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A Study of Solar Rotation and Differential Rotation

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Abstract

The Sun has solar rotation; nevertheless, many evidences have suggested that different latitude of the Sun rotates in different speed, which is now known as differential rotation. This work calculates the solar rotation speeds near the equator and 30˚ in the northern hemisphere using Fixed-Point Arithmetic method. The calculated results show a greater speed at the equator than the speed at 30˚, indicating that the speed decreases as the latitude becomes higher.

Keywords

Sun, Differential Rotation, Active Region, Fixed-Point Arithmetic Method

1. Introduction

The Sun is made up of a blazing combination of gases that are in the form of plasma. The outer layer of Sun is the Photosphere, the Chromosphere, the Transition Region and the Corona. Like most stars, the sun also rotates. The existence of the solar rotation can be confirmed by the movement of sunspots and other active objects on the solar surface, such as prominences and flares, or by the Doppler effect of spectral lines along the eastern and western edges of the sun [1]. Since sun is in plasma state, the rotation of it is not as rigid as the rotation of Earth: on Earth, the angular velocity is the same at different latitude, while on Sun, different latitudes have different rotations and different angular velocities. This difference between speeds is called differential rotation. At the solar equator, rotation is the fastest, and the higher the latitude, the slower the rotation.

The discovery of solar differential rotation is closely related to the observation of sunspots. Galileo first discovered in 1612 that sunspots accelerate and widen as they move from the east to the center of the sun, and slow and thin as they move west. The changes in the position, size and shape of sunspots were thought by Galileo to be caused by the sun's rotation. Therefore, sunspots were widely used as tracers to observe and study the rotation of the sun's surface. Differential rotation was then quantitatively described in 1863 by Carrington on the basis of sunspot observations [2]. The speed at which sunspots move over the solar surface depends on the heliolatitude at which they are observed. Differential rotation holds important clues for several fundamental solar processes. For example, solar rotation can help draw a careful description of coronal magnetic field rotation, which is necessary for a complete specification of the inner boundary condition for the solar wind [3]. After the fundamental importance of this phenomenon to any theory describing the observed evolution, time variation, and periodicity of the solar activity was realized, it was studied in detail by many authors [2].

The existence of differential rotation affects the determination of the coordinates of the sun's surface activity. In order to establish a general coordinate system of the solar surface, Carrington studied the motion of low latitude sunspots and established a coordinate system of the solar surface rotating at a constant speed (14.18439716 deg·day^{-1}), which is Carrington coordinate system. A rotation of the Carrington coordinates takes 25.38 days (sidereal cycle). In a review of solar rotation period measurements published in 2000 by John G. Beck, he summarized the methods and results of all observational studies of the solar rotation since Carrington. From John's result, the period of solar rotation is about 37 days at the poles, and about 27 days at the equator [4].

After the earliest results of Carrington's direct fitting of the three parameters of the period of sunspot reappearance [5], there were many results using different methods of sunspot observation to calculate the period. As faculae, coronal holes and other solar events have been discovered, more solar activity has been used as tracers to observe the sun's rotation. With the development of spectrograph, the method of measuring the Doppler displacement of spectral lines in the early 20th century was also formally used to measure the solar rotation. Other methods such as the use of magnetic field observation of the solar surface around 1980 and later results using magnetic field distribution, and the use of Doppler effect to calculate the rotation period were also applied. The later results were actually very similar to the results of Carrington's in the eighteenth century.

This research paper studies solar rotation and differential rotation using fixed-point Arithmetic and calculate the rotation speed of the sun by trailing the movement of solar active regions, *i.e.*, the sunspots, at different latitudes. Sunspots are magnetic structures that appear dark on the solar surface and occur on photosphere of the sun. Each sunspot is characterized by a dark core, the umbra, and a less dark halo, the penumbra [6]. Therefore, due to its convenience to be observed and tracked, it always serves as a useful tool to study Sun.

2. Calculation

First, two active regions, NOAA AR 12,738 and NOAA AR 12,880, are chosen to be tracked. AR 12,738 is located very close to the equator in the Northern hemisphere, while AR 12,880 is located around 30˚ northern latitude. Choosing two groups of active areas with a large latitude gap can more directly reflect the difference in the rotation speed of the sun at different positions of latitude.

In order to calculate the linear velocity of the surface, this formula is used:

$$v = \frac{d}{t}$$

Thus, the distance of the active region rotates in a certain time need to be obtained, which can be translated into the distance between two points on the surface of the sphere.

Given the longitude and latitude coordinates of two spherical points (Φ_1, λ_1) and (Φ_2, λ_2), the distance between the two points can be expressed by the formula:

$$d = 2 \cdot r \cdot \sin^{-1}\sqrt{\sin\left(\frac{\Phi_2 - \Phi_1}{2}\right)^2 + \cos(\Phi_1)\cdot\cos(\Phi_2)\cdot\sin\left(\frac{\lambda_2 - \lambda_1}{2}\right)^2}$$

1) converting the angular coordinates of latitude and longitude into radians, ($m\Phi_1$, $m\Phi_2$) and ($m\lambda_1$, $m\lambda_2$);

2) where r represents the radius of the Sun.

Take the active region AR 12,738 as an example. From **Figure 1** shot on Apr. 8, 2019, the active region is located at N06 E60. That is, $\Phi_1 = 6˚$, $\lambda_1 = -60˚$. In **Figure 2** shot on Apr.10, 2019, the region is located at N06 E34, which is $\Phi_2 = 6˚$, $\lambda_2 = -34˚$.

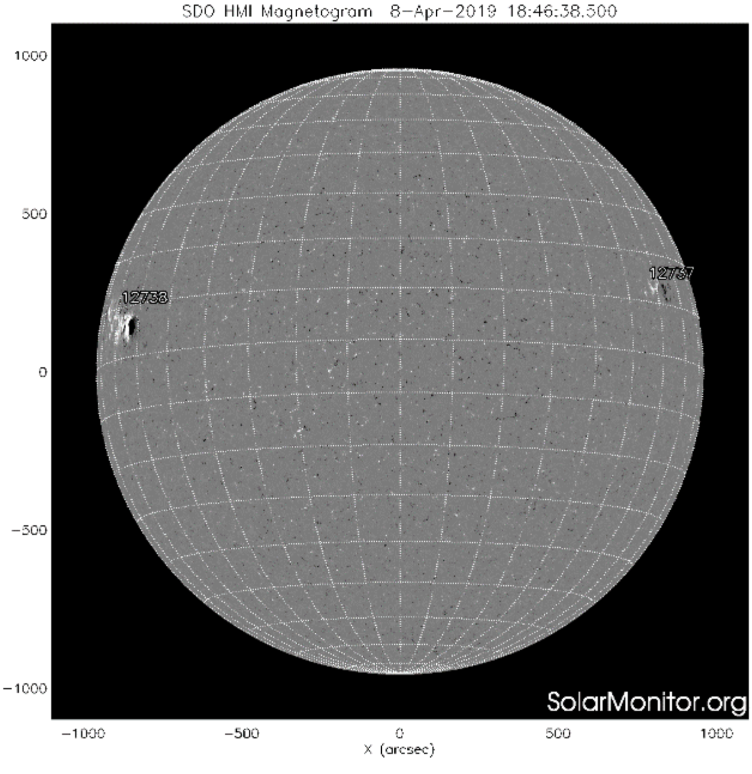

Figure 1. AR 12,738 on Apr 8th 2019.

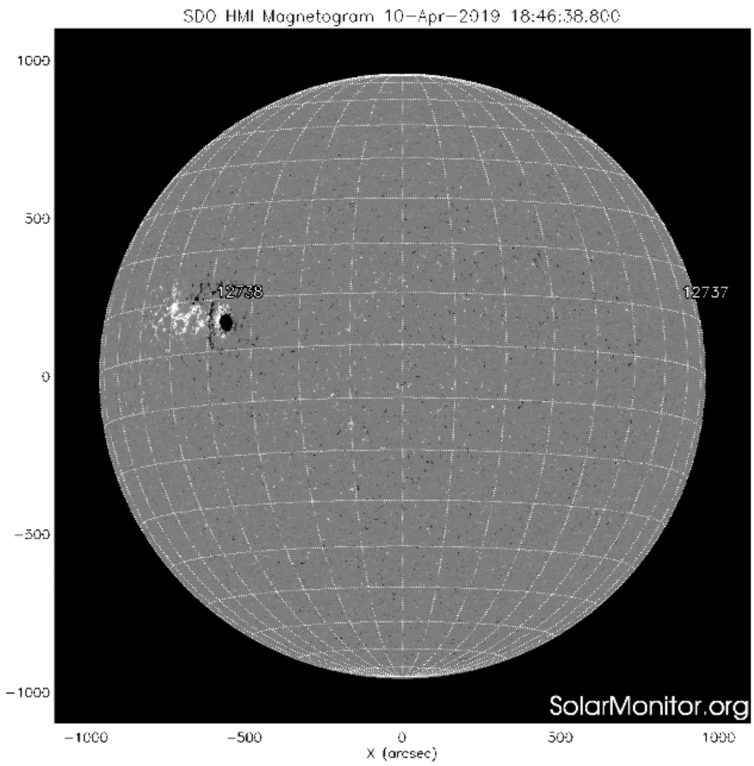

Figure 2. AR 12,738 on APR 10th 2019.

Turning degree system into the radian system, the distance is

$$d = 2\cdot 696000\cdot \sin^{-1}\sqrt{\sin\left(\frac{\frac{\pi}{30}-\frac{\pi}{30}}{2}\right)^2+\cos\left(\frac{\pi}{30}\right)\cdot\cos\left(\frac{\pi}{30}\right)\cdot\sin\left(\frac{-\frac{17\pi}{90}-\left(-\frac{\pi}{3}\right)}{2}\right)^2}$$

$$= 314074.5475\ \text{km}$$

The time is

$$2\ \text{days}\cdot 24\ \text{h}\cdot 60\ \text{min}\cdot 60\ \text{s} = 172800\ \text{s}$$

And the speed is

$$v = \frac{d}{t} = \frac{314074.5475}{172800} = 1.8176\ \text{km/s}$$

Similarly, the distance of active region AR 12,738 from date Apr. 10 to Apr.12 is

$$d = 2\cdot 696000\cdot \sin^{-1}\sqrt{\sin\left(\frac{\frac{\pi}{30}-\frac{\pi}{30}}{2}\right)^2+\cos\left(\frac{\pi}{30}\right)\cdot\cos\left(\frac{\pi}{30}\right)\cdot\sin\left(\frac{-\frac{7\pi}{180}-\left(-\frac{17\pi}{90}\right)}{2}\right)^2}$$

$$= 326151.8357\ \text{km}$$

The time is

$$t = 181441\ \text{s}$$

And the speed is

$$v = \frac{d}{t} = \frac{640017.6901}{181441} = 1.7976 \text{ km/s}$$

Using this method to calculate four times with each time interval of 2 days, 4 results are gotten: 1.8176 km/s from Apr.8 to Apr.10, 1.7976 km/s from Apr.10 to Apr.12, 1.8176 km/s from Apr.12 to Apr.14, and 1.9573 km/s from Apr.14 to Apr.16.

The mean value of these results is

$$v_{avg} = \frac{1.8176 + 1.7976 + 1.8176 + 1.9573}{4} = 1.8475 \text{ km/s}$$

Therefore, by observing the latitude and longitude of the active region 12,738 of 5 days and calculating the distance between each two points of the active region, the approximate speed of the surface of the sun near the equator is 1.8475 km/s.

Using the same measurement, the mean value of the active region 12,880 in 30˚ northern is 1.5497 km/s.

3. Differential Rotation

The average velocities in two different latitudes have been calculated, but they do not show directly that the Sun has differential rotation. Therefore, the rotation periods of the two active regions are needed.

By calculation, the radius of the Sun at 6˚ northern is 692,187.2392 km. The perimeter is

$$C = 2\pi r = 2 \times \pi \times 692187.2392 = 4349140.6910 \text{ km}$$

The period is

$$T = \frac{C}{v_{avg}} = \frac{4349140.6910}{1.8475} = 2354068.0330 \text{ s} = 27.2461 \text{ days}$$

Similarly, by calculation, the radius of the Sun at 30˚ northern is 602,753.6810 km. The perimeter is

$$C = 2\pi r = 2 \times \pi \times 602753.6810 = 3787213.0730 \text{ km}$$

The period is

$$T = \frac{C}{v_{avg}} = \frac{3787213.0730}{1.5497} = 2443836.2730 \text{ s} = 28.2851 \text{ days}$$

Based on the calculation, the rotation period in 30˚ northern is obviously greater than that in 6˚ northern, which shows that the differential rotation does occur in solar rotation.

4. Conclusions

The two calculated speeds of active regions in 6˚ and 30˚ northern latitude on the Sun show that there is a difference of rotation speed in different latitude of the surface of the Sun. From the above calculation, the average speed of active region 12,738 located in 6˚ northern latitude is obviously faster than that of 12,880 in 30˚ northern latitude. This result indicates a gradual slowing of the speed of rotation from the equator of the Sun to 30˚. Similarly, the rotation in South hemisphere also has the same situation.

The study of solar rotation and differential rotation is important for several reasons. It helps us understand the dynamics of the Sun's interior and how it affects the behavior of the Sun's magnetic field. This information is crucial for understanding phenomena such as sunspots, solar flares, and coronal mass ejections, which can have a significant impact on the Earth's environment. Moreover, the study of solar rotation and differential rotation is also important for our understanding of stars and their evolution. By comparing the rotation of the Sun to other stars, we can gain insight into the processes that govern their evolution and magnetic activity. The solar differential rotation is also crucial to the transition from poloidal magnetic field to toroidal magnetic field and is an important basis for the solar dynamo theory which studies the origin of solar magnetic activity. At present, how the differential rotation affects the variation of the solar cycle is not well understood. Therefore, the study of the variation of the solar rotation with the solar cycle and the north-south asymmetry of the solar rotation is not only helpful to understand the variation of the solar magnetic activity, the generation mechanism of the solar cycle and the solar internal dynamics, but also can provide the observation limit for the establishment of the theoretical model of the generation of the solar rotation.

Due to the difference of velocities in different latitudes, the magnetic field lines coming from the poles of the Sun will be twisted and tangled as the Sun rotates. Nevertheless, there will be a time when these magnetic field lines become smooth and orderly again. Therefore, with the calculation of the rotation velocity in two different latitudes, the period in which the magnetic field lines go from smooth to twisted and back to smooth can be calculated. Knowing more about the strength and duration of this period, also referred to solar cycle, would help people forecast space weather and predict dangerous solar activities so that less damage and loss on explorative resources would be resulted.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Stenflo, J.O. (1989) Differential Rotation of the Sun's Magnetic Field Pattern. *Astronomy & Astrophysics*, **210**, 403-409.

[2] Obridko, V.N. and Badalyan, O.G. (2019) Solar Corona as Indicator of Differential Rotation of Subphotospheric Layers. *Cosmic Research*, **57**, 407-412.

[3] Parker, G.D. (1987) Radial Variation of Differential Rotation in the Solar Electron Corona. *Solar Physics*, **108**, 77-87.

[4] Beck, J.G. (2000) A Comparison of Differential Rotation Measurements—(Invited

Review). *Solar Physics*, **191**, 47-70.

[5] Carrington, R.C. (1863) Observations of the Spots on the Sun from November 9. 1853, to March 24, 1861, Made at Redhill. Williams and Norgate, London, 248.

[6] Solanki, S. (2003) Sunspots: An Overview. *The Astron Astrophys Rev*, **11**, 153-286.