Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2024, 12, 168-180
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

The Planck Constant and Its Relation to the Compton Frequency

Espen Gaarder Haug

Tempus Gravitational Laboratory, Ås, Norway
Email: espenhaug@mac.com

How to cite this paper: Haug, E.G. (2024) The Planck Constant and Its Relation to the Compton Frequency. *Journal of Applied Mathematics and Physics*, **12**, 168-180.
https://doi.org/10.4236/jamp.2024.121013

Received: November 27, 2023
Accepted: January 23, 2024
Published: January 26, 2024

Abstract

The Planck constant is considered one of the most important universal constants of physics, and despite all we know much about it, the physical nature of it has not been fully understood. Further investigation and new perspectives on the Planck constant should therefore be of interest. We demonstrate that the Planck constant also can be directly linked to the Compton frequency of one, which again is divided by the Compton frequency in one kg. If this is right, it means also the Planck constant is linked to quantization of matter, not only energy. However, as we will show the frequency of one when expressed in relation to kg will be observational time dependent. This means the missing mass gap surprisingly both is equal to the Planck mass, which is larger than any known particle and also it is linked to a very small mass that again is equal to what has been suggested as the photon mass in the existing literature. This new view could be an important step forward in understanding the physical nature of the Planck constant as well as the mass gap and even the rest mass of a photon.

Keywords

Planck Constant, Compton Frequency, Electron, Proton Count

1. Background

The Planck constant is a corner stone in much of modern physics including quantum physics where it plays an important role in the Heisenberg's uncertainty principle [1], the Schrödinger equation [2], the Klein-Gordon equation and the Dirac equation [3]. The Planck constant was first published by Max Planck [4] around 1900. The Planck constant is linked to that energy comes in quanta, so energy is clearly quantized, and from photon energy, we have the well-known relation:

$$E = hf \tag{1}$$

A series of methods to measure the Planck constant exist, for example, Landauer quantization [5], photoemission spectroscopy technique [6], the kibble balance technique [7] [8] [9] and other methods [10]. Since 2019, the Planck constant has been fixed to an exact number that then again is linked to the kilogram through the watt balance [11] [12]. Despite many relations to the Planck constant are known, we think the Planck constant has not been fully understood from a deeper perspective. This is not only our view, but also a view that we think reflects the wider view on the Planck constant, for example, Chang [13] in 2017 writes:

> Planck's constant h is now regarded as one of the most important universal constants. The physical nature of h, however, has not been well understood.

See also [14] for a discussion on the Planck constant and its physical nature. In this paper, we will discuss how the Planck constant is likely the frequency of one divided by the Compton frequency in an arbitrary amount of matter that we call one kilogram, which again is multiplied by c^2. That is the Planck constant is linked to a frequency ratio. Most important is that the Planck constant is linked to a frequency of one. This will give us new insight into energy and matter, and also a new way to measure and define the Planck constant.

2. The Planck Constant at the Compton Frequency of One, Divided by the Compton Frequency in One Kilogram Multiplied by c^2

The reduced Compton frequency per second in a mass is simply the speed of light divided by the reduced Compton wavelength in the mass. For an electron, we have:

$$f_e = \frac{c}{\bar{\lambda}} \approx 7.76 \times 10^{20} \text{ per second} \tag{2}$$

Next, the hypoteticaly reduced Compton wavelength of one kilogram can be found by Compton's [15] wavelength formula:

$$\bar{\lambda} = \frac{\hbar}{mc} = \frac{\hbar}{1kg \times c} = \frac{\hbar}{c} = 3.52 \times 10^{-43} \text{ m} \tag{3}$$

This means the reduced Compton frequency in one kilogram is:

$$f_{1kg} = \frac{c}{\bar{\lambda}_{1kg}} \approx 8.52 \times 10^{50} \text{ per second} \tag{4}$$

All masses expressed in kilogram can be described as the reduced Compton frequency divided by the reduced Compton frequency in one kilo, see Haug [16] [17]. This gives for example the kg mass of the electron equal to:

$$m_e = \frac{f_e}{f_{1kg}} \approx \frac{7.76 \times 10^{20}}{8.52 \times 10^{50}} \approx 9.11 \times 10^{-31} \text{ kg} \tag{5}$$

A frequency divided by a frequency is a dimensionless number, which should have no output units, so how did we get it to be *kg*? Well, what exactly is one kilogram? The kilogram mass of something is the amount of matter relative to the decided-upon kilogram of matter. Our calculation above corresponds to the observed mass of the electron in terms of kg. This mass is in general also observational time independent. If we reduce the observational time window to half a second, both the reduced Compton frequency in the electron and in the one kg will be reduced in half, so their ratio will stay the same. However, if the observational window is close to, or below the Compton time then this mass is observational time dependent.

We will claim that the minimum mass above zero that one can observe inside a time interval always is linked to a reduced Compton frequency of 1. This is because observed frequencies must come in integer numbers, and a frequency of for example, a half does not make any sense from an observational point of view, even if it can be linked to probabilities of such an event, see [16] [17]. If we are operating with observational window of one second, then the smallest observable reduced Compton frequency we can observe in one second is still one. To turn this into a kg mass, we need to divide one by the reduced Compton frequency in one kg, this gives:

$$m_1 = \frac{f_1}{f_{1kg}} \approx \frac{1}{8.52\times 10^{50}} \approx 1.17\times 10^{-51}\ \text{kg} \tag{6}$$

To make the energy equivalent of this, we can use the energy mass relation of Einstein $E = mc^2$, this gives us:

$$\frac{1}{f_{1kg}}c^2 = \frac{1}{8.52\times 10^{50}}c^2 \approx 1.0545\times 10^{-34} \tag{7}$$

This is the same numerical value as the reduced Planck constant, $\hbar$. This is in our view much more than a coincident. It shows that the Planck constant indeed is linked to the quantization of energy, but also of matter. We will claim matter comes in discrete units linked to their reduced Compton frequency, and that the smallest frequency that can be observed indeed is 1. Still, so far, this is not a new way to find the Planck constant as we used the Planck constant to find the reduced Compton wavelength of the one kilogram, so one could easily think this is just that we are getting back what we were putting in, by a little algebra massage. However, we will soon see how we also can find the Compton frequency in matter without any knowledge of the Planck constant.

Inside a given time interval one can only observe integer numbers in the frequency (collisions), non integer numbers can be linked to probabilities for expecting to observe them, but observed phenomena comes in integer numbers of frequencies [16] [17]. The Planck constant has since 2019 been exactly defined as $1.054571817 \times 10^{-34}$ (NIST CODATA). Further, the output units from the Planck constant are given by Joules per second, in other words, it is a time unit there in seconds. Further Joule is $\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}$. So, the Planck constant is linked to both the kg, meters and seconds. Let's now look more precisely at the reduced Compton frequency in one kg more based on this exactly 2019 (NIST) defined value of the reduced Planck constant, we must have:

$$f_{1kg}=\frac{c}{\bar{\lambda}_{1kg}}=\frac{c}{\frac{\hbar}{1kg\times c}}=\frac{c^2}{\hbar}=\frac{89875517873681800}{1.054571817\times 10^{-34}} \approx 852246536697289379581438217023772407526798148826311.7503 \quad (8)$$

That is the reduced Compton frequency for one kg in one second is given by only the speed of light and the reduced Planck constant. Both the speed of light and the Planck constant are after 2019 defined exactly, and we see that this leads to a non-integer reduced Compton frequency in one kg during an observational time window of one second. That is a non-observable reduced Compton frequency, as only integers in relation to frequencies can be observed. As indicated earlier the mass gap seems to be related to a frequency of one, and the reduced Planck constant is this frequency of 1 divided by the frequency in one kg multiplied by c^2. As we have assumed *kg* physically represent a frequency ratio. A frequency of one divided by the reduced Compton frequency in one kilogram inside the chosen observational window is then the kg definition of the mass gap, and to turn it into the energy gap we need to multiply by c^2. An issue here is that the reduced Compton frequency that is linked to the Planck constant as defined by the 2019 NIST value never can be observed in one second, as it is liked to one 1 divided by 852246536697289379581438217023772407526798148826311.7503⋯, and the reduced Compton frequency of any clump of matter that we can observe inside a time interval must be an integer number. An alternative and exact definition of the reduced Planck constant would therefore be to decide on a new definition of the kilogram that where linked to an exact integer number in the reduced Compton frequency during a given time interval. For example we could round the current number up to exact:

$$852246536697289379581438217023772407526798148826312$$

or down to:

$$852246536697289379581438217023772407526798148826311$$

we could decide on any integer number and link it to a kilogram, but if we want it close (and almost unrecognizable) to the current kilogram definition it would make sense to round up this number indirectly given by the current definition of c and $\hbar$, but again this new definition would have to change $\hbar$ slightly or alternatively c slightly. If we hold c as it, is then the new definition of the reduced Planck constant would be:

$$\hbar=c^2\frac{1}{852246536697289379581438217023772407526798148826312} \approx 1.0545718169999999999999999999999999999999999999969\cdots\times 10^{-34} \quad (9)$$

or alternatively one could decide on rounding the reduced Compton frequency down to the integer below and get a Planck constant definition off:

$$\hbar = c^2 \frac{1}{85224653669728937958143821702377240752679814882631 1} \approx 1.054571817000000000000000000000000000000000000000928\cdots\times 10^{-34} \quad (10)$$

This later one is nice as one, as then one could use $\hbar \approx 1.054571817\times 10^{-34}$ as an increadible accurate approximation, or one could use the exact formula above when needing maximum precision.

Even if one has chosen to operate with seconds in the SI unit system, this is an arbitrary chosen time interval. If there is a fundamental time interval in nature it is likely the Planck time. The reduced Planck constant re-formulated in relation to Planck time would be:

$$\hbar_p = c^2 \frac{1}{852246536697289379581438217023772407526798148826312\frac{l_p}{c}} = \frac{c^3}{852246536697289379581438217023772407526798148826312 l_p} \quad (11)$$

or

$$\hbar_p = c^2 \frac{1}{852246536697289379581438217023772407526798148826311\frac{l_p}{c}} = \frac{c^3}{852246536697289379581438217023772407526798148826311 l_p} \approx 1954056587 = m_p c^2 \quad (12)$$

However, since there is considerably uncertainty in the Planck time because it is considerably uncertainty in the Planck length it would be considerably uncertainty in this number, but for the CODATA value of the Planck length the divisor value is approximately 45,994,327.12, and then the Planck constant is equal to the Planck mass energy. Still again the divisor number can for observational purposes likely only be an integer. If the current Planck constant as given by CODATA (2019 definition) should show itself to not give an integer number at the Planck time for the reduced Compton frequency in one kg, then the current definition is perhaps not fully consistent. Still if any correction it will be mini-scule, as it would require for example to re define 45,994,327.12 to 45,994,327. However, the main problem here is we do not know accurratley the Planck length and therefore also not the Planck time. The Planck length is normally is given by the formula first described by Max Planck [18] [19] $l_p = \sqrt{\frac{G\hbar}{c^3}}$, so it looks like one need to know the Planck constant to find the Planck length. Recent research has shown how to measure the Planck length independent on any knowledge of G, or $\hbar$, see [20] [21].

3. A New Way to Define or Find the Planck Constant

First, we measure the Compton wavelength [15] in an electron by Compton scattering, it is given by:

$$\lambda_e = \frac{\lambda_{\gamma,2} - \lambda_{\gamma,1}}{1 - \cos\theta} \tag{13}$$

That is we shoot a photon on an electron and measure the wavelength of the photon when we send it towards the electron and after it has hit the electron. Further we measure the angle between the photon we sent out and the photon reflected by the electron θ. We could also have found the electron Compton wavelength from the kg mass of the electron by the well known formula (also given by Compton in 1923):

$$\lambda_e = \frac{h}{mc} \tag{14}$$

but here, our purpose of finding the Compton wavelength is to determine the Planck constant. Therefore, this formulaic approach cannot be used for our purpose. After we find the Compton wavelength, independent of the electron kilogram mass and independent of the Planck constant, we then proceed to utilize the fact that the cyclotron frequency ratio of a proton to an electron must be identical to the ratio of their Compton wavelength. This cyclotron frequency is given by:

$$f = \frac{qB}{2\pi m} \tag{15}$$

And since the proton and electron charges are the same, we end up with:

$$\frac{f_e}{f_P} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_P}} = \frac{m_P}{m_e} = \frac{\lambda_e}{\lambda_P} \approx 1836.15 \tag{16}$$

This is more than theory; the cyclotron frequency has indeed been used as a method to find the proton-electron mass ratio, which is identical to their Compton length ratio (see, for example, [22] [23]). Now that we know the Compton wavelength of the proton, it is approximately:

$$\lambda_P = \frac{\lambda_e}{\frac{f_e}{f_P}} \approx \frac{2.42 \times 10^{-12}}{1836.15} \approx 1.32 \times 10^{-15}\ \text{m} \tag{17}$$

The CODATA 2019 values for the Compton wavelength of the electron and the proton are respectively 2.426,310,238,67 × 10^{-12} m and 1.321,409,855,39 × 10^{-15} m.

Now that we know the proton's wavelength, we can decide how many protons we want in our kilogram definition. Counting the number of atoms is nothing new; it's one of several important ways to define the Planck constant (see, for example, [24] [25] [26]). What is new here is how we link this at a deeper level to the Compton frequency in matter. We could define it as exactly 6 × 10^{26} protons plus 6 × 10^{26} electrons. This sum represents the number of these individual protons. The binding energy can also be easily treated as mass equivalent.

This means the Compton frequency per second for such a mass is:

$$6\times10^{26}\times\left(\frac{c}{\lambda_P}+\frac{c}{\lambda_e}\right)\approx1.36\times10^{50}\text{ times per second} \tag{18}$$

If we take a frequency of one per second and divide by this, we get:

$$\frac{1}{1.36\times10^{50}}\approx7.35\times10^{-51}\text{ kg} \tag{19}$$

We have put kg after it, that is we will claim the kg mass at a deeper level is the frequency of the mass (energy) of interest divided by the Compton frequency in the arbitrary mass called a kilogram, this is a view we have discussed in [16] and also will be discussing further in this paper. If we multiply this by c^2, we get energy, so we get:

$$hf=h\times1=\frac{1}{1.36\times10^{50}}c^2\approx6.6\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}^2 \tag{20}$$

and we naturally have per definition $\hbar=\frac{h}{2\pi}\times1\approx1.05\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}$. This is very close to the Planck constant and the reduced Planck constant, and we could make it as close to the current Planck constant we want, by linking the Planck constant to the Compton frequency in today's kg defenition. The main point is not exactly what the Planck constant value should be, as we see it is linked to an arbitrary quantity of matter, which we choose to call one kilogram. The Planck constant is always equal to one divided by the Compton frequency in this chosen upon quantity of matter multiplied by c^2. The multiplication of c^2 is simply needed because the Planck constant is directly linked to the minimum quantum of energy and therefore also indirectly to the minimum quantum of mass.

As we can see the Planck constant is linked to a frequency of one divided by the Compton frequency in a reference mass, and this is again multiplied by c^2. If we go back and look at the quantization of energy, this makes sense. Energy comes in quanta based on the following formula:

$$E=hf=h\frac{c}{\lambda} \tag{21}$$

That is naturally that energy is h multiplied by a frequency. The frequency is per a time unit. If we express the energy in Joule then we have to remember Joule is kg·m^2·s^{-2}. That is we see Joule is linked to time. In particular, if we only look at the frequency f it is easy to see it is time dependent, as the Planck constant just is a constant, and the frequency is a frequency per second, $f=\frac{c}{\lambda}$.

4. Can Composite Masses Have a Compton Wavelength?

In Section 3 of this paper, we determined the Compton wavelength of a proton. This was accomplished by first calculating the Compton wavelength of an electron, and subsequently utilizing a cyclotron to estimate the Compton wavelength of a proton. One might posit that a Compton wavelength is only applicable to elementary particles like electrons, rather than composite masses like protons.

We generally concur with such arguments. Nevertheless, composite masses are fundamentally composed of a series of elementary particles. It is our assumption that each of these elementary particles possesses a genuine Compton wavelength. To ascertain the Compton wavelength of a composite particle, we should be able to aggregate these individual wavelengths together.

The aggregated Compton wavelength of a composite mass must inherently align with the standard mass aggregation principle, *i.e.* it must be consistent with:

$$m = m_1 + m_2 + m_3 + \cdots + m_n \tag{22}$$

Additionally, as any kilogram mass can be expressed mathematically as $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$, we can once again rewrite it as:

$$\begin{gathered}\frac{\hbar}{\bar{\lambda}}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_2}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_3}\frac{1}{c} + \cdots + \frac{\hbar}{\bar{\lambda}_n}\frac{1}{c} \\ \frac{1}{\bar{\lambda}} = \frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \cdots + \frac{1}{\bar{\lambda}_n} \\ \bar{\lambda} = \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \cdots + \frac{1}{\bar{\lambda}_n}}\end{gathered} \tag{23}$$

This would not exhibit precisely the same Compton frequency as a composite mass composed of atoms. The underlying rationale for this phenomenon is the presence of binding energy, as illustrated in Reference [27]. Ordinarily, the mass of a composite entity is marginally less than the cumulative sum of the individual masses constituting the composite mass. Nevertheless, the nuclear binding energy typically constitutes less than 1% of the total mass-energy. Nonetheless, we can consider binding energy as having mass equivalence, leading us to the expression for a composite mass:

$$\begin{gathered}m = m_1 + m_2 + m_3 + \cdots + m_n - \frac{E_1}{c^2} - \frac{E_n}{c^2} \\ \bar{\lambda} = \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \cdots + \frac{1}{\bar{\lambda}_n}}\end{gathered} \tag{24}$$

Consequently, any mass inherently possesses a Compton wavelength, even astronomical entities such as planets, the Sun, and stars. Nevertheless, any mass greater than the Planck mass exhibits a Compton wavelength shorter than the Planck length, strongly implying that it does not constitute a physical Compton wavelength, as the Planck length is likely the shortest feasible physical Compton wavelength. However, the aggregated Compton wavelength can indeed be shorter than the Planck length, given that it does not represent a physical length but rather an amalgamated value that proves invaluable in computations and in comprehending the underlying reality. Hence, utilizing the Compton wavelength of one kilogram, as we have done earlier, is not erroneous. In fact, recent research suggests that matter resonates at the Compton frequency, as expounded in references such as [28] and [29].

Interest in the Compton wavelength of the proton (a composite mass) traces back to at least 1958 in a paper by Levitt [30]. More recently, there has been a renewed focus on the Compton wavelength of the proton. Trinhammer and Bohr [31] have shown that the Compton wavelength of the proton could potentially be directly linked to the proton's radius.

5. The Relation between Compton Wavelength and Photon Wavelength

The energy of a photon is given by:

$$E = hf = h\frac{c}{\lambda_\gamma}, \tag{25}$$

where λ_γ represents the wavelength of the photon. Einstein's [32] most renowned formula provides a crucial insight into the relationship between energy and matter:

$$E = mc^2, \tag{26}$$

which implies that we must also have:

$$h\frac{c}{\lambda_\gamma} = mc^2. \tag{27}$$

Furthermore, since all masses in kilograms can be expressed as $m = \frac{h}{\lambda}\frac{1}{c}$, which is essentially the rearranged form of the Compton wavelength formula $\lambda = \frac{h}{mc}$ solving for m, this indicates that we must also have:

$$h\frac{c}{\lambda_\gamma} = \frac{h}{\lambda}\frac{1}{c}c^2$$

$$\lambda_\gamma = \lambda \tag{28}$$

Despite the simplicity in its solution, this result has been scarcely discussed before. It reveals that the photon wavelength in terms of pure energy corresponds to the Compton wavelength in rest mass. One speculative idea is that matter might actually consist of photons moving back and forth over the Compton wavelength within material objects, undergoing collisions [33]. It is likely that such collisions between photons contribute to the creation of mass. Importantly, this concept does not conflict with the fundamental principles of the standard theory, where photon-photon collisions indeed are expected to give rise to the formation of matter (see [34]).

Matter wavelengths are typically associated with the de Broglie wavelength and not the Compton wavelength. In his Ph.D. thesis, de Broglie [35] proposed that in addition to particle properties, matter also exhibits wavelike properties. In addition he suggested that the matter wavelength takes the form $\lambda_b = \frac{h}{mv}$ in the non-relativistic case and $\lambda_b = \frac{h}{mv\gamma}$ in the relativistic case [36].

In 1927, Davisson and Germer [37] published experimental results confirming the wavelike nature of matter. Consequently, de Broglie's hypothesis that matter possesses wave-like properties was substantiated. However, it appears that the scientific community made a possibly mistake in connecting this concept directly to the de Broglie mathematical wavelength. This overlooks the fact that around the same time, Compton had determined the wavelength of electrons through Compton scattering, so that one had discovered wave like properties in matter could just as well be related to the Compton wavelength.

The de Broglie wavelength presents several challenges and peculiarities. Notably, it is mathematically invalid for a particle at rest, as it would involve division by zero when setting $v = 0$. This concern can be partially addressed by invoking Heisenberg's uncertainty principle [1], which implies that a particle can never be completely at rest. Nevertheless, even with this consideration, the de Broglie wavelength approaches infinity as the velocity v approaches zero. On the contrary, the Compton wavelength always behaves consistently and always has length of atomic and subatomic distances. This does not mean the de Broglie wavelength is wrong, but that it likely simply can be seen as a derivative of the real matter wavelength: the Compton wavelength. The de Broglie wavelength is always equal to the Compton wavelength multiplied by $\frac{c}{v}$. Everything we can do with the de Broglie wavelength we can do with the Compton wavelength, however in addition, the Compton wavelength is also valid when the particle is fully at rest, that is for rest-mass particles.

The question arises as to why matter would exhibit two distinct types of wavelengths—the Compton wavelength and the de Broglie wavelength—while photons are associated with only one type. As we have demonstrated, the wavelength of photons is linked to the Compton wavelength rather than the de Broglie wavelength. Furthermore, the Planck constant indeed appears to be the minimum quantum of energy and matter. This can be expressed as the frequency of one divided by the frequency in a chosen clump of matter used as a mass standard, such as the kilogram.

We encourage readers not to accept these notions without scrutiny, but rather to thoroughly investigate these possibilities themselves. They should ask how we can be certain that the de Broglie wavelength is the true measure of matter wavelength and not the Compton wavelength, which is the current consensus that we are challenging. For an in-depth discussion about the many implications of understanding the Compton wavelength is the real matter wavelength (see [38]).

6. Conclusion

We have shown how the Planck constant can be derived or defined as one divided by the Compton frequency in one kilogram. And the kilogram can be defined as a given number of protons. The Compton frequency in one kilogram will be the sum of the Compton frequency in all the protons and electrons making up the kilogram. We can define the Planck constant in this way, it is naturally important that we can find the Compton frequency of a proton independent of knowing the Planck constant as we have demonstrated is fully possible. We think this gives new insight into the Planck constant and even into the mass gap. The mass gap is related to a Compton frequency of one.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Heisenberg, W. (1927) Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik. *Zeitschrift für Physik*, **43**, 172-198. https://doi.org/10.1007/BF01397280

[2] Schrödinger, E. (1926) An Undulatory Theory of the Mechanics of Atoms and Molecules. *Physical Review*, **28**, 1049-1070. https://doi.org/10.1103/PhysRev.28.1049

[3] Dirac, P. (1928) The Quantum Theory of the Electron. *Proceedings of the Royal Society of London*, **117**, 610-624. https://doi.org/10.1098/rspa.1928.0023

[4] Planck, M. (1901) Ueber das Gesetz der Energieverteilung im Normalspectrum. *Annalen der Physik*, **4**, 553-563. https://doi.org/10.1002/andp.19013090310

[5] Damyanov, D.S., Pavlova, L.N., Ilieva, S.L., Gourev, V.N., Yordanov, V.G. and Mishonov, T.M. (2015) Planck's Constant Measurement by Landauer Quantization for Student Laboratories. *European Journal of Physics*, **36**, Article ID: 055047. https://doi.org/10.1088/0143-0807/36/5/055047

[6] Huang, J., *et al.* (2020) High Precision Determination of the Planck Constant by Modern Photoemission Spectroscopy. *Review of Scientific Instruments*, **91**, Article ID: 045116. https://doi.org/10.1063/1.5129140

[7] Kibble, B.P. (1975) A Measurement of the Gyromagnetic Ratio of the Proton by the Strong Field Method. In: Sanders, J.H. and Wapstra, A.H., Eds., *Atomic Masses and Fundamental Constants* 5, Springer, Berlin, 545-551. https://doi.org/10.1007/978-1-4684-2682-3_80

[8] Petley, B.W., Kibble, B.P. and Hartland, A. (1987) A Measurement of the Planck Constant. *Nature*, **327**, 605-606. https://doi.org/10.1038/327605a0

[9] Haddad, D., *et al.* (2017) Measurement of the Planck Constant at the National Institute of Standards and Technology from 2015 to 2017. *Metrologia*, **54**, 633-641. https://doi.org/10.1088/1681-7575/aa7bf2

[10] Steiner, R. (2012) History and Progress on Accurate Measurements of the Planck Constant. *Reports on Progress in Physics*, **76**, Article ID: 016101. https://doi.org/10.1088/0034-4885/76/1/016101

[11] Thomas, M.M., Espel, P.P., Ziane, D.D., *et al.* (2015) First Determination of the Planck Constant Using the LNE Watt Balance. *Metrologia*, **52**, 433-443. https://doi.org/10.1088/0026-1394/52/2/433

[12] Robinson, I.A. and Schlamminger, S. (2016) The Watt or Kibble Balance: A Technique for Implementing the New SI Definition of the Unit of Mass. *Metrologia*, **53**, A46-A74. https://doi.org/10.1088/0026-1394/53/5/A46

[13] Chang, D.C. (2017) Physical Interpretation of Planck's Constant Based on the Maxwell Theory. *Chinese Physics B*, **26**, Article ID: 040301. https://doi.org/10.1088/1674-1056/26/4/040301

[14] Quincey, P. (2013) Planck's Constant as a Natural Unit of Measurement. *Physics Education*, **48**, 597-600. https://doi.org/10.1088/0031-9120/48/5/597

[15] Compton, A.H. (1923) A Quantum Theory of the Scattering of X-Rays by Light Elements. *Physical Review*, **21**, 483-502. https://doi.org/10.1103/PhysRev.21.483

[16] Haug, E.G. (2020) Collision Space-Time: Unified Quantum Gravity. *Physics Essays*, **33**, 46-78. https://doi.org/10.4006/0836-1398-33.1.46

[17] Haug, E.G. (2021) Quantum Gravity Hidden in Newton Gravity and How to Unify It with Quantum Mechanics. In: Krasnoholovets, V., Ed., *The Origin of Gravity from the First Principles*, NOVA Publishing, New York, 133-216.

[18] Planck, M. (1899) Natuerliche Masseinheiten. Der Königlich Preussischen Akademie der Wissenschaften, Berlin. https://www.biodiversitylibrary.org/item/93034#page/7/mode/1up

[19] Planck, M. (1906) Vorlesungen über die Theorie der Wärmestrahlung. J.A. Barth, Leipzig, 163-164.

[20] Haug, E.G. (2020) Finding the Planck Length Multiplied by the Speed of Light without Any Knowledge of *G*, *c*, or *h*, Using a Newton Force Spring. *Journal Physics Communication*, **4**, Article ID: 075001. https://doi.org/10.1088/2399-6528/ab9dd7

[21] Haug, E.G. (2021) Demonstration That Newtonian Gravity Moves at the Speed of Light and Not Instantaneously (Infinite Speed) as Thought! *Journal of Physics Communication*, **50**, Article ID: 025005. https://doi.org/10.1088/2399-6528/abe4c8

[22] Gräff, G., Kalinowsky, H. and Traut, J. (1980) A Direct Determination of the Proton Electron Mass Ratio. *Zeitschrift für Physik A Atoms and Nuclei*, **297**, 35-39. https://link.springer.com/article/10.1007/BF01414243 https://doi.org/10.1007/BF01414243

[23] Van-Dyck, R.S., Moore, F.L., Farnham, D.L. and Schwinberg, P.B. (1985) New Measurement of the Proton-Electron Mass Ratio. *International Journal of Mass Spectrometry and Ion Processes*, **66**, 327-337. https://doi.org/10.1016/0168-1176(85)80006-9

[24] Becker, P. and Bettin, H. (2011) The Avogadro Constant: Determining the Number of Atoms in a Single-Crystal ^{28}Si Sphere. *Philosophical Transactions of the Royal Society A*, **369**, 3925-3935. https://doi.org/10.1098/rsta.2011.0222

[25] Bettin, H., Fujii, K., Man, J., Mana, G., Massa, E. and Picard, A. (2013) Accurate Measurements of the Avogadro and Planck Constants by Counting Silicon Atoms. *Annalen der Physik*, **525**, 680-687. https://doi.org/10.1002/andp.201300038

[26] Li, S.S., *et al.* (2014) Progress on Accurate Measurement of the Planck Constant: Watt Balance and Counting Atoms. *Chinese Physics B*, **24**, Article ID: 010601. https://doi.org/10.1088/1674-1056/24/1/010601

[27] D'Auria, S. (2018) Introduction to Nuclear and Particle Physics. Springer, Berlin. https://doi.org/10.1007/978-3-319-93855-4_7

[28] Lan, S., Kuan, P., Estey, B., English, D., Brown, J.M., Hohensee, M.A. and Müller (2013) A Clock Directly Linking Time to a Particle's Mass. *Science*, **339**, 554-557. https://doi.org/10.1126/science.1230767

[29] Dolce, D. and Perali, A. (2015) On the Compton Clock and the Undulatory Nature of Particle Mass in Graphene Systems. *The European Physical Journal Plus*, **130**, Article No. 41. https://doi.org/10.1140/epjp/i2015-15041-5

[30] Levitt, L.S. (1958) The Proton Compton Wavelength as the "Quantum" of Length. *Experientia*, **14**, 233-234. https://doi.org/10.1007/BF02159173

[31] Trinhammer, O.L. and Bohr, H.G. (2019) On Proton Charge Radius Definition. *Europhysics Letters*, **128**, Article 21001. https://doi.org/10.1209/0295-5075/128/21001

[32] Einstein, A. (1905) On the Electrodynamics of Moving Bodies. *Annalen der Physik*, **322**, 891-921. https://doi.org/10.1002/andp.19053221004

[33] Haug, E.G. (2022) Unified Quantum Gravity Field Equation Describing the Universe from the Smallest to the Cosmological Scales. *Physics Essays*, **35**, 61-71. https://doi.org/10.4006/0836-1398-35.1.61

[34] Pike, O.J., Mackenroth, F., Hill, E.G. and Rose, S.J. (2014) A Photon-Photon Collider in a Vacuum Hohlraum. *Nature Photonics*, **8**, 434-436. https://doi.org/10.1038/nphoton.2014.95

[35] de Broglie, L. (1924) Recherches sur la théorie des quanta. Ph.D. Thesis, Université de Paris, Paris.

[36] de Broglie, L. (1930) An Introduction to the Study of Wave Mechanics. Metheum & Co., Essex.

[37] Davisson, C. and Germer, L.H. (1927) Diffraction of Electrons by a Crystal of Nickel. *Physical Review*, **30**, 705-741. https://doi.org/10.1103/PhysRev.30.705

[38] Haug, E.G. (2023) The Compton Wavelength Is the True Matter Wavelength, Linked to the Photon Wavelength, While the de Broglie Wavelength Is Simply a Mathematical Derivative. *Qeios*. https://doi.org/10.32388/OZ0IRU