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Explanation of Two Important Empirical Relations for Galaxies

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Abstract

The phenomenon of "missing mass" in galaxies has triggered new theoretical exploration, forming a competition between dark matter assumption, modified Newtonian dynamics and modified gravity. Over the past forty years, various versions of the modified scenario have been proposed to simulate the effects of missing mass. These schemes replace the dynamic effect of dark matter by introducing some tiny extra force terms in the dynamic equations. Such extra forces have mainly interactions on large scales of galaxies, such as fitting the Tully-Fisher relation or asymptotically flat rotation curves. The discussion in this paper shows that the evidence of taking the modified schemes as fundamental theory is still insufficient. In this paper, we display a system of simplified galactic dynamical equations derived from weak field and low-speed approximations of Einstein field equations, and then we use it to discuss two important empirical relations in galactic dynamics, namely the Faber-Jackson relation and Tully-Fisher relation, as well as the related fundamental plane. These discussions provide a reference scheme for improving the dispersion of the empirical relations, and also provide a theoretical foundation to analyze the properties of dark matter and galactic structures.

Keywords

Galactic Dynamics, Faber-Jackson Relation, Tully-Fisher Relation, Dark Matter, MOND

1. Introduction

The general theory of relativity has achieved great success in gravitational redshift, starlight bending, Mercury's perihelion precession, radar echo delay, modern astrophysics such as the properties of dense stars, gravitational wave detection, and cosmology. However, it is still a weak link in the study of galactic structure, and there are some unsolved mysteries, such as "missing mass" in galaxies, resulting in competing new theories in galactic astronomy [1] [2]. In 1934, Swiss astronomer Zwicky found that the total gravitational mass of a galaxy calculated by the gravitational virial theorem was much larger than the luminous mass measured by optics, so most of the gravitational mass in galaxies was invisible [3] [4]. According to Newton's law of universal gravitation, the rotational velocity of stars in a galaxy should gradually decrease as the radius increases. But in the 1970s, American female astronomer Rubin *et al.* found that the rotation velocity of the gases around the center of a spiral galaxy at different radii is almost a constant, which is the mystery of the asymptotic flat rotation curve of galaxies [5] [6]. Obviously, such rapidly rotating stars in a galaxy, also need the strong gravity of "missing mass" to maintain balance. This mystery is also called "Dark Matter" mystery. Except for the gravitational force, this dark matter has no other interaction with ordinary matter, including electromagnetic interaction, accounting for more than 90% of the gravitational mass of the galaxy [7] [8].

In the framework of Newton's universal gravity, if the total radial mass distribution of the galaxy is $M(r)$, then the equation of motion of ordinary matter in the galaxy is given by:

$$\frac{GM(r)}{r^2} = f(a), \quad a = \left|\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}\right|. \tag{1.1}$$

For the uniform circular motion, the centripetal acceleration is $a = v^2/r$. Zwicky's calculations show that [3] [4], the optical mass in the galaxy is too small and Equation (1.1) does not hold. In order to solve this contradiction, physicists and astronomers have proposed three scenarios: first, the existence of non-baryon dark matter, second, the Modified Newtonian Dynamics (MONDs), and third, the Modified Gravity (MOG). The first scenario is in the framework of conventional mechanics and gravity theory, so it needs a large amount of non-luminous dark matter in galaxies, involved in gravitational interactions, binding the entire galaxy or cluster together. Non-baryon dark matter is the Weakly Interacting Massive Particles (WIMPs). At present, it is generally believed that WIMP candidates include supersymmetric particles, axions, noble neutrinos, quantum zero-point fluctuation black holes, magnetic monopoles, Kaluza-Klein high-dimensional particles, and so on [9]. Ordinary neutrinos with small mass can also be dark matter candidates in the universe. But when galaxies formed in the early universe, neutrinos were hot dark matter and could not form a gravitational potential well to generate galaxy. Only cold dark matter particles can produce galaxies, so ordinary neutrinos do not meet the requirements of cold dark matter. WIMP has been a hot issue in astrophysics but has not been detected [10] [11].

The second scenario is MOND. This scenario does not require the dark matter assumption, so it requires a modification of Newtonian mechanics [12] [13] [14] [15] [16], that is, to modify the $f(a)$ in the equation of motion (1.1). The initial scheme of Milgrom was $f(a) \to a^2/a_0$ at a very small acceleration of $a \ll a_0 = 1.2 \times 10^{-10}\ \mathrm{m/s^2}$. In the MOND scenario, the squared galaxy rotational velocity is given by:

$$v^2 = \sqrt{GM(r)a_0}. \tag{1.2}$$

Many researchers believe that the above equation can well explain some phenomena of galaxies, such as baryon Tully-Fisher relation in galaxies [17], radial acceleration relation [18] [19] [20], asymptotically flat rotation curve, etc. [21] [22]. Thus, these may indicate that the MOND can be viewed as a universal phenomenological description of galaxies. However, most data for galaxies have substantial uncertainty. For example, the high-quality rotation curve data (SPARC) used to support the MOND has an average of 7.1% observational uncertainty [23]. In theory, there are some problems contradicting the expectations of Milgrom. For example, the above formula only holds for a spherically symmetric density or some very special density distribution like homoeoid [24] (Ch.2 Section 1.1). MOND considers that a spherically symmetric dark halo does not exist, so the mass distribution of the spiral galaxy must be disk-like. Unless the surface density satisfies the special Mestel disk $\Sigma \propto r^{-1}$, generally the equation of motion of the particles in disk is different from the above formula [24] (Ch.2 Section 6.3). The Newtonian gravitational acceleration produced by the homogeneous disk is given by $g_N \approx GMr/R^3$, and the equation of motion is very different from (1.2) [25]. The $M(r)$ of ordinary matter in the galaxy disk is generally a function of r, so the idea that MOND can explain the flat rotation curve and Tully-Fisher relation may be insufficient, and doubts have always existed [26] [27] [28] [29].

The third scenario is the Modified Gravity (MOG). In such schemes, any dark matter assumption is also unneeded [30] [31] [32] [33], but the Newtonian theory of gravity no longer holds at the large scale of galaxy. Since Newtonian gravity is the first-order approximation to Einstein's theory of gravity, to modify Newtonian theory, we need to modify general relativity first. Although the MOND theory also requires modifying the general relativity theory of gravity, it is not the same as that stated here for modified gravity. Modified gravity has encountered difficulties in explaining the galaxy kinematics and the Tully-Fisher relation [34].

Due to the complexity of the nonlinear partial differential equations, general relativity is rarely used in the study of galactic structures. In the paper [25], the author simplifies the galactic dynamical equations by introducing the following three working hypotheses. These three assumptions are as follows: 1) Einstein field equation is correct, and the galactic dynamics should be its weak field and low-speed approximation. In the large-scale structure, the retarded potential of the gravitational field cannot be ignored. 2) In a well-developed galaxy, ordinary matter such as stars is pressure-free and inviscid perfect fluid, which moves along geodesics. 3) The structure of galaxies is mainly controlled by the total mass density distribution. The simplified equations of galactic dynamics are decoupled from each other, and the equations are well-structured and convenient for analysis and solving.

In this paper, we first introduce the weak field and low-speed approximation of Einstein field equation, which is the simplified galactic dynamics. According to the dynamics, then we explain two important empirical relations in galaxy astronomy, namely the Faber-Jackson relation and the Tully-Fisher relation. Finally, we analyze the shortcomings of the modified theories. The arguments of the paper depend on that general relativity is still true at the galaxy scale. The simplified dynamical equations provide a rational and practical theoretical framework for studying galactic structure, capable of solving many practical problems.

2. Simplified Galactic Dynamics

General theory of relativity is a successful theory repeatedly verified by astronomical observation, with profound philosophical principles and mathematical foundations. Therefore, it is the fundamental theory that should be selected first in astrophysics. Einstein field equation is essentially a wave equation, and the measurements of gravitational waves show that the waves propagate at the speed of light c with a relative error of no more than 10^{-15}. On October 16th, 2017, Group of Gravitational Wave Observatories in the United States and Europe, and including Chinese Space X-ray Astronomical Satellite Eye Telescope and the Antarctic Survey Telescope, more than 70 astronomical observatories around the world have jointly announced that, at 17 Aug 17, 2017 12:41:04 (UTC) from the NGC4993 galaxy about 130 million light years away from Earth, Gravitational wave signals (GW170817) generated by the merger of two neutron stars with 0.86 and 2.26 times of solar mass were detected by two laser interferometric gravitational wave detectors at Advanced LIGO in the United States and Advanced Virgo in Italy [35] [36] [37]. This is the first time in human history that a gravitational wave signal from a binary neutron star merger has been detected. At 1.7 seconds after the gravitational wave signal arrived, the gamma-ray burst monitoring system GBM on the NASA Fermi gamma-ray Space Telescope received the gamma-ray burst signal (GRB170817A) associated with the double neutron star merger event [38]. This shows that the gravitational waves and electromagnetic waves from the merger of two neutron stars reach the Earth through 130 million light-years of distance. The gravitational wave signal and electromagnetic wave signal spread in space for 4×10^{15} seconds, and the two reach the Earth only 1.7 seconds apart. This difference should be caused by the refractive index of charged interstellar dust, so the velocity of gravitational waves should travel at the exact speed of light. Thus, before encountering clear problems in theory and observation, the most reliable basic theory is still general relativity, and the safest way is to study Einstein's field equation.

It is generally believed that relativistic effects need to be considered only when high-speed motion and severely curved space-time are involved. In fact, for large-scale problems, relativistic effects should also be considered due to the gravitational retarded potential. The galactic diameter is from 100,000 to a million light years, so galactic dynamics must include the effects of the retarded potential. Unless the galaxy is a static structure, ignoring the retarded potential will inevitably lead to distorted conclusions. The galactic diameter exceeds 100,000 light years, so the influence of the gravitational retarded potential should be considered when solving the dynamic equations. Stars near the center of the galaxy have circled the center several times before the corresponding change of gravitational field reaches the edge of the galaxy. Newton's gravity is an action at a distance, which ignores this time delay. By Newton's gravity we cannot get the stable spiral structure of a galaxy.

In the context of general relativity, the whole system of the dynamic equations for galactic evolution should be Einstein field equations:

$$G^{\mu\nu} \equiv \mathcal{R}^{\mu\nu} - \frac{1}{2} g^{\mu\nu}\mathcal{R} = -\bar{\kappa} T^{\mu\nu}, \quad \left(\bar{\kappa} = 8\pi G c^{-4}\right), \tag{2.1}$$

combined with the law of energy-momentum conservation and the equation of state of the gravitating source. In which $\mathcal{R} = g_{\mu\nu}\mathcal{R}^{\mu\nu}$ is the scalar curvature. In the classical approximation for the total energy-momentum, tensor takes the following form [25] [39] [40] [41]:

$$T^{\mu\nu} = (\rho + P)\mathcal{U}^{\mu}\mathcal{U}^{\nu} + (W - P) g^{\mu\nu}, \tag{2.2}$$

where W corresponds to the nonlinear potentials, which acts as negative pressure or dark energy, and leads to the deviation from geodesic. According to the law of energy-momentum conservation $T^{\mu\nu}_{;\nu} = 0$, we can derive the continuity equation $\mathcal{U}_{\mu} T^{\mu\nu}_{;\nu} = 0$ and the equation of motion for the source as follows:

$$\mathcal{U}^{\mu}\partial_{\mu}(\rho + W) = -(\rho + P)\mathcal{U}^{\mu}_{;\mu}, \tag{2.3}$$

$$(\rho + P)\mathcal{U}^{\nu}\mathcal{U}^{\mu}_{;\nu} = \left(g^{\mu\nu} - \mathcal{U}^{\mu}\mathcal{U}^{\nu}\right)\partial_{\nu}(P - W). \tag{2.4}$$

In the case of $W \sim \rho \gg P$, such as for the nonlinear spinors, we find that the stream lines are quite different from the geodesics $\mathcal{U}^{\nu}\mathcal{U}^{\mu}_{;\nu} = 0$. Therefore, a fully relativistic simulation of the galactic evolution should include such terms. However, in the following non-relativistic approximation, the effects of (P, W) can be merged into an effective mass-energy density ρ, which will greatly simplify the galactic dynamics.

For convenience, we take $c = 1$ as the unit of velocity. Noting the facts that collisions between stars rarely occur, the trajectories of the ordinary matter, such as atoms, are almost geodesics, so for stars, the following pressure-free and inviscid energy-momentum tensor holds $T_s^{\mu\nu} = \rho_s U^{\mu} U^{\nu}$, in which ρ_s is the comoving mass density of stars, and U^{μ} is the 4-vector speed of the stellar flow. For energy-momentum tensor of a compound system, the energy-momentum of any independent subsystem is conserved respectively [42]. Thus, the ordinary matter satisfies the law of energy-momentum conservation independent of that of the dark halo, so we have $T^{\mu\nu}_{s;\nu} = 0$. Expressing it in the form of equations of continuity and motion, we obtain the dynamic equations for the stars:

$$U^{\mu}\partial_{\mu}\rho_s+\rho_s U^{\mu}_{;\mu}=0,\quad U^{\nu}U^{\mu}_{;\nu}=0. \tag{2.5}$$

The total energy-momentum tensor of the galaxy is still given by (2.2), and satisfies the dynamic Equations (2.3) and (2.4). Using (2.1) and (2.2), we obtain:

$$\mathcal{R}=\bar{\kappa}\left(\rho+4W-3P\right). \tag{2.6}$$

Substituting (2.6) into (2.1), we obtain:

$$\mathcal{R}^{\mu\nu}=-\bar{\kappa}\left(\rho+P\right)\mathcal{U}^{\mu}\mathcal{U}^{\nu}+\frac{1}{2}\bar{\kappa}\left(\rho+2W-P\right)g^{\mu\nu}, \tag{2.7}$$

where $\mathcal{U}^{\mu}$ is the average 4-vector speed of all gravitating source.

In order to make a weak-field approximation, we choose the harmonic coordinate system, which leads to the usual Cartesian coordinate system when linearizing of metric. Then we have the de Donder coordinate condition:

$$\Gamma^{\mu}\equiv g^{\alpha\beta}\Gamma^{\mu}_{\alpha\beta}=-\frac{1}{\sqrt{g}}\partial_{\nu}\left(\sqrt{g}g^{\mu\nu}\right)=0,$$

where $g=\left|\det\left(g\right)\right|$. Denote the Minkowski metric by $\eta_{\mu\nu}=\eta^{\mu\nu}=\operatorname{diag}\left(1,-1,-1,-1\right)$. For the weak-field approximation, we have the linearization for the metric:

$$g_{\mu\nu}\equiv\eta_{\mu\nu}+h_{\mu\nu},\quad g^{\mu\nu}\doteq\eta^{\mu\nu}-h^{\mu\nu},$$

$$h^{\mu\nu}=\eta^{\mu\alpha}\eta^{\nu\beta}h_{\alpha\beta},\quad h=h^{\mu}_{\mu}=\eta^{\mu\nu}h_{\mu\nu},$$

$$g\doteq 1+h,\quad \sqrt{g}\doteq 1+\frac{1}{2}h.$$

For convenience, we directly use = to replace $\doteq$. By straightforward calculation, we obtain the linearization for other parameters:

$$\Gamma^{\mu}_{\alpha\beta}=\frac{1}{2}\eta^{\mu\nu}\left(\partial_{\alpha}h_{\nu\beta}+\partial_{\beta}h_{\alpha\nu}-\partial_{\nu}h_{\alpha\beta}\right),$$

$$\Gamma^{\mu}=\partial_{\nu}\left(h^{\mu\nu}-\frac{1}{2}\eta^{\mu\nu}h\right),$$

$$\mathcal{R}_{\mu\nu}=\frac{1}{2}\partial_{\alpha}\partial^{\alpha}h_{\mu\nu}-\frac{1}{2}\left(\eta_{\mu\alpha}\partial_{\nu}\Gamma^{\alpha}+\eta_{\nu\alpha}\partial_{\mu}\Gamma^{\alpha}\right),$$

$$\mathcal{R}^{\mu\nu}=\frac{1}{2}\partial_{\alpha}\partial^{\alpha}h^{\mu\nu}-\frac{1}{2}\left(\eta^{\mu\alpha}\partial_{\alpha}\Gamma^{\nu}+\eta^{\nu\alpha}\partial_{\alpha}\Gamma^{\mu}\right),$$

$$\mathcal{R}=\frac{1}{2}\partial_{\alpha}\partial^{\alpha}h-\partial_{\alpha}\Gamma^{\alpha}.$$

In which $\partial_{\alpha}\partial^{\alpha}=\partial_t^2-\nabla^2$ is the d'Alembert operator.

In the harmonic coordinate system, we have:

$$\Gamma^{\mu}=\partial_{\nu}\left(h^{\mu\nu}-\frac{1}{2}\eta^{\mu\nu}h\right)=0, \tag{2.8}$$

$$\mathcal{R}_{\mu\nu}=\frac{1}{2}\partial_{\alpha}\partial^{\alpha}h_{\mu\nu},\quad \mathcal{R}^{\mu\nu}=\frac{1}{2}\partial_{\alpha}\partial^{\alpha}h^{\mu\nu}, \tag{2.9}$$

$$\mathcal{R}=\frac{1}{2}\partial_{\alpha}\partial^{\alpha}h,\quad G^{\mu\nu}=\frac{1}{2}\partial_{\alpha}\partial^{\alpha}\left(h^{\mu\nu}-\frac{1}{2}\eta^{\mu\nu}h\right). \tag{2.10}$$

From (2.8) and (2.10), we find that if $\Gamma^\mu = 0, \partial_t \Gamma^\mu = 0$ at any given time $t = t_0$, it will always hold due to the Bianchi identity $G^{\mu\nu}_{;\nu} = 0$.

To compare with electromagnetism and to understand the physical meaning of the parameters, denote:

$$\Phi = \frac{1}{2} h_{tt} = \frac{1}{2} h^{tt}, \quad \boldsymbol{A} = \left(h^{tx}, h^{ty}, h^{tz}\right) = -\left(h_{tx}, h_{ty}, h_{tz}\right), \tag{2.11}$$

$$H = \left(h_{ab}\right) = \left(h^{ab}\right), \left(\{a,b\} \in \{1,2,3\}\right), \quad \boldsymbol{B} = \nabla \times \boldsymbol{A}. \tag{2.12}$$

In the International System of Units, we have the order of magnitude of the metric components:

$$c^2 \left|h_{ab}\right| \sim c\left|A_k\right| \sim |\Phi| \ll 1, \ (a \neq b), \tag{2.13}$$

which means $|h_{ab}| \ll |A_k| \ll |\Phi| \ll 1$ if we take $c = 1$ as the unit.

For the present purpose, we define the stellar speed $\boldsymbol{V}$ by:

$$\boldsymbol{V} \equiv \frac{1}{U^0}\left(U^1, U^2, U^3\right), \tag{2.14}$$

which is approximately equivalent to the usual definition. For galaxies, we have the following order of magnitude:

$$|\boldsymbol{V}| \sim 300 \text{ km/s} = 10^{-3} c, \quad \boldsymbol{A} \sim \bar{\kappa} \boldsymbol{V}, \quad h_{ab} \sim \bar{\kappa} |\boldsymbol{V}|^2, (a \neq b),$$

in which the coefficient $\bar{\kappa}$ is also a number of small value. Then, according to

$$1 = \sqrt{g_{\mu\nu} U^\mu U^\nu} = \left(1 + 2\Phi - 2\boldsymbol{A}\cdot\boldsymbol{V} + g_{ab} V^a V^b\right)^{\frac{1}{2}} U^0,$$

by omitting the $O\left(V^2\right)$ terms, the low-speed assumption gives:

$$U^0 = 1 - \Phi + \boldsymbol{A}\cdot\boldsymbol{V}. \tag{2.15}$$

Substituting (2.14) and (2.15) into (2.5) and omitting the higher-order terms, we obtain the continuity equation and motion equation for the stars:

$$\left(\partial_t + \boldsymbol{V}\cdot\nabla\right)\rho_s = -\rho_s\left[\nabla\cdot\boldsymbol{V} + \left(\partial_t\Phi + \nabla\cdot\boldsymbol{A}\right)\right], \tag{2.16}$$

$$\left(\partial_t + \boldsymbol{V}\cdot\nabla\right)\boldsymbol{V} = -\nabla\Phi + \left(-\partial_t\boldsymbol{A} + \boldsymbol{V}\partial_t\Phi\right) + \boldsymbol{V}\times\boldsymbol{B} + \boldsymbol{V}\cdot\partial_t H. \tag{2.17}$$

In (2.16), we used the de Donder condition $\Gamma^0 = 0$ in the form:

$$\frac{1}{2}\partial_t\left(h_{xx} + h_{yy} + h_{zz}\right) = -\left(\partial_t\Phi + \nabla\cdot\boldsymbol{A}\right). \tag{2.18}$$

The equation of motion (2.17) is similar to electrodynamics. From it, we learn that, Φ gives the Newtonian gravitational potential, and $\boldsymbol{A}$ leads to the gravimagnetic field $\boldsymbol{B}$.

By (2.6) and (2.10), we have:

$$\partial_\alpha\partial^\alpha h = 2\bar{\kappa}\left(\rho + 4W - 3P\right). \tag{2.19}$$

By (2.19), (2.7) and (2.9), we obtain the dynamic equations for $h^{\mu\nu}$:

$$\partial_\alpha\partial^\alpha h^{\mu\nu} = -2\bar{\kappa}\left(\rho + P\right)\mathcal{U}^\mu\mathcal{U}^\nu + \bar{\kappa}\left(\rho + 2W - P\right)\eta^{\mu\nu}, \tag{2.20}$$

$$\partial_\alpha\partial^\alpha \chi^{\mu\nu} = -2\bar{\kappa}\left[\left(\rho + P\right)\mathcal{U}^\mu\mathcal{U}^\nu + \left(W - P\right)\eta^{\mu\nu}\right], \tag{2.21}$$

where $\chi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h$. If the average speed of the dark halo is also small, omitting $O\left(\vec{\mathcal{U}}^2\right)$ from (2.20), we obtain $h^{xx} = h^{yy} = h^{zz} \equiv 2\Psi$, $h^{ab} = 0, (a \neq b)$ and:

$$\partial_\alpha \partial^\alpha \Phi = \frac{1}{2}\partial_\alpha \partial^\alpha h^{00} = -4\pi G \rho, \tag{2.22}$$

$$\partial_\alpha \partial^\alpha \Psi = \frac{1}{2}\partial_\alpha \partial^\alpha h^{kk} = -4\pi G \tilde{\rho}, \tag{2.23}$$

where ρ and $\tilde{\rho}$ are the effective mass densities with little difference. Their zeroth-order approximation gives:

$$\rho = \rho\left[2\left(\mathcal{U}^0\right)^2 - 1\right] - 2W + P\left[2\left(\mathcal{U}^0\right)^2 + 1\right] \doteq \rho - 2W + 3P. \tag{2.24}$$

$$\tilde{\rho} \doteq \rho + 2W - P. \tag{2.25}$$

In the following discussion, only the zeroth-order approximation of (2.16), (2.17) and (2.22) are involved. Thus, in large-scale structures we have:

Conclusion 1. *The equations of galactic dynamics at low speed and weak field approximation are given by*:

$$\partial_\alpha \partial^\alpha \Phi = -\kappa \rho, \quad \rho = \rho_d + \rho_s, \tag{2.26}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{V} \equiv \left(\partial_t + \boldsymbol{V} \cdot \nabla\right)\boldsymbol{V} = -\nabla \Phi, \tag{2.27}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_s \equiv \left(\partial_t + \boldsymbol{V} \cdot \nabla\right)\rho_s = -\rho_s \nabla \cdot \boldsymbol{V}, \tag{2.28}$$

in which $\kappa = 4\pi G$ *is the interaction coefficient,* $\boldsymbol{V}(t, \boldsymbol{x}) = \dot{\boldsymbol{X}}$ *is the speed of ordinary matter such as stars and dust.* ρ_d *is the density of dark halo*, *and* $\rho_s = \sum_k m_k \delta\left(\boldsymbol{x} - \boldsymbol{X}_k\right)$ *is the density of ordinary matter*.

Dark matter is an unknown superfluid, with special properties, very different from the equation of motion of ordinary matter, and has a decisive role in the formation of galactic structure and the evolution of the universe. Except for the bulge, we should have $\rho_s \ll \rho_d$ in the vast region of a galaxy. In this case, ρ_s in (2.26) can be ignored. As a reasonable approximation, the equations of galactic dynamics are decoupled from each other, which brings great convenience for analysis and calculation. According to different cases, Equation (2.27) can be regarded as either hydrodynamics of pressure-free and inviscid perfect fluid, or as an equation of motion of mass points. Corresponding to the dynamical Equations (2.26) and (2.27), we have:

Conclusion 2. *The total Lagrangian of a galaxy is the sum of Lagrangians of all subsystems*:

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_s + \mathcal{L}_d, \quad \mathcal{L}_\Phi = \frac{1}{2\kappa}\partial_\mu \Phi \partial^\mu \Phi - \rho_d \Phi, \tag{2.29}$$

$$\mathcal{L}_s = \sum_k \left(\frac{1}{2}m_k \boldsymbol{V}_k^2 - m_k \Phi\right)\delta\left(\boldsymbol{x} - \boldsymbol{X}_k\right), \tag{2.30}$$

and $\mathcal{L}_d\left(\rho_d, U_d^\mu, P_d\right)$ is the Lagrangian of the dark halo.

Since the specific form of $\mathcal{L}_d$ is not clear, in the following calculation, ρ_d can only be limited by observed data. The main component of dark matter is non-baryon dark particles, *i.e.* weakly interacting massive particles WIMP that do not participate in electromagnetic interactions. Despite many theoretical candidates for dark matter particles, the author proposes that the dark halo may be composed of nonlinear spinor fields with mass [25] [41]. A nonlinear spinor field is a superfluid without electromagnetic interaction, whose trajectory is very different from that of ordinary matter. If the effect of stellar motion on the dark halo is negligible, the star is equivalent to moving in a stationary gravitational field. In the case of $\Phi_k=\Phi\left(\boldsymbol{r}_k\right)$ independent of time, then many conclusions in Newton's gravitational field, such as the conservation law of mechanical energy, the virial theorem, are all valid, which greatly simplifies the analysis and calculation.

By Noether theorem, we have the total energy of the system:

$$E=\int_{R^3}\left(\sum_f \frac{\partial \mathcal{L}}{\partial \dot{f}} \dot{f}-\mathcal{L}\right) \mathrm{d}^3 x=E_s+E_d,$$

in which $\left(E_s, E_d\right)$ are the energy of ordinary matter and dark halo in the galaxy respectively:

$$E_s=\int_{R^3} \rho\left(\frac{1}{2} \boldsymbol{V}^2+\Phi\right) \mathrm{d}^3 x=\sum_k m_k\left(\frac{1}{2} \dot{\boldsymbol{X}}_k^2+\Phi_k\right),$$

$$E_d=K_d+\int\left[\frac{1}{2 \kappa}\left(\left(\partial_t \Phi\right)^2+(\nabla \Phi)^2\right)+\rho_d \Phi\right] \mathrm{d}^3 x,$$

where $\Phi_k=\Phi\left(t, \boldsymbol{X}_k\right)$ is the potential energy of k-th particle. The summation is calculated only once for each particle from the center to the edge of the galaxy. K_d is the equivalent kinetic energy of the dark halo. For isolated systems, E is a conserved quantity.

Astronomical observations show that a fully developed spiral galaxy has a stable structure, including the bulge, disk, spiral arm and dark halo. Except for the bulge, the stars and the interstellar dust are located in the disk. The spiral arm consists of the density waves of ordinary matter. In 1942, Swedish astronomer Lindblad proposed the concept of density wave. In 1964, Lin and Shu established the density wave theory of spiral galaxy [43]. The velocity and density are varied as the star rotates around the center of the galaxy. The density of ordinary matter in the spiral arm is larger and the gas is compressed to form new stars. The pattern rotational speed Ω is approximately a constant. About 2/3 of spiral galaxies have bar structure, which is not density waves but composed of fixed stars and rotating at pattern speed. If the effect of the motion of ordinary matter on the dark halo is negligible, namely, at large scale $\rho_s \ll \rho_d$, then both E_s and E_d can be regarded as conserved quantities.

In stationary galaxies, the density of dark halo ρ is a given function independent of time in the comoving rotational frame. Denoting the pattern speed by Ω, in the comoving rotating coordinate system $(t, r, \theta, \varphi = \phi - \Omega t)$, assuming (ρ, Φ) is independent of time, then we have the following transformation:

$$\boldsymbol{V} = \left(\dot{r}, r\dot{\theta}, r\sin\theta\left(\Omega + \dot{\varphi}\right)\right), \quad \partial_\mu \Phi = \left(c^{-1}\Omega\partial_\varphi, \partial_r, \partial_\theta, \partial_\varphi\right)\Phi,$$

$$\partial_\mu \Phi \partial^\mu \Phi = c^{-2}\Omega^2\left(\partial_\varphi \Phi\right)^2 - \left(\partial_r \Phi\right)^2 - \frac{1}{r^2}\left(\partial_\theta \Phi\right)^2 - \frac{1}{\left(r\sin\theta\right)^2}\left(\partial_\varphi \Phi\right)^2.$$

The mechanical energy of the gases is given by:

$$E_s = \sum_k m_k\left(\frac{1}{2}\left[\dot{r}_k^2 + r_k\dot{\theta}_k^2 + \left(r_k \sin\theta_k\right)^2\left(\Omega + \dot{\varphi}_k\right)^2\right] + \Phi_k\right). \tag{2.31}$$

In the case without confusion, we take the speed of light as the speed unit $c = 1$ for simplicity.

For stationary spiral galaxies, the spiral structure as a whole rotates around the *z*-axis with a constant angular speed Ω. Therefore, under the coordinate transformation $\varphi = \phi - \Omega t$, the structure equation in the new coordinate system is close to static, and the solution is independent of time *t*. Thus, we get:

Conclusion 3. *For non-warped stationary spiral galaxies, the galactic disks satisfy the structural equation*:

$$\left(\partial_r^2 + \frac{2}{r}\partial_r + \frac{1}{r^2}\hat{L}^2 - \frac{\Omega^2}{c^2}\partial_\varphi^2\right)\Phi - \kappa\rho = 0, \tag{2.32}$$

$$\left(V_r\partial_r + V_\varphi\partial_\varphi\right)V_r - r\left(V_\varphi + \Omega\right)^2 + \partial_r\Phi = 0, \tag{2.33}$$

$$\left(V_r\partial_r + V_\varphi\partial_\varphi\right)V_\varphi + \frac{2}{r}V_r\left(V_\varphi + \Omega\right) + \frac{1}{r^2}\partial_\varphi\Phi = 0, \tag{2.34}$$

$$\left(V_r\partial_r + V_\varphi\partial_\varphi\right)\Sigma + \left(\partial_r V_r + \partial_\varphi V_\varphi + \frac{1}{r}V_r\right)\Sigma = 0, \tag{2.35}$$

in which $\hat{L}^2 = \partial_\theta^2 + \cot\theta\partial_\theta + \frac{1}{\sin^2\theta}\partial_\varphi^2$. The physical significance of the terms related with Ω in the above formula is that $r\left(V_\varphi + \Omega\right)^2$ is the centrifugal force, and $\frac{2}{r}V_r\Omega$ is Coriolis force.

Due to the gravitational field background of the dark halo, (2.32)-(2.34) is represented in the spherical coordinate system, but (2.35) is represented in the polar coordinate system. Equations (2.32)-(2.35) is the structural equation for a stationary galaxy with bar and spiral structure, by which the details of the density and velocity distribution of ordinary matter can be solved [25]. Strictly speaking, the density wave theory of galaxy structure is still phenomenological, and only the solution of dynamical Equations (2.32)-(2.35) is an essential explanation of the nature of galactic structure. The exact details of the motion are very complicated and time-dependent, such as the velocity dispersion of stars. The pattern speed may be weakly correlated with radius *r* and time *t* and so on. If these details are entangled in the beginning, it will be difficult to study deeply and reveal the nature of galaxies. Based on the understanding of dynamics (2.26)-(2.28), these details can be computed and analyzed by numerical simulations. In the next, two main empirical relations in astronomy, namely Faber-Jackson relation and Tully-Fisher relation, will be explained according to the structural Equations (2.32)-(2.35).

3. Theoretical Explanation of Two Empirical Relations

3.1. Faber-Jackson Relation

Elliptical galaxies have no disk, the stars do not follow a regular rotation, and most of the kinetic energy of stars is in the random motion. The gravitational potential $\Phi(\boldsymbol{x})$ in (2.31) is independent of time, and the virial theorem holds. The Faber-Jackson relation is the relation between the luminosity of elliptical galaxies and the velocity dispersion of the luminous matter therein, discovered by Faber and her student Jackson in 1976 [44]. This correlation was first qualitatively proposed qualitatively by Morgan and Mayall in 1957 [45], “For progressively fainter Virgo cluster ellipticals, the spectral lines tend to become narrower, as if there were a line width-absolute magnitude effect for the brighter members”. However, the explicit relation of this effect $L \propto \sigma^{\gamma}$ is found by Faber and Jackson, where L is the luminosity of the elliptical galaxy, σ is the velocity dispersion, and power index $3 \le \gamma \le 5$. The relation can be used to determine the mass and gravitational distribution in ellipticals and the distance of galaxy.

Since the two relations are the relationship between the average velocity and the centralized parameters, we mainly discuss the integrals of the parameters. The spherical average of the gravitational potential within a stable elliptical galaxy has a power-law form ([24], Section 2.1.2):

$$\bar{\Phi} = \frac{1}{4\pi}\oint \Phi \sin\theta \mathrm{d}\theta \mathrm{d}\varphi = Ar^n, \quad nA > 0,$$

where (n, A) are constants, but it is related with galactic structure. By (2.32) and $\Omega = 0$, we have the mass distribution in the galaxy as:

$$\bar{\rho} = \frac{1}{\kappa}\nabla^2\bar{\Phi} = \frac{1}{\kappa}\left(\partial_r^2\bar{\Phi} + \frac{2}{r}\partial_r\bar{\Phi}\right) = \frac{n(n+1)A}{\kappa}r^{n-2}. \tag{3.1}$$

Corresponding to the potential energy of mass point we have $n = -1$, while for a homogeneous ball, we have $n = 2$, so there must be $-1 < n < 2$. Thus, the total mass of the galaxy is given by:

$$M = 4\pi\int_0^R \bar{\rho} r^2 \mathrm{d}r = \frac{4\pi nA}{\kappa}R^{n+1}, \tag{3.2}$$

where R is the dynamical radius of the galaxy, which is proportional to the optical radius and scale length. Thus, the kinetic energy of ordinary matter in the galaxy is as:

$$K = \frac{1}{2}M_s\sigma^2 = \frac{1}{2}\xi_m M\sigma^2 = \frac{2\pi\xi_m nA}{\kappa}R^{n+1}\sigma^2, \tag{3.3}$$

where σ denotes velocity dispersion, $\xi_m = M_s/M$ is the partition ratio of ordinary matter to dark matter in the universe, and therefore has little relation to the galactic structure. All such coefficients can be finally merged together and then determined by observational data. The potential energy of ordinary matter is about:

$$U = 4\pi\int_0^R \xi_m \bar{\rho}\bar{\Phi} r^2 \mathrm{d}r = \frac{4\pi n(n+1)\xi_m A^2}{(2n+1)\kappa} R^{2n+1} \tag{3.4}$$

$$= \frac{\kappa \xi_m M^2}{4\pi R} \cdot \frac{n+1}{n(2n+1)}. \tag{3.5}$$

For a balanced system, there is a proportional relationship between various energies, this is virial theorem [46] [47] [48]. For the ordinary matter in a stable galaxy, we have:

Conclusion 4. *For a stable galaxy with power-law potential* $\Phi \propto r^n$, *the ordinary matter satisfies virial theorem in the centroid coordinate system*:

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\sum_k m_k \boldsymbol{r}_k \cdot \boldsymbol{v}_k = \int_{R^3} \rho_s \left(\boldsymbol{v}^2 - \boldsymbol{r}\cdot\nabla\Phi\right)\mathrm{d}^3 x = 2K - nU. \tag{3.6}$$

The above equation shows that the application of $2K + U$ to galaxies is not always correct.

Substituting (3.3) and (3.4) into the above equation, we have:

$$R = \sqrt[n]{\frac{2n+1}{An(n+1)}}\,\sigma^{\frac{2}{n}}, \quad -\frac{1}{2} < n < 2. \tag{3.7}$$

Substituting (3.7) into (3.2), we obtain:

$$M = \frac{4\pi}{\kappa\sqrt[n]{nA}}\sqrt[n]{\left(\frac{2n+1}{n+1}\right)^{n+1}}\,\sigma^{\frac{2(n+1)}{n}}. \tag{3.8}$$

In addition, by luminosity formula:

$$L = \int_0^\infty 2\pi I_0 \exp\left(-\frac{r}{R_d}\right) r\mathrm{d}r = 2\pi I_0 R_d^2, \tag{3.9}$$

where R_d is the luminous scale radius of the galaxy, which is proportional to the dynamical radius $R_d = \xi_d R$. I_0 is the surface brightness at the center of the galaxy, with some correlation with the galaxy structure. By (3.7), we obtain the relation between luminosity and velocity dispersion as:

$$L = 2\pi I_0 \xi_d^2 R^2 = 2\pi I_0 \xi_d^2 \sqrt[n]{\left(\frac{2n+1}{An(n+1)}\right)^2}\,\sigma^{\frac{4}{n}}, \tag{3.10}$$

The parameter A has some correlation with the radius R. From (3.7) and (3.2), among the parameters with dimension (R, M, σ, A), only two are independent variables. If A is related to galactic structure, it should have the form $A \propto R^k$, thus we have:

Conclusion 5. *Assuming that the spherical mean potential function of an elliptical galaxy has the following form*:

$$A = \frac{C}{n} R^{m-n}, \quad \bar{\Phi} = \frac{CR^m}{n}\left(\frac{r}{R}\right)^n, \tag{3.11}$$

in which ($-\frac{1}{2}<n<2, m>0, C>0$) are constants, which have little correlation with R. Then we have relations:

$$\bar{\rho}=\frac{C(n+1)}{\kappa} R^{m-n} r^{n-2}, \tag{3.12}$$

$$R=B \sigma^{\frac{2}{m}}, \quad B=\sqrt[m]{\frac{2n+1}{C(n+1)}}, \tag{3.13}$$

$$M=\frac{4\pi C}{\kappa} R^{m+1}=\frac{4\pi C}{\kappa} B^m R \sigma^2=\frac{4\pi C}{\kappa} B^{m+1} \sigma^{\frac{2(m+1)}{m}}, \tag{3.14}$$

$$L=2\pi I_0 \xi_d^2 R^2=2\pi I_0 \xi_d^2 B^2 \sigma^{\frac{4}{m}}. \tag{3.15}$$

The conclusion can be verified by substituting (3.11) into (3.1) and (3.7)-(3.10). The conclusion shows that:

1) The power index $-\frac{1}{2}<n<2$ has only influence on the coefficient of Faber-Jackson relation, but $m \sim 1$ has mainly influence on the power index.

2) In general cases, $L \propto M$ only holds approximately, and the accurate relations between the two involves other physical principles beyond classical mechanics.

3) The coefficients (B, C) and m should be determined by observational data.

The above conclusion provides a theoretical basis for studying the intrinsic properties of ellipticals and improving the precision of Faber-Jackson relation. According to the fundamental plane relation [49] [50], the parameters (R, σ, I_0) satisfy the following relation with small dispersion:

$$\lg R=a \lg \sigma+b \lg I_0+\lambda, \tag{3.16}$$

where λ depends on the definition and measuring method of (R, σ, I_0), that is, it relates to the coefficients ξ_d, ξ_m and so on. We can determine the relation between M and L according to the observed data (a, b, λ). Suppose $M=kL^\alpha$, by substituting it into (3.14) and then combining with (3.13) and (3.15), we can solve for $(\lg R, \lg \sigma, \lg I_0)$ represented by $\lg L$. Putting the 3 expressions into Equation (3.16), we get:

$$[\alpha(am-4b-2)+2(m+1)b] \lg L+(am-4b-2) \lg k+\cdots=0, \tag{3.17}$$

in which the omitted items are independent of (k, L). Since $\lg L$ is a variable, its coefficient must vanishes. Thus, we have solution:

$$\alpha=-\frac{2(m+1)b}{am-4b-2}, \tag{3.18}$$

$$k=\frac{\pi C}{\kappa}\left(4^{m(a+b)-3b-2} \pi^{2b(m+1)} \mathrm{e}^{-2\lambda(m+1)} B^{am(m+1)}\right)^{\frac{1}{am-4b-2}}. \tag{3.19}$$

where k is related to the coefficient λ, depending on how the radius is defined and measured, but α is independent of the proportional coefficient. By the observational data of r_G band [49], converting the absolute magnitude into brightness, we get coefficients:

$$a = 1.39, \quad b = -0.9, \quad \lambda = -6.71. \tag{3.20}$$

Again by $m > 0$, we have:

Conclusion 6. *For ellipticals, we have* $M = kL^{\alpha}, (1.125 < \alpha < 1.295)$. *In the case* $m = 1$,

$$\alpha = \frac{6}{5}, \quad k = \frac{10881C}{\kappa} B^{0.93}, \quad L \propto \sigma^{\frac{10}{3}}, \quad M \propto \sigma^4.$$

Due to the level of observation, the fundamental planes of different bands are usually obtained by different samples, galactic parameters are not obtained by unified measurements, and each sample has different selection conditions, which will affect the study of the parameters of fundamental plane [51]. In 2003, Bernardi *et al.* analyzed about 9000 ellipticals of samples of SDSS in different bands, showing that the change of fundamental plane from g to z bands is small [52]. In 2008, La Bernardi *et al.* found no significant changes in fundamental plane [53], probably because of the narrow observed band range of (0.4 - 1.0 μm). In 2009, Jeong *et al.* studied the fundamental plane of near-UV and far-UV [54], and the parameter *a* values fit the generally believed trend, but the *b* value was significantly different from −0.8. In 2012, Focardi and Malavasi analyzed the effect of galaxy environment and brightness on the Faber-Jackson relation by using a sample of 384 nearby ellipticals [55]. The analysis shows that the intrinsic dispersion of the Faber-Jackson relation is significantly reduced when ellipticals in high-density environment are compared to those in low-density environment. For high luminosity galaxies $\gamma > 4$, but for faint galaxies $\gamma < 4$. The larger the central mass of the galaxy, the higher the velocity dispersion, and the deeper its gravitational potential well. By representing the coefficients as functions of the dynamical radius *R*, we obtain the relations with less dispersion.

3.2. Tully-Fisher Relation

The Tully-Fisher relation was first proposed by Tully and Fisher in 1977 [56]. They found that the width of HI line and the absolute magnitude of B band photometry in the disk galaxies (including the galaxy group, M81, M101, etc.) have reliable power-law relation from the observational data at that time. The width of HI line reflects the rotational velocity and is independent of distance. The magnitude of galaxies is easy to measure, and when the magnitude is certain, the absolute magnitude is a single variable function of distance. Therefore, Tully-Fisher relation becomes a ranging tool for extragalactic disk galaxies, and Tully and Fisher try to calculate the distance of galaxies in the Virgo cluster and the Ursa Major cluster. Soon, it was found that near infrared metering was advantages over optical metering. In 1980, Mould *et al.* used H-band (1.65 mm) metering for M81, M101, etc. [57]. In 1983, Aaronson and Mould found that compared with B band (blue light) photometry, the Tully-Fisher relation obtained by infrared photometry does not depend on the morphological characteristics of the galaxy, and the dispersion of galaxy samples is smaller, indicating that the Tully-Fisher relation obtained by infrared photometry is a better ranging tool [58].

For spiral galaxies, denote the circumferential mean potential energy and the tangential mean velocity of the gases at radius r in the disk $\theta=\frac{\pi}{2}$ respectively by:

$$\bar{\Phi}=\frac{1}{2\pi}\int_0^{2\pi}\Phi\mathrm{d}\varphi,\quad v^2=\frac{1}{2\pi}\int_0^{2\pi}r^2\left(V_\varphi+\Omega\right)^2\mathrm{d}\varphi.$$

Observations show that the average rotation velocity of stars in the disk is close to a constant $v\in(200,400)\mathrm{m/s}$. By (2.33), the relation between the mean velocity and the mean gravitational potential in the disk is given by:

$$-\frac{v^2}{r}+\partial_r\bar{\Phi}=0\quad\Rightarrow\quad\bar{\Phi}=v^2\ln\left(\frac{r}{r_0}\right). \tag{3.21}$$

Thus, we have the mean value of the total mass density distribution in the disk as:

$$\kappa\bar{\rho}=\left(\partial_r^2+\frac{2}{r}\partial_r\right)\bar{\Phi}=\frac{v^2}{r^2}. \tag{3.22}$$

In spiral galaxies, ordinary matter is concentrated in the disk, while the dark halo of superfluid is close to a spherically symmetric distribution, so the spherical average of the density of the galaxy should be also approximately (3.22). Compared the above equation with (3.1), we find that the mass distributions of ellipticals and spiral galaxies are different. In 1974, Einasto *et al.* stated that [59], the mass of a spherically symmetric galaxy can be estimated from (3.22). However, for nonsymmetric galaxy, $M(r)$ has 10% error [25] [60]. By (3.22), we get the total dynamic mass of the galaxy:

$$M=4\pi\int_0^R\bar{\rho}r^2\mathrm{d}r=G^{-1}Rv^2. \tag{3.23}$$

For stable spiral galaxies, the ordinary matter in the disk forms two-dimensional density wave, and the circumferential average density $\bar{\Sigma}=\Sigma_0 f(r/R)$ is of a stationary structure, thus we have:

$$M_s=2\pi\Sigma_0\int_0^R f\left(\frac{r}{R}\right)r\mathrm{d}r=bR^2, \tag{3.24}$$

where the coefficient b has less correlation with the radius R. Observational data show that we approximately have:

$$M\propto M_s\propto L\propto R^2, \tag{3.25}$$

Taking v as a known parameter, and solving (3.25) and (3.23), we get:

Conclusion 7. *For a stable spiral galaxy, if the rotation curve outside the bulge is almost flat, then we have Tully-Fisher relation*:

$$R\propto v^2,\quad L\propto M\propto v^4. \tag{3.26}$$

The Tully-Fisher relation is an empirical relation between the luminosity or total mass and rotational velocity of a spiral galaxy, where the galaxy samples are distributed near a straight line in double logarithmic coordinates and the slope of the line is about 3 - 4. The dispersion of the baryonic Tully-Fisher relation in galaxy samples is smaller than that of traditional Tully-Fisher relation, possibly caused by the difference in the motion law of the dark halo from ordinary matter [25] [41]. The Tully-Fisher relation is an effective ranging tool for spiral galaxies. The Interference Array telescope, now used to observe galaxies, greatly improves the spatial resolution, captures the details of the characteristics of galaxy motion, and better avoids the problem of other sources interfering with target sources. Therefore, in addition to providing more reliable data for the baryon Tully-Fisher relation study of galaxies, the interference array can also study the intergalactic medium. Much of the work on the baryonic Tully-Fisher relation and the traditional Tully-Fisher relation has various different definitions of rotational velocity [17] [61], such as $V_{\text{flat}}, V_{\max}, W_{50}, W_{20}$, where the smallest dispersion of the baryonic Tully-Fisher relation corresponds to V_{flat}. Similar to the case of Faber-Jackson relation, the slope of the line deviation from 4 may be caused by the correlation between (b, I_0) and R, which can only be explained by detailed dynamical calculations.

4. Discussion and Conclusions

The phenomenon of "missing mass" in galaxies sparked debate between the dark matter hypothesis, MOND and MOG schemes. From the above discussion, we learn that general relativity is a successful theory with profound philosophical thought and mathematical foundation, and has been verified by a large number of experiments and observations, so it is a basic theory that cannot be easily abandoned and denied. Since Einstein field equation is a complicated system of nonlinear partial differential equations, theoretical analysis and solution are very difficult to achieve, they are rarely used directly in the calculation of galactic structure, and galactic dynamics become a weak link for the application of general relativity. If the weak field and low-speed approximations are used to simplify Einstein field equation, we can obtain simple and practical equations for galactic dynamics (2.26)-(2.28) to study the details of galactic structure and motion of matter, such as spiral structure, pattern rotation speed and asymptotically flat rotational curves. The analysis in this paper shows that the Faber-Jackson relation and Tully-Fisher relation do not violate relativity and Newtonian mechanics, only because we have not deeply studied the application of general relativity in galactic structure.

It is not completely unacceptable to modify traditional fundamental theories, but there must be sufficient philosophical reasons and extensive verification of experimental data. Modified schemes such as MOND or MOG only use a class of special functions containing parameters to fit the observed data. Although this is also one of the scientific methods, it only derives a plausible result by several assumptions, rather than gives an essential explanation. The idea that the modified schemes can compete with the classical theory is very fragile if only by fitting the simplified models of concentrated parameters such as Tully-Fisher relation or flat rotation curve. If the modified schemes can calculate the distribution details of the mass and velocity in a spiral pattern, this will greatly increase their credibility. However, according to the author's experience, the modified schemes can hardly achieve this goal for the following reasons: Galactic spiral arms are places in which stars are dense, where the stars have the lowest velocity, the least kinetic energy, and thus the highest potential energy [25]. But this phenomenon is contradictory to the assumption that there is no dark matter, because if the dark halo is absent, then the place where matter is dense must have low potential energy and high speed. This shows that dark matter must not only exist, but also move along different orbits from that of ordinary matter. Dark matter and luminous matter in a galaxy must be separated automatically. Dark matter provides a dynamic background for the generation and evolution of stars, equivalent to the amniotic fluid for a fetus. The modified theories make it impossible to explain the spiral structure of galaxies, and therefore they cannot be the fundamental theory of galaxies. A stable solution of the spiral structure of galaxies can be obtained only by introducing the gravitational theory with retarded potential.

There is a tendency to deepen and expand the correlation study of concentrated parameters of galaxies, which is a matter requiring vigilance. Statistical models with concentrated parameters have their limitations. For example, there is a positive correlation between body weight and height of humans. If we introduce parameters such as gender, shoulder width and waist circumference, we can get quite accurate fitting formulae. But this correlation is obviously impossible to have a precise result, and the dispersion always exists. Therefore, correlation studies will never replace causal explanations. The final solution of dark matter properties and galaxy structure depends on a thorough study of the relativistic theory of gravity. All scientific achievements constantly remind that nature is far wiser than humans. However, the research method of opposing philosophical thinking and emphasizing subjective model and data calculation advocated by the physics society in the last century has been weakening the sensitive nerves of people's communication and dialogue with nature. Only by changing this situation, can we promote substantial progress in fundamental physics.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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