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# Corresponding Prime Number Distribution Equation

**Dan Liu, Chenglong Liu**

School of Mathematics, Neijiang Normal University, Neijiang, China
Email: 576672568@qq.com, 30068721@qq.com

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## Abstract

The conjecture of twin prime numbers is a mathematical problem. Proving the twin prime conjecture using traditional modern number theory is extremely profound and complex. We propose an elementary research method for corresponding prime number, proved that the conjecture of twin prime numbers and obtain the corresponding prime distribution equation. According to the distribution rate of corresponding prime numbers, the distribution pattern of twin prime numbers was proved the distribution rate theorem. This is the distribution rate of prime numbers corresponding to composite numbers, which approaches the distribution rate of prime numbers corresponding to integers. Based on the corresponding prime distribution equation, obtain the twin prime inequality function. Then, the formula for calculating twin prime numbers was discussed. There is also the Hardy Littlewood conjecture. This provides a practical and feasible approach for studying the distribution of twin prime numbers.

## Keywords

Prime Number, Composite Number, Twin Prime Numbers, Corresponding Prime Distribution Equation, Twin Prime Inequality Function

## 1. Introduction

The problem of the distribution of prime numbers is profound and complex. The Goldbach conjecture, the twin prime number conjecture, the prime number theorem, and the Riemann conjecture are all famous problems of prime number distribution. Both can be studied using the "sieve method". Twin prime numbers are two prime numbers separated by 2 [1] [2] [3] [4].

(3, 5), (5, 7), (11, 13), (59, 61)

There are infinitely many such prime numbers, called The conjecture of twin prime numbers.

Assuming the prime number $p$, the twin prime conjecture is to prove that $p + 2$ has infinitely many prime numbers.

Mathematicians use complex "screening methods" to study the conjecture of twin prime numbers.

In 2013, Zhang Yitang proved using the sieve method that there are infinitely many prime numbers for

$$p + n \leq 70000000$$

has infinitely many prime numbers.

By 2022, Tao Zhexuan mentioned that another simple method can be used to prove that

$$p + n \leq 6$$

has infinitely many prime numbers.

We propose an elementary method corresponding to prime numbers to study the twin prime number conjecture. Famous mathematicians' evaluation of elementary methods:

It takes genius and wit to discover clever and useful elementary mathematical methods. This is much more difficult than discovering profound mathematical methods to prove.

Refer to: Pan Chengdong, Pan Chengbiao. Elementary proof of the prime number theorem [M]. Shanghai Science and Technology Press. Page 20.

In this article, we use a simple elementary method to prove that for any $x$.

Corresponding prime distribution equation:

$$\pi(x) = c(x) + L(x) \tag{1.1}$$

This provides a practical and feasible approach for studying the distribution of twin prime numbers.

Based on the (1.1) and using limits, the upper limit of $c(x)$ can be proven, thereby obtaining the lower limit of the number of twin prime $L(x)$.

Twin prime inequality function:

$$L(x) > \frac{x}{(\log x)^3}, \quad (x \to \infty), \tag{1.2}$$

**For example**

| $x$ | $c(x)$ | $L(x)$ | $\frac{x}{(\log x)^3}$ |
|---|---|---|---|
| $10^4$ | 1024 | 205 | 12 |
| $10^{15}$ | 28667361180365 | 1177209242304 | 24270524216 |

Before discussing (1.2), let's take a look at the elementary screening method.

Set the integer $k \leq 16$, the first line $k$, the second line $k + 2$,

$k$_____ 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |10|11|12|13|14,

$k + 2$__ 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |10|11|12|13|14|15|16, Blue numbers, including 0, 1 and composite numbers 4, 6, 8, 9, 10, 12, and 14, are not prime numbers. These numbers, which are not prime numbers, are collectively referred to as sum numbers.

The red numbers 2, 3, 5, 7, 11, and 13 are all prime numbers.

Screen out the blue sum $h$ in the first row, while also filtering out the number in the second row:

$h$_____ 0 | 1 | 4 | 6 | 8 | 9 |10|12|14

$h+2$__ 2 | 3 | 6 | 8 |10|11|12|14|16

After screening, the remaining amount is:

$p$_____ 2 | 3 | 5 | 7 |11|13

$p+2$__ 4 | 5 | 7 | 9 |13|15

Screen out the sum of blue numbers in the second row, while also filtering out the numbers in the first row, leaving only twin prime numbers:

$p$_____ 3 | 5 |11

$p+2$__ 5 | 7 |13

According to the screening process, we can obtain the formula for calculating twin prime numbers:

$$L(x) \sim \frac{1}{2} \prod_{p \le \sqrt{x}} \left(1 - \frac{2}{p}\right)$$

Among them, $p$ is an odd prime number. In addition, there the Hardy Littlewood hypothesis:

$$L(x) \sim c_x \frac{x}{(\log x)^2}$$

Among them, constant

$$c_x = 2 \times 0.66016\cdots = 1.32032\cdots$$

The difficult problem with the screening method is:

It is difficult to confirm the number of prime numbers for $h+2$.

If you don't know the number of prime numbers in $h+2$,
then you also don't know the number of prime numbers in $p+2$.

Therefore, it may be difficult to prove the twin prime conjecture using the sieve method.

Therefore, we propose a research method for corresponding prime numbers.

## 2. Corresponding Prime Distribution Equation

We prove the corresponding prime distribution equation.

prove

Let's take a look at the corresponding prime numbers.

**For example**

Assuming $x$=16, we examine the corresponding number,

1____0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |10|11|12|13|14,

2____2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |10|11|12|13|14|15|16.

In the first row, the numbers in blue and red correspond to the prime numbers in the second row

0 | 1 | 3 | 5 | 9 |11

2 | 3 | 5 | 7 |11|13

The number of prime numbers $\pi$ (16) = 6, which is the number of prime numbers that do not exceed 16.

In the first row, the sum of blue numbers corresponds to the prime numbers in the second row

0 | 1 | 9

2 | 3 | 11

The number of prime numbers $c$ (16) = 3, which is the number of prime numbers for the previous screening method $h + 2$.

The first line, the red prime number, corresponds to the prime number in the second line

3 | 5 |11

5 | 7 |13

The number of prime numbers corresponding to prime numbers $L$ (16) = 3, *i.e.* the number of twin prime numbers [3] [4].

Obviously,

$$\pi(16) = c(16) + L(16)$$

For any $x$, we can obtain:

$$\pi(x) = c(x) + L(x)$$

This proves that the twin prime distribution Equation (1.1) is correct.

Now, let's take a look at the corresponding prime distribution rate.

Definition:

The ratio of $\pi(x)$ to the number of integers $x$ is called the distribution rate of prime numbers corresponding to integers. That is, the distribution rate of prime numbers

$$\rho_k = \frac{\pi(x)}{x}$$

The ratio of $c(x)$ to the number of sums $F = x - \pi(x)$ is called the distribution rate of prime numbers corresponding to sums. That is, the distribution rate of prime numbers for the previous screening method h + 2

$$\rho_h = \frac{c(x)}{F}$$

The ratio of $L(x)$ to the number of prime numbers $\pi(x)$ is called the distribution rate of prime numbers corresponding to prime numbers. The distribution rate of twin prime numbers

$$\rho_p = \frac{L(x)}{\pi(x)}$$

**For example**

| $x$ | $\frac{\pi(x)}{x}$ | $\frac{c(x)}{x-\pi(x)}$ | $\frac{L(x)}{\pi(x)}$ |
|---|---|---|---|
| $10^4$ | 0.1229 | 0.1117 | 0.1668 |
| $10^8$ | 0.05761455 | 0.05646461 | 0.07642375 |

Obviously, the distribution rates of sums and integers are first-order.

Let's take a look at the ratio of sum to integer:

$$\frac{F}{x}=\frac{x-\pi(x)}{x}=1-\frac{\pi(x)}{x}$$

Assuming $x$ tends towards infinity, it can be confirmed that:

$$\frac{F}{x}=0.99999999\cdots \quad \text{and} \quad F\sim x$$

Almost all integers are composite numbers. From this, it can be confirmed that the distribution rate of composite numbers and integers is equivalent. We can obtain the theorem for the distribution rate of prime numbers:

$$\frac{c(x)}{x-\pi(x)}\sim\frac{\pi(x)}{x},\ (x\to\infty) \tag{2.1}$$

Show: The distribution rate of prime numbers corresponding to sum numbers approaches the distribution rate of prime numbers corresponding to integers.

Now, according to (2.1), prove the lower limit of the distribution of twin prime numbers.

prove

Let's discuss the distribution rate of twin prime numbers. Including these aspects:

If $\frac{L(x)}{\pi(x)}=\frac{\pi(x)}{x}$, so $L(x)=\frac{\pi^2(x)}{x}$

If $\frac{L(x)}{\pi(x)}>\frac{\pi(x)}{x}$, so $L(x)>\frac{\pi^2(x)}{x}$

If $\frac{L(x)}{\pi(x)}\sim\frac{\pi(x)}{x}$, so $L(x)\sim\frac{\pi^2(x)}{x}$

In these cases, it can be confirmed that there are infinitely many twin prime numbers.

If $\frac{L(x)}{\pi(x)}<\frac{\pi(x)}{x}$, so $L(x)<\frac{\pi^2(x)}{x}$

In these cases, $L(x)$ may be infinite or finite, which is not clear.

According to:

$$\frac{L(x)}{\pi(x)}<\frac{\pi(x)}{x}$$

It can be confirmed that:

$$\frac{\pi(x)}{x} \to 0 \quad \text{and} \quad \frac{L(x)}{\pi(x)} \to 0$$

By (1.1) have $c(x) = \pi(x) - L(x)$ and (2.1) transformation

$$\frac{c(x)}{\pi(x)} \Big/ \frac{F}{x} = \frac{\pi(x) - L(x)}{\pi(x)} \Big/ \frac{x - \pi(x)}{x} = \frac{1 - \dfrac{L(x)}{\pi(x)}}{1 - \dfrac{\pi(x)}{x}}$$

Assuming the coefficient $k(x)$, we obtain:

$$\frac{c(x)}{x - \pi(x)} = \frac{\pi(x)}{x} k(x) \tag{2.2}$$

According to (2.2), it can be obtained that

$$c(x) = k(x)\pi(x) - k(x)\frac{\pi^2(x)}{x}$$

Obviously, it can be inferred that

$$c(x) < k(x)\pi(x) - k(x)\frac{\pi^2(x)}{x}\frac{\pi(x)}{x}$$

We can get

$$c(x) < k(x)\pi(x) - k(x)\frac{\pi^3(x)}{x^2}$$

Assuming $x$ approaches infinity, it can be confirmed that

$$k(x) \to 1$$

From this, it can be concluded that

$$c(x) < \pi(x) - \frac{\pi^3(x)}{x^2}$$

By (1.1) get

$$c(x) = \pi(x) - L(x)$$

Can get

$$\pi(x) - L(x) < \pi(x) - \frac{\pi^3(x)}{x^2}$$

From this, we can obtain

$$L(x) > \frac{\pi^3(x)}{x^2}$$

According to the prime number theorem, from (2.2) we can obtain

$$L(x) > \frac{x}{(\log x)^3}, \quad (x \to \infty)$$

This proves that (1.2) is correct.

The above proof is based on distribution rate. Can we prove (1.2) without using this theorem?

This is very difficult. We can discuss this issue.

## 3. Proof of Inequality Function

Based on the corresponding prime distribution equation, prove the lower limit of the twin prime distribution.

prove

According to the corresponding prime numbers mentioned earlier, for large numbers, $L(x)$ has many.

According to (1.1)

$$\pi(x) = c(x) + L(x)$$

obviously, it is estimated that

$$\pi(x) > c(x)$$

Transformation

$$\frac{\pi(x)}{c(x)} > 1$$

Multiply both sides by the same term:

$$\frac{\pi(x)}{c(x)}\frac{x-\pi(x)}{x} > \frac{x-\pi(x)}{x}$$

$$\frac{x\pi(x)-\pi^2(x)}{c(x)x} > 1-\frac{\pi(x)}{x}$$

Add the same items on both sides:

$$\frac{x\pi(x)-\pi^2(x)}{c(x)x}+\frac{\pi(x)}{x+\pi(x)} > 1-\frac{\pi(x)}{x}+\frac{\pi(x)}{x+\pi(x)}$$

$$\frac{x\pi(x)-\pi^2(x)}{c(x)x}+\frac{\pi(x)}{x+\pi(x)} > 1-\left(\frac{\pi(x)}{x}-\frac{\pi(x)}{x+\pi(x)}\right)$$

Wherein

$$\frac{\pi(x)}{x}-\frac{\pi(x)}{x+\pi(x)} = \frac{\pi(x)}{x}\frac{\pi(x)}{x+\pi(x)}$$

Can get

$$\frac{x\pi(x)-\pi^2(x)}{c(x)x}+\frac{\pi(x)}{x+\pi(x)} > 1-\frac{\pi(x)}{x}\frac{\pi(x)}{x+\pi(x)} \tag{3.1}$$

Assuming $x$ approaches infinity, it can be confirmed that second-order infinitesimal

$$\frac{\pi(x)}{x}\frac{\pi(x)}{x+\pi(x)} \to 0$$

Can get

$$1-\frac{\pi(x)}{x}\frac{\pi(x)}{x+\pi(x)}=0.99999999\cdots$$

According to arithmetic theory

$$0.99999999\cdots=1$$

By (3.1), it can be concluded that [5] [6]

$$\frac{x\pi(x)-\pi^2(x)}{c(x)x}+\frac{\pi(x)}{x+\pi(x)}>1$$

$$\frac{x\pi(x)-\pi^2(x)}{c(x)x}>1-\frac{\pi(x)}{x+\pi(x)}=\frac{x}{x+\pi(x)}$$

$$\frac{x\pi(x)-\pi^2(x)}{x}\frac{x+\pi(x)}{x}>c(x)$$

$$\frac{x^2\pi(x)-x\pi^2(x)+x\pi^2(x)-\pi^3(x)}{x^2}>c(x)$$

We can get

$$\frac{x^2\pi(x)-\pi^3(x)}{x^2}>c(x)$$

According to (1.1) get

$$c(x)=\pi(x)-L(x)$$

We get

$$\frac{x^2\pi(x)-\pi^3(x)}{x^2}>\pi(x)-L(x)$$

From this, we can obtain:

$$L(x)>\frac{\pi^3(x)}{x^2},\ (x\to\infty) \tag{3.2}$$

**For example**

| $x$ | $\pi(x)$ | $L(x)$ | $\frac{\pi^3(x)}{x^2}$ |
|---|---|---|---|
| $10^4$ | 1229 | 205 | 18 |
| $10^8$ | 5761455 | 440312 | 19124 |

According to the theorem of prime numbers, substitute (3.2) to obtain：

$$L(x)>\frac{x}{(\log x)^3},\ (x\to\infty)$$

This proves that (1.2) is correct.

We used the corresponding prime distribution Equation (1.1) to prove the lower limit of twin prime numbers. This indicates that the corresponding prime distribution equation is fundamental to studying the distribution of twin prime numbers.

It is practical and feasible.

## 4. Hardy Littlewood Conjecture

Set constant $c_x = 1.32032\cdots$.

Based on the distribution rate of prime numbers corresponding to sums.

We obtain the equivalent distribution rate [4] [7]

$$\frac{c(x)}{x-\pi(x)} \sim \frac{c(x)}{x-c_x\pi(x)}$$

According to (1.1) $c(x)=\pi(x)-L(x)$, We can get

$$\frac{c(x)}{x-c_x\pi(x)} = \frac{\pi(x)-L(x)}{x-c_x\pi(x)} = \frac{\pi(x)}{x}\frac{1-\dfrac{L(x)}{\pi(x)}}{1-\dfrac{c_x\pi(x)}{x}}$$

Can get:

$$\frac{\pi(x)-L(x)}{x-c_x\pi(x)} \Big/ \frac{\pi(x)}{x} = k(x) \tag{4.1}$$

**For example**

| $x$ | $\pi(x)$ | $L(x)$ | $k(x)$ |
|---|---|---|---|
| $10^4$ | 1229 | 205 | 0.99458663 |
| $10^{15}$ | 2984457042 2669 | 117720924 2304 | 0.99995806 |

Assuming $x$ approaches infinity, it can be confirmed that

$$1-\frac{L(x)}{\pi(x)} = 0.99999999\cdots \quad \text{and} \quad 1-\frac{c_x\pi(x)}{x} = 0.99999999\cdots$$

From this, it can be confirmed that

$$k(x) \to 1$$

By (4.1) we can get

$$\frac{\pi(x)-L(x)}{x-c_x\pi(x)} \sim \frac{\pi(x)}{x}$$

From this, it can be concluded that

$$\pi(x)-L(x) \sim \frac{x\pi(x)-c_x\pi^2(x)}{x}$$

Can get

$$L(x) \sim \pi(x) - \frac{x\pi(x)-c_x\pi^2(x)}{x}$$

From this, it can be concluded that

$$L(x) \sim c_x\frac{\pi^2(x)}{x}$$

Substitute the prime number theorem, we can get Hardy Littlewood hypothesis.

$$L(x) \sim c_x \frac{x}{(\log x)^2},\ (x \to \infty) \tag{4.2}$$

wherein $c_x = 1.32032\cdots$, (4.2) this is a strong twin prime conjecture.

## 5. Super Strong Twin Prime Conjecture

Let's discuss another Hardy Littlewood conjecture. The Super Strong Twin Prime Conjecture.

Assuming $x$ tends to infinity, it can get

$$\frac{x-\pi(x)}{x} \to 1$$

be confirmed based on the theorem of prime numbers, it can get

$$\frac{x}{\pi(x)\log x} \to 1$$

We can get [4] [5] [6] [8]

$$\frac{x-\pi(x)}{x} \sim \frac{x}{\log x \pi(x)}$$

Transform according to (4.2)

$$\frac{\pi^2(x)}{x} = \pi(x) - \pi(x)\frac{x-\pi(x)}{x}$$

Can get:

$$\frac{\pi^2(x)}{x} = \pi(x) - \pi(x)\frac{x}{\pi(x)\log x} = \pi(x) - \frac{x}{\log x}$$

Substituting this into (4.2) yields the twin prime calculation formula.

$$L(x) \sim k\left(\pi(x) - \frac{x}{\log x}\right),\ (x \to \infty) \tag{5.2}$$

**For example**

| $x$ | $\pi(x)$ | $L(x)$ | $k\left(\pi(x)-\frac{x}{\log x}\right)$ |
|---|---|---|---|
| $10^{12}$ | 37607912018 | 1870585220 | 1870504200 |
| $10^{15}$ | 29844570422669 | 1177209242304 | 1177203864025 |

In addition, the integral function $li(x)$ can be used to obtain

$$\pi(x) \sim li(x)$$

which can be substituted into (5.2) to obtain

$$L(x) \sim k\left(li(x) - \frac{x}{\log x}\right),\ (x \to \infty)$$

This is the super strong Hardy Litwood conjecture.

## 6. Conclusions

We obtained through the study of the distribution of corresponding prime numbers.

Corresponding Prime Number Distribution Equation

$$\pi(x) = c(x) + L(x) \tag{6.1}$$

The Distribution Rate Theorem of Prime Numbers

$$\frac{c(x)}{x-\pi(x)} \sim \frac{\pi(x)}{x}, \ (x \to \infty)$$

According to (6.1), it is obtained that Twin prime inequality function [6] [8]

$$L(x) > \frac{x}{(\log x)^3}, \ (x \to \infty) \tag{6.2}$$

According to (6.2), it is confirmed that there are infinitely many twin prime numbers. Also

Hardy Litwood Conjecture

$$L(x) \sim c_x \frac{x}{(\log x)^2}, \ (x \to \infty)$$

**For example**

| $x$ | $L(x)$ | $\frac{x}{2(\log x)^3}$ | $c_x \frac{x}{(\log x)^2}$ |
|---|---|---|---|
| $10^4$ | 1870585220 | 6 | 155 |
| $10^{15}$ | 1177209242304 | 12135262108 | 1106790203548 |

Formula for calculating twin prime numbers

$$L(x) \sim k\left(\pi(x) - \frac{x}{\log x}\right), \ (x \to \infty)$$

This is a super strong calculation formula. From this, it can be concluded that

Super Strong Twin Prime Conjecture

$$L(x) \sim k\left(li(x) - \frac{x}{\log x}\right), \ (x \to \infty)$$

This is the same value as the calculation formula for twin prime numbers.

The above is a discussion on the elementary proof of the twin prime conjecture.

The elementary method corresponding to the prime distribution equation is practical and feasible.

Thank you:

Inspired by Professor Bell, a number theory expert at Manchester University in the United States.

Commentary by Professor Wang Maoze, Visiting Scholar at Peking University

and North Star Institute of Basic Mathematics:

Your principle and formulation are particularly simple, making it easy for anyone to understand and draw conclusions, which is worth learning from. On this basis, the conclusion that there are infinite pairs of twin prime numbers is reached by proving the monotonic increasing property of the formula for the number of twin prime numbers, and proving the conjecture of twin prime numbers.

Prove the conjecture of twin prime numbers there is hope.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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