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The Accelerated Expansion of the Universe in the Light of the Maximum Ordinality Principle

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Abstract

The main aim of the paper is to present (and at the same time offer) a different perspective for the analysis of the accelerated expansion of the Universe. A perspective that can surely be considered as being "in parallel" to the traditional ones, such as those based, for example, on the hypotheses of "Dark Matter" and "Dark Energy", or better as a "com-possible" perspective, because it is not understood as being "exclusive". In fact, it is an approach that, when confirmed by experimental results, always keeps its validity from an "operative" point of view. This is because, in analogy to the traditional perspectives, on the basis of Popper's Falsification Principle the corresponding "Generative" Logic on which it is based has not the property of the perfect induction. The basic difference then only consists in the fact that the Evolution of the Universe is now modeled by considering the Universe as a Self-Organizing System, which is thus analyzed in the light of the Maximum Ordinality Principle.

Keywords

Accelerated Expansion of the Universe, Maximum Ordinality Principle, Incipient Differential Calculus

1. Introduction

The Maximum Ordinality Principle [1] allows us to show that the Accelerated Expansion of the Universe is nothing but the "Emerging Exit", or more properly an "Emerging Quality", of the *Ordinal Nature*, of a *Generative* Process, specifically of the Universe as a Whole, which at the same time is understood as a Self-Organizing System, and thus is characterized by that above-mentioned "Emerging Quality", as precisely described by the Maximum Ordinality Principle.

To this purpose, it is worth preliminarily recalling the *basic aspects* pertaining to the Maximum Ordinality Principle and its subjacent perspective.

2. The Maximum Ordinality Principle, its "Emerging Quality" and the Correlative Formal Language Based on the "Incipient Derivative"

The Maximum Ordinality Principle (MOP), presented in 2010 at the 6th Biennial Emergy Conference, Univ. of Florida [1], is a Principle that is apt for describing the "Emerging Quality" of Self-Organizing Systems. Its verbal enunciation asserts that "*Every System tends to maximize its Ordinality*, *including that of its surrounding habitat*", and it is formulated by means of *two fundamental equations*, which are so *strictly related to each other* so as to form *a Whole* [2] [3], as they are repurposed in the successive points 2 and 3.

The specific introduction of the MOP is directly referable to the fact that Self-Organizing Systems always show an unexpected "*excess*" with respect to their phenomenological premises. So they usually say: "*The Whole is much more than its parts*".

Such an "excess" can be termed as *Quality* (with a capital Q) because it cannot be understood as being a simple "property" of a given phenomenon. This is because it is *never reducible* to its phenomenological premises in terms of traditional mental categories: *efficient causality*, *logical necessity*, *and functional relationships*. Consequently, it cannot be described in terms of the *traditional derivative* that, at the level of formal language, represents the perfect reflex of such "a priori" mental categories.

This evidently suggests a *radically new* gnoseological perspective and, *in adherence*, the adoption of "*new mental categories*"[1]: *Emerging Causality*, *Generative Logic*, *and Ordinal Relationships*. These, in turn, suggest the development of a completely *new formal language*, in order to formulate *one sole Reference Principle* [3], the Maximum Ordinality Principle, so that the description of Self-Organizing Systems could possibly result in faithfully conforming to their "Emerging Quality" [4].

In this perspective, the approach based on the MOP is perfectly *Com-possible* with those more traditional ones because, as anticipated, the *Generative Logic*on which it is based has not, by itself, the property of "perfect induction". Consequently, the present approach cannot be considered as being "exclusive". How-

[1]These "*new mental categories*" can no longer be termed as "pre-suppositions", because they are not defined "a priori" (as in the case of Traditional Approach). In fact, they are adopted "a posteriori", only on the basis of the "Emerging Quality" previously recognized. "*Emerging Causality*", in fact, refers to the *capacity* of a Self-Organizing System to manifest an "irreducible excess"; "*Generative Logic*" correspondently refers to the capacity of our mind to draw "*emergingconclusions*". That is, "conclusions" whose information content is much higher than the information content corresponding to their logical premises, although they are persistently "adherent" to the latter. "Ordinal Relationships", in turn, refer to particular relationships of *genetic nature*, like in the case of "brothers". The latter in fact are termed as such not because of their "direct reciprocal relationships", but because their *direct reference* to *the same genetic principle*: their father (or their mother or both).

ever, the same happens for all the various approaches based on *Classical Logic* also termed as *necessary logic*, because this Logic has not, by itself, the property of "perfect induction". This means that no approach among those until now adopted can be considered as being "exclusive", precisely because all of them are based on the *necessary logic* previously mentioned. Nonetheless, when confirmed by experimental results, they always keep their validity from an "*operative*" point of view.

The *only difference* then consists in the fact that in the traditional approaches, the reference mental categories are adopted "a priori", while in the present Ordinal Approach, they are adopted "a posteriori". In fact, they are based on the previously recognized "*Emerging Quality*" of Self-Organizing Systems.

This is why *a new concept of derivative* was introduced, that is the "*Incipient Derivative*" [5], which is defined as follows

$$\left(\frac{\tilde{d}}{\tilde{dt}}\right)^{\tilde{q}} f(t) = \underset{\tilde{\Delta}t:0\to 0^{+}}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{\tilde{q}} \circ f(t) \quad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n} \tag{1}.$$

A definition that clearly shows that the "*Incipient Derivative*" is not an "operator", like the derivative (d/dt) in the Traditional Differential Calculus (TDC), but it can be termed as a "*generator*", because it describes the *Generativity* of a given Process, *in its same act of being born*. In fact:

1) The sequence of the symbols is now interpreted according to the *direct priority* of the three elements that constitute its definition (from left to right). This is why they acquire a completely different meaning with respect to the traditional definition;

2) The three symbols, in fact, do not represent "three" distinct operations, but a *unique and sole* Generative Process;

3) The symbol $\tilde{Lim}$, whose etymological origin comes from the Latin word "Limen" (which means a "threshold"), represents the "threshold" of that "ideal window" from which we observe and describe the considered phenomenon;

4) The symbol $\tilde{\Delta}t:0\to 0^{+}$ now indicates not only the initial time of our registration but also the proper "*origin*" (in its etymological sense) of *something new* which we observe (and describe) in its proper act of being born, as a *Generative Process* [6];

5) It is then evident that the "symbol" $\tilde{\delta}$ now indicates (and registers) the *variation* of the observed property $f(t)$, not only in terms of quantity but also, and especially, in terms of Quality (as the symbol "tilde" would expressly remind that). Thus, the ratio that appears in Equation (3) indicates not only a quantitative variation in time but both the variation in Quality and quantity;

6) Consequently, when we take the incipient (or "prior") derivative of Ordinality $\tilde{q}$ of any $f(t)$, the *Exit* of such a process keeps "memory" of its genetic origin because, besides its quantity, it will result as being structured according to the indication of such an exponent. The latter in fact is properly termed *Ordinality*, because it precisely expresses how each part of the output is *genetically*

Ordered to the Whole and, at the same time, *how each part is related to all the others* in terms of *Ordinal Relationships* (as shown by Equations (A1) and (A5) in Appendix);

7) In this way the "incipient" derivative represents the *Generativity of the considered Process*, that is the output "excess" (per unit time) characterized by both its Ordinality and its related cardinality, while the sequence of the symbols in its definition (Equation (1)) can be interpreted as representing a *unique inter-action process* between the same;

8) The above-mentioned reasons clearly show why the "incipient" derivative is able to *unify* (and, at the same time, to specify) the description of the various Self-Organizing Processes when explicitly understood in terms of Quality;

9) This also means that the Incipient Derivative has an "*Exit*" that is generally different from the *necessary* "*result*" of the corresponding derivative in TDC, even when its Ordinality is reduced to *a mere cardinality*. For example, the traditional derivative of order n of the function $e^{\alpha(t)}$, evaluated according to Faà di Bruno's formula, when compared with the corresponding *same order* incipient derivative, respectively give, in the first case

$$\left(\frac{d}{dt}\right)^n e^{\alpha(t)} = e^{\alpha(t)} \sum \frac{n!}{k_1!k_2!\cdots k_n!} \cdot \left(\frac{\dot{\alpha}}{1!}\right)^{k_1} \left(\frac{\ddot{\alpha}}{2!}\right)^{k_2} \cdots \left(\frac{\alpha^{(n)}}{n!}\right)^{k_n} \tag{2}$$

and, in the second case

$$\left(\frac{\tilde{d}}{\tilde{dt}}\right)^n e^{\alpha(t)} \overset{*}{=} e^{\alpha(t)} \cdot \left[\overset{\circ}{\alpha}(t)\right]^n \tag{3}$$

where $\overset{\circ}{\alpha}(t)$ represents the first-order "incipient" derivative. And even if in some cases the two derivatives coincide (for instance when $\alpha(t)$ is linear) such a coincidence has to be seen in the light of the symbol $\overset{*}{=}$ in Equation (3), which reminds us that any "*incipient derivative*" is always the "*Exit*" of a *Generative-Logical Process* and not the "*result*" of a *necessary* logical process.

3. The First Fundamental Equation of the Maximum Ordinality Principle (MOP)

On the basis of the previous concept of "incipient" derivative, the *First* Fundamental Equation of the MOP is formulated as follows

$$\left(\tilde{d}/\tilde{dt}\right)_s^{(\tilde{m}/\tilde{n})} \{\tilde{r}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{4}$$

$$(\tilde{m}/\tilde{n}) \rightarrow Max \rightarrow \{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\} \tag{4.1}$$

where $\{\tilde{r}\}$ is the *Relational Space* of the System under consideration (see Equation (A2) in Appendix), while $(\tilde{m}/\tilde{n})$ represents its corresponding *Ordinality*, characterized by $\tilde{m}$ Ordinal Co-productions and $\tilde{n}$ Ordinal Interactions, and such an *Ordinality* reaches its *maximum* when it equals $\{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\}$ (as indicated in Equation (1.1)).

In this respect, it is worth noting that:

1) The *underlined* symbol $\left(\tilde{d}/\tilde{dt}\right)_s$ explicitly indicates that the *Generative capacity* of the System (more appropriately termed *Generativity*) is *"internal" to the same System*. This is because it is precisely that which gives origin to the Self-Organization of the System as a Whole;

2) The symbol "$\overset{[\rightarrow}{=}\left\{\tilde{0}\right\}$" represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact, it now represents, at the same time:

- the specific "*origin and habitat*" *conditions* associated with the considered Ordinal Differential Equation (4);
- while the symbol "$\overset{[\rightarrow}{=}$" indicates that the System, during its Generative Evolution, is persistently "adherent" to its "origin and habitat" conditions.

4. The Second Fundamental Equation of the Maximum Ordinality Principle

Itis formulated as follows

$$\left(\tilde{d}/\tilde{dt}\right)^{\left(\tilde{2}/\tilde{2}\right)}\left\{\left\{\tilde{r}\right\}\circledR\left(\tilde{d}/\tilde{dt}\right)^{\left(\tilde{2}/\tilde{2}\right)}\left\{\tilde{r}\right\}\right\}\overset{[\rightarrow}{=}\left\{\tilde{0}\right\} \tag{5}$$

and it can be considered as representing a *global* Feed-Back Process of *Ordinal Nature*, which is "*internal*" to the same System. Equation (5), in fact, asserts that the *Relational Space* $\left\{\tilde{r}\right\}$ of the System, which "emerges" as a solution from the First Equation, interacts in the form of the Relational Product ® (defined in the Appendix) with *its proper Generative Capacity* $\left(\tilde{d}/\tilde{dt}\right)^{\left(\tilde{2}/\tilde{2}\right)}\left\{\tilde{r}\right\}$. In such a way as to originate, in turn, a *comprehensive* Generative Capacity, which is particularly important for the *Ordinal Stability* of the System, especially when the latter interacts with other surrounding Systems understood as being its proper habitat.

The Maximum Ordinality Principle, in its two fundamental equations, always presents an *explicit solution*, which is illustrated in Appendix A1. In this case, however, the general explicit solution to the MOP is also presented and structured in a more *operative form* (in Appendix A2), so that it may result as being more directly and easily adopted in analyzing the Ordinal Evolution of the System under consideration.

5. The Accelerated Expansion of the Universe Described in Terms of the MOP

The accelerated expansion of the Universe could be analyzed by considering, for example, as *a case study*, a Galaxy Cluster made up of 1000 Galaxies. In such a case the Galaxy Cluster, if described in the light of the MOP, it is modeled as being a Self-Organizing System of Ordinality $\tilde{1000}/\tilde{1000}$, that is an Ordinality that is characterized by 1000 Ordinal Co-Productions and 1000 correlative Ordinal Interactions. In such a case, however, even in the presence of an *explicit solution* of the MOP (as given in Appendix A1), such a general approach would require the adoption of an appropriate computer and, at the same time, a correlative sophisticated computer code.

However, in order to show the process of an *accelerated expansion* inside such a Galaxy Cluster, it is also possible *to limit* the analysis to a significantly reduced portion of the same.

For example, it is possible to consider a self-organizing system made up of 12 Galaxies contained in a cone of 5 Mpc in length, with its vertex in the correspondence of the origin of the Cluster, and a circular section 1.5 Mpc in diameter.

The Ordinal *topological distribution* of such 12 Galaxies can be obtained on the basis of the *operative* Ordinal Relationships obtained from the explicit solution of the MOP, as illustrated in Appendix A2 (Equations from (A12) to (A16)), here reproduced for the sake of clarity:

1) $$\tilde{\rho}_{1j}\left(t_0\right)=\tilde{A}\cdot e^{\tilde{S}_l\left(t_0\right)} \tag{6}$$

where

$$\tilde{S}_l\left(t_0\right)=\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right] \quad \text{for} \quad l=1,2,\cdots,N-1 \tag{6.1}$$

2) $$\tilde{\varphi}_{1j}\left(t_0\right)=\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Phi}_0-C_l\cdot\tilde{\Sigma}_0\right] \tag{7}$$

3) $$\tilde{\theta}_{1j}\left(t_0\right)=\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Theta}_0-C_l\cdot\tilde{\Sigma}_0+C_l\left(\tilde{\Phi}_0-\tilde{\Theta}_0\right)\right] \tag{8}$$

where

$$B_l=\cos\left(\sqrt{2}\cdot\psi_l\right),\quad C_l=D_l=\frac{1}{\sqrt{2}}\sin\left(\sqrt{2}\cdot\psi_l\right) \tag{9}$$

and

$$E_l=\frac{\varepsilon_1+4\pi\cdot l}{N-1},\quad \psi_l=\psi_2\cdot\frac{\varepsilon_2+2\pi\cdot l}{N-1}. \tag{10}$$

Such *operative* Relationships, according to the *Ordinal Deductive* Process shown in Appendix A2, represent an operative "*Emerging Solution*" from the Maximum Ordinality Principle, which is valid for *any* Claster made up of *arbitrary* number N of Galaxes. Consequently, even if such Relationships are formulated with reference to each single generic couple "*1j*", and thus they are expressed in terms of the three distinct variables $\tilde{\rho}_{1j},\tilde{\varphi}_{1j},\tilde{\vartheta}_{1j}$, the latter do not represent a simple traditional "vector", but an "*Ordinal* vector". That is a *unique and sole Relational Entity*, which is usually represented *in the form* $\left\{\tilde{\rho}_{1j},\tilde{\varphi}_{1j},\tilde{\vartheta}_{1j}\right\}$, because understood as a Whole.

This means that the three variables $\tilde{\rho}_{1j},\tilde{\varphi}_{1j},\tilde{\vartheta}_{1j}$, although recognizable as being "distinct", are not conceptually "separable" between them.

Such an assertion is even truer, and in *particular way*, with reference to the triples of variables pertaining to *all the couples* of the System, which are not conceptually "separable" between them precisely because the System is understood *as a Whole*.

In other words, the Fundamental Relations previously shown do not only furnish the N *single Ordinal vectors* $\left\{\tilde{\rho}_{1j}\left(t_0\right),\tilde{\varphi}_{1j}\left(t_0\right),\tilde{\vartheta}_{1j}\left(t_0\right)\right\}$ that characterize each single couple of the System, but they represent, even more, a *Unified Ordinal Description* of the System understood as Whole.

A direct and correlative consequence is that, even if at a "preliminary and intuitive" interpretation such Ordinal Relationships could be thought as giving the "distances" between couples of Galaxies of the analyzed System, in reality, in adherence to the MOP, such an interpretation (and the corresponding "terminology") should be substantially modified. In particular, by adopting a more appropriate term, that is the one of "*Uniance*", instead of that of "distance".

This is because the concept of "*distance*" tends more to *divide* than to *unify*. In fact, the same *etymology* of the word (from Latin "*dis-stant*") indicates that "one element *stays here* and the other one *stays there*" or, equivalently, "*one is here and the other one is there*".

Consequently, in the case of an *Ordinal* Perspective the term "distance" should preferably be replaced by a different term, possibly able to indicate the concept of "*union*" of two elements, more than their "distance".

In this respect, by introducing a *neologism* (that "rhymes" with the term "distance", but it exactly indicates the opposite meaning), we could say that the same value that in a "functional" perspective represents a "distance", in an *Ordinal* Perspective indicates a "*uni-ance*". That is, it indicates that the two elements form "*one sole Entity*" of *Ordinal Nature*, because they are the "*Exit*" of the same *Generative* Process. So that the term "Uniance" expresses an *Ordinal concept*, and not a mere cardinal concept, such as that of "distance". Any "*Uniance*", in fact, is characterized by *its own Ordinality*, which is the "Exit" of a *Generative Process*.

As a simple example, let us think of a Binary-Duet of Ordinality 2/2. Such a specific and proper *Ordinality* is precisely that which represents *the Ordinal Generative* "Unity" between two elements of the System. While, at the same time, its "associated cardinality" only indicates their topological distribution in the *Relational Space* of the System (defined by Equation (A2) in Appendix).

Consequently, when all the various "Uniances" are considered in the context of such *Harmony Relationships*, they reveal that the System is a *Whole of Ordinal Generative Nature*, in perfect adherence to the Maximum Ordinality Principle. In addition, such an assertion has also an *even more general sense*, that is: it is precisely *the specific Generativity* of the Self-Organizing System the one that, with its proper *Diffusivity through the System*, characterizes all the elements of the System in terms of "Ordinal" and, at the same time, of "Harmony" Relationships. This is because such Ordinal Relationships are all of *genetic nature*, like in the case of "brothers". In fact, as previously anticipated, "brothers" are termed as such not because of their "direct reciprocal functional relationships", but because of their *direct and harmonic reference* to *the same genetic principle*: their father (or their mother or both). In this sense, the term "*Uniance*" correlatively *synthesizes* the concept of an *Ordinal Unity of a Harmonic Genetic Origin*.

6. Analysis of the Considered Case Study on the Basis of the Previous Ordinal Generative Relationships

The Ordinal Relationships previously shown are able to describe the Ordinal

distribution of *any* given number N of Galaxies belonging to *any* arbitrary selected Galaxy Cluster. To this aim, it is sufficient to define the values of the characteristic Ordinal parameters pertaining to the Self-Organizing System analyzed, in terms of their "associated" cardinalities.

In this respect, a self-organizing system made up, for instance, of $N = 12$ Galaxies, can be modeled by means of the associated cardinal values of the coordinates of the *reference couple* $\left\{\tilde{\sigma}_{12}(t_0), \tilde{\varphi}_{12}(t_0), \tilde{\vartheta}_{12}(t_0)\right\}$, which, at the time t_0, can be synthetically represented as $\Sigma_0, \Phi_0, \Theta_0$, and can respectively be assumed equal to $\Sigma_0 = 0.02$, $\Phi_0 = 0.006$, $\Theta_0 = 0.006$. This is because, although such values represent only the "associated cardinalities", the values of Φ_0 and Θ_0 cannot be assumed as being both identically null, otherwise the corresponding Ordinality of the *reference couple* will be completely "denied".

The Ordinal Topological Distribution of the considered Galaxy Claster is given in Table 1, whose corresponding values were obtained by assuming that:

- the parameters B_l and C_l are characterized by the value $\varepsilon_2 = 0$ and are "modulated" by an internal periodicity $\psi_2 = 0.05$.
- E_l is characterized by the value $\varepsilon_1 = 0$ and it is "modulated" (see Equations (6.1), (7), (8))) by a "periodicity" $\psi_1 = 16.5$ (defined in Appendix A2, point 2).
- and by assuming the *Internal Scale Factor* (see Appendix A2) equal to A = 0.2.

Such distribution is then clearly represented in Table 1 in terms of the "*associated cardinalities*" of the *Uniances* $\left\{\tilde{\rho}_{1j}(t_0), \tilde{\phi}_{1j}(t_0), \tilde{\vartheta}_{1j}(t_0)\right\}$ of the various Galaxies, at a given time t_0, distributed in a cone which, as anticipated, is 5 Mpc

Table 1. Ordinal topological distribution of the galaxies.

Couple	Index l	B_l	C_l	E_l	$\tilde{\rho}_{1j}(t_0)$ [Mpc]	$\tilde{\varphi}_{1j}(t_0)$ [rad]	$\tilde{\vartheta}_{1j}(t_0)$ [rad]
1-2	1	0.999184	0.020192	1.14241	0.29016	0.105393	0.105393
1-3	2	0.996739	0.040368	2.28479	0.41637	0.195021	0.195021
1-4	3	0.992668	0.060511	3.42719	0.58989	0.268368	0.268368
1-5	4	0.986978	0.080604	4.56959	0.82362	0.324951	0.324951
1-6	5	0.979677	0.100632	5.71199	1.13134	0.364308	0.364308
1-7	6	0.970779	0.120577	6.85438	1.53618	0.386161	0.386161
1-8	7	0.960298	0.140425	7.99678	2.01856	0.389678	0.389678
1-9	8	0.948250	0.160157	9.13918	2.61327	0.374944	0.374944
1-10	9	0.934655	0.179760	10.2816	3.30629	0.341453	0.341453
1-11	10	0.919536	0.199215	11.4241	4.08117	0.288948	0.288948
1-12	11	0.902917	0.218508	12.5664	4.90959	0.217161	0.217161

in length, with its vertex in correspondence of the reference couple "12", and a circular section of 1.5 Mpc in diameter.

The values in Table 1 show that there is a progressive increase of the *radial* Uniances between the various Galaxies, although such a trend is represented in terms of the sole their "associated cardinalities". Nonetheless, such a particular aspect could be seen as being a "preliminary indication" of the different increasing recessions of the velocity between the various Galaxies.

At the same time, the values of the *angular distributions* of the various Galaxies with respect to the reference "couple 12", after an initial progressive tendency to increase, are characterized by the fact that the last four values *tend to decrease*.

In this respect, it is worth pointing out that the trend of the two *angular distributions* $\tilde{\varphi}_{1j}(t_0)$ and $\tilde{\vartheta}_{1j}(t_0)$, that will be reconsidered successively in more detail, result as being equal between them only because of the assumption of identical initial values of Φ_0 and Θ_0 at the initial time t_0.

The Accelerated Expansion of Galaxies of this Claster will benow analyzed in three successive steps, each one characterized by a progressively increasing level of generality.

6.1. First Phase of Analysis of the Evolution of the Considered Galaxy Claster

Let us now assume that Galaxy 2 has a recession velocity, with respect to Galaxy 1, which is equal to χ [Km/sec/Mpc].

This means that, in a given time interval Δt, the corresponding *increase of their correlative radial Uniance* (expressed in Mpc) will be the following one

$$\tilde{\rho}_{12}(t_0+\Delta t)=\tilde{A}\cdot e^{\psi_1\cdot E_1\cdot\left[B_1\cdot\tilde{\Sigma}_0-C_1\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right]}\cdot(1+\chi\cdot\Delta t) \tag{11}$$

in which, because of the limited time interval Δt initially considered in this analysis, the value of the astronomical recession velocity χ can be assumed as being substantially constant.

Equation (11) is particularly interesting because it offers the possibility of a wider and more general interpretation.

In fact, Equation (11) can also be rewritten in the following form

$$\tilde{\rho}_{12}(t_0+\Delta t)=\left\{\tilde{A}\cdot(1+\chi\cdot\Delta t)\right\}\cdot e^{\psi_1\cdot E_1\cdot\left[B_1\cdot\tilde{\Sigma}_0-C_1\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right]}. \tag{12}$$

Such a reformulation, in fact, even if from a "functional" point of viewcanbe considered as being perfectly "equivalent" to Equation (11), in reality, in an Ordinal *Generative Context* it is not understood as a "functional" equation but, more properly, as being a *Generative Equation*. In such a *Generative* context, the "*Translation*" of the *Internal Scale Factor* $\tilde{A}$, expressed by the term $\left\{\tilde{A}\cdot(1+\chi\cdot\Delta t)\right\}$ in Equation (12), do not simply and specifically refer to the single couple "12", but it can be considered as being referred *to all the System* as a Whole. This is because, the definition of the Scale Factor $\tilde{A}$ (as illustrated in

Appendix A2), is understood as an *Ordinal* parameter that is *"Internal"* to the System, and it exactly "reflects" the fact that the radial Uniances $\tilde{\rho}_{1j}(t_0)$, given by the Fundamental Relationship (6), do not onlyrefer to each *single and specificcouple* of the System, but they represent a *unified Ordinal* description of the System understood as Whole.

Consequently, *the same* Translation of the *Internal* Scale Factor $\tilde{A}$, in Equation (12), in reality pertains to *all the Couples of Galaxies*, according to the following general reformulation of the same Equation (12), now rewritten with reference to the generic Uniance $\tilde{\rho}_{1j}(t_0+\Delta t)$ and the same time interval Δt

$$\tilde{\rho}_{1j}(t_0+\Delta t)=\tilde{A}\cdot(1+\chi\cdot\Delta t)\cdot e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot(\tilde{\Phi}_0+\tilde{\Theta}_0)\right]}. \tag{13}$$

If we now take the "Incipient Derivative" of Equation (13), precisely because we are describing a *Generative Process*, its "Exit" can be structured as follows

$$\overset{\circ}{\tilde{\rho}}_{1j}(t_0+\Delta t)=\tilde{A}\cdot e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot(\tilde{\Phi}_0+\tilde{\Theta}_0)\right]}\cdot\chi. \tag{14}$$

where the notation $\overset{\circ}{\tilde{\rho}}_{1j}$, as we previously said, now represents the *first order incipient derivative* of the generic Uniance of Equation (13).

Equation (14) then shows that *each Galaxy* of the Claster has a *recession velocity* that is equal to *the product* between *its specific Uniance* $\tilde{\rho}_{1j}(t_0)$ with respect to Galaxy 1 and the assumed value of χ.

Equation (14) then allows us to assert that, in adherence to astronomical observations about the Accelerated Expansion of the Universe, which is characterized by a recession velocity of the Galaxies equal 73.2 Km/sec/Mpc, the *recession velocities* of the Galaxies of the considered Claster can write in the form

$$\overset{\circ}{\tilde{\rho}}_{1j}(t_0+\Delta t)=\tilde{A}\cdot e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot(\tilde{\Phi}_0+\tilde{\Theta}_0)\right]}\cdot 73.2\left[\mathrm{Km/sec/Mpc}\right], \tag{15}$$

where the term $\tilde{A}\cdot e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot(\tilde{\Phi}_0+\tilde{\Theta}_0)\right]}$ represents the *radial Uniance* between *any* given Galaxy and Galaxy 1, and each Unianceis expressed in Mpc.

Such an "Exit" then clearly shows that the expansion of the Galaxies is nothing but *one sole Harmonious Process*, as a "reflex" of the *Harmony Relationships* (6) and (6.1) that give their Uniances, which are all "coordinated" by the *Ordinal Generativity* of the considered Self-Organizing System, understood as a Whole.

This interpretation, however, represents only the first modality of showing the time Expansion of the considered Galaxy Cluster.

6.2. Second Phase of Analysis of the Evolution of the Galaxy Cluster

A more general approach consists in considering the *radial translation* of the Ordinal parameter Σ_0 *in the time* Δt. In such a case, being such an Ordinal parameter an exponent of the exponential number "*e*", its translation, in the time Δt, has to be formulated in logarithmical terms, that is

$$\Sigma_0+\ln(1+\chi\cdot\Delta t) \tag{16}$$

and, at the same time, it has to be reduced by the product $\psi_1 \cdot E_l \cdot B_l$, so that it might be referred to the translation of *the sole* couple "12".

In this way the translation of the Uniance of the *generic couple*, given by Equation (6) and (6.1), becomes

$$\tilde{\rho}_{1j}\left(t_0+\Delta t\right)=\tilde{A}\cdot e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\left(\tilde{\Sigma}_0+\frac{\ln\left(1+\chi\cdot\Delta t\right)}{\psi_1\cdot E_l\cdot B_l}\right)-C_l\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right]}, \tag{17}$$

which can also be rewritten as

$$\tilde{\rho}_{1j}\left(t_0+\Delta t\right)=\tilde{A}\cdot e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right]+\ln\left(1+\chi\cdot\Delta t\right)}. \tag{18}$$

If we now take the "Incipient Derivative" of Equation (18), we get

$$\overset{\circ}{\tilde{\rho}}_{1j}\left(t_0+\Delta t\right)=\tilde{A}\cdot e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right]+\ln\left(1+\chi\cdot\Delta t\right)}\cdot\frac{\chi}{1+\chi\cdot\Delta t}. \tag{19}$$

However, if Equation (18) is previously rewritten in the form

$$e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right]}\cdot e^{\ln\left(1+\chi\cdot\Delta t\right)}=e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right]}\cdot\left(1+\chi\cdot\Delta t\right), \tag{20}$$

the previous "Incipient" Derivative given by Equation (19) now becomes

$$\overset{\circ}{\tilde{\rho}}_{1j}\left(t_0+\Delta t\right)=\tilde{A}\cdot e^{\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right]}\cdot\chi, \tag{21}$$

that is, it gives *the same* Ordinal "Exit" obtained in the previous phase 1 of the analysis (see Equation (14)).

This approach, however, results as being more appropriate because, as we will see in the next phase of analysis, it allows us to evaluate, in addition, the correlative trends of the angular phases $\tilde{\varphi}_{1j}\left(t\right)$ and $\tilde{\theta}_{1j}\left(t\right)$ characteristics of each specific Uniance.

6.3. Third Phase of Analysis of the Evolution of the Galaxy Cluster

As already anticipated, the approach just shown as phase 2 of the analysis, even if it leads to the same "Exit" as phase 1 as far as the *radial expansions* of the various Uniances are concerned, it also offers the possibility of a more general perspective.

In fact, in addition to the previous "Exit", it is also able to point out the *variations* of the correlative angular phases $\tilde{\phi}_{1j}\left(t_0+\Delta t\right)$ and $\tilde{\theta}_{1j}\left(t_0+\Delta t\right)$ evaluated in correspondence of various time intervals Δt of the translation of the correlative Uniances.

In fact, by introducing the same radial translation of the Ordinal parameter Σ_0 given by Equation (16) into the Equations (7) and (8), we get

$$\tilde{\phi}_{1j}\left(t_0+\Delta t\right)=\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Phi}_0-C_l\cdot\tilde{\Sigma}_0\right]-\frac{C_l}{B_l}\cdot\ln\left(1+\chi\cdot\Delta t\right) \tag{22}$$

$$\tilde{\theta}_{1j}\left(t_0+\Delta t\right)=\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Theta}_0-C_l\cdot\tilde{\Sigma}_0+C_l\left(\tilde{\Phi}_0-\tilde{\Theta}_0\right)\right]-\frac{C_l}{B_l}\cdot\ln\left(1+\chi\cdot\Delta t\right). \tag{23}$$

Consequently, by taking into account that the values of the coefficients C_l in Table 1 progressively tend to increase with *l*, while on the contrary the coeffi-

cients B_l progressively tend to decrease, this means that the values of Equations (22) and (23), in correspondence of the translation of the Ordinal parameter Σ_0 and the interval Δt each time considered, they all progressively decrease, by tending toward the value of *zero*. Consequently, the values of the angular recessions of all the various Galaxies almost uniformly tend to progressively decrease, so that the *general* evolutiontendsto reduce to a simple "*radial*" expansion, with respect the trend represented by the original conditions in Table 1.

Such an assertion, however, is evidently true only in correspondence to a *short time intervals* Δt considered *with respect to the initial time* t_0, that is time intervals "comparable" with the value Δt preliminary considered in phase 1 and 2.

In fact, *in the long run*, that is for values of the interval Δt sufficiently high, such a general trend can evidently lead to a correlative possible "*inversion*" of the values of the *angular components* of the Uniances (from positive to negative values). This means that, *in the long run*, Equations (22) and (23) can *present a progressively* increasing *trend of negative values*, in spite of their positive values corresponding to the *initial* Ordinal conditions Φ_0 and Θ_0.

This means that, while the various Ordinal Uniancescontinue to progressively expand "radially", their *angular components* generally show a progressive "retrograde" trend (with respect to the initial conditions). And even if the "radial" expansion of the Uniances given by Equation (15) represents *the prevailing trend*, its *combined* consideration with the angular components of the Uniancesmay contextually represent a "*preliminary*" *indication* of that very particular phenomenon which is generally termed as "*The Synchrony of Galaxies*". An aspect that, however, will be analyzed in more detail in a successive paper.

7. Conclusions

On the basis of the previous modalities of analysis, the Recession Velocity of the Galaxes and the correlative Accelerated Expansion of *any* Galaxy Cluster can be seen as an "Emerging Ordinal Exit" of the same and unique Astronomical System considered, when understood as a Self-Organizing System of *Generative Nature*, and thus analyzed in the light of the Maximum Ordinality Principle.

Such an "Emerging Ordinal Exit" may also suggest a more general perspective, always based on the MOP, with reference to the *entire Universe*. In fact, it "reveals" that the same Accelerated Expansion of the *entire Universe*, can be seen as a direct "Emerging Exit" of an *Ordinal Generativity*, which is *specific* of the same Universe, when the latter is modeled as a unique Self-Organizing System understood as Whole.

A Generativity that diffuses its "*Harmonious Exits*" on all the various Galaxy Clusters of the Universe and, at the same time, on *each single* Galaxy included in them.

The previous Ordinal Analysis in fact is not understood as being a sort of "*re*

compositio ad unum", but it is exactly understood in the opposite sense.

In fact, from the *very beginning* of the Ordinal Analysis, the "Attention" is devoted to the "Emerging Quality" of *any* specific Self-Organizing System. Where such an "Emerging Quality", understood as an "Irreducible Excess", among other aspects manifests *its presence* through the *Harmony Relationships* of the same System. While such *Harmony Relationships*, in turn, represent the "Exceeding" Ostension of the specific Generativity of the System, when the latter is, at the same time, *Self-Organizing*, of *Ordinal Nature*, and understood *as a Whole*.

This also means that all the Relationships considered in the Analysis are all of *Ordinal Nature*, and not simply of "functional" nature. Thisevidently, and in particular, reflects on the adoption of the previous concept of "*Uniance*", instead of that of "distance".

In such a perspective, the "*Exit*" so obtained is not the "result" of a mere "algebraic" analysis, not even of a simple "functional" nature. This is because all the Relationships adopted, in particular those based on the concept of *Relational Space* $\{\tilde{r}\}_s$ (see Equation (A2) in Appendix), are all of *Generative* Nature. In fact, they are all obtained and expressed by means of the innovative concept of the "Incipient" Derivative.

Consequently, the Recession of each Galaxy does not reduce to a simple "sum" of "local geometrical aspects", specific and characteristic of each Galaxy, but it is pertaining to *all the System* understood as *a Whole*, according to which all the different modalities of "*Recession*" are faithfully described by its *Harmony Relationships*.

This also means that *any* System, when understood as a Whole, cannot be considered as being "divisible in parts", "distinct and separated", as they were related between them in "functional" terms. In other words, inside a System understood as a Whole, each single "variation" of one "part" is not, by itself, an "efficient cause" that can "influence" all the other ones. This is because it is exactly true the "opposite": any "variation" of a single Galaxy is, by itself, only a simple Ordinal "Indicator" of a *global variation* of the Whole. Where such a "comprehensive variation", in turn, "reflects" in *Ordinal terms* on the same considered Galaxy, precisely because the latter is "intimately" related to the Whole.

This also shows that any Ordinal Self-Organizing System, and in particular the *Entire Universe*, does not "evolve" or "expand" in a "geometrical space", defined "a priori" and of "absolute nature". But, in adherence to the same concept of *Relational Space*, each Ordinal System, and in particular the *Entire Universe*, "expands" its *Proper Relational Space* on the basis of its *specific* Generativity, according to a Process in which the same System, as already shown in [7], contextually "evolves" in *its Proper Time*.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A1. General Explicit Solution of the MOP in its Two Fundamental Equation Sunderstood as a Whole

The first Fundamental Equation (4) always presents an *explicit solution* (Ref. [7]), which can always be written in the general form

$$\left\{\tilde{r}\right\} = e^{\left\{\tilde{\alpha}(t)\right\}} = e^{\left\{\begin{pmatrix}\tilde{\alpha}_{11}(t)\\ \tilde{\alpha}_{21}(t)\\ \vdots\\ \tilde{\alpha}_{N1}(t)\end{pmatrix}\begin{pmatrix}\tilde{\alpha}_{12}(t)\\ \tilde{\alpha}_{22}(t)\\ \vdots\\ \tilde{\alpha}_{N2}(t)\end{pmatrix}\begin{pmatrix}\cdots\\ \cdots\\ \vdots\\ \cdots\end{pmatrix}\begin{pmatrix}\tilde{\alpha}_{1N}(t)\\ \tilde{\alpha}_{2N}(t)\\ \vdots\\ \tilde{\alpha}_{NN}(t)\end{pmatrix}\right\}} \tag{A1}$$

where the *Relational Space* $\left\{\tilde{r}\right\} = e^{\left\{\tilde{\alpha}(t)\right\}}$ depends on the Nature of the System analyzed, while the corresponding structure of each term of the Ordinal Matrix depends on the *Specific Generativity* $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s$ of the System.

For example, if the *Relational Space* of the System is represented by $\left\{\tilde{\sigma},\tilde{\varphi},\tilde{\vartheta}\right\}$, that is by means three Ordinaltopological coordinates always considered as *the exit of a Generative Process*, we can assume

$$\left\{\tilde{r}\right\}_s = e^{\tilde{\alpha}(t)} = e^{\left\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\right\}}. \tag{A2}$$

This is because, on the basis of a generalized form of De Moivre representation, it is always possible to write

$$\begin{aligned}\left\{\tilde{r}\right\}_s &= \left\{\tilde{\rho}\circledR\tilde{i}\circledR e^{\tilde{\varphi}\circledR\tilde{j}}\circledR e^{\tilde{\vartheta}\circledR\tilde{k}}\right\} = \left\{e^{\tilde{\sigma}\circledR\tilde{i}}\circledR e^{\tilde{\varphi}\circledR\tilde{j}}\circledR e^{\tilde{\vartheta}\circledR\tilde{k}}\right\}\\ &= e^{\left\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\right\}} = e^{\tilde{\alpha}(t)},\end{aligned} \tag{A3}$$

where the traditional versors $\vec{i},\vec{j},\vec{k}$ are now replaced by three unit *spinors* $\tilde{i},\tilde{j},\tilde{k}$, which are defined in such a way as to satisfy the following *Relational Product* Rules:

$$\tilde{i}\circledR\tilde{i} = \oplus 1 \quad \tilde{i}\circledR\tilde{j} = \tilde{j} \quad \tilde{i}\circledR\tilde{k} = \tilde{k} \tag{A4.1}$$

$$\tilde{j}\circledR\tilde{i} = \tilde{j} \quad \tilde{j}\circledR\tilde{j} = \Theta 1 \quad \tilde{j}\circledR\tilde{k} = \tilde{k} \tag{A4.2}$$

$$\tilde{k}\circledR\tilde{i} = \tilde{k} \quad \tilde{k}\circledR\tilde{j} = \tilde{k} \quad \tilde{k}\circledR\tilde{k} = \Theta 1 \tag{A4.3}$$

where the symbols $\oplus$ and $\circledR$ express more intimate relationships between the same: both in terms of sum ($\oplus$) and in terms of (relational) product ($\circledR$) with respect to the case of traditional versors $\vec{i},\vec{j},\vec{k}$.

So that representation (A3) is similar (albeit not strictly equivalent) to a system of three complex numbers, characterized by one "real" *Ordinal* unit ($\tilde{i}$) and two "imaginary" *Ordinal* units ($\tilde{j}$ and $\tilde{k}$).

Equation (A1) thus describes the *Generative Evolution* of the System as the exit of an *Ordinal Cooperation* of *NCo-Productions* (vertical brackets) and their associated *N Inter-Actions* (horizontal brackets). At the same time, when the Process has reached its Maximum Ordinality, each term $\tilde{\alpha}_{ij}(t)$ of the Ordinal Matrix, as a consequence of such a Maximization Process, will transform into a Binary-Duet Relationship, represented as $\left\{\tilde{\alpha}_{ij}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$.

At the same time, the adoption of the brackets "{}" in Equation (A1) is explicitly finalized to remind us that the Ordinal Matrix represents the *Ordinal* Structure of the System understood as a *Whole*.

In fact, all the elements of the Ordinal Matrix (in Equation (A1)) satisfy the following "*Ordinal Relationships*"

$$\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_j \circledR \left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \quad \text{for } j=1,2,3,\cdots,N-1 \tag{A5}$$

where the additional terms $\left\{\tilde{\lambda}_{i,j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$ explicitly account for the inter-relations between the correlative couples $\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$ and their associated habitat conditions.

Equations (A5) can also be termed as "*Harmony Relationships*" precisely because they show that all the elements $\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$ of the Ordinal Matrix can be obtained by means of *one sole* arbitrary couple $\left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$, assumed as reference, and the N-1 Ordinal Roots $\left(\sqrt[N-1]{\{\tilde{1}\}}\right)_j$ of the Ordinal Unity $\{\tilde{1}\}$.

Consequently, if each element of the Ordinal Matrix (in Equation (A1)) is expressed in terms of the reference couple $\left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$, the solution to the first Fundamental Equation (1) assumes the following form

$$\left\{\tilde{r}\right\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ\left\{\begin{pmatrix}\left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{11}\\ \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{21}\\ \vdots\\ \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{N1}\end{pmatrix},\begin{pmatrix}\left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{12}\\ \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{22}\\ \vdots\\ \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{N2}\end{pmatrix},\begin{pmatrix}\cdots\\ \cdots\\ \vdots\\ \cdots\end{pmatrix},\begin{pmatrix}\left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{1N}\\ \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{2N}\\ \vdots\\ \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{NN}\end{pmatrix}\right\}} \tag{A6}$$

where, for the sake of simplicity, the term $\left\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\right\}$ stands for $\left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$.

The same Ordinal Matrix, in addition, may always be represented in a more synthetic form by means of one sole symbol, by adopting the following synthetic notation

$$\left\{\left(\sqrt[N-1]{\{\tilde{1}\}}\right)_{ij}\right\}^{\uparrow\{\tilde{N}/\tilde{N}\}}, \tag{A7}$$

where the arrow "↑" explicitly reminds us that the Ordinality $\{\tilde{N}/\tilde{N}\}$ has always to be considered as being a particular form of *Over-Ordinality*.

In this way the explicit solution to the first Fundamental Equation (1) can synthetically be expressed as follows

$$\left\{\tilde{r}\right\} = e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\right\}\circ\left(\sqrt[N-1]{\{\tilde{1}\}}\right)^{\uparrow\{\tilde{N}/\tilde{N}\}}}. \tag{A8}$$

Consequently, if such a solution is introduced into the Global Feed-Back Process represented by the second Fundamental Equation (2), the latter transforms into a typical Riccati's Equation of *Ordinal Nature*, whose explicit solution is given by

$$\left\{\tilde{r}\right\} = e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\tilde{B}(t)\right\}\circ\left(\sqrt[N-1]{\{\tilde{1}\}}\right)^{\uparrow\{\tilde{N}/\tilde{N}\}}}, \tag{A9}$$

where

$$\tilde{B}(t)=\left\{\begin{pmatrix}\oplus\tilde{A}(t)\\ \Theta\tilde{A}(t)\end{pmatrix},\begin{pmatrix}\Theta\tilde{A}(t)\\ \oplus\tilde{A}(t)\end{pmatrix}\right\} \tag{A10}$$

and

$$\tilde{A}(t)=\left\{\left\{\tilde{\alpha}_{12}(0)\right\}^{\{\tilde{2}/\tilde{2}\}}\oplus\left\{\tilde{\lambda}_{12}(0)\right\}^{\{\tilde{2}/\tilde{2}\}}\right\}\circ\left\{\left(\sqrt[N-1]{\{\tilde{1}\}}\right)^{\uparrow\{\tilde{N}/\tilde{N}\}}\right\}^{(\tilde{2}/\tilde{2})}\oplus\ln\left(\tilde{c}_1\oplus\{\tilde{c}_2,t\}\right), \tag{A11}$$

in which the term $\ln\left(\tilde{c}_1\oplus\{\tilde{c}_2,t\}\right)$ accounts for the *origin and habitat conditions* of the Feed-Back Equation and, at the same time, also represents an *Over-Ordinality* contribution specifically due to the same Feed-Back Process. A contribution which is particularly important for *the System stability* when the System interacts with a System of its Habitat.

Equation (A9) then represents the Explicit "Emerging Solution" to the Maximum Ordinality Principle, formulated in two "Incipient" Differential Equations ((4) and (5)), considered as being a Whole.

Appendix A2. The Explicit Solution Reformulated in Operative Terms with Reference to a Given Initialtime t_0

In order to get an explicit solution which may result much easier to program on a computer and, in particular, on a PC, the Solution (A8) can be restructured in more operative termsas follows, which are the same Relationships that characterize the Emerging Quality Simulator (EQS), as illustrated in more details in [8], and formulated with initial reference to a given time t_0

1) $$\tilde{\rho}_{1j}(t_0)=\tilde{A}\cdot e^{\tilde{S}_l(t_0)} \tag{A12}$$

where

$$\tilde{S}_l(t_0)=\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Sigma}_0-C_l\cdot\left(\tilde{\Phi}_0+\tilde{\Theta}_0\right)\right] \tag{A12.1}$$

2) $$\tilde{\varphi}_{1j}(t_0)=\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Phi}_0-C_l\cdot\tilde{\Sigma}_0\right] \tag{A13}$$

3) $$\tilde{\theta}_{1j}(t_0)=\psi_1\cdot E_l\cdot\left[B_l\cdot\tilde{\Theta}_0-C_l\cdot\tilde{\Sigma}_0+C_l\left(\tilde{\Phi}_0-\tilde{\Theta}_0\right)\right] \tag{A14}$$

where

$$B_l=\cos\left(\sqrt{2}\cdot\psi_l\right),\quad C_l=D_l=\frac{1}{\sqrt{2}}\sin\left(\sqrt{2}\cdot\psi_l\right) \tag{A15}$$

and

$$E_l=\frac{\varepsilon_1+4\pi\cdot l}{N-1},\quad \psi_l=\psi_2\cdot\frac{\varepsilon_2+2\pi\cdot l}{N-1}. \tag{A16}$$

In the previous Relationships:

1) $\tilde{\Sigma}_0,\tilde{\Phi}_0,\tilde{\Theta}_0$ represent the Ordinal coordinates of the reference couple, generally termed as "couple 12",considered at the time t_0. Then the symbols Σ_0,Φ_0,Θ_0 synthetically stand for $\left\{\tilde{\sigma}_{12}(t_0),\tilde{\varphi}_{12}(t_0),\tilde{\vartheta}_{12}(t_0)\right\}$

2) the Ordinalfactor $\psi_1\cdot E_l$ originates from the assumption that, in Equations (A5), which represent the *Harmony Relationships*, here reproduced for the sake of clearness

$$\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_j \circledR \left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \quad \text{(A17)}$$

for $j = 1, 2, 3, \cdots, N-1$

the terms $\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}$, after a previous reduction of the Ordinalities $\{\tilde{2}/\tilde{2}\} \to 1$, can be directly expressed in terms of a *specific periodicity*

$$E_l = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1} \quad \text{(A18)}$$

which, at the same time, ismodulated by the Ordinal factor ψ_1;

3) Then, after having rewritten the Ordinal Relationships (Equation (A8)) in the exponential form

$$Exp\left\{\tilde{\sigma}_{1j}(t_0), \tilde{\varphi}_{1j}(t_0), \tilde{\vartheta}_{1j}(t_0)\right\} \overset{*}{=} Exp\left[\left(\sqrt[N-1]{\tilde{1}}\right)_l \circledR \left\{\tilde{\sigma}_{12}(t_0), \tilde{\varphi}_{12}(t_0), \tilde{\vartheta}_{12}(t_0)\right\}\right] \quad \text{(A19)}$$

4) and after having assumed the explicit expression of the Ordinal Roots of the Unity asfollow (see Ref. [8])

$$\left(\sqrt[N-1]{\tilde{1}}\right)_l \overset{*}{=} Exp\left\{\tilde{\alpha} \circledR \tilde{i} \oplus \tilde{\beta} \circledR \tilde{j} \oplus \tilde{\gamma} \circledR \tilde{k}\right\}, \quad \text{(A20)}$$

where

$$\alpha = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1}, \quad \beta = \frac{\varepsilon_2 + 2\pi \cdot l}{N-1}, \quad \gamma = \frac{\varepsilon_3 + 2\pi \cdot l}{N-1}, \quad \text{(A21)}$$

the expansion series of Equation (A20) inside Equation (A19), together with the contextual adoption of the Rules of the Ordinal Product (A4.1), (A4.2), (A4.3), leads to the Ordinal Relationships (A12), (A12.1), (A13), (A14) initially introduced, with the associated coefficients expressed by Equations (A15), (A16).

For the sake of completeness it is worth adding that:

- The symbol $\{\tilde{1}\}$ represents *the Unity of the System* (understood as a Whole) by means the representation of the Ordinal *Unity* of its *Proper Space of Relations*;
- $\varepsilon_1, \varepsilon_2, \varepsilon_3$ characterize the *spatial orientation* of the System as a Whole, with reference to its Ordinal Proper Space;
- in Equations (A.21) the "periodicity" of the "spinor" $\tilde{i}$ is assumed equal to 4π, because it is expressed in steradians
- while the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to 2π radians, because these spinors are always "orthogonal", both among them and with respect to the spinor $\tilde{i}$. An "orthogonality" that can be seen as a form of reciprocal "irreducibility" (as also indicated by the same Relational Products);
- and the Factor "$\tilde{A}$" in Equation (6) represents an *Internal Ordinal Factor* according to which *all theradial* Uniances of the various Couples are appropriately referred to the *radial* Uniance of the Reference Couple "12". So that, on the basis of its "associated" cardinality, *all theradial* Uniances of the various Couples are all expressed in terms of an appropriate *scale measure* (e.g. in Mpc).