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# Cosmological Model in Four Time and Four Space Dimensions

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## Abstract

We develop a cosmological model in a physical background scenario of four time and four space dimensions ((4+4)-dimensions or (4+4)-universe). We show that in this framework the (1+3)-universe is deeply connected with the (3+1)-universe. We argue that this means that in the (4+4)-universe there exists a duality relation between the (1+3)-universe and the (3+1)-universe.

## Keywords

Cosmological Model, (4+4)-Dimensions, Duality Symmetry

## 1. Introduction

Recent discoveries from the James Webb Space Telescope [1] [2] [3] (JWST) have provided evidence that the standard cosmological model needs to be extended with an alternative theory, which is surely unexpected. Among these discoveries is the observation of 6 galaxies that are so close to the supposed big bang that they force us to rethink the usual ideas about the big bang and the evolution of the universe. In fact, these galaxies are larger than the Milky Way, but in the cosmological sense, they formed too early. In other words: these galaxies are where no one would expect them to be. This unexpected observation from the James Webb Space Telescope is framed by famous physicist Roger Penrose who says; "There is a big bang, but the big bang was not the beginning". In the same way, the also famous theoretical physicist Michio Kaku said: "Suddenly we realize that we need to rewrite all text books about the beginning of the universe. Now it takes thousands of millions of years to recreate a galaxy, like the Milky Way galaxy, with one hundred thousand millions of stars. But the James Webb Space Telescope identifies 6 galaxies that exist around 500 million years after the big bang, that are 10 times larger than the Milky Way galaxy" (see Ref.

[4] and references therein).

We believe that the above observations can be seen as a theoretical opportunity to propose such an alternative unexpected cosmological model. For theoretical reasons explained below, in this work, we would like to propose a cosmological model with background scenario in four time and four space dimensions ((4+4)-universe). We show that in this framework the (1+3)-universe is deeply connected with the (3+1)-universe. We argue that this means that in the (4+4)-universe there exists a duality relation between the (1+3)-universe and the (3+1)-universe. Indeed, a solution of our cosmological model allows us to choose the relation $a^{(+)}a^{(-)}=\frac{l^2}{3}$, with $a^{(+)}$ the scale factor of the (1+3)-universe and $a^{(-)}$ the scale factor associated with (3+1)-universe. Here, $l$ is some fundamental constant, such as the Plank length. An interesting aspect of this relation is that there is a duality expansion/contraction correspondence between the universes (1+3)-universe and the (3+1)-universe. In particular, if $a^{(+)}\to 0$ we have that $a^{(-)}\to\infty$ and *vice versa*. This proves that in fact "the big bang is not the beginning" as Roger Penrose said.

It seems to us it is necessary to give an argument about the relationship between theory and observations of the JWST. It is clear to any physicist that theory and observations must go hand in hand. But sometimes the theory comes forward and sometimes the experiment. Curiously, the JWST remarkable observations (6 galaxies that are so close to the big bang) are so recently that surely will require a lot of time to be able propose a correct cosmological theory. At least in this work, as mentioned in the previous paragraph, we have found a solution in which the singularity $a^{(+)}\to 0$ is not the beginning.

Let us now mention some theoretical reasons why ((4+4)-universe) may be interesting. In Refs. [5] [6] it is shown that there exists a triality relation between the signatures (9+1), (5+5) and (1+9). This means that from the (5+5)-universe we may obtain, *via* triality, the (9+1)-universe or the (1+9)-universe. At the same time it is known that (5+5)-universe is a common signature in both type *IIA* strings and type *IIB* strings [7] [8].

Since the (4+4)-universe (see Refs. [9]-[16]) can be understood as the transverse scenario of the (5+5)-universe we may conclude that just as by imposing a coordinate constraint in the (1+4)-universe it leads to a de Sitter space associated with the (1+3)-universe, with positive cosmological constant, and (2+3)-universe to the anti-de Sitter space-time associated with the (3+1)-universe, with the negative cosmological constant, a coordinate constraint in the (4+4)-universe may lead to a kind of (1+3)-universe/(3+1)-universe (de Sitter/anti-de Sitter) correspondence.

It turns out that due to a number of remarkable results, the (4+4)-universe, by itself, may be considered, mathematically and physically interesting. Physically, the Dirac equation in (4+4)-dimensions is consistent with Majorana-Weyl spinors which give exactly the same number of components as the complex spinor of $\frac{1}{2}$-spin particles such as the electron or quarks (see Refs. [10] and [15]). Second, the most general Kruskal-Szekeres transformation of a black-hole coordinates in (1+3)-dimensions leads to 8-regions (instead of the usual 4-regions), which can be better described in (4+4)-dimensions [17]. Third, it also has been shown [11] that duality $\sigma^2 \leftrightarrow \frac{1}{\sigma^2}$, of a Gaussian distribution in terms of the the standard deviation $\sigma$ of 4-space coordinates associated with the de Sitter space (anti-de Sitter) and the vacuum zero-point energy yields a Gaussian of 4-time coordinates of the same vacuum scenario. Moreover, loop quantum gravity in (4+4)-dimensions [13] admits a self-duality curvature structure analog to the traditional self-duality in (1+3)-dimensions. Furthermore, it has been shown [13] that there existed a duality symmetry in a black hole in (4+4)-dimensions. Finally, in Ref. [18] it is proved that a 4-rank totally antisymmetric field strength $F^{\hat{\mu}\hat{\nu}\hat{\alpha}\hat{\beta}}$ in (4+4)-dimensions can be broken into two electromagnetic field strengths; the antisymmetric field strength $F^{\mu\nu}$ associated with the (1+3)-universe and the antisymmetric field strength $G^{ij}$ associated with the (3+1)-universe. An interesting aspect of this result is that there is a hidden duality symmetry feature between the fields $F^{\mu\nu}$ and $G^{ij}$ in the sense that $G^{ij}$ contribute to the source of $F^{\mu\nu}$ and *vice versa*. Mathematically, it has been suggested that the mathematical structures of oriented matroid theory (see Refs. [19]-[26] and references therein) and surreal number theory (see also Refs [27] [28] and references therein) may provide interesting routes for a connection with the (4+4)-universe.

## 2. A Short Review of Cosmology in a (1+3)-Universe

We shall start considering one of the simplest cosmological models which is determined by the line element [29];

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\tilde{g}_{ij}(\xi^k)\mathrm{d}\xi^i\mathrm{d}\xi^j. \tag{1}$$

Here, we use geometric units such that the speed of light in vacuum, *c*, and the gravitational constant, *G*, are set equal to unity, *i.e.* $c=1$ and $G=1$. We also consider that $\tilde{g}_{ij}$ is a known quantity and refers to a 3-dimensional homogeneous space. In this context, the only unknown variable is $a(t)$, which may be interpreted as the radius of a 3-sphere and its evolution in time must be determined by the relativistic gravitational field equations. Moreover, $\xi^i$ denotes the three angles $(\psi,\theta,\phi)$. The ansatz associated with (1) is

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^2(t)\tilde{g}_{ij}(\psi,\theta,\phi) \end{pmatrix}, \tag{2}$$

which inverse is given by

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^{-2}(t)\tilde{g}^{ij}(\psi,\theta,\phi) \end{pmatrix}. \tag{3}$$

Using (2) and (3), we obtain that the non-vanishing Christoffel symbols are

$$\Gamma_{ij}^{1} = a\dot{a}\tilde{g}_{ij}, \quad \Gamma_{1j}^{i} = \frac{\dot{a}}{a}\delta_{j}^{i}, \\ \Gamma_{ij}^{k} = \tilde{\Gamma}_{ij}^{k}, \tag{4}$$

where $\dot{a} \equiv \frac{\mathrm{d}a}{\mathrm{d}t}$. Furthermore, from (4), we find that the non-vanishing components of the Riemann tensor are

$$R_{i1j}^{1} = a\ddot{a}\tilde{g}_{ij}, \quad R_{1j1}^{i} = -\frac{\ddot{a}}{a}\delta_{j}^{i}, \\ R_{jkl}^{i} = \tilde{R}_{jkl}^{i} + \dot{a}^{2}\left(\delta_{k}^{i}\tilde{g}_{jl} - \delta_{l}^{i}\tilde{g}_{jk}\right). \tag{5}$$

Thus, from (5) we get that non-vanishing components of the Ricci tensor $R_{\mu\nu} = R_{\mu\alpha\nu}^{\alpha}$ are given by

$$R_{11} = -3\frac{\ddot{a}}{a}, \\ R_{ij} = \tilde{R}_{ij} + \left(a\ddot{a} + 2\dot{a}^{2}\right)\tilde{g}_{ij}, \tag{6}$$

while the scalar curvature $R = g^{\mu\nu}R_{\mu\nu}$ becomes

$$R = 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^{2}}{a^{2}} + 6\frac{k}{a^{2}}, \tag{7}$$

where we had set $\tilde{R} = 6k$, with $k = \{-1,0,1\}$, which is true for a 3-dimensional homogeneous space. Our task is now to substitute (6) and (7) into the relativistic gravitational field equations [29]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}. \tag{8}$$

Here, $T_{\mu\nu}$ is the energy-momentum tensor. We get the splitting equations

$$R_{11} - \frac{1}{2}g_{11}R = 8\pi T_{11}, \\ R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij}. \tag{9}$$

Since we have only one unknown quantity $a(t)$, we can focus on the first of these two equations. Thus, substituting (6) and (7) into the the first equation of (9) gives us

$$-3\frac{\ddot{a}}{a} - \frac{1}{2}(-1)\left(6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^{2}}{a^{2}} + 6\frac{k}{a^{2}}\right) = 8\pi\rho, \tag{10}$$

where the variable

$$\rho \equiv T_{11}$$

shall be identified with the matter-energy density of the system. Simplifying (10) we end up with

$$\frac{\dot{a}^{2}}{a^{2}} + \frac{k}{a^{2}} = \frac{8\pi}{3}\rho. \tag{11}$$

Hence, given $\rho$ as a function of $a(t)$, in principle from (11), we may be able to determine $a$ as a function of time $t$ (see Ref. [28] for details).

## 3. Cosmological Model in (4+4)-Universe

Let us now consider the ansatz for the metric $g_{\mu\nu}$ in (4+4)-dimensions:

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \left(a^{(+)}\right)^2 \tilde{g}^{(+)}_{ij} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\left(a^{(-)}\right)^2 \tilde{g}^{(-)}_{AB} \end{pmatrix}, \tag{12}$$

where $\tilde{g}^{(+)}_{ij}$ is a function of three parameters, let's say $\psi^{(+)}$, $\theta^{(+)}$ and $\phi^{(+)}$, while $\tilde{g}^{(-)}_{AB}$ is a function of other three parameters, let's say $\psi^{(-)}$, $\theta^{(-)}$ and $\phi^{(-)}$. It turns out that, for a non-singular metric, $\det\left(g_{\mu\nu}\right) \neq 0$, we can always find a transformation such that this metric can also be written as

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \left(a^{(+)}\right)^2 \tilde{g}^{(+)}_{ij} & 0 \\ 0 & 0 & 0 & -\left(a^{(-)}\right)^2 \tilde{g}^{(-)}_{AB} \end{pmatrix}, \tag{13}$$

or

$$g_{\mu\nu} = \begin{pmatrix} -\eta_{ab} & 0 & 0 \\ 0 & \left(a^{(+)}\right)^2 \left(t^a\right) \tilde{g}^{(+)}_{ij} & 0 \\ 0 & 0 & -\left(a^{(-)}\right)^2 \left(t^a\right) \tilde{g}^{(-)}_{AB} \end{pmatrix}, \tag{14}$$

with $\eta_{ab} = diag\left(1,-1\right)$.

We find that the non-vanishing Christoffel symbols are

$$\begin{aligned} &\Gamma^a_{ij} = \eta^{ac} a^{(+)} \partial_c a^{(+)} \tilde{g}^{(+)}_{ij}, \quad \Gamma^i_{aj} = \frac{\partial_a a^{(+)}}{a^{(+)}} \delta^i_j, \\ &\Gamma^k_{ij} = \tilde{\Gamma}^k_{ij}, \end{aligned} \tag{15}$$

and also

$$\begin{aligned} &\Gamma^a_{AB} = -\eta^{ac} a^{(-)} \partial_c a^{(-)} \tilde{g}^{(-)}_{AB}, \quad \Gamma^A_{aB} = \frac{\partial_a a^{(-)}}{a^{(-)}} \delta^A_B, \\ &\Gamma^A_{BC} = \tilde{\Gamma}^A_{BC}. \end{aligned} \tag{16}$$

Furthermore, from (15) and (16) we learn that the non-vanishing components of the Riemann tensor become

$$\begin{aligned} &R^a_{ibj} = \eta^{ac} a^{(+)} \partial_b \partial_c a^{(+)} \tilde{g}^{(+)}_{ij}, \quad R^i_{ajb} = -\frac{\partial_a \partial_b a^{(+)}}{a^{(+)}} \delta^i_j, \\ &R^i_{jkl} = \tilde{R}^{(+)i}_{jkl} + \eta^{ab} \partial_a a^{(+)} \partial_b a^{(+)} \left(\delta^i_k \tilde{g}_{jl} - \delta^i_l \tilde{g}_{jk}\right). \end{aligned} \tag{17}$$

and

$$R^{a}_{AbB} = -\eta^{ac} a^{(-)} \partial_b \partial_c a^{(-)} \tilde{g}^{(-)}_{AB}, \quad R^{A}_{aBb} = -\frac{\partial_a \partial_b a^{(-)}}{a^{(-)}} \delta^{A}_{B},$$
$$R^{A}_{BCD} = \tilde{R}^{(-)A}_{BCD} - \eta^{ab} \partial_a a^{(-)} \partial_b a^{(-)} \left( \delta^{A}_{C} \tilde{g}_{BD} - \delta^{A}_{D} \tilde{g}_{BC} \right). \tag{18}$$

Similarly, we learn that the non-vanishing components of the Ricci tensor are

$$R_{ab} = -\frac{3\partial_a \partial_b a^{(+)}}{a^{(+)}} - \frac{3\partial_a \partial_b a^{(-)}}{a^{(-)}}, \tag{19}$$

$$R_{ij} = \eta^{cd} a^{(+)} \partial_c \partial_d a^{(+)} \tilde{g}^{(+)}_{ij} + 2\eta^{cd} \partial_c a^{(+)} \partial_d a^{(+)} \tilde{g}^{(+)}_{ij} + \tilde{R}^{(+)}_{ij} \tag{20}$$

and

$$R_{AB} = -\eta^{cd} a^{(-)} \partial_c \partial_d a^{(-)} \tilde{g}^{(-)}_{AB} - 2\eta^{cd} \partial_c a^{(-)} \partial_d a^{(-)} \tilde{g}_{AB} + \tilde{R}^{(-)}_{AB}. \tag{21}$$

Therefore, the scalar curvature $R$ becomes

$$R = \frac{6\eta^{cd} \partial_c \partial_d a^{(+)}}{a^{(+)}} + \frac{6\eta^{cd} \partial_c a^{(+)} \partial_d a^{(+)}}{\left(a^{(+)}\right)^2} + \frac{6\eta^{cd} \partial_c \partial_d a^{(-)}}{a^{(-)}}$$
$$+ \frac{6\eta^{cd} \partial_c a^{(-)} \partial_d a^{(-)}}{\left(a^{(-)}\right)^2} + \frac{6k^{(+)}}{\left(a^{(+)}\right)^2} - \frac{6k^{(-)}}{\left(a^{(-)}\right)^2}. \tag{22}$$

Here, we denote $\tilde{R}^{(+)} = 6k^{(+)}$ and $\tilde{R}^{(-)} = 6k^{(-)}$. Consequently, the *ab* relativistic gravitational field equation becomes

$$-\frac{3\partial_a \partial_b a^{(+)}}{a^{(+)}} - \frac{3\partial_a \partial_b a^{(-)}}{a^{(-)}} + \frac{1}{2}\eta_{ab} \left( \frac{6\eta^{cd} \partial_c \partial_d a^{(+)}}{a^{(+)}} + \frac{6\eta^{cd} \partial_c a^{(+)} \partial_d a^{(+)}}{\left(a^{(+)}\right)^2} \right.$$
$$\left. + \frac{6\eta^{cd} \partial_c \partial_d a^{(-)}}{a^{(-)}} + \frac{6\eta^{cd} \partial_c a^{(-)} \partial_d a^{(-)}}{\left(a^{(-)}\right)^2} + \frac{6k^{(+)}}{\left(a^{(+)}\right)^2} - \frac{6k^{(-)}}{\left(a^{(-)}\right)^2} \right) = 8\pi T_{ab}, \tag{23}$$

which can be reduced to

$$-\left( \frac{\partial_a \partial_b a^{(+)}}{a^{(+)}} - \eta_{ab} \frac{\eta^{cd} \partial_c \partial_d a^{(+)}}{a^{(+)}} \right) - \left( \frac{\partial_a \partial_b a^{(\cdot)}}{a^{(-)}} - \eta_{ab} \frac{\eta^{cd} \partial_c \partial_d a^{(-)}}{a^{(-)}} \right)$$
$$+\eta_{ab} \left( \frac{\eta^{ab} \partial_a a^{(+)} \partial_b a^{(+)}}{\left(a^{(+)}\right)^2} + \frac{\eta^{ab} \partial_a a^{(-)} \partial_b a^{(-)}}{\left(a^{(-)}\right)^2} + \frac{k^{(+)}}{\left(a^{(+)}\right)^2} + \frac{k^{(-)}}{\left(a^{(-)}\right)^2} \right) = \frac{8\pi}{3} T_{ab}. \tag{24}$$

A further simplification of (24) is

$$-\left( \delta^{c}_{a} \delta^{d}_{b} - \eta_{ab} \eta^{cd} \right) \left( \frac{\partial_c \partial_d a^{(+)}}{a^{(+)}} + \frac{\partial_c \partial_d a^{(-)}}{a^{(-)}} \right)$$
$$+\eta_{ab} \left( \frac{\eta^{cd} \partial_c a^{(+)} \partial_d a^{(+)}}{\left(a^{(+)}\right)^2} + \frac{\eta^{cd} \partial_c a^{(-)} \partial_d a^{(-)}}{\left(a^{(-)}\right)^2} + \frac{k^{(+)}}{\left(a^{(+)}\right)^2} - \frac{k^{(-)}}{\left(a^{(-)}\right)^2} \right) = \frac{8\pi}{3} T_{ab}. \tag{25}$$

We see that a possible solution of (25) depends on $T_{ab}$, which we may assume it has the form $T_{ab} = diag\left(\rho^{(+)}, \rho^{(-)}\right)$, with the quantities $\rho^{(+)}$ and $\rho^{(-)}$ associated with the mass-energy density of the (1+3)-universe and (3+1)-universe, respectively.

## 4. Searching for a Cosmological Duality in (4+4)-Universe

Since $\partial_d\left(\ln a^{(+)}\right)=\frac{\partial_d a^{(+)}}{a^{(+)}}$ and $\partial_d\left(\ln a^{(-)}\right)=\frac{\partial_d a^{(-)}}{a^{(-)}}$ we observe that (25) can also be written as

$$\begin{aligned}&-\left(\delta_a^c\delta_b^d-\eta_{ab}\eta^{cd}\right)\left(\partial_c\partial_d\left[\ln\left(a^{(+)}a^{(-)}\right)\right]+\frac{\partial_c a^{(+)}\partial_d a^{(+)}}{\left(a^{(+)}\right)^2}+\frac{\partial_c a^{(-)}\partial_d a^{(-)}}{\left(a^{(-)}\right)^2}\right)\\&+\eta_{ab}\left(\frac{\eta^{cd}\partial_c a^{(+)}\partial_d a^{(+)}}{\left(a^{(+)}\right)^2}+\frac{\eta^{cd}\partial_c a^{(-)}\partial_d a^{(-)}}{\left(a^{(-)}\right)^2}+\frac{k^{(+)}}{\left(a^{(+)}\right)^2}-\frac{k^{(-)}}{\left(a^{(-)}\right)^2}\right)=\frac{8\pi}{3}T_{ab}.\end{aligned}\tag{26}$$

Hence, if we set

$$a^{(+)}a^{(-)}=\frac{l^2}{3},\tag{27}$$

with $l$ a fixed constant, we have $\partial_c\partial_d\ln\left(a^{(+)}a^{(-)}\right)=0$ and consequently (26) is reduced to

$$\frac{3\eta^{cd}\partial_a a^{(+)}\partial_b a^{(+)}}{\left(a^{(+)}\right)^2}+\frac{3\eta^{cd}\partial_a a^{(-)}\partial_b a^{(-)}}{\left(a^{(-)}\right)^2}+\frac{2k^{(+)}}{\left(a^{(+)}\right)^2}-\frac{2k^{(-)}}{\left(a^{(-)}\right)^2}=\frac{8\pi}{3}T,\tag{28}$$

where

$$T=\eta^{ab}T_{ab}.\tag{29}$$

Since $T_{ab}=diag\left(\rho^{(+)},\rho^{(-)}\right)$ and $\eta_{ab}=diag(1,-1)$ we discover that $T$ becomes

$$T=\rho^{(+)}-\rho^{(-)}.\tag{30}$$

A further simplification of (28) is achieved if we use (27). This is because in such a case we find

$$\frac{3\eta^{cd}\partial_a a^{(+)}\partial_b a^{(+)}}{\left(a^{(+)}\right)^2}+\frac{k^{(+)}}{\left(a^{(+)}\right)^2}-\frac{9}{l^4}\left(a^{(+)}\right)^2k^{(-)}=\frac{4\pi}{3}T.\tag{31}$$

A remarkable aspect of this procedure is that the expression (27) can be interpreted as a duality relation. In the sense that if the radius $a^{(+)}$ of the (1+3)-universe increase (expansion) the radius $a^{(-)}$ of the (3+1)-universe decreases (contraction) and *vice versa*.

## 5. Cosmological Constant in a (4+4)-Universe

Except for the last term in the left-hand side of the expression (31), this formula can be understood as the analogue of Equation (11) of the traditional cosmological model in (1+3)-dimensions. In fact, such a kind of term arises when the cosmological constant is included in the field equations of general relativity (8). This is a good example of how our cosmological formalism in a (4+4)-universe may motivate further work. For this purpose, let us shorty develop how a cosmological constant can be considered in a universe of (4+4)-dimensions. A flat line element in the (4+4)-dimensions can be written as

$$ds^2 = \eta^{(+)}_{\hat{i}\hat{j}} dx^{(+)\hat{i}} dx^{(+)\hat{j}} + \eta^{(-)}_{\hat{A}\hat{B}} dx^{(-)\hat{A}} dx^{(-)\hat{B}}, \tag{32}$$

where $\eta^{(+)}_{\hat{i}\hat{j}}$ is a flat metric in a (1+3)-universe;

$$\eta^{(+)}_{\hat{i}\hat{j}} = diag(-1,1,1,1) \tag{33}$$

and $\eta^{(-)}_{\hat{A}\hat{B}}$ is a flat metric in the (3+1)-universe;

$$\eta^{(-)}_{\hat{A}\hat{B}} = diag(1,-1,-1,-1). \tag{34}$$

We shall now assume the coordinate constraint

$$\eta^{(+)}_{\hat{i}\hat{j}} x^{(+)\hat{i}} x^{(+)\hat{j}} + \eta^{(-)}_{\hat{A}\hat{B}} dx^{(-)\hat{A}} dx^{(-)\hat{B}} = l^2, \tag{35}$$

with $l$ a fixed constant. Thus, redefine (32) and (35) in the form

$$ds^2 = \eta^{(+)}_{\hat{i}\hat{j}} dx^{(+)\hat{i}} dx^{(+)\hat{j}} + dw^{(-)2} \tag{36}$$

and

$$\eta^{(+)}_{\hat{i}\hat{j}} x^{(+)\hat{i}} x^{(+)\hat{j}} + w^{(-)2} = l^2, \tag{37}$$

with

$$dw^{(-)2} = \eta^{(-)}_{\hat{A}\hat{B}} dx^{(-)\hat{A}} dx^{(-)\hat{B}} \tag{38}$$

and

$$w^{(-)2} = \eta^{(-)}_{\hat{A}\hat{B}} x^{(-)\hat{A}} x^{(-)\hat{B}}, \tag{39}$$

after a usual, but long, computations we end up with the line element

$$ds^2 = -\left(1 - \frac{\Lambda^{(+)}}{3} r^{(+)2}\right) dt^{(+)2} + \frac{dr^2}{1 - \frac{\Lambda^{(+)}}{3} r^{(+)2}} + r^{(+)2}\left(d\theta^{(+)2} + \sin^2\theta^{(+)} d\phi^{(+)2}\right), \tag{40}$$

with positive cosmological constant

$$\Lambda^{(+)} = \frac{3}{l^2}. \tag{41}$$

This means that we have reduced the (4+4)-universe to the de Sitter (1+3)-universe with positive cosmological constant $\Lambda^{(+)}$.

Now, we would like to discuss the case of anti-de Sitter space-time, which we can say is associated with negative cosmological constant $\Lambda^{(-)}$ in the (3+1)-universe. The idea is now to redefine (32) in the alternative form

$$ds^2 = dw^{(+)2} + \eta^{(-)}_{\hat{A}\hat{B}} dx^{(-)\hat{A}} dx^{(-)\hat{B}} \tag{42}$$

and

$$w^{(+)2} + \eta^{(-)}_{\hat{A}\hat{B}} x^{(-)\hat{A}} x^{(-)\hat{B}} = l^2, \tag{43}$$

with

$$dw^{(+)2}=\eta^{(+)}_{\hat{i}\hat{j}}dx^{(+)\hat{i}}dx^{(+)\hat{j}} \tag{44}$$

and

$$w^{(+)2}=\eta^{(+)}_{\hat{i}\hat{j}}x^{(+)\hat{i}}x^{(+)\hat{j}} \tag{45}$$

Since $\eta^{(-)}_{\hat{A}\hat{B}}\leftrightarrow-\eta^{(+)}_{\hat{A}\hat{B}}$, we learn that (43) can also be written as

$$-w^{(+)2}+\eta^{(+)}_{\hat{A}\hat{B}}x^{(-)\hat{A}}x^{(-)\hat{B}}=-l^2. \tag{46}$$

This, of course, is the typical form of the coordinate constraint that determines the anti-de Sitter space-time. Making usual computations at the end we must obtain the line element of the form

$$ds^2=\left(1-\frac{\Lambda^{(-)}}{3}r^{(-)2}\right)dt^{(-)2}-\frac{dr^2}{1-\frac{\Lambda^{(-)}}{3}r^{(-)2}}-r^{(-)2}\left(d\theta^{(-)2}+\sin^2\theta^{(-)}d\phi^{(-)2}\right), \tag{47}$$

with the negative cosmological constant $\Lambda^{(-)}$ given by

$$\Lambda^{(-)}=-\frac{3}{l^2}. \tag{48}$$

This means that the (4+4)-universe implies the anti-de Sitter (3+1)-universe with negative cosmological constant $\Lambda^{(-)}$.

Hence, we have shown that from the (4+4)-universe we can obtain both the de Sitter space-time, associated with (1+3)-universe and corresponding to positive cosmological constant $\Lambda^{(+)}$, and the anti-de Sitter space-time, associated with (3+1)-universe and corresponding to the negative cosmological constant $\Lambda^{(-)}$. From (43) and (46) we note that

$$w^{(+)2}+w^{(-)2}=l^2. \tag{49}$$

Therefore, since the quantity $l$ is a commune constant for both (1+3)-universe and (3+1)-universe we must conclude that the cosmological constants $\Lambda^{(+)}$ and $\Lambda^{(-)}$ must be related. In fact, if we add (41) and (48) we find the expression

$$\Lambda^{(+)}+\Lambda^{(-)}=0, \tag{50}$$

which proves that in the (4+4)-universe there exists an additive duality relation between the cosmological constants $\Lambda^{(+)}$ and $\Lambda^{(-)}$.

## 6. Final Remarks

In the previous section, it has been shown that it makes sense to consider the cosmological constant structure in the scenario of (4+4))-universe. In this context, we can say that the third term in the left hand of (31) can be interpreted as a cosmological term with $\Lambda^{(+)}=\frac{3}{l^2}k^{(-)}$ leading

$$\frac{\eta^{cd}\partial_a a^{(+)}\partial_b a^{(+)}}{\left(a^{(+)}\right)^2}+\frac{k^{(+)}}{3\left(a^{(+)}\right)^2}-\Lambda^{(+)}\frac{\left(a^{(+)}\right)^2}{l^2}=\frac{4\pi}{9}T. \tag{51}$$

Similarly, analysis from (27) and (28) we must obtain that

$$\frac{\eta^{cd}\partial_a a^{(-)}\partial_b a^{(-)}}{\left(a^{(-)}\right)^2}-\frac{k^{(-)}}{6\left(a^{(-)}\right)^2}+\Lambda^{(-)}\frac{\left(a^{(-)}\right)^2}{l^2}=\frac{4\pi}{9}T, \tag{52}$$

with $\Lambda^{(-)}=-\frac{3}{l^2}k^{(+)}$.

Hence, in both cases, the cosmological radius $a^{(+)}$ and $a^{(-)}$ associated with the (4+4)-universe describe increase/decrease (or expansion/contraction) correspondence and the cosmological constants $\Lambda^{(+)}$ and $\Lambda^{(-)}$ determine positive/negative correspondence. This proves that in (4+4)-universe duality can be seen as an underlying symmetry.

In Ref. [30] it was shown that, *via S*-duality in linearized gravity, there exists the possibility of duality transformation of the form $\Lambda\leftrightarrow\frac{1}{\Lambda}$. It may be interesting to explore this small/large duality correspondence of the cosmological constant with the duality (50) positive/negative correspondence associated with the (4+4)-universe.

Further, it has been proved that the Grassmann-Plücker relations [31] [32] [33] link seemingly unrelated concepts such as qubits, oriented matroids, and twistors. In turn, qubits and oriented matroids admit a connection with the (4+4)-universe [22] [25] and [27]. Consequently, it may be attractive for further research to connect the present work with such mathematical concepts.

Moreover, in the recent work Ref. [34], Lagrangians on split octonionic fields in (4+4)-dimensions that generalize Dirac and Maxwell systems are constructed using group invariant forms. We believe that this work may be useful for many potential directions for future research on (4+4)-universe.

Finally, the duality expression (27) allows us to say that the beginning in the (1+3)-universe corresponds to the end of the (3+1)-universe and *vice versa*. This establishes a beginning/end correspondence between the (1+3)-universe and the (3+1)-universe. An implication of this result is that there is also expansion/contraction correspondence. Thus, Penrose expression in connection with the James Webb Space Telescope "There is a big bang, but the big bang is not the beginning" can also be expressed as "there is expansion of the universe but the maxima expansion is not the end".

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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