Biomechanical Considerations in the Unruptured Cerebral Aneurysm Study (UCAS Japan): Rupture Risk and True Stress of Wall

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Abstract

When an unruptured aneurysm is found, deciding whether to operate or follow up is one of the most important issues. There are guidelines for making the best final decision on treatment, taking into account the effectiveness of diagnostic and therapeutic devices and the risk-benefit ratio of patients, caregivers, and healthcare professionals. The guidelines evidence-based of large clinical data for this purpose are presented by national medical societies. As one of the rupture risk indicators, there is the hazard risk ratio derived by the UCAS Japan research group based on the statistical method of 6697 aneurysms in 5720 patients with cerebral aneurysms of 3 mm or more. Therefore, we investigated the biomechanical significance of this hazard risk ratio using a spherical aneurysm model. It was revealed that 1) the reason why the frequency of aneurysm rupture is relatively high up to about 10 mm, 2) the UCAS hazard risk ratio corresponds to stress of the aneurysm wall, and the true stress can be calculated by multiplying the patient’s blood pressure, and 3) the factors that cause the daughter’s sac (irregular protrusion of the aneurysm wall). In addition, our two methods for measuring the strength of the
blood vessel wall of an individual patient were described.

**Keywords**
Cerebral Aneurysm, Rupture Risk, UCAS, Hazard Ratio, Biomechanics

### 1. Introduction

An aneurysm occurs in locally weakened part of the blood vessel wall, and the part that expands to more than 1.5 times the size of normal arterial diameter is called aneurysms [1] [2] [3]. And aneurysms can occur in any part of the body, but are most common in the brain, thoracic aneurysms, abdominal aortic aneurysms and legs [4] [5]. According to recent reports, diagnostic imaging using high-resolution magnetic resonance imaging (MRI) scans often accidentally finds a few millimeters of unruptured aneurysms. And it has become commonly known that a few percent of adults have an aneurysm, an average frequency of about 3 - 5 percent, mean age 50 years [6] [7] [8] [9]. The risk assessment of rupture, that is, evidence-based examination decisions for unruptured aneurysm, and whether surgery is necessary will take into account age related, smoking, hypertension, size, location, growth rate 5 years after diagnosis, family history of the aneurysm and so on [10]-[16].

In recent years, several guidelines including the latest issues have been published that provide comprehensive evidence-based recommendations for managing patients with unruptured aneurysms by the American Heart Association/Stroke Association [6], the European Society of Cardiology (ESC) [17] [18], and the Unruptured Cerebral Aneurysm Study (UCAS) in Japan [19] [20] [21]. These guidelines aim to provide guidance for making the best final decision taking into consideration the effects of specific diagnostic or therapeutic measures and the risk-benefit ratio by patients, caregivers and responsible healthcare workers.

On the other hand, biomechanical considerations [22]-[30] are one of tools to understand the actual rupture risk assessment and to predict its occurrence. In particular, the aneurysm shape model obtained from the XCT and MRI images were analyzed by finite element analysis. And then the wall stress criterion [24] [26], rupture potential index (RPI) [27] [28], and the Peak Wall Rupture Risk Index (PWRI) [29] [30] based on mechanical and mathematical consideration [31] [32] have been proposed. However, in these reports, arterial wall stress is mainly discussed using nominal stress. The vessel wall is a soft material that deforms significantly when it ruptures. Therefore, it should use the true stress by the cross-sectional area at the time of rupture. Yangkun et al. reported [33] that the importance of the relationship between the wall and rupture risk based on the data measured skillfully the sphere model size of saccular aneurysm in real-time using a dielectric elastomer capacitance sensor. The authors also re-
ported on the relativity of shape, size and wall strength for the rupture risk assessment of aneurysm [34]. And the three important physical quantities were shown to be the shape of the aneurysm, the true stress due to blood pressure, and the strength of the patient’s artery wall.

In this paper, we demonstrated that the hazard risk value derived based on the clinical data statistics published by the UCAS Japan investigators corresponds to the stress of a spherical aneurysm model’s wall. In addition, true stress of the vessel wall can calculate by multiplying the blood pressure. Therefore, the risk of rupture can be quantitatively examined by comparing it with the rupture strength in vivo of the patient’s vessel wall. We also discussed the factors that cause the daughter sac (irregular protrusion of the aneurysm wall) from a mechanical point of view. It was also described how to measure the mechanical strength of a patient’s vessel wall by using the echo image of the arterial wall and by using the natural frequency of the blood vessel wall due to the pulse wave.

2. Materials and Methods

Data Source

The UCAS Investigators examined about the natural course of unruptured cerebral aneurysms in a Japanese Cohort from 2001 for 8 years, follow-up period [19]. And out of 6697 studied (5720 patients) aneurysms of 3 mm or larger, average aneurysm size 5.7 ± 3.6 mm, mean age 62.5 ± 10.3 years, % ruptured patients 0.93/year, and risk of rupture (the hazard ratio) by five size-category has shown as follows: 1) 3 to 4 mm, 2) 5 to 6 mm, 3) 7 to 9 mm, 4) 10 to 24 mm, and 5) 25 mm or larger.

Figure 1(a) shows the relation between aneurysm size and the hazard risk that

![Figure 1](image-url)

**Figure 1.** (a). Baseline characteristics of size and rupture risk, the classification derived by the Unruptured Cerebral Aneurysm Study (UCAS) based on 5720 patients (6697 aneurysms) over the age of 20. The hazard ratio is the ratio with an aneurysm with a diameter of 3 mm as the denominator. The figure was quoted from the report [20] and partially revised. (b) Changes in true stress of the wall with the expansion of an aneurysm under internal pressure. Note that as the sphere expands, the wall thickness (r*) decreases by 1/(ni)², creating very high stresses.
have derived by the UCAS study [20] [21]: the aneurysm illustration was partially modified from the reference [20].

The hazard ratio has been quantified by the frequency of rupture, which means the rupture rate when the size category: 3 - 4 mm is set to one, from which the ratio abruptly increases. Annual rupture rate is the percentage of patients who ruptured in each size category. In this report below, the graphs were made with the data for the value of a 3 mm aneurysm as a ratio of one.

**Figure 1(b)** shows the expansion ratio ($n_i$), thickness reduction, and true stress ratio of sphere model under internal pressure. Note that the wall thickness ($t$) of the expanded model is reduced by $1/(n_i)^2$, resulting in very high stress.

### 3. Results

**Figure 2** shows the relationship between aneurysm size and wall thickness reduction ratio calculated from the data of **Figure 1(a)**. Here, the shape of the aneurysm was approximated by a sphere and a spheroid.

From the graph, it can be seen that the decrease in wall thickness is inversely proportional ($1/n_i^2$) to the square of the $n_i(=d_i/d_1)$ for sphere and about $(1.06/n_i^2)$ for spheroid (in case of $a = 1, 2, 3, 4, 5, 6, b/a = 0.4, 0.5, 0.6, 0.8$, and $c/a = 0.2, 0.5$ [34]). Where $a, b, c$ are the length of three-axis of a spheroid.

It should be noted that the thickness decrease rapidly between the size of $d_i = 3 - 8$ mm. These results indicate the presence of large stresses on the vessel wall. Thus, it is essential to examine the problem of rupture with the true stress using the actual thickness.

In **Figure 1(b)**, the expansion ratio ($n_i$) and thickness reduction ($t$) are shown.

![Graph of wall thickness reduction ratios due to expansion of sphere and spheroid models.](image)

**Figure 2.** Wall thickness reduction ratios due to expansion of sphere and spheroid models. The thickness decreases in inverse proportion to the square of the size ratio ($n = d_i/d_1$). From 1 to 10 mm ($n = 10/3 = 3.3$ times), the wall thickness is significantly reduced (stress is increased). A 3 mm sphere ($a = 1.5$ mm) is under stressed of $0.75p/t'$; $p$ is blood pressure, $t'$ is actual wall thickness.
Next, we discuss the hazard risk value derived by UCAS researchers based on biomechanics.

3.1. Biomechanical Considerations

**True stress of arterial wall**

*Figure 3* shows the stress expression for sphere and spheroid models. The wall of aneurysm is subjected to three stresses due to internal blood pressure: circumferential stress ($\sigma_\theta$), longitudinal stress ($\sigma_z$), and radial stress ($\sigma_r$). In fact, these stresses need to be calculated by using true stress to solve the rupture problem, however other reports often discussed by using nominal stresses [24]-[32]. The true stress can be calculated from the specimen length at the fracture point and the constant volume (incompressible) condition: true stress $\sigma_u = \sigma_o \left(1 + \epsilon_u\right) = \sigma_o \lambda$, $\sigma_o$ is nominal stress, $\lambda = (l/l_o)$ is elongation ratio.

1) Sphere ($a = b = c$, $d = 2a$): for all location

$\sigma_u = \sigma_o \left(1 + \epsilon_u\right) = \sigma_o \lambda$, $\sigma_o$ is nominal stress, $\lambda = (l/l_o)$ is elongation ratio.

thickness reduction $t' = 1/n^2$, expansion ratio $n = d_d/d_o$

For example, as in *Figure 1* and *Figure 2*, the stress of an early detected 3 mm spherical aneurysm is $0.75p/t'$.

2) Spheroid ($b = c$): at the top T

*Figure 3*. Coordinate representation of stresses acting on aneurysm wall: true stress can be calculated by using the actual thickness ($t$) at the rupture.
\[
\sigma_\varphi = \sigma_z = \frac{pa^2}{2h'}, \text{ which is } (a/b) \text{ time of the sphere.}
\]

\[
\sigma_r = -p, \quad t' = 1.06/n^2
\]

(2)

The coefficient (1.06) was obtained in case that \(a = 1, 2, 3, 4, 5, 6\), \(b/a = 0.4, 0.5, 0.6, 0.8\), and \(c/a = 0.2, 0.5\).

At the equator E: when \(a/b = \sqrt{2}\), hoop stress \(\sigma_\varphi = 0\).

\[
\sigma_\varphi = \frac{pa/t'}{1 - \frac{1}{2} (a^2/b^2)}
\]

\[
\sigma_z = \frac{pa}{2t'}: \text{ the same as sphere}
\]

\[
\sigma_r = -p, \quad t' = 1.06/n^2
\]

(3)

For material failure problems under multi-axial load conditions, an equivalent uniaxial tensile stresses (such as the von Mises stress) are commonly used as a criterion for isotropic and ductile materials. Blood vessels, on the other hand, are viscoelastic materials, but the dilation of aneurysms is a very slow phenomenon (the function of time is approximately zero and the viscosity term is small).

Therefore, we used the von Mises stress fracture criteria as in a textbook of solid mechanics.

The von Mises stress is expressed by

\[
\sigma_{\text{Mises}} = \left[\frac{1}{2} \left( \sigma_1 - \sigma_2 \right)^2 + \left( \sigma_2 - \sigma_3 \right)^2 + \left( \sigma_3 - \sigma_1 \right)^2 \right]^{1/2}
\]

(4)

And the rupture criterion is \(\sigma_{\text{Mises}} \geq \sigma_{\text{u, arterial tensile strength}}\)

(5)

where \(\sigma_1, \sigma_2, \text{ and } \sigma_3\) are three principal stresses: \(\sigma_1 = \sigma_\varphi, \sigma_2 = \sigma_z, \text{ and } \sigma_3 = \sigma_r\) at the crown top and at the equator. That is, if the von Mises stress is equal to or greater than the patient’s arterial strength (true tensile stress), the arterial wall cracks and the aneurysm will rupture.

As an example for a spherical aneurysm with the initial diameter \(d_i = 3\) mm, \(t_i = 1\) mm, and dilated diameter \(d' = 9\) mm, \(t' = (1/3)^2\), using Equation (1) the stresses are

\[
\sigma_1 = \sigma_2 = \sigma_\varphi = \sigma_z = 20.25 \rho = 323.2 \text{ kPa}
\]

and \(\sigma_3 = \sigma_r = -p = -16 \text{ kPa} \) (when \(p = 120\) mmHg).

From Equation (4), \(\sigma_{\text{Mises}} = 339.2 \text{ kPa}\), which is smaller than arterial strengths 0.5 - 2.1 MPa (these data were measured using the common carotid artery of 9-autopsy of 80’s by authors), therefore this aneurysm will not rupture. However, since strength varies from person to person, it is necessary to measure the strength in vivo. As an example, author’s methods for measuring strength using echo images and the natural vibration analysis [35] [36] are described in the Appendix.

3.2. Hazard Ratio in the UCAS Study

Figure 4 shows the relationship between the hazard ratio by the UCAS and the aneurysm size \((d_i)\), and also the relationship between the stress increase ratio, \((\sigma/\sigma_{3\text{mm}})\), of 3-mm-aneurysm and \(d_i\).
Figure 4. The relationship between the hazard ratio of UCAS and aneurysm size, and the relationship between the stress ratios ($y$) based on 3-mm aneurysm and aneurysm size: $y$-value curve (red dashed line) is equivalent to the hazard ratio value, and the wall stress is $\sigma_\theta = nyp$.

The true stress of the wall with increasing $d_i$ is as follows using Equation (1).

$$\sigma_\theta = \sigma_z = pa'/2t' = \left( a_1n^2/2t_1 \right) np = ynp$$  \hspace{1cm} (7)

where $a' = a_1n$, $t' = t_1/n^2$, and $y = a_1n^2/2t_1$, $2a_i = d_i$.

The $y$-curve (red dashed line) in Figure 4 represents the true stress of the dilated aneurysm, and the true stress is given by multiplying by $np$. Therefore, this figure indicates that there is a good relationship between the hazard ratio of UCAS and the $y$-curve. The correlation equation for the $y$-curve is as follows:

$$y = 0.0833d_i^2, \hspace{1cm} R^2 = 0.99.$$  \hspace{1cm} (8)

Therefore, the hazard ratio corresponds to the magnitude of true stress, which is very informative to assess rupture of aneurysm. For example, when $d_i = 2a' = 9$ mm, and $n = 9/3 = 3$, the ratio ($y$) is 6.74, then $\sigma_\theta = ynp = 20.24p$, which is the same with 20.25$p$ of the result using Equation (1). Therefore, we can quickly obtain the true stress of the wall from the aneurysm size ($d_i$) and Equation (8).

Daughter sac

Figure 5 shows the relation between true stresses ($\sigma_\theta$ and $\sigma_z$) and size ($d_i$) at the crown top (T) and at the equator (E) of sphere and spheroid models.

For the sphere, the stress $\sigma_\theta = \sigma_z$ is the same uniform stress state. It is shown by the dashed red line for comparison with other stresses.

For the spheroid, $\sigma_\theta = \sigma_z$ at the T, and $\sigma_\theta$ and $\sigma_z$ increase as the value of $(b/a)$ decreases, this is $(a/b)$ times that of the sphere.

At the E, $\sigma_z$ is the same value as the sphere. The $\sigma_\theta$ is zero when $b/a = 1/\sqrt{2}$, and it is greater than $b/a = 1/\sqrt{2}$, $\sigma_\theta$ becomes negative. Therefore, the spheroid
Figure 5. The relationship between true stress ($\sigma_\theta$ and $\sigma_z$) of arterial wall and size ($d_0$) at the crown top (T) and at the equator (E) of the sphere and spheroid models: when $b/a$ is less than 0.707, negative hoop stress is applied to the equator, which can cause the sphere to become concave.

dehorns inward on the equator and the deformation depends on the stiffness of the material.

In addition, since $\sigma_\theta$ and $\sigma_z$ at T are large, the deformation of the crown top area is promoted, and it may become a cocoon-like shape.

Figure 6(a) shows the hazard risk ratio for both types with and without of daughter sac, quoted from the UCAS study [19] [20]. In the figure, the hazard ratio was 1.64, but no aneurysm size data was shown.

Figure 6(b) shows the relationship between ($b/a$) and the stress ratio (spheroid/sphere) by using Equation (2). The stress ratio increases with the value of $z = (a/b)$, and the ratio of 1.64 correspond to a spheroid of ($b/a$) = 0.61. That is, the stress at T of spheroidal aneurysm is 1.64 times the stress of the sphere. Therefore, the above example ($d' = 9$ mm, $\sigma_{\text{Mises}} = 339.2$ kPa) is $339.2 \times 1.64 = 556.3$ kPa, which increases the possibility of rupture. This is the difference between the two types, sphere and spheroid. As shown in the figures, these stresses related to size and shape are expected to be useful data for assessing rupture.

3.3. Rupture Risk Assessment Based on Biomechanics

As mentioned above, aneurysm ruptures when the true stress of the vessel wall due to blood pressure equals the strength of the patient’s artery. Therefore, major tasks are location, shape, size, and wall stress, and blood vessel strength from a biomechanical point of view (Figure 7).

(a) Location and shape: types of aneurysms and size change
(b) True stress: wall stress due to reduced thickness
(c) In vivo strength: strength of each patient’s artery by non-invasive measurement
Figure 6. (a) The hazard ratio for both types with and without of daughter sac which derived by the UCAS Japan study [20]: the ratio suggests that the stress is 1.64 times larger than that of a spherical without daughter sac aneurysm. (b) Change in the stress ratio (spheroid/sphere) vs. (b/a): the ratio increases with \( z = \frac{a}{b} \) value.

Figure 7. Major tasks required to assess the risk of aneurysm rupture from biomechanical viewpoint.
In this consideration, we used a spherical model to examine the hazard risk of UCAS analyzed statistically. However, in actually, shape of the aneurysm is irregular and locally discontinuous and individual stress analysis of the aneurysm requires considerable time.

Then, once again, we give attention to Figure 5, the difference between the data of UCAS and the spherical model can be considered as the difference between clinical data and model geometry. For example, at $d_i = 25$ mm, the hazard ratio = 76.3 (black dot) and 52 on the $y$-curve (Equation (8), red dash line), and this difference is about 1.5 times (76.3/52). Therefore, this is one of differences between clinical data and the results of the spherical model. Therefore, if the actual shape of the aneurysm is irregular, the hazard risk can be predicted to be about 1.5 times. In the future, it will be necessary to consider quantitatively based on further clinical data.

This report mainly described the risk of rupture due to blood pressure and dilation of the aneurysm. Hemodynamics, on the other hand, are thought to play an important role in the pathogenesis and treatment of head aneurysms [34] [35], but it is difficult to measure the hemodynamics of interest in vivo. This study does not address the problem of impaired blood flow to the lining of blood vessels, hemodynamic analysis.

4. Conclusions

The biomechanical signification of the hazard risk ratio of 6697 unruptured aneurysms reported by the UCAS Japan research group based on statistical method was discussed using a spherical aneurysm model. It is summarized below.

1) As the unruptured aneurysm expands, the wall thickness decreases by a square of the aneurysm expansion ratio, therefore the true stress on the wall increases by the square. Especially the size is up to 10 mm, the change is large, and so it is important to manage the aneurysm size.

2) It was found that the hazard risk ratio of UCAS corresponds to stress of the aneurysm wall, and multiplying this value by the patient’s blood pressure gives the true stress value.

3) When size of aneurysm was about 10 to 25 mm, the difference in hazard risk ratio between the spherical model and the clinical aneurysm was about 1.5 times (difference between actual and spherical shape). It is important to manage the true stress of the aneurysm wall and the strength of the patient’s blood vessels to quantitatively determine the likelihood of rupture.

4) In case of shape of the aneurysm is close to a thin ellipsoid, it is highly possible that a daughter sac will be formed.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

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Appendix

1) A method using echo images of the CCA [36] [37]

Measurement principle: Blood vessels have the property that their elongation and strength with aging shows non-linearity (viscoelasticity), therefore, the stiffness was defined by the linear gradient ($\text{Eth} = \Delta \sigma/\Delta \varepsilon$) between changes in resting blood pressure ($p_{d} - p_{s}$).

The diameter variation of $d_i$ and $d_o$ with time is measured from B-mode moving image of a couple of heartbeats. And we assumed that $d_o = d_i + \text{wall thickness}$ ($t = 0.5 - 0.6$) from $r = 0.4 - 0.8 \text{ mm}$ of the age of 40 s - 80 s in an anatomical textbook. The trial software developed for analyzing the variation of $d_i$ allows real-time measurement. The stiffness is

$$\text{Eth} = \sigma_{io}/\varepsilon_{io}$$

$$\sigma_{io} = \left( (d_o^2 + d_i^2) p_s - (d_o^2 + d_i^2) p_o \right) \left( d_o^2 - d_i^2 \right)$$

From the continuity condition (incompressibility, no volume-change between $p_s$ and $p_o$), it is given as, $d_o^2 - d_i^2 = \bar{d}_o^2 - \bar{d}_i^2$, where $\sigma_{io}$ and $\varepsilon_{io}$ are circumferential stress and strain at the inner wall (crack occurs on the inner surface), and underline means dilated diameter. The values of $d_i$ and $\bar{d}_i$ can be measured using ultrasonic B-mode video.

Next, an empirical correlation between Eth and fracture strength ($\sigma_s$) was obtained from the tensile and internal pressure burst test data: common carotid artery (CCA) of 80 s and animals (young sheep), step-by-step loading and finally burst testing.

Figure A1 shows a correlation curve between the stiffness Eth and burst pressure (Bp) as well as fracture strength ($\sigma_s$). A correlation curve obtained (Figure A1) is,

$$\text{Bp} = 7.0 \text{ Eth}^6 - 109.7 \text{ Eth}^5 + 689.2 \text{ Eth}^4 - 2234.5 \text{ Eth}^3 + 3951.8 \text{ Eth}^2 - 3727.9 \text{ Eth} + 1975.4, \text{ mmHg (133.3 Pa)}$$

This Bp can calculate the predicted burst pressure of CCA from the Eth value using the echo images. Here, the size of CCA was calculated with the average values of test pieces $d_o = 8.2$, $d_i = 6.8 \text{ mm}$, that is, $\sigma_s$ depends on the different diameters of each individual.

Figure A2 shows a trial system for estimating sclerosis of in vivo artery using B-mode image. A linear-type ultrasound transducer (7.5 MHz, 540 × 420 pixel, 0.0713 mm/pixel) was used. The example was 16 years male, and the estimated in vivo stiffness Eth = 0.154 MPa, and Bp = 1487 mmHg (198 Pa) from Equation (a2).

Because sclerosis of the artery proceeds in localized areas, we also measured stiffness distribution by moving the ultrasound transducer linearly and intervals of 3 - 5 mm for nine positions were analyzed.

Figure A3 shows the results of in vivo Eth in a 64-year-old woman (here, 5
From this figure, in vivo Eth has slight dependence on location, with the maximum Eth = 1.01 MPa. The ruptured blood

**Figure A1.** A correlation curve of measured data (CCA specimens of human and sheep, we measured at the UNSW, Australia).

16-year male
pulse rate: 71/min.
Ps: 113mmHg
Pd: 70mmHg

outer diameter:
do = 5.35+1.6 mm at Pd
do’ = 6.15+1.6 mm at Ps
inner diameter:
di = 5.35 mm at Pd
di’ = 6.15 mm at Ps

wall thickness
\( t = (d_o - d_i)/2 = 0.8 \text{ mm} \),
\( \Delta d_o = d_o’ - d_o \), \( \Delta d_i = d_i’ - d_i \)

The maximum stress \( (\sigma_0) \) occurs at the inner wall of the artery.

**Figure A2.** A trial system for estimating sclerosis of in vivo artery using B-mode image, and the result of young student data.
pressure of the CCA from Equation (a2) is $B_p = 548$ mmHg (73.1 kPa). We believe that it is effective to use as an index of arteriosclerosis.

2) A method using the natural frequency of near artery [38]

Measurement principle: This is a new technique to measure stiffness ($E_{th}$) without the use of ultrasound equipment. The measurement principle is to use the relationship between the natural frequency of the blood vessel wall due to the heartbeat pulse wave and the stiffness. All structure has a natural frequency which is depend on mass ($m$) and stiffness ($k$, spring constant) of the system as response after applied force. Therefore, it can measure the natural frequency of bulge vibration generated.

A simple mass-spring system was used as in Figure A4, the equation of motion is

$$m \left( \frac{d^2x}{dt^2} \right) + kx = 0, \quad \omega_n^2 = \frac{k}{m},$$

that is, the frequency is $f = \omega_n / 2\pi$.

A prototype measurement device was made to analyze the vibration of the arterial wall. After confirming the performance of the measuring instrument using several tuning forks, the correlation equation between $E_{th}$ (CCA, superficial temporal artery near the wall of the external auditory canal, brachial artery, and radial artery) and the natural frequency ($f_n$) was obtained (Figure A5 and Equation (a3)).

$$E_{th} = 0.014 (f_n) - 0.13, \quad R^2 = 0.66. \quad \text{(a3)}$$

Figure A6 shows a typical measurement example. Vibration waveforms were recorded at 5 points including the heart at the same time, and the natural frequency was obtained by FFT analysis. The heart waveform was used as the basic time axis.
Figure A4. Mass and spring model.

Figure A5. A correlation between the natural frequencies measured (fn) on the skin surface and stiffness (Eth) of the CCA using echo images.

Figure A6. An example of free vibration of the arterial wall recorded simultaneously at five locations, age 65.
Figure A7 shows the vision of a new earphone for testing for arteriosclerosis. This can be achieved by inserting a pair of sensors into both ear canals and measuring for a few seconds in a relaxed state. The plan is promising as a new earphone that can be measured while listening to music, is easy to use, is inexpensive, and has data recording and telemedicine via smart phone. Long-term recording is possible from young adulthood and lifelong vascular health can be managed.

Figure A7. Vision of a new earphone for examining arteriosclerosis between music and next music, which allows that long-term recording is possible from young adulthood and lifelong vascular health can be managed.