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# A New Form of Quantum Mechanical Wave Equation Unifying Quantum Mechanics and Electro-Magnetics

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## Abstract

The Schrödinger differential equation is what we usually solve for the microscopic particles in non-relativistic quantum mechanics. Niels Bohr suggested the power two of the (usually) complex answer shows the probability of the particle's existence at a point of space. Also, the time dependence of Schrodinger wave equation is one whereas for light in electromagnetism is two. In this paper, we show a solution for both problems. We derive a Wave Equation for the energy of every system. This electromagnetic wave equation is shown to convert to those classical (*i.e.* the Schrodinger) and special relativistic (*i.e.* Klein-Gordon) quantum mechanical equations. Also, accordingly there definitely is a physical meaning to answer to this wave equation. And therefore, switching the probabilistic interpretation of quantum mechanics to a deterministic one as (Albert) Einstein demanded.

## Keywords

Quantum Mechanics, Electricity & Magnetism, Wave Equations, Probabilistic vs Deterministic Presentations

## 1. Introduction

The problem of the ultimate nature of matter, or that of the elementary particle structure, is much distant from having been resolved [1] [2] [3]. To unify the structure model of the elementary particles, one of the early models suggested for it has been a protuberance in space made by wave packets *i.e.* a bell shaped function representing a particle, made of waves [4]. However, the main problem with this model is spreading (in classical case) and contracting (in relativistic case) as a function of time, meaning it can easily be shown that in this model the group ($v_g$) and phase ($v_p$) velocities of each wave (making supposedly the elementary particle, using the Fourier seris) are never equal [4]. Consequently, the duality in the elementary particle structure model is remained. This unsuccessful model of the experimental results that could either be interpreted by particle or by waves, convinced physicists that there is no single-structural model of the elementary particles [4]. Thus, physicists have accepted the wave-particle structure of the microscopic world, as the ultimate model. So, the dual wave-particle, model is now the accepted one for the elementary particles in quantum mechanics.

Also, the square of particle state function $\psi^2$ (where $\psi$ is the solution of quantum mechanical wave equation) is interpreted as equal to the probability of the particle's existence at any point of space and any time [4]. This probabilistic model of the microscopic world is called the (Niels) Bohr model.

In this paper, we first derive the _A_ngha _W_ave _E_quation (*AWE*) for a particle [5] (in a potential field). Next, we show that the *AWE* is the same as Schrödinger and Klein-Gordon equations of quantum mechanics; therefore, it indeed is a quantum mechanical wave equation. Finally, we explicitly show that the *AWE* and the *E & M* wave equations are the same.

We look at the field of a stationary charged particle and start from infinity. Advancing toward the particle center, we know that the *E & M* energy of that particle (*i.e.* $m_0c^2 - e^2/2r$) decreases as the distance from the center ($r$) decreases. The core of the present model is as follows. Attribute this decrease in energy to an increase in the permittivity [$\varepsilon = \varepsilon(r)$] of space around the particle. We take the permittivity, $\varepsilon$ as [6]:

$$\varepsilon = \frac{W_0}{W}. \tag{1}$$

where, $W_0$ is the particle's rest energy ($m_0c^2$) at infinity, and $W$ is the content of its energy at a point $r$ from the center.

We also can write Equation (1) as:

$$W = W_0 \frac{v_{lt}^2}{c^2}. \tag{2}$$

Here, we have used that $W_0/W = c^2/v_{lt}^2$, where $v_{lt}$ is the velocity of light [5] in the field of the particle.

## 2. The Angha Wave Equation

Let us now derive the *E & M* fields equation with a permittivity other than one. Starting with the Maxwell's equations in a space with charge density $\rho(r)$ and no current density (in cgs system) [6]:

$$\nabla \cdot \boldsymbol{D} = 4\pi\rho(r), \tag{3-a}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}, \tag{3-b}$$

$$\nabla \cdot \boldsymbol{B} = 0 , \tag{3-c}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} . \tag{3-d}$$

Taking, $\boldsymbol{E} = \boldsymbol{E}(r)\mathrm{e}^{-i\omega t}$, $\boldsymbol{B} = \boldsymbol{B}(r)\mathrm{e}^{-i\omega t}$, $W_0 = \hbar\omega_0$, and $W = \hbar\omega$, and performing the partial time derivatives, we get:

$$\nabla \cdot \boldsymbol{D} = 4\pi\rho , \tag{4-a}$$

$$\nabla \times \boldsymbol{E} = \frac{iW}{\hbar c}\boldsymbol{B} \tag{4-b}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{4-c}$$

$$\nabla \times \boldsymbol{B} = -\frac{iW}{\hbar c}\varepsilon \boldsymbol{E} \tag{4-d}$$

where all $\boldsymbol{D}$, $\boldsymbol{E}$, *W*, $\boldsymbol{B}$, and $\varepsilon$ are functions of *r*. Since $\varepsilon W = W_0$ then,

$$\nabla \times \boldsymbol{B} = -\frac{iW_0}{\hbar c}\boldsymbol{E} \tag{4-d'}$$

And since

$$\nabla \cdot \boldsymbol{D} = (\nabla \mathcal{E}) \cdot \boldsymbol{E} + \mathcal{E}\nabla \cdot \boldsymbol{E} = 4\pi\rho . \tag{5}$$

But since for any vector $\boldsymbol{A}$ we can write $\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$ then, from Equation (4-d'), the divergence of its left side is zero. Thus:

$$0 = \nabla \cdot \boldsymbol{E} . \tag{6}$$

In this case, and therefore Equation (5) is:

$$\nabla \cdot \boldsymbol{D} = (\nabla \mathcal{E}) \cdot \boldsymbol{E} = 4\pi\rho . \tag{7}$$

Or we can write:

$$\rho = \frac{1}{4\pi}\left[(\nabla \mathcal{E}) \cdot \boldsymbol{E}\right]. \tag{8}$$

So, the existence of an electric charge (at a point of space) is equivalent to a variable permittivity *ε*, at that point. In essence, according to Equation (8), a variable permittivity (*ε*) indeed is the cause of an electric charge density and vice versa.

For example, let us show that indeed for a particle of charge *e*, the $\int \frac{1}{4\pi}\left[(\nabla \mathcal{E}) \cdot \boldsymbol{E}\right]\mathrm{d}V$ is also the same as *e*. Using Equation (1) and entering the *E* *&M* of the particle of rest energy

$W_0$, (since we are going to find the charge of the particle itself) we can write:

$$\varepsilon = \frac{w_0}{w} = \frac{w_0}{w_0 - \frac{e^2}{2r}} = \frac{1}{1 - \frac{e^2}{2w_0 r}} . \tag{9}$$

Defining: $r_0 \equiv e^2/2W_0$, Equation (9) changes to

$$\varepsilon = \frac{r}{r - r_0} \Rightarrow \nabla\varepsilon = -\frac{r_0}{(r - r_0)^2}\hat{e}_r \tag{10}$$

where $\hat{e}_r$ is unit vector in the $r$ direction.

For this particle the Coulomb electric field [at $r(\gg r_0)$] is $\boldsymbol{E} = e \cdot \hat{e}_r / r^2$. And Equation (7) results in:

$$(\nabla \mathcal{E}) \cdot \boldsymbol{E} = 4\pi\rho \Rightarrow \rho = -\frac{1}{4\pi} \frac{r_0}{(r - r_0)^2} \frac{e}{r^2}. \tag{11}$$

At $r(\gg r_0)$ we have $\rho(r) = -er_0 / 4\pi r^4$. Integrating this $\rho$ we have:

$$er_0 \int^{r_0} -\frac{\mathrm{d}r}{r^2} = e \tag{12}$$

So, the total charge of this particle is $Q = e$, *whereas in traditional physics a particle is extended from* $r = 0$ (*where almost everything goes to infinity*), *to* $r_0$, *in our model the picture of a particle is from*

Let us go back to deriving the wave equation now: since

$$\nabla \times (\nabla \times \boldsymbol{V}) = \nabla(\nabla \cdot \boldsymbol{V}) - \nabla^2 \boldsymbol{V} \tag{13}$$

(where $\boldsymbol{V}$ is any twice differentiable vector), we take the curl of the Equation (4-d)

$$\nabla \times (\nabla \times \boldsymbol{B}) = -\frac{iW_0}{\hbar c} (\nabla \times \boldsymbol{E}).$$

$$\nabla(\nabla \cdot \boldsymbol{B}) - \nabla^2 \boldsymbol{B} = -\frac{iW_0}{\hbar c} (\nabla \times \boldsymbol{E}). \tag{14}$$

Using Equations (4-c) and (4-b) we can write:

$$-\nabla^2 \boldsymbol{B} = -\frac{iW_0}{\hbar c} (\nabla \times \boldsymbol{E}) \Rightarrow \nabla^2 \boldsymbol{B} = \frac{iW_0}{\hbar c} \left( \frac{iW}{\hbar c} \boldsymbol{B} \right) \Rightarrow \nabla^2 \boldsymbol{B} = -\frac{WW_0}{\hbar^2 c^2} \boldsymbol{B}. \tag{15}$$

On the other hand, finding the curl of the Equation (4-b) and using the identity (13) we have

$$\nabla \times (\nabla \times \boldsymbol{E}) = \frac{i}{\hbar c} \nabla \times (W\boldsymbol{B})$$

$$\nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} = \frac{i}{\hbar c} [\nabla W \times \boldsymbol{B} + W \nabla \times \boldsymbol{B}]$$

Using, $\nabla \cdot \boldsymbol{E} = 0$ and $\nabla \times \boldsymbol{B} = (-iW\varepsilon / \hbar c) \boldsymbol{E}$ we have:

$$\nabla^2 \boldsymbol{E} = -\frac{i}{\hbar c} \nabla W \times \boldsymbol{B} - -\frac{WW_0}{\hbar^2 c^2} \boldsymbol{E} \tag{16}$$

Using the identity, $\nabla^2 (\boldsymbol{r} \cdot \boldsymbol{A}) = \boldsymbol{r} \cdot \nabla^2 \boldsymbol{A} + 2\nabla \cdot \boldsymbol{A}$, that $\nabla \cdot \boldsymbol{E} = 0$, and $\nabla \cdot \boldsymbol{B} = 0$ we can convert Equations (15) and (16) respectively to:

$$\nabla^2 (\boldsymbol{r} \cdot \boldsymbol{B}) = -\frac{WW_0}{\hbar^2 c^2} (\boldsymbol{r} \cdot \boldsymbol{B}) \tag{17}$$

$$\nabla^2 (\boldsymbol{r} \cdot \boldsymbol{E}) = -\frac{i}{\hbar c} \boldsymbol{r} \cdot (\nabla W \times \boldsymbol{B}) - \frac{WW_0}{\hbar^2 c^2} (\boldsymbol{r} \cdot \boldsymbol{E}) \tag{18}$$

But, since $W$ is a function of $r$, then, its gradient, $\nabla W$ is in the direction of $\boldsymbol{r}$ thus, (no matter what the direction of $\boldsymbol{B}$ is) the term $\nabla W \times \boldsymbol{B}$ is perpendicular to $\boldsymbol{r}$, resulting in $(i/\hbar c)\boldsymbol{r}\cdot(\nabla W\times\boldsymbol{B})=0$. Therefore, from Equation (18) we have:

$$\nabla^2(\boldsymbol{r}\cdot\boldsymbol{E})=-\frac{WW_0}{\hbar^2c^2}(\boldsymbol{r}\cdot\boldsymbol{E}). \tag{19}$$

Taking, $\boldsymbol{r}\cdot\boldsymbol{E}(r)=rE_r\equiv\xi$ and/or $\boldsymbol{r}\cdot\boldsymbol{B}=rB_r\equiv\xi$, we have:

$$\nabla^2\xi=-\frac{WW_0}{\hbar^2c^2}\xi \tag{20}$$

This is the $AWE$ for the $E\,\&\,M$ fields of a particle [when $\mathcal{E}=\mathcal{E}(r)$].

## 3. The Cohesion of *AWE* with Schrodinger and with Klein-Gordon Equations

We take the energy of a system in general $W=m_0\mathcal{V}^2$ [5]. Classically (where $\mathcal{E}\ll W_0$) $W_0=m_0c^2+\mathcal{E}\approx m_0c^2$, and $\mathcal{E}=T+V$. Also, since the time dependence of the $E\,\&\,M$ fields of the particle is $\boldsymbol{E}(r,t)=\boldsymbol{E}(r)\cdot\mathrm{e}^{-i\omega t}$, $\boldsymbol{B}(r,t)=\boldsymbol{B}(r)\cdot\mathrm{e}^{-i\omega t}$ thus, $\xi=\xi(r)\cdot\mathrm{e}^{-i\omega t}$. The $AWE$ for the above is

$$\begin{gathered}\nabla^2\xi=-\frac{WW_0}{\hbar^2c^2}\xi\Rightarrow\nabla^2\xi=-\frac{2Tm_0c^2}{\hbar^2c^2}\xi\Rightarrow-\frac{\hbar^2}{2m_0}\nabla^2\xi=T\xi\\ \Rightarrow-\frac{\hbar^2}{2m_0}\nabla^2\xi+V\xi=\mathcal{E}\xi\Rightarrow-\frac{\hbar^2}{2m_0}\nabla^2\xi+V\xi=i\hbar\frac{\partial}{\partial t}\xi\end{gathered} \tag{21}$$

This is the Schrödinger wave Equation (7). Note that $\psi=\xi$ because the differential equations are the same.

From Equation (2) we have $\mathcal{V}_{lt}^2/c^2=W/W_0$. For relativistic cases the energy $W_0$ is now equal to the total energy of the system (*i.e.* $W_0=\mathcal{E}$ so $\mathcal{E}=m_0c^2\big/\left(1-\mathcal{V}_{lt}^2/c^2\right)^{1/2}$ reference [7]. We find $\mathcal{V}_{lt}^2/c^2=1-\left(m_0c^2/\mathcal{E}\right)^2$. We also find $W=\left[1-\left(m_0c^2/\mathcal{E}\right)^2\right]\mathcal{E}$. Inserting these into $AWE$ we arrive at:

$$\nabla^2\xi=-\frac{WW_0}{\hbar^2c^2}\xi=-\frac{1}{\hbar^2c^2}\left[1-\left(m_0c^2/\mathcal{E}\right)^2\right]\mathcal{E}\cdot\mathcal{E}\xi$$

$$\nabla^2\xi=-\frac{1}{\hbar^2c^2}\left(\mathcal{E}^2-m_0^2c^4\right)\xi$$

$$\Rightarrow-\hbar^2c^2\nabla^2\xi-\mathcal{E}^2\xi+m_0^2c^4\xi=0$$

$$-\hbar^2c^2\nabla^2\xi+\hbar^2\frac{\partial^2}{\partial t^2}\xi+m_0^2c^4\xi=0 \tag{22}$$

which is the Klein-Gordon equation [8].

Finally, considering that $\xi(\tau)=\xi(r)\cdot\mathrm{e}^{-i\omega t}$ we can convert the $AWE$ to the following:

$$\nabla^2\xi=\frac{W_0}{Wc^2}\frac{\partial^2}{\partial t^2}\xi. \tag{23}$$

But,

$$\frac{c^2}{v_{lt}^2}=\frac{W_0}{W}\Rightarrow\nabla^2\xi=\frac{1}{v_{lt}^2}\frac{\partial^2}{\partial t^2}\xi. \tag{24}$$

This is the $E\ \&\ M$ wave equation. Therefore, the Schrödinger, Klein-Gordon and the $E\ \&\ M$, wave equations are all derivable from the $AWE$.

## 4. Discussions

We have derived the $E\ \&\ M$, equation (for $\mathcal{E} = \mathcal{E}(r)$), and shown that it is the same as Schrödinger (and Klein-Gordon) equations. Also this solves the problem of the difference in the time dependence of Quantum mechanical and $E\ \&\ M$, wave equations. The $\psi = \xi = rE_r$ (or $rB_r$) is deterministic now. And as we will discuss in detail in future papers the $AWE$ is compatible with relativity.

This solves many problems of quantum mechanics including: it gives a deterministic version for its solution. Plus the fact that the $AWE$ is the same as the Schrödinger (and Klein-Gordon) equations means nothing is changed in quantum mechanics.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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