An Analysis of the Maximum Lifespan in the World and Japan

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Abstract

Extreme value theory provides methods to analyze the most extreme parts of data. We used the generalized extreme value (GEV) distribution to predict the human lifespan in the world and Japan. The diagnostic plots, which assessed the accuracy of the GEV model, were fitted to the human lifespan, validating the model. The human lifespan in the world and Japan had shape parameters of $-0.1623$ and $-0.2949$ and had an upper limit. The calculated upper limit in the world was 128.7 years. The world’s oldest record holder, Jeanne Calment’s age of 122.45 years, was close to the 260-year return level and was far from the calculated upper limit. The calculated upper limit in Japan was 120.4 years. Japan’s oldest record holder, Kane Tanaka’s age of 119 years in 2022, was the 500-year return level and was close to the upper limit. In the world, achieving the calculated limit was difficult, but the human lifespan will soon reach the upper limit in Japan.

Keywords

Human Lifespan, Extreme Value Theory, GEV Model

1. Introduction

Both for and against a limit on the human lifespan were argued [1]. For humans, the maximum reported age at death is generally set at 122.45 years, the age at death of Jeanne Calment (France) in 1997. An estimation method for maximum human lifespan was developed and about 126 years was obtained from periodic life tables for Swedish females between 1950 and 2005. The age at death of the world’s oldest person has not increased since the 1990s, and the maximum lifespan is fixed and subject to natural constraints [2]. In western countries and Japan and after age 110, the risk of dying is constant and is about 47% per year. Human life is unlimited, and anyone will live longer than 128 years in western
countries and Japan [3]. Our study contributes to this discussion from a statistical point of view and predicts the human lifespan in the world and Japan using the extreme value theory (EVT).

EVT has emerged as one of the most important statistical disciplines in applied science. Extreme value techniques are also widely used in other disciplines, such as financial market risk assessment and telecommunications traffic prediction [4]. EVT deals with statistical problems concerning the far tail of the probability distribution and is unique as a statistical tool since it develops models and techniques to describe the unusual event rather than the usual. Using EVT, the theoretical distribution and its population parameter that the maximum value follows are estimated from long-term observation data. Additionally, the maximum or large value that occurs once every period can be predicted based on the calculated results. Statistical approaches focused on extreme values have shown promising results in forecasting unusual events in earth science [5] [6], medicine [7] [8], finance [9], and sports [10].

In our study, we predict the human lifespan in the world and Japan using the extreme value theory.

2. Data and Method of Analysis

2.1. Data

We used the human lifespan in the world and Japan from 1953 to 2018 [11] as shown in Figure 1.

2.2. Method

The extreme value theory concerns the phenomena of extreme data. We used the block maxima method. A method for modeling the extremes of a stationary time series is block maxima, in which consecutive observations are grouped into non-overlapping blocks of length n, generating a series of m block maxima, Mn, 1, ..., Mn, m, to which the Generalized Extreme Value (GEV) distribution can be fitted for some large value of n. The usual approach considers blocks of a given time length, thus yielding maxima at regular intervals [4]. Here a block

Figure 1. Plot of the human lifespan in the world and Japan.
was considered as a year, i.e., annual maxima values were used. Although the block maxima method is suitable for analyzing maximum value data, it has the disadvantage of being easily affected by one realization value and having a large variance of the estimator.

When data are taken to be the maxima (or minima) over certain blocks of time (such as annual maximum precipitation), it is appropriate to use the GEV distribution:

\[
G(z) = \begin{cases} 
\exp \left[- \left(1 + \frac{\xi (z - \mu)}{\sigma}\right)^{-\frac{1}{\xi}} \right] , & \xi \neq 0, \\
\exp \left[- \exp \left(-\frac{z - \mu}{\sigma} \right) \right] , & \xi = 0,
\end{cases}
\]

where \( z \) are extreme values from blocks, \( \mu \) a location parameter, \( \sigma \) a scale parameter, and \( \xi \) a shape parameter. \( G(z) \) is defined for all \( z \) such that \((1 + \xi (z - \mu)/\sigma) > 0 \) for \( \xi \neq 0 \), and all \( z \) for \( \xi = 0 \). Three families of GEV distributions are defined depending on the value of \( \xi \). We get the Fréchet distribution with a heavy tail for \( \xi > 0 \), the Gumbel distribution with a lighter tail for \( \xi = 0 \), and the Weibull distribution with a finite tail for \( \xi < 0 \).

If a GEV distribution is fitted to observations, it becomes possible to estimate the probability of an event that has not yet been observed. Estimates of extreme quantiles of the annual maximum distribution are obtained by inverting Equation (1):

\[
z_p = \begin{cases} 
\mu - \frac{\sigma}{\xi} \left\{1 - \left(1 - \log (1 - p)\right)^{-\frac{1}{\xi}}\right\} , & \xi \neq 0, \\
\mu - \sigma \log \left(1 - p\right) , & \xi = 0,
\end{cases}
\]

where \( G(z_p) = 1 - p \). \( z_p \) is the return level associated with the return period \( 1/p \), since \( z_p \) is expected to be exceeded on average once every \( 1/p \) year with a reasonable degree of accuracy. More accurately, \( z_p \) is exceeded by the annual maximum in any particular year with probability \( p \) [4].

Modeling was performed using the evd package in R for the GEV calculations. We also tried a non-stationary model in the GEV, but it did not work.

3. Results

3.1. Human Lifespan in the World

The human lifespan in the world from 1953 to 2018 is shown in Figure 1. Estimates of the GEV parameters, which were the results of the GEV modeling on the human lifespan in the world using the block maxima method, are shown in Table 1. The GEV parameters were estimated using the maximum likelihood estimation (MLE). The model has three parameters: location parameter, \( \mu \); scale parameter, \( \sigma \); and shape parameter, \( \xi \). Because \( \xi \) was \(-0.1623 \) with a 95% confidence interval, CI, \((-0.3298, 0.005280)\), the human lifespan in the world has a finite upper limit.
Estimates of the maximum return levels for the return periods of 10, 20, 50, 100, and 500 years along with their respective 95% CI are shown in Table 2. We estimated the 10-year return level to be 118.0 years, with a 95% CI (116.7, 119.3). We estimated the 100-year return level to be 121.4 years, with a 95% CI (118.9, 123.9). We explain it differently, so it means that there is approximately a 1% chance (1/100) each year that the human lifespan will exceed 121.4 years. There is approximately a 10% chance (1/10) each year that the human lifespan will exceed 118 years.

The diagnostic plots for assessing the accuracy of the GEV model fitted to the human lifespan in the world are shown in Figure 2. Straight lines and curves in the solid lines represent estimated functions. Each point plot and short-dashed line are a realization value. The points on both sides represent the 95% CI. Probability and quantile plots show the validity of the proposed model: each set

![Figure 2. Diagnostic plots for GEV fit to the human lifespan in the world.](image)

<table>
<thead>
<tr>
<th>Parameter estimate</th>
<th>μ</th>
<th>σ</th>
<th>ξ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard error</td>
<td>0.4292</td>
<td>0.2964</td>
<td>0.08698</td>
</tr>
<tr>
<td>95% CI</td>
<td>(112.4, 114.1)</td>
<td>(1.923, 3.084)</td>
<td>(−0.3298, 0.005280)</td>
</tr>
</tbody>
</table>
Table 2. GEV return level estimates in the human lifespan in the world.

<table>
<thead>
<tr>
<th>Return period (years)</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return level (years)</td>
<td>118.0</td>
<td>119.2</td>
<td>120.5</td>
<td>121.4</td>
<td>123.1</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.6483</td>
<td>0.7894</td>
<td>1.046</td>
<td>1.283</td>
<td>1.917</td>
</tr>
<tr>
<td>95% CI</td>
<td>(116.7, 119.3)</td>
<td>(117.6, 120.7)</td>
<td>(118.5, 122.6)</td>
<td>(118.9, 123.9)</td>
<td>(119.3, 126.8)</td>
</tr>
</tbody>
</table>

of points follows a near-linear behavior. The corresponding density estimate is consistent with the data. In the return level curve, the estimated curve is nonlinear with a downward convex shape due to the negative $\xi$. Consequently, all diagnostic plots supported the fitted GEV model.

### 3.2. Human Lifespan in Japan

The human lifespan in Japan from 1953 to 2018 is shown in Figure 1. Estimates of the GEV parameter, which were the results of the GEV modeling on the human lifespan in Japan using the block maxima method, are shown in Table 3. Because $\xi$ was $-0.2949$ with a 95% CI ($-0.6350, 0.04516$), the human lifespan in the world has a finite upper limit.

Estimates of the maximum return levels for the return periods of 10, 20, 50, 100, and 500 years along with their respective 95% CI are shown in Table 4. We estimated the 10-year return level to be 115.9 years, with a 95% CI (114.7, 117.0). We estimated the 100-year return level to be 118.1 years, with a 95% CI (115.6, 120.7). Another way to interpret the plot is to say that there is approximately a 1% chance (1/100) each year that the human lifespan will exceed 118.1 years. There is approximately a 10% chance (1/10) each year that the human lifespan will exceed 115.9 years.

The diagnostic plots for assessing the accuracy of the GEV model fitted to the human lifespan in Japan are shown in Figure 3. In the return level curve, the estimated curve is nonlinear with a downward convex shape due to the negative $\xi$. All diagnostic plots supported the fitted GEV model.

### 4. Discussion

The human lifespan in the world and Japan from 1953 to 2018 is shown in Figure 1. The two tendencies are similar, but that in the world is more variable. The human lifespan in the world and Japan had shape parameters, $\xi$, of $-0.1623$ with a 95% CI ($-0.3298, 0.005280$) and $-0.2949$ with a 95% CI ($-0.6350, 0.04516$). Since $\xi$ was negative, it had an upper limit.

The return levels at each return period for the human lifespan in the world and Japan are shown in Figure 4. In the case of $\xi < 0$, the plots deviated from the straight line and were convex upward.
Figure 3. Diagnostic plots for GEV fit to the human lifespan in Japan.

Figure 4. Return level at each return period for the human lifespan in the world and Japan.

The return levels at each return period for the human lifespan in the world and Japan in a log-log plot are shown in Figure 5. Approximately straight lines are also shown. Table 5 shows the results when the approximate straight line is $y = bx^a$. Both cases were well approximated to the straight lines, following a power
Figure 5. Return level at each return period for the human lifespan in the world and Japan in a log-log plot.

Table 3. GEV parameter estimates in the human lifespan in Japan.

<table>
<thead>
<tr>
<th>Parameter estimate</th>
<th>μ</th>
<th>σ</th>
<th>ξ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard error</td>
<td>0.5310</td>
<td>0.4090</td>
<td>0.1735</td>
</tr>
<tr>
<td>95% CI</td>
<td>(110.6, 112.7)</td>
<td>(1.774, 3.377)</td>
<td>(−0.6350, 0.04516)</td>
</tr>
</tbody>
</table>

Table 4. GEV return level estimates in the human lifespan in Japan.

<table>
<thead>
<tr>
<th>Return period (years)</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return level (years)</td>
<td>115.9</td>
<td>116.7</td>
<td>117.6</td>
<td>118.1</td>
<td>119.0</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.5771</td>
<td>0.7162</td>
<td>1.030</td>
<td>1.307</td>
<td>1.949</td>
</tr>
<tr>
<td>95% CI</td>
<td>(114.7, 117.0)</td>
<td>(115.3, 118.1)</td>
<td>(115.6, 119.6)</td>
<td>(115.6, 120.7)</td>
<td>(115.2, 122.8)</td>
</tr>
</tbody>
</table>

Table 5. The results when the approximate straight line is $y = bx^a$ in the human lifespan in the world and Japan. The correlation coefficient is indicated by $r$.

<table>
<thead>
<tr>
<th></th>
<th>World</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$1.014 \times 10^{-2}$</td>
<td>$6.102 \times 10^{-3}$</td>
</tr>
<tr>
<td>$b$</td>
<td>115.6</td>
<td>114.5</td>
</tr>
<tr>
<td>$r$</td>
<td>0.9935</td>
<td>0.9817</td>
</tr>
</tbody>
</table>

law. A power low, known as a scaling law, is a relation of the type $y = bx^a$, where $y$ and $x$ are variables of interest, $a$ is called the power exponent, and $b$ is a constant. This indicates a correlation between the return level and the return period.
The inclination of the world \((1.014 \times 10^{-2})\) is larger than that of Japan \((6.102 \times 10^{-3})\). The human lifespan in the world will grow more in the future.

The calculated upper limit in the world was 128.7 years with a 95% CI (114.2, 143.2). The world’s oldest record holder, Jeanne Calment’s age of 122.45 years, was close to the 260-year return level and was far from the calculated upper limit. Based on a tendency for survival probability to be maximized in modern human survival curves, they developed an estimation method for maximum human lifespan and obtained about 126 years for Swedish females for 1950-2005 [1].

The calculated upper limit in Japan was 120.4 years with a 95% CI (112.5, 128.3). Japan’s oldest record holder, Kane Tanaka’s age of 119 years in 2022, was the 500-year return level and was close to the upper limit. Hence, in the world, achieving the calculated limit was difficult. However, in Japan, the human lifespan will soon reach the upper limit.

5. Conclusions

Extreme value theory can provide methods to analyze the most extreme parts of data. We used the generalized extreme value (GEV) distribution to predict the human lifespan in the world and Japan. The results are summarized as follows:

1) The diagnostic plots, which assessed the accuracy of the GEV model, were fitted to the human lifespan, validating the model.

2) The human lifespan in the world and Japan had shape parameters of \(-0.1623\) and \(-0.2949\) and had an upper limit.

3) The calculated upper limit in the world was 128.7 years. The world’s oldest record holder, Jeanne Calment’s age of 122.45 years, was close to the 260-year return level and was far from the calculated upper limit.

4) The calculated upper limit in Japan was 120.4 years. Japan’s oldest record holder, Kane Tanaka’s age of 119 years in 2022, was the 500-year return level and was close to the upper limit. In the world, achieving the calculated limit was difficult, but the human lifespan will soon reach the upper limit in Japan.

5) The relationship between the return level of the human lifespan and the return period follows a power law.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References


