

Design of Two-Stage Comb-Based Decimation Filter with High Aliasing Rejection and Low Passband Droop

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Abstract

This paper presents a simple method for the design of comb-based decimation filter with a high aliasing rejection and low passband droop. The method is based on the application of the mathematical result on zeros of symmetric polynomials on the unit circle. The application of this result leads to the introduction of the four additional single zeros into comb folding bands, where aliasing occur. As a result, the folding bands become wider and the aliasing rejection is increased. The passband droop is compensated by a simple compensator who works at low rate. The comparisons with the most recent methods from literature confirmed the advantages of this proposal regarding magnitude responses and the required complexity.

Keywords

Decimation, Comb Filter, Aliasing, Compensator, Passband Droop

1. Introduction

Decimation is the process of decreasing the sampling rate in the digital domain by an integer, called downsampling factor. This process is composed of two stages. In the first stage is the decimation filter while in the second stage the sampling rate is decreased. The decimation filter is needed to prevent aliasing which occurs during the decreasing the sampling rate. Decimation has many applications in communications, software defined radio, Sigma-Delta Analog-Digital Converters, among others [1].

The most simple decimation filter is a comb filter, which has all its coefficients equal to unity. The system function of comb filter is given as:

$$H(z) = \frac{1 - z^{-M}}{M(1 - z^{-1})}, \quad (1)$$

where M is the decimation factor.

Comb filter naturally provides the attenuation in the frequency bands around comb zeros, called folding bands. However, this attenuation is not enough in many applications. This attenuation could be increased by cascading K combs, where K is called the comb order. However, the increase of K increases the droop in the passband, which may result in the deterioration of the decimated signal.

Different methods have been proposed to increase the aliasing rejection in comb filters [2]-[11]. Presti in [2] proposed Rotated Sinc (RS) filter obtained by the rotation of the comb zeros around their original positions in the folding bands. As a result, the folding bands become wider, thus providing increased attenuation in the folding bands. The drawback of this approach is the introduction of two multipliers, and possible instability when the coefficients of RS filter are presented with finite precision. The authors in [3] [4] [5] used simple multiplierless filters to get the similar effect as in RS filters but without the mentioned drawbacks. Laddomada in [3] used the cyclotomic polynomials, while the authors in [4] [5] explored symmetric polynomials to separate comb zeros [4], or to introduce the additional zeros into comb folding bands [5]. Similarly, the sharpening techniques were proposed in [6]-[11]. Willson at all introduced in [6] the simple sharpening polynomial to improve the passband and the stopband comb magnitude characteristic. The authors in [7] proposed a two-stage structure where the sharpening is performed only at the second stage. Coleman in [8] proposed sharpening using Chebyshev polynomials. In [9] [10] are proposed sharpened polynomials to increase comb aliasing rejection and the novel compensators to compensate high passband droop in sharpened comb filters. In [11] was proposed a linear programming optimization to find the sharpening coefficients and improve comb magnitude response in a two stage structure.

The goal of this work is to propose an alternative method to improve comb magnitude characteristic. The proposed method is based on the modification of work in [5] and the simple compensator working at low rate.

The rest of the paper is organized as follows. Next Section introduces the proposed filter. The design of compensators is elaborated in Section 3. Finally, the comparisons with some recent methods from literature are given in Section 4.

2. Proposed Filter

In [5] was proposed the following decimation filter,

$$H_m(z) = H^{K-2}(z)G(z), \quad (2)$$

where K is the order of the equivalent comb filter, $H(z)$ is the comb filter given in (1) and $G(z)$ is given as:

$$G(z) = \left[1 + \lambda_1 \sum_{k=1}^{M-2} z^{-k} + z^{-(M-1)} \right] \times \left[1 + \lambda_2 \sum_{k=1}^{M-2} z^{-k} + z^{-(M-1)} \right], \quad (3)$$

where $\lambda_1 = 2^{-k}$ and $\lambda_2 = 1 + \lambda_1$, and k is an integer.

The filter (3) introduces two additional zeros into comb folding bands. However, for high values of M and K , the effect of the introduced zeros is low.

We propose here a two-stage structure, in which the modified filter (3) is applied only in the second stage as described in the following section.

2.1. Increasing Aliasing Rejection

It is supposed that the decimation factor M can be presented as a product of two integers, $M = M_1 M_2$. The system function of the comb (1), can be presented as:

$$H(z) = H_1(z)H_2(z^{M_1}), \quad (4)$$

where:

$$H_1(z) = \frac{1 - z^{-M_1}}{M_1(1 - z^{-1})}; H_2(z^{M_1}) = \frac{1 - z^{-M}}{M_2(1 - z^{-M_1})}. \quad (5)$$

We consider a two-stage comb structure in which a comb filter $H_1(z)$ is decimated by M_1 in the first stage, while in the second stage is a comb filter $H_2(z)$ decimated by M_2 .

We propose to introduce a modified comb filter, derived from (3) and denoted as $H_m(z)$, only at the second stage:

$$H_m(z) = \left[1 + 2^{-1} \sum_{i=1}^{M_2-2} z^{-i} + z^{-(M_2-1)} \right]^2 \times \left[1 + (1 + 2^{-1}) \sum_{i=1}^{M_2-2} z^{-i} + z^{-(M_2-1)} \right]^2. \quad (6)$$

Using (5) and (6), the system function of the proposed filter $H_p(z)$ is given as:

$$H_p(z) = H_1^{K_1}(z) H_2^{K_2}(z^{M_1}) H_m(z^{M_1}), \quad (7)$$

where K_1 and K_2 are orders of the combs $H_1(z)$ and $H_2(z)$, respectively. The values of K_1 and K_2 are chosen to give high aliasing attenuation, and are given in **Table 1**. The method is illustrated in the following example.

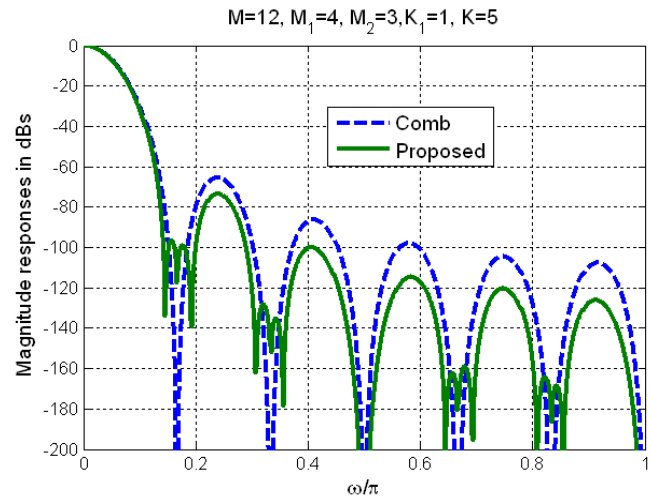
Example 1: We consider $M = 12$, $K_1 = 6$, $K_2 = 1$, and three different values for M_2 : 3, 4 and 6. The magnitude responses of the proposed filter and the equivalent comb of the order $K = 5$, are shown in **Figure 1**.

2.2. Choice of M_1 and M_2

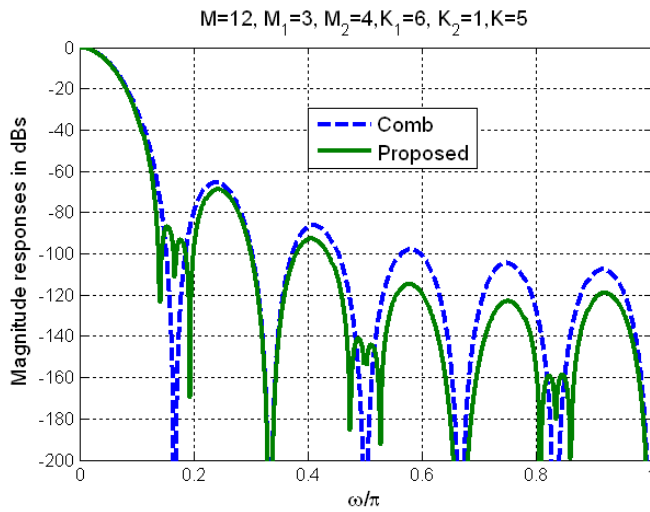
Observing **Figure 1** we can see that the choice of M_2 affects the magnitude response.

Table 1. Values of K_1 and K_2 .

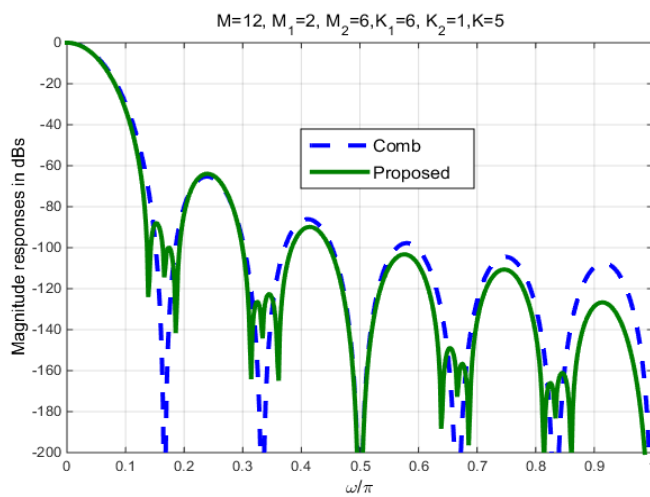
K_1	K_2
5	1
6	1, 2
7	2



(a)



(b)



(c)

Figure 1. Magnitude responses in Example 1. (a) $M_1 = 4, M_2 = 3$; (b) $M_1 = 3, M_2 = 4$; (c) $M_1 = 2, M_2 = 6$.

If M_2 is odd, then each M_2 -th folding band is not improved. However, if M_2 is even, then each $M_2/2$ -th folding band is not improved.

Therefore, the value M_2 should be chosen in order that more folding bands are improved. As a result, in Example 1 the choice $M_2 = 3$ and $M_2 = 6$ (every third band is not improved) are better than the choice $M_2 = 4$ (every second band is not improved).

From the other side, the value of M_1 affects the complexity. Higher values of M_1 result that the second stage works at lower rate.

Therefore the choice of M_1 and M_2 must be a compromise between the high aliasing rejection and low complexity.

In Example 1 the best choice is $M_1 = 4$ and $M_2 = 3$.

2.3. Complexity

The complexity is expressed in number of *APOS* (adders per output sample), [12].

The number of *APOS* for the proposed filter is presented in the following equation:

$$APOS = K_1 M + (K_1 - K_2) M_2 + M_2 (2M_2 - 1) + K_2. \quad (8)$$

Using (8) we find the following values of *APOS* for given values of M_1 and M_2 in the Example 1.

$$M_1 = 4, M_2 = 3: APOS = 103.$$

$$M_1 = 3, M_2 = 4: APOS = 121.$$

$$M_1 = 2, M_2 = 6: APOS = 169.$$

Note that the increase of M_1 results in the decrease of number of *APOS*.

3. Compensation

Since the passbands in the proposed filter and the equivalent comb are similar, we propose to adapt the compensator from [13]. The compensator in [13] provides a good compensation of the comb passband droop requiring a low number of adders. The magnitude response of the compensator is given as:

$$C(e^{j\omega M}) = [1 + A \sin^4(\omega M/2)] \times [1 + B \sin^2(\omega M/2)], \quad (9)$$

where A and B are the amplitudes of sinusoidal functions.

The total number of adders of compensator N_a is given as [13]:

$$N_a = 9 + N_A + N_B, \quad (10)$$

where N_A and N_B are the total number of adders for the parameters A and B , respectively, which are presented in a SPT form. The compensator works at low rate, *i.e.* after the decimation by M . The system function of the proposed compensated filter is given as:

$$H_{pc}(z) = H_p(z) C(z^M) = H_1^{K_1}(z) H_2^{K_2}(z^{M_1}) H_m(z^{M_1}) C(z^M), \quad (11)$$

where $H_p(z)$ is given in (7) and $C(z)$ is the system function of the compensator.

Using (8), the total number of $APOS_C$ in the proposed compensated filter is

equal to:

$$APOS_C = APOS + N_A = K_1M + (K_1 - K_2)M_2 + M_2(2M_2 - 1) + K_2 + N_a. \quad (12)$$

where N_a is given in (10).

As a difference to [13], the values A and B are obtained here using the Particle Swarm Optimization (PSO) of MATLAB. Particularly, the MATLAB function *particleswarm.m* is used. The procedure of using *particleswarm.m* to get the parameters A and B for a given M , and K is presented in [14], and is not repeated here. The only difference is that here the optimization is performed for the proposed filter (11) and in [14] for the comb filter.

As an example, **Table 2** presents the values of A and B in the SPT form for $M = 12$ and the values K_1 and K_2 from **Table 1**. The total number of adders N_a and the absolute value of the maximum passband deviation δ in dBs, are also shown.

Next example compares the magnitude responses of the proposed filter from Example 1, and the proposed compensated filter.

Example 2. In this example all parameters are equal as in Example 1. The parameters of compensator are obtained from the second row in **Table 2** and are equal: $A = 2^0 - 2^{-3} - 2^{-7}$, $B = 2^0 - 2^{-2} + 2^{-4}$, requiring 13 adders. From (12) the number of $APOS_C$ is equal to 116. The magnitude responses are shown in **Figure 2**.

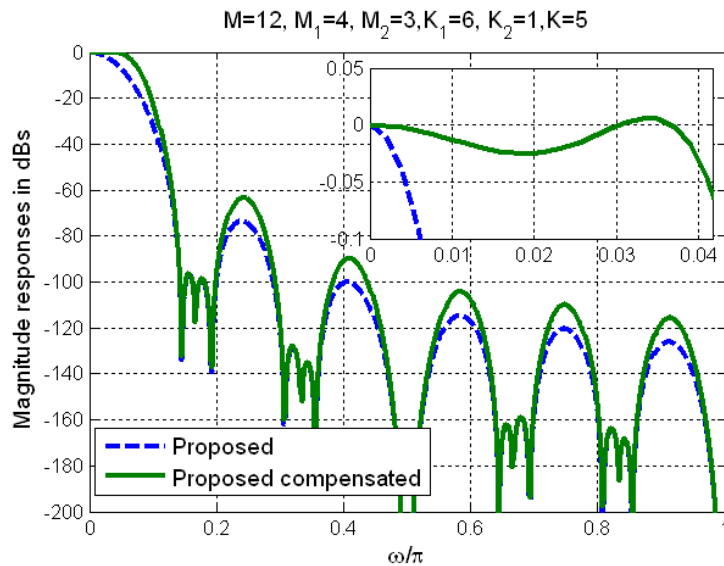


Figure 2. Magnitude responses of the compensated and the original proposed filter.

Table 2. Values of A , B , N_a and δ .

K_1	K_2	A	B	N_a	δ
5	1	$2^0 - 2^{-5}$	$2^0 - 2^{-2}$	11	0.054
6	1	$2^0 - 2^{-3} - 2^{-7}$	$2^0 - 2^{-2} + 2^{-4}$	13	0.05
6	2	$2^0 + 2^{-7}$	2^0	10	0.062
7	2	$2^0 + 2^{-6}$	$2^0 + 2^{-5}$	11	0.068

4. Comparisons

In this section we compare the proposed filter with the methods from literature regarding magnitude response and complexity expressed in APOS.

4.1. Comparison with Method in [5]

We pick up $M = 12$ and $K = 4$ in the method in [5]. In the proposed method $M_1 = 4$, $M_2 = 3$, $K_1 = 5$ and $K_2 = 1$. The parameters of compensator are used from the first row in **Table 2**.

The overall magnitude responses and the passband zooms are contrasted in **Figure 3**. Observe that the proposed compensated filter provides better aliasing rejection and an improved passband.

Additionally, the proposed method has low complexity, requiring the number of $APOS_C = 99$, while the method in [5] requires 302 numbers of APOS.

4.2. Comparison with Method in [4]

For a sake of comparison we chose $M = 15$ and $K = 5$ in the method proposed in [4]. In the proposed method $M_1 = 5$, $M_2 = 3$, $K_1 = 6$, and $K_2 = 2$. The parameters of compensator are $A = 2^0 + 2^{-5}$ and $B = 2^0$.

The number of APOS in method [4] is equal to 123, while in the proposed method is equal to 119 (without compensator), and 129 (with compensator).

The magnitude responses are compared in **Figure 4**. Note that the proposed method provides much better magnitude characteristic.

4.3. Comparison with Method in [9]

The authors in [9] proposed various sharpened polynomials to improve the

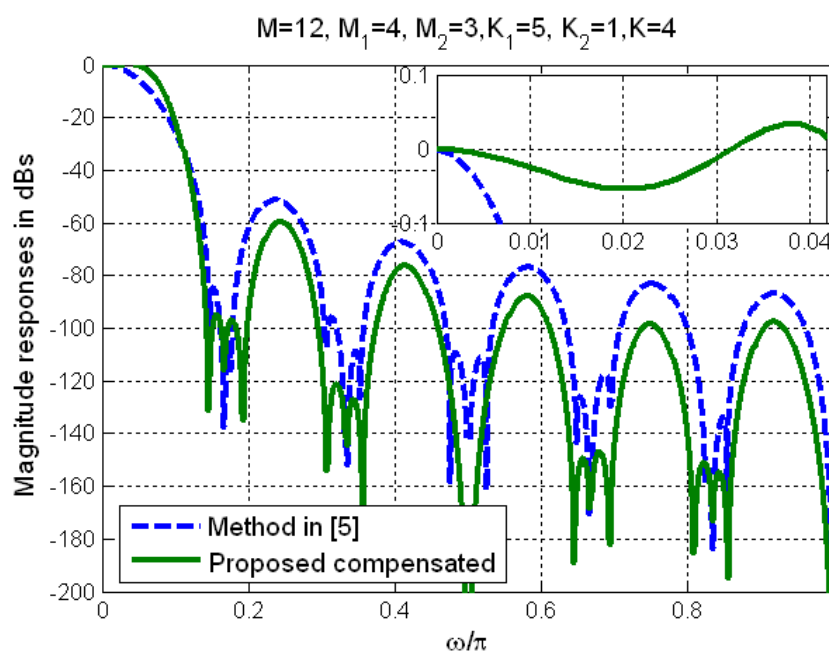


Figure 3. Comparison with method in [5].

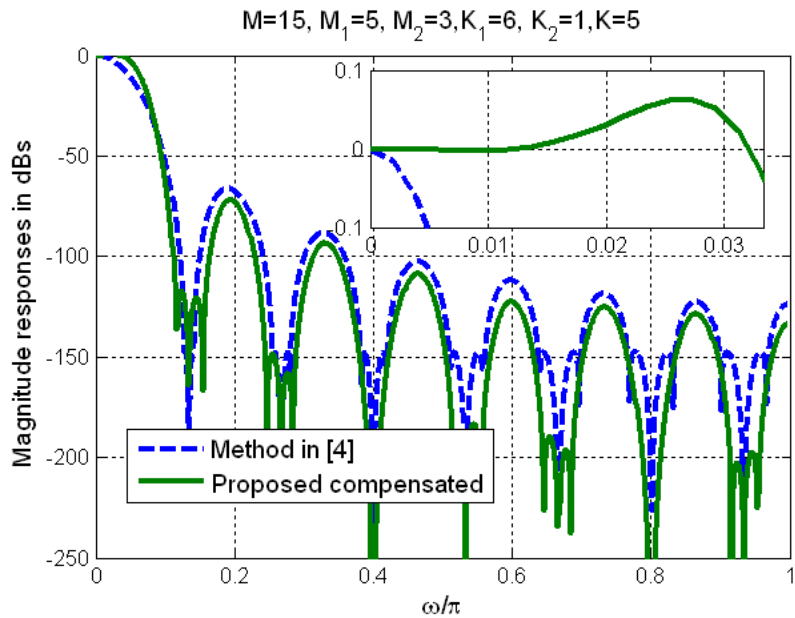


Figure 4. Comparison with method in [4].

comb aliasing rejection. Since sharpened combs exhibit a high passband droop, the designs of the compensators were also proposed in [9].

In this comparison we chose $M = 32$ and the sharpening polynomial in [9] $p(x) = -2^{-7}x^2 + x^4$, where x is the comb filter. The coefficients of compensator are given in Table in [9].

In the proposed filter $M_1 = 8$, $M_2 = 4$, $K_1 = 5$, and $K_2 = 1$. The parameters of compensator are $A = 2^0 - 2^{-8}$, and $B = 2^0 - 2^{-2}$, requiring $N_a = 11$ adders.

The number of *APOS* in the filter proposed in [9] is equal to 233, while in the proposed method is equal to 216.

The magnitude responses are shown in **Figure 5**. We can observe that the proposed filter provides better aliasing rejection and better passband characteristic, requiring lesser number of *APOS*.

4.4. Comparison with Method in [11]

Authors in [11] proposed a two-stage decimation filter. In the first stage is a comb filter of order Q , while in the second stage is a sharpened comb. The coefficients of the sharpened polynomial are obtained using LP optimization and are given in Table in [11]. The parameters of design in this comparison in method [11] are: $M = 32$ and $M_1 = 4$, $M_2 = 8$, $Q = 8$. In the proposed method $M_1 = 8$, while $M_2 = 4$, $K_1 = 7$, $K_2 = 2$. The parameters of compensator are $A = 2^0 + 2^{-4}$, and $B = 2^0 + 2^4$, requiring 11 adders.

The magnitude responses are compared in **Figure 6**.

The number of *APOS* in [11] is equal 403, while in the proposed method is equal to 285.

We can see that the proposed method provides better aliasing rejection, as well as better passband characteristic, requiring lesser number of *APOS*.

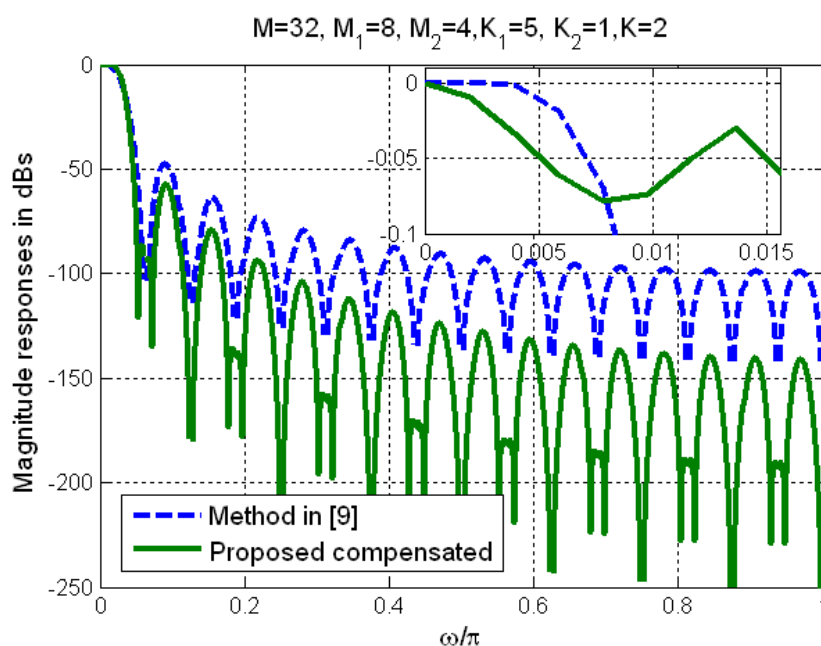


Figure 5. Comparison with method in [9].

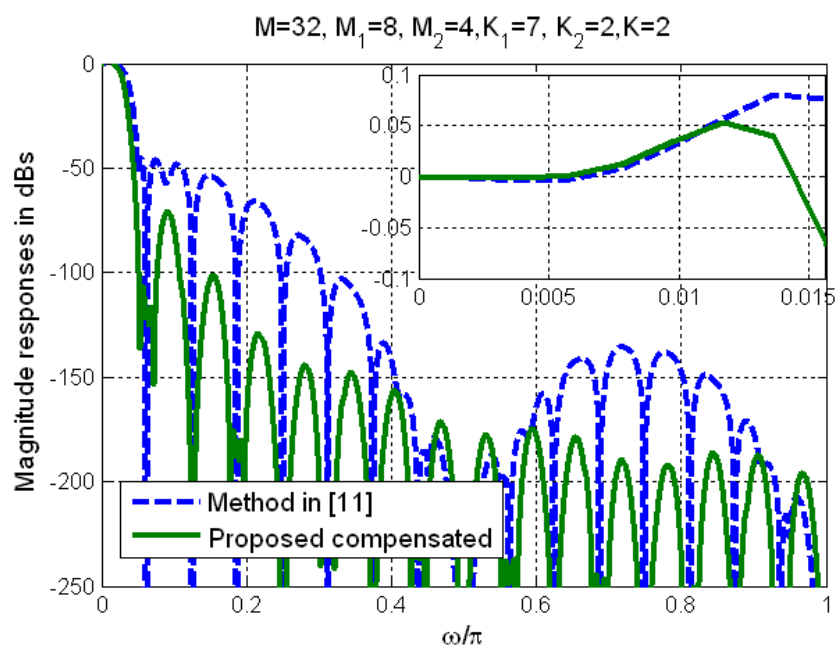


Figure 6. Comparison with method in [11].

5. Conclusions

We present a simple design of a two stage decimation filter with high aliasing rejection and good passband compensation.

The aliasing rejection is increased by introducing the modified comb at the second stage. As a result, four additional zeros are inserted into certain comb folding bands, thus providing increases of the folding bands widths, and increases of aliasing rejection in folding bands. The attenuation in folding bands is

higher than 80 dBs.

The decimation factor at the second stage determines which folding band gets additional zeros. On the other side, the complexity of the filter, expressed in *APOS* is determined by the decimation factor at the first stage.

Usually, there is a compromise in choosing the decimation factors in the first and second stages.

The passband is improved by the simple multiplierless compensator working at low rate. The parameters of compensator are obtained by PSO. The absolute value of the maximum passband deviation is lesser than 0.08 dBs, and depends on the values of decimation factors and the orders of comb filters, at the first and second stages.

The comparisons with methods from literature demonstrated that the proposed filter provides better magnitude characteristic, while requires lesser number of *APOS*.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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