alignment delay and T_p represents the transmission delay or transmission time interval (TTI) needed to transmit the packet. In URLLC, since the event to deliver is usually very emergency, its data can preempt the channels which have already been allocated to other traffics. So, T_q and T_f could be ignored. From the communication point of view, both T_t and T_r can be thought of as constants when a specific air interface technique is applied. Then, all the left is T_p which is composed of block length and the travel time of the wireless radio signal in space. Since the latter

must follow the physical law in nature, it can not be changed by any artificial means. As a result, shortening block length is a feasible method to reduce latency.

However, as mentioned above, packet error probability ε , packet length n and coding rate R are the three fundamental parameters which are highly correlated to each other and no one is dispensable, just like the vertexes of a triangle. So, shortening block length will lead to an increase of packet error which breaks the promise of high reliability unexpectedly. In order to remain the block length unchanged, the other physical resources should be used. In current 4G/5G standards, orthogonal frequency division multiplexing (OFDM) technique is adopted as the main air interference for its advantages in high bit rate transmissions over frequency-selective fading channel. In these systems, the frequency resources in term of subcarriers can be used for accommodating data information for channel coding. However, frequency and time are just both sides of the transform domain. They are highly correlated to each other. So, increasing the number of subcarriers in the frequency domain inevitably makes the signal duration become longer in the time domain. Moreover, the larger size of IDFT/DFT operation will, at the same time, increase the processing delays at both sides of the communication link. Consequently, spreading the data block into the space domain is only the most effective way to reduce latency for future 6G URLLC applications, since there is a new space time exchanging theory emerging with the rapid development of massive MIMO.

3.2. Space Time Exchanging Theory

So far, massive MIMO has been adopted in 5G as a key physical-layer technology to meet the demand for higher data capacity for mobile networks. It is supposed that the massive MIMO techniques will continue to evolve and more antennas are considered to employ, as the Tera Hertz frequency band is envisioned for future 6G. More antennas would provide more augmented degrees of freedom which leads to some remarkable properties of massive MIMO, such as channel hardening and decorrelation. That is, as the number of antennas in the system increases, the variations of channel gain decrease in both the time- and frequency domain and the channel vectors become asymptotically orthogonal.

The rapidly development of massive MIMO techniques companied with theoretical achievement in finite block length channel coding give rise to a new theory on space time exchanging which is firstly proposed in [19] and [20]. According to the MIMO theory, after singular value decomposition (SVD) of eigenvalue decomposition, the MIMO channel can be transformed into multiple parallel orthogonal links. It can be viewed as that the radio propagation environment is sampled by a so-called MIMO sampler at a certain sampling period and all the sampled elements compose the space domain, just like the conventional sampling behavior in the time domain. In conventional MIMO systems, the channel

coding is conducted in the time domain along each link. However, space time exchanging theory tell us that channel coding can also be performed in space domain, since it is capable of providing the same coding rate. In specific,

- for a certain number of space samples, space domain coding can achieve the same coding rate as that in the time domain,
- when the number of antennas approaches infinite, the Shannon capacity can also be attained in the space domain.

The space time exchanging theory points out an exciting and feasible direction for us to design 6G URLLC. For example, we can model it as a MIMO problem with short packets and pursuit the optimal number of the antennas which are used to support the desired data rate, for a given ε and n .

4. Some Preliminary Numerical Results

For easy understanding, we present some preliminary numerical results in this section to give an overview to the space time exchanging theory. **Figure 1** shows the channel dispersion comparison for an 8×4 MIMO as a function of SNR, under various code-word constraints. From Fig. 1, we can see that the analytical results coincide well with their Monte-Carlo simulated values which validates the Polyanskiy's theory in [11] [12] [13]. For comparison, none-constraint code-word is also simulated. As seen from the figure, the dispersion of code-word without constraint is much larger than those of the code-word with constraints and the eigen mode spherical constraint can achieve the minimal dispersion, by now.

Figure 2 gives the average achievable rate for a finite bloklength of 2560

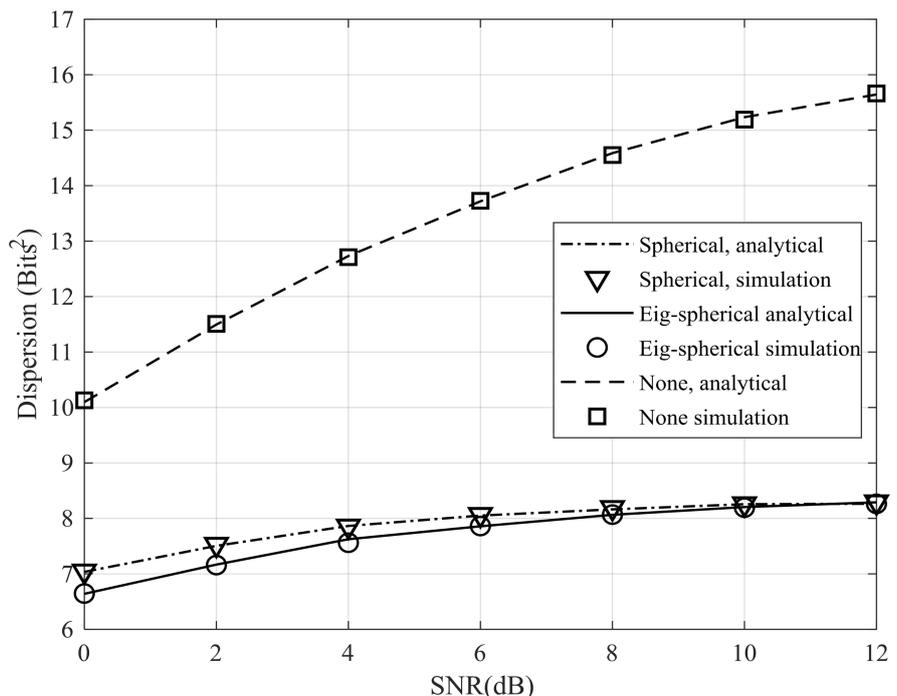


Figure 1. MIMO channel dispersion comparison.

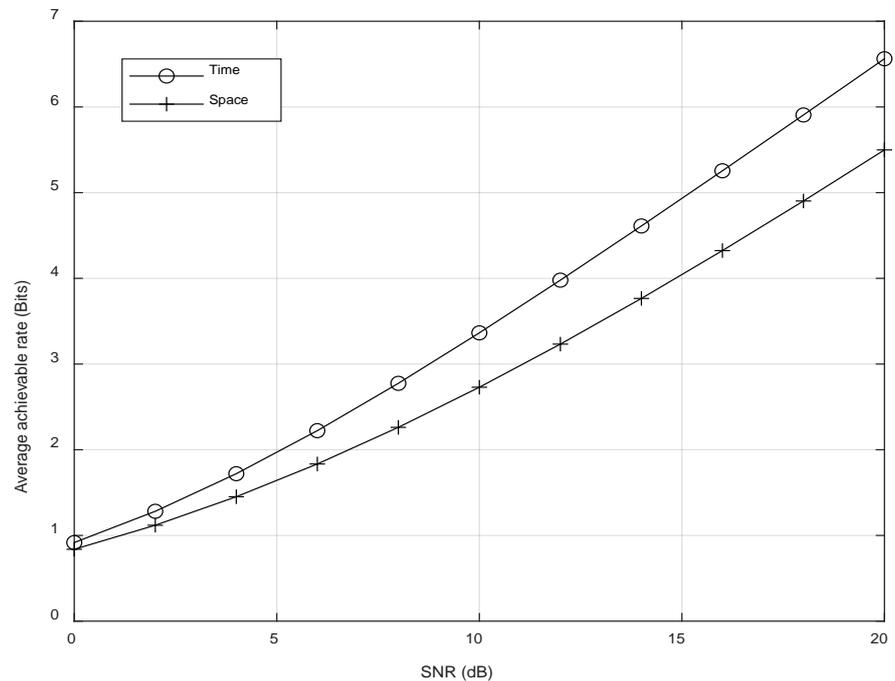


Figure 2. Average achievable rate comparison.

channel uses. For comparison, the coding rate for an AWGN channel in the conventional time domain is also simulated which can be viewed as the upper bound of achievable rate according to Polyanskiy's theory. When there is a 128×128 MIMO channel, we can conduct channel coding along the antennas to use the space channel instead of the time channel. For the sake of fair comparison, we do not consider the multiplexing gain and obtain the curve, labelled by "space", in **Figure 2**. As can be seen from **Figure 2**, the curve "space" approaches the bound of "Time" which validates the theory of space time exchanging. It also be noted that the performance gap is mainly introduced due to the lack of minimal dispersion which drives us to search the optimal code structure in the future.

5. Conclusion

In this paper, we roughly review the Shannon information theory and focus on channel coding rate in finite blocklength regime. Then a new theory on space time exchanging is introduced which is developed based on the massive MIMO techniques and the theoretical achievements in finite blocklength. From this theory, we learn that performing channel coding in space domain can achieve the same coding rate as that in time domain. By exchanging the time by space, the latency can be reduced dramatically which provides a feasible way to design 6G URLLC.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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