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Quantum Computingvia Entanglement in Geometric Algebra Approach

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Abstract

The superiority of hypothetical quantum computers is not due to faster calculations but due to different scheme of calculations running on special hardware. At the same time, one should realize that quantum computers would only provide dramatic speedups for a few specific problems, for example, factoring integers and breaking cryptographic codes in the conventional quantum computing approach. The core of quantum computing follows the way a state of a quantum system is defined when basic things interact with each other. In the conventional approach, it is implemented through the tensor product of qubits. In the suggested geometric algebra formalism simultaneous availability of all the results for non-measured observables is based on the definition of states as points on a three-dimensional sphere, which is very different from the usual Hilbert space scheme.

Keywords

Geometric Algebra, Wave Functions, Entanglement, Maxwell Equations, Three-Dimensional Sphere

1. Introduction. What Is Entanglement in Conventional Approach and in the Geometric Algebra Approach

In conventional quantum mechanics entanglement means that multiple objects share a single quantum state [1] [2]. Entanglement is the amount by which multiple objects share a quantum state. They remain indeterminate until they are disentangled by a measurement.

The simplest quantum mechanical state, qubit, reads:

$$C^2 \ni \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = z_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + z_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = z_1 |0\rangle + z_2 |1\rangle$$

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2, \quad \|x_1 + iy_1\|^2 + \|x_2 + iy_2\|^2 = 1$$

It has just two observable "things" after measurement, say "up" for $|0\rangle$ and "down" for $|1\rangle$, with probabilities z_1^2 and z_2^2. The qubit dimension two is the number of different observable things available after making a measurement on the particle.

In the case of two particles the vector space C^2 is generalized to density matrix defined on tensor product $C^2 \otimes C^2$. The appropriateness of tensor products is that the tensor product itself captures all the ways that basic things can "interact" with each other. Quantum entanglement means that aspects of one particle of an entangled pair depend on aspects of the other particle.

To understand the quantum entanglement, it is important to, first, consider quantum superposition. Quantum superposition is the idea that particles exist in multiple states at once. When a measurement is performed, it is as if the particle selects one of the states in the superposition. The quantum entanglement emerges from the reality of quantum superposition.

Consider the decay of a particle called the pi meson. When this particle decays, it produces an electron and a positron that have opposite spin and are moving away from each other. If the electron spin is measured to be up, then the measured spin of the positron could only be down, and vice versa. Because of quantum mechanics, the spin of each particle is both part up and part down until it is measured. Only when the measurement occurs does the quantum state of the spin "collapse" into either up or down—instantaneously collapsing the other particle into the opposite spin.

That's crucial words: "instantaneously collapsing". Common wisdom considers that as the physically real transformation of the other particle's unknown state into the known one.

The scheme suggested in the geometric algebra approach is based on the manipulation and transfer of quantum states as operators acting on observables. Wave functions act in that context on static G_3^+ elements through measurements, creating "particles." [3]

Normalized wave functions as elements of G_3^+ are naturally mapped onto unit sphere $\mathbb{S}^3$ (see **Figure 1**), [4] [5]. *Two-state system is then just a couple of points on* $\mathbb{S}^3$, *say*

$$\mathrm{e}^{I_{S_1}\varphi_1} = \alpha_1 + I_{S_1}\beta_1 = \alpha_1 + \beta_1 b_1^1 B_1 + \beta_1 b_1^2 B_2 + \beta_1 b_1^3 B_3$$

$$\mathrm{e}^{I_{S_2}\varphi_2} = \alpha_2 + I_{S_2}\beta_2 = \alpha_2 + \beta_2 b_2^1 B_1 + \beta_2 b_2^2 B_2 + \beta_2 b_2^3 B_3$$

where

$$(\alpha_1)^2 + (\beta_1)^2\left((b_1^1)^2 + (b_1^2)^2 + (b_1^3)^2\right) = (\alpha_1)^2 + (\beta_1)^2 = 1$$

$$(\alpha_2)^2 + (\beta_2)^2\left((b_2^1)^2 + (b_2^2)^2 + (b_2^3)^2\right) = (\alpha_2)^2 + (\beta_2)^2 = 1$$

in some bivector basis $B_1 B_2 B_3 = 1$, with multiplication rules $B_1 B_2 = -B_3$, $B_1 B_3 = B_2$, $B_2 B_3 = -B_1$.

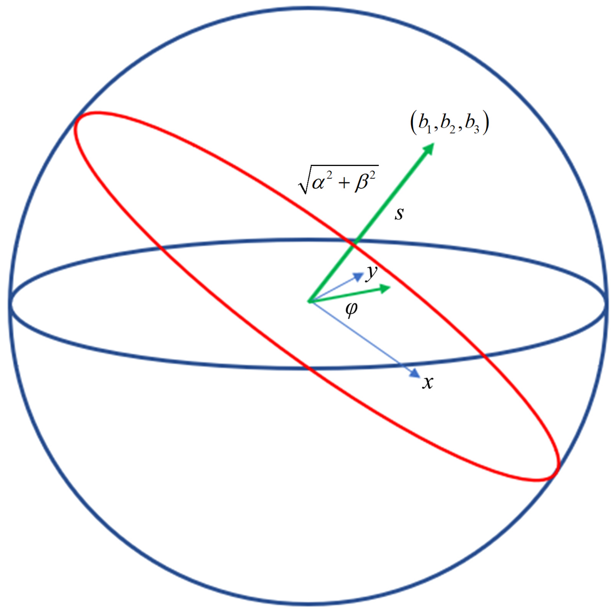

Figure 1. Geometric representation of a G_3^+ element.

Then it follows that two wave functions of an arbitrary two-function system are, in any case, connected by the Clifford translation[1]:

$$\mathrm{e}^{I_{S_2}\varphi_2} = \left(\mathrm{e}^{I_{S_2}\varphi_2}\mathrm{e}^{-I_{S_1}\varphi_1}\right)\mathrm{e}^{I_{S_1}\varphi_1} \equiv Cl\left(S_2,\varphi_2,S_1,\varphi_1\right)\mathrm{e}^{I_{S_1}\varphi_1}$$

The product of exponents $\mathrm{e}^{I_{S_2}\varphi_2}\mathrm{e}^{-I_{S_1}\varphi_1}$ *is trivial in the case* $S_1 = S_2 \equiv S$ (*the case of an unspecified imaginary unit in conventional quantum mechanics,*) *namely*: $\mathrm{e}^{I_{S_2}\varphi_2}\mathrm{e}^{-I_{S_1}\varphi_1} = \mathrm{e}^{I_S(\varphi_2-\varphi_1)}$. *In the general case we have more complicated result, see* (1.2.2) *in* [6]:

$$\begin{aligned} Cl\left(S_2,\varphi_2,S_1,\varphi_1\right) &\equiv \mathrm{e}^{I_{S_2}\varphi_2}\mathrm{e}^{-I_{S_1}\varphi_1} \\ &= \cos\varphi_1\cos\varphi_2 + \left(s_1\cdot s_2\right)\sin\varphi_1\sin\varphi_2 + I_3 s_2\cos\varphi_1\sin\varphi_2 \\ &\quad + I_3 s_1\cos\varphi_2\sin\varphi_1 + I_3\left(s_2\times s_1\right)\sin\varphi_1\sin\varphi_2 \end{aligned} \qquad (1.1)$$

where s_1 and s_2 are vectors dual to planes S_1 *and* S_2 *matching orientation of* I_3.

Take a general definition:

$$g_1g_2 = \alpha_1\alpha_2 - \left(s_1\cdot s_2\right)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 - I_3\left(s_1\times s_2\right)\beta_1\beta_2$$

The bivector part of the product g_1g_2 belongs to the plane orthogonal to the associated vector as seen in **Figure 2**.

To get the module $\left|g_1g_2\right|$ and angle of rotation in the plane of the bivector part of g_1g_2 we need the scalar part:

$$\alpha_1\alpha_2 - \left(s_1\cdot s_2\right)\beta_1\beta_2 = \alpha_1\alpha_2 - \beta_1\beta_2\left(b_1^1b_2^1 + b_1^2b_2^2 + b_1^3b_2^3\right)$$

and representation of $I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 - I_3\left(s_1\times s_2\right)\beta_1\beta_2$ as linear combination of the basis bivectors $\left\{B_1, B_2, B_3\right\}$:

[1]It is universally possible due to the hedgehog theorem.

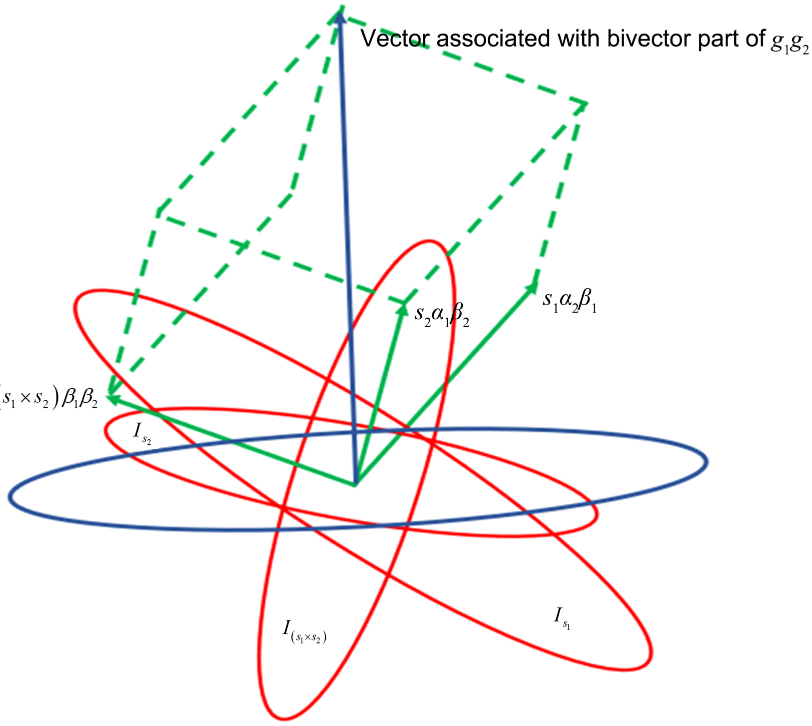

Figure 2. Constructing the bivector plane of the product.

$$\begin{aligned}
&I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 - I_3\left(s_1 \times s_2\right)\beta_1\beta_2 \\
&= \alpha_2\beta_1\left(b_1^1B_1 + b_1^2B_2 + b_1^3B_3\right) + \alpha_1\beta_2\left(b_2^1B_1 + b_2^2B_2 + b_2^3B_3\right) \\
&\quad + \beta_1\beta_2\left[\left(b_1^2b_2^3 - b_1^3b_2^2\right)B_1 + \left(b_1^3b_2^1 - b_1^1b_2^3\right)B_2 + \left(b_1^1b_2^2 - b_1^2b_2^1\right)B_3\right] \\
&= B_1\left(\alpha_2\beta_1b_1^1 + \alpha_1\beta_2b_2^1 + \beta_1\beta_2\left(b_1^2b_2^3 - b_1^3b_2^2\right)\right) \\
&\quad + B_2\left(\alpha_2\beta_1b_1^2 + \alpha_1\beta_2b_2^2 + \beta_1\beta_2\left(b_1^3b_2^1 - b_1^1b_2^3\right)\right) \\
&\quad + B_3\left(\alpha_2\beta_1b_1^3 + \alpha_1\beta_2b_2^3 + \beta_1\beta_2\left(b_1^1b_2^2 - b_1^2b_2^1\right)\right)
\end{aligned}$$

We get:

$$\begin{aligned}
g_1g_2 = {}& \alpha_1\alpha_2 - \beta_1\beta_2\left(b_1^1b_2^1 + b_1^2b_2^2 + b_1^3b_2^3\right) \\
&+ \beta\left(B_1\frac{\alpha_2\beta_1b_1^1 + \alpha_1\beta_2b_2^1 + \beta_1\beta_2\left(b_1^2b_2^3 - b_1^3b_2^2\right)}{\beta}\right. \\
&+ B_2\frac{\alpha_2\beta_1b_1^2 + \alpha_1\beta_2b_2^2 + \beta_1\beta_2\left(b_1^3b_2^1 - b_1^1b_2^3\right)}{\beta} \\
&\left.+ B_3\frac{\alpha_2\beta_1b_1^3 + \alpha_1\beta_2b_2^3 + \beta_1\beta_2\left(b_1^1b_2^2 - b_1^2b_2^1\right)}{\beta}\right)
\end{aligned}$$

where

$$\beta = \sqrt{\alpha_1^2\beta_2^2 + \alpha_2^2\beta_1^2 + \beta_1^2\beta_2^2\left|s_1 \times s_2\right|^2 + 2\alpha_1\alpha_2\beta_1\beta_2\left(s_1 \cdot s_2\right)}$$

Thus, the module $\left|g_1g_2\right|$ is:

$$
\begin{aligned}
|g_1 g_2| &= \sqrt{\left(\alpha_1\alpha_2 - \beta_1\beta_2\left(s_1\cdot s_2\right)\right)^2 + \alpha_1^2\beta_2^2 + \alpha_2^2\beta_1^2 + \beta_1^2\beta_2^2\left|s_1\times s_2\right|^2 + 2\alpha_1\alpha_2\beta_1\beta_2\left(s_1\cdot s_2\right)} \\
&= \sqrt{\alpha_1^2\alpha_2^2 + \beta_1^2\beta_2^2\left|s_1\cdot s_2\right|^2 - 2\alpha_1\alpha_2\beta_1\beta_2\left(s_1\cdot s_2\right) + \alpha_1^2\beta_2^2 + \alpha_2^2\beta_1^2 + \beta_1^2\beta_2^2\left|s_1\times s_2\right|^2 + 2\alpha_1\alpha_2\beta_1\beta_2\left(s_1\cdot s_2\right)} \\
&= \sqrt{\alpha_1^2\alpha_2^2 + \beta_1^2\beta_2^2\cos^2\left(s_1,s_2\right) + \alpha_1^2\beta_2^2 + \alpha_2^2\beta_1^2 + \beta_1^2\beta_2^2\sin^2\left(s_1,s_2\right)} \\
&= \sqrt{\alpha_1^2\alpha_2^2 + \beta_1^2\beta_2^2 + \alpha_1^2\beta_2^2 + \alpha_2^2\beta_1^2} = \sqrt{\alpha_1^2\left(g_2^2\right) + \beta_1^2\left(g_2^2\right)} \\
&= \sqrt{\left(g_2^2\right)\left(g_1^2\right)} = |g_1||g_2|
\end{aligned}
$$

If $I_{S_1} = I_{S_2}$ we also have usual formulas valid for complex numbers when formal imaginary unit i is replaced by I_{S_1} (or I_{S_2}.) Particularly, angles are added in multiplication, subtracted in division, and de Moivre's formula takes place:

If $g = |g|\left(\cos\varphi + \sin\varphi I_S\right)$ then $g^n = |g|^n\left(\cos n\varphi + \sin n\varphi I_S\right)$ for positive integer n.

In general case, when $I_{S_1} \neq I_{S_2}$, the angle of rotation $g_1 g_2$ is:

$$
\cos^{-1}\left(\frac{\alpha_1\alpha_2 - \beta_1\beta_2\left(s_1\cdot s_2\right)}{|g_1||g_2|}\right)
$$

and the plane of the rotation normal has directional cosines:

$$
\left\{\frac{\alpha_2\beta_1 b_1^1 + \alpha_1\beta_2 b_2^1 + \beta_1\beta_2\left(b_1^2 b_2^3 - b_1^3 b_2^2\right)}{\beta}, \frac{\alpha_2\beta_1 b_1^2 + \alpha_1\beta_2 b_2^2 + \beta_1\beta_2\left(b_1^3 b_2^1 - b_1^1 b_2^3\right)}{\beta}, \right.
$$
$$
\left. \frac{\alpha_2\beta_1 b_1^3 + \alpha_1\beta_2 b_2^3 + \beta_1\beta_2\left(b_1^1 b_2^2 - b_1^2 b_2^1\right)}{\beta}\right\}
$$

The result of Clifford translation (1.1) is an G_3^+ element. From knowing Clifford translation connecting any two wave functions as points $\mathbb{S}^3$ *it follows that the* result of measurement of any observable C by wave function $\mathrm{e}^{I_{S_1}\varphi_1}$, for example $\mathrm{e}^{I_{S_1}\varphi_1} C \mathrm{e}^{-I_{S_1}\varphi_1} \equiv C\left(S_1,\varphi_1\right)$, immediately gives the result of (not made) measurement by $\mathrm{e}^{I_{S_2}\varphi_2}$:

$$
\begin{aligned}
\mathrm{e}^{I_{S_2}\varphi_2} C \mathrm{e}^{-I_{S_2}\varphi_2} &= \mathrm{e}^{I_{S_2}\varphi_2}\mathrm{e}^{-I_{S_1}\varphi_1}\mathrm{e}^{I_{S_1}\varphi_1} C \mathrm{e}^{-I_{S_1}\varphi_1}\mathrm{e}^{I_{S_1}\varphi_1}\mathrm{e}^{-I_{S_2}\varphi_2} \\
&= \mathrm{e}^{I_{S_2}\varphi_2}\mathrm{e}^{-I_{S_1}\varphi_1} C\left(S_1,\varphi_1\right)\mathrm{e}^{I_{S_1}\varphi_1}\mathrm{e}^{-I_{S_2}\varphi_2} \\
&= Cl\left(S_2,\varphi_2,S_1,\varphi_1\right) C\left(S_1,\varphi_1\right)\overline{Cl\left(S_2,\varphi_2,S_1,\varphi_1\right)}
\end{aligned}
$$

When assuming that observables are also identified by points on $\mathbb{S}^3$ *and thus are connected by the formulas similar to the above one we get that measurements of any amount of observables by arbitrary set of wave functions are simultaneously available.*

This is a geometrically clear and unambiguous explanation of strict connectivity of the results of measurements instead of quite absurd "entanglement" in conventional quantum mechanics.

2. Maxwell Equation in Geometric Algebra

Without charges and currents, the Maxwell equation is:

$$\left(\partial_t + \nabla\right) F = 0 \tag{2.1}$$

The circular polarized electromagnetic waves are the only type of waves following from the solution of Maxwell equations in free space done in geometric algebra terms. Indeed, let's take the electromagnetic field in the form:

$$F = F_0 \exp\left[I_S \left(\omega t - k \cdot r \right) \right] \tag{2.2}$$

requiring that it satisfies (2.1)

Element F_0 in (2.2) is a constant element of geometric algebra G_3 and I_S is unit value bivector of a plane S in three dimensions, generalization of the imaginary unit [7]. The exponent in (2.2) is the unit value element of G_3^+:

$$\mathrm{e}^{I_S \varphi} = \cos\varphi + I_S \sin\varphi, \quad \varphi = \omega t - k \cdot r$$

Solution of (2.1) should be the sum of a vector (electric field e) and bivector (magnetic field $I_3 h$):

$$F = e + I_3 h$$

with some initial conditions:

$$e + I_3 h\big|_{t=0,\vec{r}=0} = F_0 = e\big|_{t=0,\vec{r}=0} + I_3 h\big|_{t=0,\vec{r}=0} = e_0 + I_3 h_0$$

Substitution of (2.2) into Maxwell's (2.1) will show us what the solution looks like.

The derivative by time gives

$$\frac{\partial}{\partial t} F = F_0 \mathrm{e}^{I_S \varphi} I_S \frac{\partial}{\partial t}\left(\omega t - k \cdot r \right) = F_0 \mathrm{e}^{I_S \varphi} I_S \omega = F I_S \omega$$

The geometric algebra product ∇F is:

$$\nabla F = F_0 I_S \mathrm{e}^{I_S \varphi} \nabla\left(\omega t - k \cdot r \right) = -F_0 \mathrm{e}^{I_S \varphi} I_S k = -F I_S k$$

or

$$\nabla F = F_0 \mathrm{e}^{I_S \varphi} \nabla\left(\omega t - k \cdot r \right) I_S = -F_0 \mathrm{e}^{I_S \varphi} k I_S = -F k I_S,$$

depending on do we write $I_S \left(\omega t - k \cdot r \right)$ or $\left(\omega t - k \cdot r \right) I_S$. The result should be the same because $\omega t - k \cdot r$ is a scalar.

Commutativity $I_S k = k I_S$ is valid only if $k \times I_3 I_S = 0$. The following agreement takes place between orientation of I_3, orientation of I_S and direction of vector k. The vector $I_3 I_S = I_S I_3$ is orthogonal to the plane of I_S and its direction is defined by orientations of I_3 and I_S. Rotation of right/left hand screw defined by orientation of I_S gives the movement of right/left hand screw. This is the direction of the vector $I_3 I_S = I_S I_3$. That means that the matching between $\hat{k}$ and I_S should be $\hat{k} = \pm I_3 I_S \Rightarrow \hat{k} I_S = \mp I_3$ (see **Figure 3**).

Assume first that orientation is $I_3 = \hat{k} I_S$. Then Maxwell equation becomes:

$$F\left(I_S \omega - I_3 |k| \right) = F\left(\omega I_S - |k| \hat{k} I_S \right) = 0$$

or

$$\left(e + I_3 h \right) \omega = \left(e + I_3 h \right) k$$

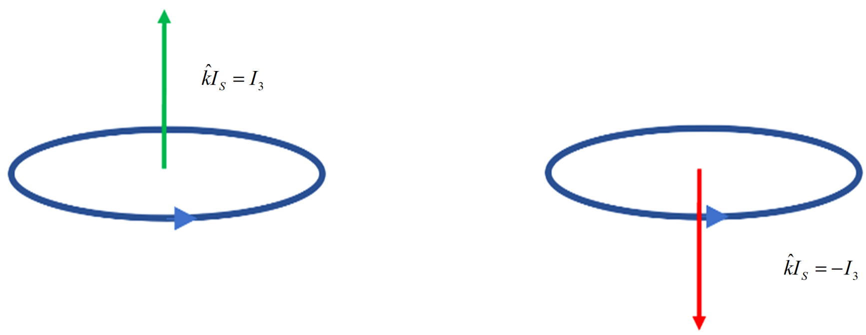

Figure 3. Two possible orientations of the wave vector.

Left hand side of equation is sum of vector and bivector, while right hand side is scalar $e \cdot k$ plus bivector $e \wedge k$, plus pseudoscalar $I_3(h \cdot k)$, plus vector $I_3(h \wedge k)$. It follows that both e and h lie on the plane of I_S and then:

$$\omega e = I_3 hk\ ,\quad \omega I_3 h = ek \to \frac{\omega^2}{|k|^2} I_3 hk = \omega e$$

Thus, $\omega = |k|$ and we get equation $I_3 h\hat{k} = e$ from which particularly follows $|e|^2 = |h|^2$ and $\hat{e}\hat{k}\hat{h} = I_3$.

The result for the case $I_3 = \hat{k}I_S$ is that the solution of (2.1) is

$$F = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k \cdot r)\right]$$

where e_0 and h_0 are arbitrary mutually orthogonal vectors of equal length, lying on the plane *S*. Vector k should be normal to that plane, $\hat{k} = -I_3 I_S$ and $|k| = \omega$.

In the above result, the sense of the I_S orientation and the direction of k were assumed to agree with $I_3 = \hat{k}I_S$. Opposite orientation, $-I_3 = \hat{k}I_S$, that's k and I_S compose left hand screw and $\hat{k} = I_3 I_S$, will give solution $F = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k \cdot r)\right]$ with $\hat{e}\hat{k}\hat{h} = I_3$.

<u>Summary:</u>

For a plane *S* in three dimensions Maxwell equation (2.1) has two solutions

- $F_+ = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k_+ \cdot r)\right]$, with $\hat{k}_+ = I_3 I_S$, $\hat{e}\hat{k}\hat{h}_+ = I_3$, and the triple $\{\hat{e}, \hat{k}, \hat{h}_+\}$ is right hand screw oriented, that's rotation of $\hat{e}$ to $\hat{h}$ by π/2 gives the movement of *right hand screw* in the direction of $k_+ = |k| I_3 I_S$.
- $F_- = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k_- \cdot r)\right]$, with $\hat{k}_- = -I_3 I_S$, $\hat{e}\hat{k}\hat{h}_- = -I_3$, and the triple $\{\hat{e}, \hat{k}, \hat{h}_-\}$ is left hand screw oriented, that's rotation of $\hat{e}$ to $\hat{h}$ by π/2 gives movement of *left hand screw* in the direction of $k_- = -|k| I_3 I_S$ or, equivalently, movement of *right hand screw* in the opposite direction, $-k_-$.
- e_0 and h_0, initial values of e and h, are arbitrary mutually orthogonal vectors of equal length, lying on the plane *S*. Vectors $k_\pm = \pm|k_\pm| I_3 I_S$ are normal to that plane. The length of the "wave vectors" $|k_\pm|$ is equal to the angular frequency ω.

Maxwell Equation (2.1) is a linear one. Then any linear combination of F_+ and F_- saving the structure of (2.2) will also be a solution.

Let's write:

$$\begin{cases} F_+ = (e_0 + I_3 h_0)\exp\left[I_S \omega\left(t - (I_3 I_S)\cdot r\right)\right] \\ \quad = (e_0 + I_3 h_0)\exp[I_S \omega t]\exp\left[-I_S\left[(I_3 I_S)\cdot r\right]\right] \\ F_- = (e_0 + I_3 h_0)\exp\left[I_S \omega\left(t + (I_3 I_S)\cdot r\right)\right] \\ \quad = (e_0 + I_3 h_0)\exp[I_S \omega t]\exp\left[I_S\left[(I_3 I_S)\cdot r\right]\right] \end{cases} \tag{2.3}$$

Then for arbitrary (real[2]) scalars λ and μ:

$$\lambda F_+ + \mu F_- = (e_0 + I_3 h_0)\mathrm{e}^{I_S \omega t}\left(\lambda \mathrm{e}^{-I_S\left[(I_3 I_S)\cdot r\right]} + \mu \mathrm{e}^{I_S\left[(I_3 I_S)\cdot r\right]}\right) \tag{2.4}$$

is solution of (2.1). The item in the second parenthesis is a weighted linear combination of two states with the same phase in the same plane but opposite sense of orientation. The states are strictly coupled, entangled if you prefer, because the bivector plane should be the same for both, does not matter what happens with that plane.

Arbitrary linear combination (2.4) can be rewritten as:

$$\lambda \mathrm{e}^{I^+_{Plane}\varphi^+} + \mu \mathrm{e}^{I^-_{Plane}\varphi^-} \tag{2.5}$$

where

$$\varphi^{\pm} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\cos\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)\right),$$

$$I^{\pm}_{Plane} = I_S \frac{\sin\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}} + I_{B_0}\frac{\cos\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}} + I_{E_0}\frac{\sin\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}}$$

The triple of unit value basis orthonormal bivectors $\{I_S, I_{B_0}, I_{E_0}\}$ is comprised of the I_S bivector, dual to the propagation direction vector; I_{B_0} is dual to an initial vector of the magnetic field; I_{E_0} is dual to the initial vector of the electric field. The expression (2.5) is a linear combination of two geometric algebra states, g-qubits.

Linear combination of the two equally weighted basic solutions of the Maxwell equation F_+ and F_-, $\lambda F_+ + \mu F_-$ with $\lambda = \mu = 1$ reads:

$$\lambda F_+ + \mu F_-\big|_{\lambda=\mu=1} = 2\cos\omega\left[(I_3 I_S)\cdot r\right]\left(\frac{1}{\sqrt{2}}\cos\omega t + I_S\frac{1}{\sqrt{2}}\sin\omega t + I_{B_0}\frac{1}{\sqrt{2}}\cos\omega t + I_{E_0}\frac{1}{\sqrt{2}}\sin\omega t\right) \tag{2.6}$$

where $\cos\varphi = \frac{1}{\sqrt{2}}\cos\omega t$ and $\sin\varphi = \frac{1}{\sqrt{2}}\sqrt{1+(\sin\omega t)^2}$. It can be written in

[2]Remember, in the current theory scalars are real ones. "Complex" scalars have no sense.

standard exponential form $\cos\varphi + \sin\varphi I_B = e^{I_B \varphi}$.[3]

I will call such g-qubits ***spreons*** because they spread over the whole three-dimensional space for all values of time and instantly change under Clifford translations over the whole three-dimensional space for all values of time, along with the results of measurement of any observable.[4]

3. Measurement of Observables by Spreons

Measurement of any observable $C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3$ (actually Hopf fibration) by a state $\alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3$ in the current formalism is, see, for example, [6]:

$$
\begin{aligned}
&C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3 \xrightarrow{\alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3} \\
&C_0 + \left(C_1\left[\left(\alpha^2 + \beta_1^2\right) - \left(\beta_2^2 + \beta_3^2\right)\right] + 2C_2\left(\beta_1\beta_2 - \alpha\beta_3\right) + 2C_3\left(\alpha\beta_2 + \beta_1\beta_3\right)\right) B_1 \\
&+\left(2C_1\left(\alpha\beta_3 + \beta_1\beta_2\right) + C_2\left[\left(\alpha^2 + \beta_2^2\right) - \left(\beta_1^2 + \beta_3^2\right)\right] + 2C_3\left(\beta_2\beta_3 - \alpha\beta_1\right)\right) B_2 \\
&+\left(2C_1\left(\beta_1\beta_3 - \alpha\beta_2\right) + 2C_2\left(\alpha\beta_1 + \beta_2\beta_3\right) + C_3\left[\left(\alpha^2 + \beta_3^2\right) - \left(\beta_1^2 + \beta_2^2\right)\right]\right) B_3
\end{aligned}
$$

In the case of spreon (2.6):

$$B_1 = I_S, \quad B_2 = I_{B_0}, \quad B_3 = I_{E_0},$$

$$
\begin{aligned}
\alpha &= 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\left(\cos\gamma\cos\omega t - \gamma_1 \sin\gamma\cos\omega t\right. \\
&\qquad \left. - \gamma_2 \sin\gamma\cos\omega t - \gamma_3 \sin\gamma \sin\omega t\right) \\
\beta_1 &= 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\left(\cos\gamma\sin\omega t + \gamma_1 \sin\gamma\cos\omega t\right. \\
&\qquad \left. - \gamma_2 \sin\gamma\sin\omega t + \gamma_3 \sin\gamma \cos\omega t\right) \\
\beta_2 &= 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\left(\cos\gamma\cos\omega t + \gamma_1 \sin\gamma\sin\omega t\right. \\
&\qquad \left. + \gamma_2 \sin\gamma\cos\omega t - \gamma_3 \sin\gamma \sin\omega t\right) \\
\beta_3 &= 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\left(\cos\gamma\sin\omega t - \gamma_1 \sin\gamma\cos\omega t\right. \\
&\qquad \left. + \gamma_2 \sin\gamma\sin\omega t + \gamma_3 \sin\gamma \cos\omega t\right),
\end{aligned}
$$

and we get a G_3^+ element spreading through the three-dimensional space for all values of the time parameter *t*. It is G_3^+ element spreading through the three-dimensional space for all values of time parameter *t*:

$$
\begin{aligned}
&4\cos^2\omega\left[\left(I_3 I_S\right)\cdot r\right]\left[C_0 + C_3 I_S + \left(C_1 \sin 2\omega t + C_2 \cos 2\omega t\right) I_{B_0}\right. \\
&\left. + \left(C_2 \sin 2\omega t - C_1 \cos 2\omega t\right) I_{E_0}\right]
\end{aligned}
\tag{3.1}
$$

[3]Good to remember that the two basic solutions F_+ and F_- differ only by the sign of $I_3 I_S$, which is caused by orientation of I_S that in its turn defines if the triple $\{\hat{E}, \hat{H}, \pm I_3 I_S\}$ is right-hand screw or left-hand screw oriented.

[4]The two received solutions are similar in their form to the Majorana operators, though with a bivector instead of formal imaginary unit. Thus, not surprising that the following computational scheme represents more general software simulation which gets around the problem of physical implementation of Majorana fermions.

Geometrically, that means that the measured observable is rotated by π/2 in the I_{B_0} plane, such that the $C_3 I_S$ component becomes orthogonal to the plane I_S and remains unchanged. Two other components became orthogonal to I_{B_0} and I_{E_0} and continue rotating in I_S with angular velocity $2\omega t$. The factor $4\cos^2\omega\left[\left(I_3 I_S\right)\cdot r\right]$ defines the dependency of those transformed values through all points of the three-dimensional space.

The current approach transcends common quantum computing schemes since the latter are principally based on qubit entanglement (whatever it is) and thus have tough problems of creating large sets of entangled qubits. In the current scheme, any test observable can be placed anywhere into the continuum of the (t,r) dependent values of the spreon state. The above formula (3.1) gives the result of measurements simultaneously at all points (t,r).

Let us get back to the geometric algebra entanglements explained at the end of Sec.1. Assume we have two observables, $C_0^1 + C_1^1 B_1 + C_2^1 B_2 + C_3^1 B_3$ and $C_0^2 + C_1^2 B_1 + C_2^2 B_2 + C_3^2 B_3$. Write them in exponential form:

$$\begin{aligned} C^1 &\equiv C_0^1 + C_1^1 B_1 + C_2^1 B_2 + C_3^1 B_3 \\ &= \left|C^1\right|\left(\frac{C_0^1}{\left|C^1\right|} + \frac{\left|C_B^1\right|}{\left|C^1\right|}\frac{C_1^1 B_1}{\left|C_B^1\right|} + \frac{\left|C_B^1\right|}{\left|C^1\right|}\frac{C_2^1 B_2}{\left|C_B^1\right|} + \frac{\left|C_B^1\right|}{\left|C^1\right|}\frac{C_3^1 B_3}{\left|C_B^1\right|}\right) \\ &= \left|C^1\right|\left(\cos\varphi_1 + \sin\varphi_1 S_1\right) = \left|C^1\right| \mathrm{e}^{S_1\varphi_1} \end{aligned}$$

where $\left|C^1\right| = \sqrt{\left(C_0^1\right)^2 + \left(C_1^1\right)^2 + \left(C_2^1\right)^2 + \left(C_3^1\right)^2}$, $\left|C_B^1\right| = \sqrt{\left(C_1^1\right)^2 + \left(C_2^1\right)^2 + \left(C_3^1\right)^2}$,

$$\cos\varphi_1 = \frac{C_0^1}{\left|C^1\right|},\quad \sin\varphi_1 = \frac{\left|C_B^1\right|}{\left|C^1\right|},\quad S_1 = \frac{C_1^1 B_1}{\left|C_B^1\right|} + \frac{C_2^1 B_2}{\left|C_B^1\right|} + \frac{C_3^1 B_3}{\left|C_B^1\right|}$$

$$\begin{aligned} C^2 &\equiv C_0^2 + C_1^2 B_1 + C_2^2 B_2 + C_3^2 B_3 \\ &= \left|C^2\right|\left(\frac{C_0^2}{\left|C^2\right|} + \frac{\left|C_B^2\right|}{\left|C^2\right|}\frac{C_1^2 B_1}{\left|C_B^2\right|} + \frac{\left|C_B^2\right|}{\left|C^2\right|}\frac{C_2^2 B_2}{\left|C_B^2\right|} + \frac{\left|C_B^2\right|}{\left|C^2\right|}\frac{C_3^2 B_3}{\left|C_B^2\right|}\right) \\ &= \left|C^2\right|\left(\cos\varphi_2 + \sin\varphi_2 S_2\right) = \left|C^2\right| \mathrm{e}^{S_2\varphi_2} \end{aligned}$$

where $\left|C^2\right| = \sqrt{\left(C_0^2\right)^2 + \left(C_1^2\right)^2 + \left(C_2^2\right)^2 + \left(C_3^2\right)^2}$, $\left|C_B^2\right| = \sqrt{\left(C_1^2\right)^2 + \left(C_2^2\right)^2 + \left(C_3^2\right)^2}$,

$$\cos\varphi_2 = \frac{C_0^2}{\left|C^2\right|},\quad \sin\varphi_2 = \frac{\left|C_B^2\right|}{\left|C^2\right|},\quad S_2 = \frac{C_1^2 B_1}{\left|C_B^2\right|} + \frac{C_2^2 B_2}{\left|C_B^2\right|} + \frac{C_3^2 B_3}{\left|C_B^2\right|}$$

The observable C^2 can be written as:

$$C^2 = \frac{\left|C^2\right|}{\left|C^1\right|} C^1 \mathrm{e}^{-S_1\varphi_1} \mathrm{e}^{S_2\varphi_2},$$

and then its measurement by any g-qubit $\mathrm{e}^{S\varphi}$ reads:

$$\mathrm{e}^{-S\varphi} C^2 \mathrm{e}^{S\varphi} = \mathrm{e}^{-S\varphi} \frac{\left|C^2\right|}{\left|C^1\right|} C^1 \mathrm{e}^{S\varphi} \mathrm{e}^{-S\varphi} \mathrm{e}^{-S_1\varphi_1} \mathrm{e}^{S_2\varphi_2} \mathrm{e}^{S\varphi}$$

that is, up to the factor $\frac{|C^2|}{|C^1|}$, the result of measurement of C^1, multiplied by the result of measurement of $e^{-S_1\varphi_1}e^{S_2\varphi_2}$.

4. Software Simulation of Analog Computing

The hardware creating superfields requires special implementation of a photonic/laser device that does not exist yet. Instead, we have a very convenient equivalent simulation scheme where the amount of simultaneously available space/time points of observable measured values is only restricted, for example, by the overall available Nvidia GPU number of threads.

Formula (3.1) gives the result of measuring any type of G_3^+ element $C_0 + C_1B_1 + C_2B_2 + C_3B_3$ by the sprefield state.

Let us consider pieces of code for possible implementation.

The CUDA code begins with allocating an initial array of (3.1) on the host at $t = 0$. For example:

```
size_t memsize = 0;
struct cudaDeviceProp deviceProp;
checkCudaErrors(cudaGetDeviceProperties(&deviceProp,
0));
memsize = deviceProp.totalGlobalMem;
// We can never use all the memory so to keep things simple
we aim to
// use around half the total memory
memsize /= 2;

int defaultDim = (int)floor(pow((memsize / (2.0 * si-
zeof(float))), 1.0 / 3.0));
// above is the size of cube for all considered points
float step = volumeSize.width / defaultDim;
float4* r = (float3*)malloc(memsize * sizeof(float4));

r[0].x = r[0].y = r[0].z = r[0].w = 0.0;
for (int k = 0; k < defaultDim; k++)
{
    r[k].x = r[0].x + k * step;
    r[k].y = r[0].y + k * step;
    r[k].z = r[0].z + k * step;
}
float omega = 12560000.0;   // an option from laser beams
float* factor = (float*)malloc(defaultDim * si-
zeof(float));
for (int k = 0; k < defaultDim; k++)
{
```

```
    // below from formula (3.1)
    factor[k] = 4. * (cosf(omega * r[k].x)) * (cosf(omega
* r[k].x));
    r[k].x = factor[k] * C3;
    r[k].y = factor[k] * C2;
    r[k].z = -factor[k] * C1;
    r[k].w = factor[k] * C0;
}
```

Now, copy to the device memory:

```
Float4* d_r = NULL;
checkCudaErrors(cudaMalloc(&d_r, memsize));
checkCudaErrors(cudaMemcpy(d_r, r, memsize, cudaMemcpyHostToDevice));
```

Run the CUDA part of calculations and graphics output:

```
// create VBO
createVBO(&vbo, &cuda_vbo_resource, cudaGraphicsMapFlagsWriteDiscard);
// run the cuda part
runCuda(&cuda_vbo_resource);
// start rendering mainloop
glutMainLoop();
```

with specially written kernel function

```
vbo_kernel <<< grid, block >>> (pos, mesh_width,
mesh_height, mesh_depth, time);
```

running inside the runCuda. The vbo_kernel executes in the parallel recalculation of all measured observable values sitting in the cube, changing with time and their graphics output.

5. Conclusions

The geometric algebra lift of conventional quantum mechanics qubits is the game-changing quantum leap forward potentially kicking from the quantum computing market big fishes (IBM, Microsoft, Google, dozens of smaller ones) investing billions in elaborating quantum computing devices. The approach brings into reality a kind of physical field spreading through the whole three-dimensional space and values of the time parameter. The fields can be modified instantly in all points of space and time values. All measured observable values are simultaneously available all together, not through looking one by one. In this way, the new type of quantum computer appeared to be a kind of analog computer keeping and instantly processing information by and on sets of objects possessing an infinite number of degrees of freedom. As a practical implementation, the multithread GPUs with the CUDA language functionality allow the creation of software simulating that kind of field processing numbers of space/time discrete points only restricted by the GPU threads capacity.

Specific problems that can get huge speedups with the suggested analog quantum computers are mainly related to the continuous media calculations and blockchain schemes [8].

Thus, the innovative scheme of quantum computing with its availability in the near future, incomparable low cost value and scalable simulation software platform is becoming a tremendous challenge for companies like those mentioned above.

The corresponding anticipated objective is to create simulating software running on a GPU, preferably programmed in the Nvidia CUDA language. In that way, we will get a quantum computer as an analog computer simulating instantly parallelizable processing, and transformation of states identified by points on the three-dimensional spheres, that replace the Hilbert space formalism. The amount of simultaneously available space/time points of the observable measured values is then only restricted by the overall available GPU number of threads.

The simulating software functionality will follow the requirements of further developing quantum computing in all areas where its paradigm will give tremendous effectiveness, first of all in speed increasing by orders, from running years to seconds: material science, finances, drug development, cryptography, weather forecasting, to mention just some immediately seen ones.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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