Journal of Applied Mathematics and Physics, 2023, 11, 3203-3211
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

The Electromagnetic Operator of Mass

Hosam Mahmoud Maksoud

Faculty of Science, Tishreen University, Lattakia, Syria
Email: hosmak1982@gmail.com

How to cite this paper: Maksoud, H.M. (2023) The Electromagnetic Operator of Mass. *Journal of Applied Mathematics and Physics*, **11**, 3203-3211.
https://doi.org/10.4236/jamp.2023.1110207

Received: March 19, 2023
Accepted: October 28, 2023
Published: October 31, 2023

Abstract

Studying the source of particle properties is the most important goal for scientists, so it was necessary to use the means available to us, which is physical logic to study these properties. In this paper, you will examine how the type of coordinates in which electromagnetic fields are distributed can have a role in detecting particle properties, specifically using the Riemann-Silberstein vector. Because electromagnetism it deals with electric and magnetic fields together for any electromagnetic sentence, and when we study it according to multiple coordinates and study its derivation by changing coordinates, we discover how the electromagnetic sentences are transformed from one particle to another.

Keywords

Electromagnetic Potential, Annihilation of Pairs, Creating of Pairs, Proca's Equation, Dirac's Equation, Riemann-Silberstein Vector

1. Introduction

In the creation of pairs experiment, we notice the transformation of a massless particle into a massive particle, and if we remind that the creator photon is energy that has only two properties which are an electric field and a magnetic field only, we will find that electron (positron) has only these two fields, so those fields are responsible only for all the characteristics of the new particle. We will study the free massless photon in vacuum (without interaction) according to Maxwell's equation abbreviated according to Lorentz's Gauge in the following form:

$$\partial_\mu A^\mu = 0 \quad (1.1: \text{A})$$

$$\partial_t \varphi + \nabla \boldsymbol{A} = 0 \quad (1.1: \text{B})$$

And we can write it as the following

$$\partial_\mu F^{\mu\nu} = \Box A^\mu = 0 \tag{1.2}$$

In addition, we can write it as the following:

$$\Box A^\mu = \begin{pmatrix} \Box\varphi \\ \Box \boldsymbol{A} \end{pmatrix} = \begin{pmatrix} \partial_t^2\varphi - \nabla^2\varphi \\ \partial_t^2 \boldsymbol{A} - \nabla^2 \boldsymbol{A} \end{pmatrix} = \begin{pmatrix} 0 \\ \boldsymbol{0} \end{pmatrix} \tag{1.3}$$

where $\boldsymbol{A}$ is the magnetic potential, φ is the electric potential.

And always $\nabla^2\varphi = \partial_t^2\varphi \neq 0$, also $\partial_t^2 \boldsymbol{A} = \nabla^2 \boldsymbol{A} \neq \boldsymbol{0}$.

In order to know the importance of the previous equations, we must remind that electromagnetic potential of photon which is distributed in the Cartesian coordinates, and when we study the massive photon, we will find that its motion equation is given according to the following [1]:

$$\partial_\mu F^{\mu\nu} = \Box A^\mu = -m^2 A^\mu \tag{1.4}$$

So, we must discover the reason which causes changing $\Box A^\mu$ from 0, into nonzero which is $-m^2 A^\mu$, then the last reason will be the reason off mass, because the value $-m^2 A^\mu$ comes from mass, thus, we will study the electromagnetic potential according to curved coordinates.

2. Studying of Electromagnetic Potential with Curved Coordinates

When we study the massive particle of the weak force (Proca's particle), we will find that its equation has the following [2]:

$$\Box A^\mu = -m^2 A^\mu \tag{2.1}$$

where $\nabla^2\varphi \neq \partial_t^2\varphi \neq 0$, $\partial_t^2 \boldsymbol{A} \neq \nabla^2 \boldsymbol{A} \neq \boldsymbol{0}$.

We can easily note that two different particles have the same kinetic motion, so the only reason, which makes the difference in property, is the sort of coordinates, then, for facilitating the study, we will study $\Box A^\mu$ according to the following curved coordinates which have a constant radius (because the studied particle have a classical radius) [3]:

In cylindrical coordinates (r,θ,z,t):

The radius $r \in]0,\infty[$, $\theta \in [0,2\pi]$, $z \in]0,\infty[$, t is the time

$$\begin{aligned} \nabla^2 \boldsymbol{A}(r,\theta,z) = &\left(\frac{\partial^2 A^r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A^r}{\partial \theta^2} + \frac{\partial^2 A^r}{\partial z^2} + \frac{1}{r}\frac{\partial A^r}{\partial r} - \frac{2}{r^2}\frac{\partial A^\theta}{\partial \theta} - \frac{A^r}{r^2} \right)\hat{r} \\ &+ \left(\frac{\partial^2 A^\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A^\theta}{\partial \theta^2} + \frac{\partial^2 A^\theta}{\partial z^2} + \frac{1}{r}\frac{\partial A^\theta}{\partial r} + \frac{2}{r^2}\frac{\partial A^r}{\partial \theta} - \frac{A^\theta}{r^2} \right)\hat{\theta} \\ &+ \left(\frac{\partial^2 A^z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A^z}{\partial \theta^2} + \frac{\partial^2 A^z}{\partial z^2} + \frac{1}{r}\frac{\partial A^z}{\partial r} \right)\hat{z} \end{aligned} \tag{2.2: A}$$

$$\nabla^2\varphi(r,\theta,z) = \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{\partial^2\varphi}{\partial z^2} \tag{2.2: B}$$

Now, if we checked the previous curved coordinates, we will find that the electromagnetic potential distributed according to cylindrical coordinates changes around a moving coordinate center, and this is the most important property of the boson which stops only at the receiving particle, so we can say that Proca equation in cylindrical coordinates study the weak force particle (and massive photon) as the following:

$$\square A^{\mu}\left(r,\theta,z,t\right)=-m^{2}A^{\mu} \tag{2.3}$$

In spherical coordinates $\left(r,\theta,\phi,t\right)$:

The radius $r\in\left]0,\infty\right[$, $\theta\in\left[0,2\pi\right]$, $\phi\in\left[0,\pi\right]$, t is the time

$$\begin{aligned}\nabla^{2}\boldsymbol{A}\left(r,\theta,\phi\right)=&\left(\frac{1}{r}\frac{\partial^{2}rA^{r}}{\partial r^{2}}+\frac{1}{r^{2}}\frac{\partial^{2}A^{r}}{\partial\theta^{2}}+\frac{\partial^{2}A^{r}}{r^{2}\left(\sin\theta\right)^{2}\partial\phi^{2}}+\frac{\cot\theta}{r^{2}}\frac{\partial A^{r}}{\partial\theta}\right.\\&\left.-\frac{2}{r^{2}}\frac{\partial A^{\theta}}{\partial\theta}-\frac{2}{r^{2}\sin\theta}\frac{\partial A^{\phi}}{\partial\theta}-\frac{2A^{r}}{r^{2}}-\frac{2\cot\theta A^{\theta}}{r^{2}}\right)\hat{r}\\&+\left(\frac{1}{r}\frac{\partial^{2}rA^{\phi}}{\partial r^{2}}+\frac{1}{r^{2}}\frac{\partial^{2}A^{\phi}}{\partial\theta^{2}}+\frac{\partial^{2}A^{\phi}}{r^{2}\left(\sin\theta\right)^{2}\partial\phi^{2}}+\frac{\cot\theta}{r^{2}}\frac{\partial A^{\phi}}{\partial\theta}\right.\\&\left.+\frac{2}{r^{2}\sin\theta}\frac{\partial A^{r}}{\partial\phi}+\frac{2\cot\theta}{r^{2}\left(\sin\theta\right)^{2}}\frac{\partial A^{\theta}}{\partial\phi}-\frac{A^{\phi}}{r^{2}\sin^{2}\theta}\right)\hat{\theta}\\&+\left(\frac{1}{r}\frac{\partial^{2}rA^{\phi}}{\partial r^{2}}+\frac{1}{r^{2}}\frac{\partial^{2}A^{\phi}}{\partial\theta^{2}}+\frac{\partial^{2}A^{\phi}}{r^{2}\left(\sin\theta\right)^{2}\partial\phi^{2}}+\frac{\cot\theta}{r^{2}}\frac{\partial A^{\phi}}{\partial\theta}\right.\\&\left.+\frac{2}{r^{2}\sin\theta}\frac{\partial A^{r}}{\partial\phi}+\frac{2\cot\theta}{r^{2}\left(\sin\theta\right)^{2}}\frac{\partial A^{\theta}}{\partial\phi}-\frac{A^{\phi}}{r^{2}\left(\sin\theta\right)^{2}}\right)\hat{\phi}\end{aligned} \tag{2.4: A}$$

$$\nabla^{2}\varphi=\frac{\partial^{2}\varphi}{\partial r^{2}}+\frac{2}{r}\frac{\partial\varphi}{\partial r}+\frac{1}{r^{2}\left(\sin\theta\right)^{2}}\frac{\partial^{2}\varphi}{\partial\theta^{2}}+\frac{\cos\phi}{r^{2}\sin\phi}\frac{\partial\varphi}{\partial\phi}+\frac{1}{r^{2}}\frac{\partial^{2}\varphi}{\partial\phi^{2}} \tag{2.4: B}$$

And if we checked the previous curved coordinates, we will find that the electromagnetic potential distributed according to spherical coordinates changes around a fixed coordinate center, and this is the most important property of the fermion which don't move without an external operator, so we can say that Proca equation in spherical coordinates study Dirac particles (electron-positron) as the following:

$$\square A^{\mu}\left(r,\theta,\phi,t\right)=-m^{2}A^{\mu} \tag{2.5}$$

When we note that the radius of the electron is fixed, then $\frac{\partial^{2}rA^{r}}{\partial r^{2}}=0$, $\frac{\partial^{2}rA^{\phi}}{\partial r^{2}}=0$, $\frac{\partial^{2}\varphi}{\partial r^{2}}=0$, $\frac{\partial\varphi}{\partial r}=0$, so, the changing of A^{μ} will be only by the angles and this confirms that the electron do not move without an external operator, and then the subjective electromagnetic equation describing the change in the electromagnetic potential of an electron takes the following form:

$$\square A^{\mu}\left(r,\theta,\phi,t\right)=-m^{2}A^{\mu} \tag{2.6}$$

Moreover, the Equation (2.6) studies the Dirac particle (electron-positron) in the static state (before any photon is absorbed) which is the Equations (2.7) [4]:

$$\psi_{+,\frac{1}{2}}(\text{at rest}) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}\mathrm{e}^{-imt},\ \psi_{+,-\frac{1}{2}}(\text{at rest}) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}\mathrm{e}^{-imt}$$

$$\psi_{-,\frac{1}{2}}(\text{at rest}) = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\0\\1\\0\end{pmatrix}\mathrm{e}^{imt},\ \psi_{-,-\frac{1}{2}}(\text{at rest}) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}\mathrm{e}^{imt}$$

where the sign + in $\psi_{+,\frac{1}{2}}$ means (positive energy = electron) and the sign − in $\psi_{-,\frac{1}{2}}$ means (negative energy = positron).

3. The Use of Riemann-Silberstein Vector in the Study of Mass

In order to understand the role of the electric field in shaping the characteristics of the elementary particle such as charge and the polarization of those fields that cause spin, we have to study Equation (2.6) in the field method, and the most important field vector for studying the work of the fields is the Riemann-Silberstein vector, then, we will introduce the Riemann-Silberstein (RS) vector which is defined as the complex sum of the electric and magnetic field vectors as the following [5]: $F = \boldsymbol{E} + ic\boldsymbol{B}$. It appeared in 1907 in an article by Silberstein, and it is discovered that the (RS) vector may become a matrix operator as the following:

$$\gamma^{\mu}A_{\mu} = \begin{pmatrix} 0 & A^0 + \sigma\boldsymbol{A} \\ A^0 - \sigma\boldsymbol{A} & 0 \end{pmatrix} \tag{3.1}$$

where σ is Pauli matrices, A^0 electric potential, $\boldsymbol{A}$ is magnetic potential.

In addition, we know:

$$\gamma^{\mu}\partial_{\mu} = \begin{pmatrix} 0 & \partial_t + \sigma\partial_i \\ \partial_t - \sigma\partial_i & 0 \end{pmatrix} \tag{3.2}$$

then, it follows from the definition of $\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A} - \nabla A^0$ and $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ in terms of A^{μ} which are equivalent to the homogeneous Maxwell equations.

So, we can write:

$$-\left(\partial_{\mu}\gamma^{\mu}\right)\left(\gamma^{\mu}A_{\mu}\right) = \begin{pmatrix} \sigma\left(\boldsymbol{E} + i\boldsymbol{B}\right) & 0 \\ 0 & -\sigma\left(\boldsymbol{E} - i\boldsymbol{B}\right) \end{pmatrix} = \mathbb{F}:\ c = 1 \tag{3.3}$$

If the Lorentz gauge $\partial_t\varphi + \nabla\boldsymbol{A} = 0$.

Also [6] defining the invariants of the field

$$\boldsymbol{E}^2 - \boldsymbol{B}^2 \equiv \mathcal{E}^2 - \mathcal{B}^2,\ \boldsymbol{E}\cdot\boldsymbol{B} \equiv \mathcal{E}\mathcal{B} \tag{3.4}$$

Then we have $\boldsymbol{F}^2 = \left(\mathcal{E} + i\mathcal{B}\right)^2$ and it follows

$$\mathbb{F}^2 = \begin{pmatrix} (\mathcal{E}+i\mathcal{B})^2 I & 0 \\ 0 & (\mathcal{E}-i\mathcal{B})^2 I \end{pmatrix} \tag{3.5}$$

Thus $\mathbb{F}^2$ is diagonal, in the general case, the matrix $\mathbb{F}$ satisfies the equation:

$$\mathbb{F}^4 - 2(\mathcal{E}^2 - \mathcal{B}^2)^2 \mathbb{F}^2 + (\mathcal{E}^2 + \mathcal{B}^2)\mathbb{J} = 0 \tag{3.6}$$

Thus, the Riemann-Silberstein matrix becomes a Hermitian operator of the Dirac function as the following:

$$\mathbb{F}\psi_i = \lambda_i \psi_i \tag{3.7}$$

where

$$\lambda_i = \pm(\mathcal{E} \pm i\mathcal{B}) \tag{3.8}$$

Therefore, if the RS matrix was operator then $\Box\mathbb{F}$ is operator, so we can write the last subjective personal electromagnetic equation for electron (2.4) as the following:

$$\Box A_\mu(r,\theta,\phi,t) = -m^2 A_\mu(r,\theta,\phi,t) \tag{3.9}$$

$$\partial^\mu \partial_\mu A_\mu(r,\theta,\phi,t) = -m^2 A_\mu(r,\theta,\phi,t) \tag{3.10}$$

$$(\gamma^\mu \gamma^\mu)\partial^\mu \partial_\mu A_\mu(r,\theta,\phi,t) = -m^2 \gamma^\mu \gamma^\mu A_\mu(r,\theta,\phi,t) \tag{3.11}$$

$$\partial^\mu (\partial_\mu \gamma^\mu)(\gamma^\mu A_\mu) = -m^2 \gamma^\mu (\gamma^\mu A_\mu) \tag{3.12}$$

$$-\partial^\mu \mathbb{F}(r,\theta,\phi,t) = -m^2 \gamma^\mu (\gamma^\mu A_\mu) \tag{3.13}$$

$$\partial^\mu \mathbb{F}(r,\theta,\phi,t) = m^2 \gamma^\mu (\gamma^\mu A_\mu) \tag{3.14}$$

$$\partial_\mu \partial^\mu \mathbb{F} = m^2 (\partial_\mu \gamma^\mu)(\gamma^\mu A_\mu) \tag{3.15}$$

$$\Box\mathbb{F}(r,\theta,\phi,t) = m^2 \mathbb{F}(r,\theta,\phi,t) \tag{3.16}$$

Then, the changing of (RS) vector as an operator at the wave function of Dirac particle:

$$\Box\mathbb{F}(r,\theta,\phi,t)\psi_i = m^2 \mathbb{F}(r,\theta,\phi,t)\psi_i = m^2 \lambda_i \psi_i \tag{3.17}$$

The last operator that expresses the relationship between the mass of Dirac particle and the changed electromagnetic fields of this particle on its surface with the time.

4. The Origin of the Spin and the Charge

We can easily note from the last equation (3.17) the following

The first note:

Before creation of pairs, there is a no massive electromagnetic system with Cartesian coordinates (the photon) that has two pulses (positive-negative).

After creation of pairs, there are two massive electromagnetic systems with spherical coordinates which have two charges (positive-negative), because λ_i

in the Equation (3.17) equals to $\pm(\mathcal{E} \pm i\mathcal{B})$ in the Equation (3.8) that makes the two direction $\pm\mathcal{E}$ which causes the two charges $\pm e$, more precisely, we will get on the positive pulse of wave phase when $\omega t \in]-\}\pi/2,+\pi/2[$, and we will get on the negative pulse of wave phase when $\omega t \in]+\}\pi/2,-\pi/2[$, **Figure 1**.

The second note:

We must study the RS vector of electron (positron) $\mathbb{F}(r,\theta,\phi)$, then we will expand the study to discover what this matrix can offer in the new dimensions of the studied fermion (electron-positron) which was belonging to the creator photon, so we will consider the creator photon at the creation point is circularly polarized, and when it becomes Dirac pairs must be studied in the helicity coordinates, then we will study the RS vector propagating in Z direction in closed path by the helicity basis vectors (e^{+1},e^{-1}) instead of the spherical coordinates ($e_\theta + e_\varphi$) for knowing its importance, the relation between the helical and the spherical for one vectors is

$$F = f_r e_r + f_\theta e_\theta + f_\varphi e_\varphi = f_{+1}e^{+1} + f_0 e^0 + f_{-1}e^{-1} \tag{4.1}$$

$$F_{+1} = -\frac{1}{\sqrt{2}}\left(f_\theta + if_\varphi\right), F_0 = f_r, F_{-1} = \frac{1}{\sqrt{2}}\left(f_\theta - if_\varphi\right) \tag{4.2}$$

Finally, the equation of one (RS) vector spherical harmonic.

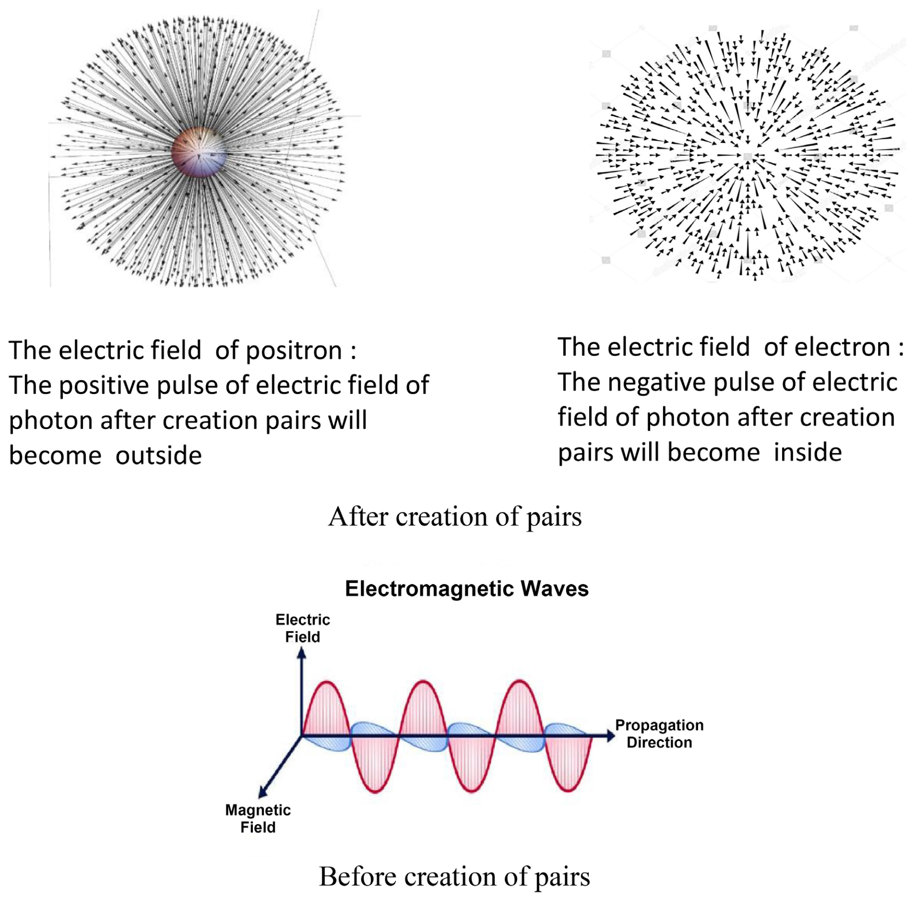

Figure 1. The origin of charge.

$$\boldsymbol{F}_{\pm}(\theta,\varphi)=\begin{pmatrix}F_{+1}(\mp ik)\\ F_0\\ F_{-1}(\pm ik)\end{pmatrix}=\mathrm{e}^{-im\varphi}\begin{pmatrix}P_{m,1}^{n}(\cos\theta)\left(\dfrac{\mathrm{d}}{\mathrm{d}r}+\dfrac{1}{r}\mp ik\right)\\ P_{m,0}^{n}(\cos\theta)\sqrt{\dfrac{n(n+1)}{2}}\dfrac{1}{r}\\ P_{m,-1}^{n}(\cos\theta)\left(\dfrac{\mathrm{d}}{\mathrm{d}r}+\dfrac{1}{r}\pm ik\right)\end{pmatrix}z_n(kr) \tag{4.3}$$

where $z_n(kr)$ Bessel spherical functions, and the functions $\left\{\mathrm{e}^{-im\varphi}P_{m,l}^{n}(\cos\theta)\right\}$ form an orthogonal basis for components of an arbitrary square integrable vector function defined on the sphere, (remind Legendre polynomials $\mathrm{e}^{-im\varphi}P_{m,l}^{n}(\cos\theta)$). F_{+1} is the component of the right-hand circular polarized field propagating in Z direction, F_{-1} the right-hand circular polarized field propagating in −Z direction, and F_0 the radial component. The functions can be interpreted as generalized spherical harmonics.

Then the equation of all circular polarized RS vector of electron (on the sphere r_0) Will be as the following [7]

$$\boldsymbol{F}_{+}=2\frac{\mathrm{e}^{-ikr}}{r}\sum_{n=1}^{\infty}\sum_{m=-n}^{m=+n}i^n\begin{pmatrix}P_{m,1}^{n}(\cos\theta)\\ 0\\ 0\end{pmatrix} \tag{4.4}$$

$$\boldsymbol{F}_{-}=2\frac{\mathrm{e}^{-ikr}}{r}\sum_{n=1}^{\infty}\sum_{m=-n}^{m=+n}i^n\begin{pmatrix}0\\ 0\\ P_{m,-1}^{n}(\cos\theta)\end{pmatrix} \tag{4.5}$$

The last equation expresses the rotation of all the RS vectors of electron, that acts rotation of electron around itself which makes the spin as in **Figure 2**, **Figure 3**. The instantaneous change of the coordinates of the creator photon at

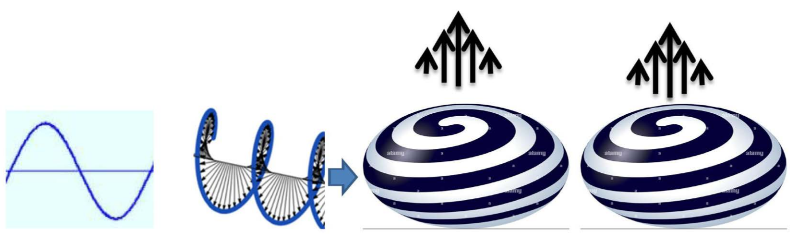

Figure 2. The photon (s = 1 Up) at the creation point becomes right-handed polarized and then ready to be spherically distributed giving two Dirac particles their spin are +1/2.

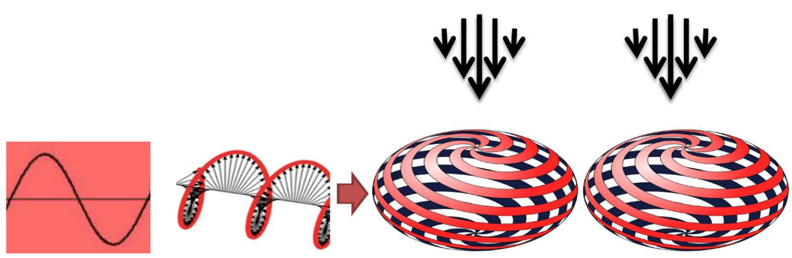

Figure 3. The photon (s = −1 Down) at the creation point becomes Left-handed polarized and then ready to be spherically distributed giving two Dirac particles their spin are −1/2.

the point of creation from Cartesian to spherical starts with changing of the polarization of the photon to circular, the right-handed polarization of the photon (spin: Up) gives in the new spherical coordinates two particles their spin +1/2, and the left polarization of the photon (s:Down) gives in the new spherical coordinates two particles their spin −1/2.

5. Conclusions

1) The task of the wave function is only to perform all measurements of the electron (positron), while determining the structure of the electron (positron) is only done by an equation of motion that contains the electromagnetic potential of the electron.

2) It is now understood how the coordinates can affect the distribution of the electromagnetic potential of the particle so that the studied particle has properties related to this distribution, and through a vector (RS) we are able to study the effect of changing the electromagnetic fields with space-time in the appearance of the property that allows the electron to have its properties (charge-spin) and allows to interact with the Higgs particle to show mass.

3) We have found that the source of gravity is the change in the values of the electric field on the surface of the elementary mass particle (spatial change) and the pulsation of the electric field at every point (temporal change), which makes it give a field similar to dark matter, as it could essentially be the gravitational field acting as a dark matter field in systems. Certain [8] likewise, dark matter can be considered a modified gravitational field, where the modified theory of gravity F(R, T) is adopted to explain the effects of dark matter in spiral galaxies, as inferred from flat rotation curves [9].

4) The Equation (3.17) in this shape (5.1):

$$\Box\mathbb{F} = m^2\mathbb{F}$$

$$\Box\mathbb{F}\left(r,\theta,\phi,t\right)\psi_i = m^2\lambda_i\psi_i$$

can study all real particles (fermions-bosons), when we study the boson, we will choose an suitable coordinate without wave function, and when we study the fermions, we preserve the wave function according to exclusively spherical coordinates, it is **SHAAM Equation**.

5) After we knew the origin of mass and charge and spin, we can now put the electromagnetic model of electron (positron).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Thide, B. (2000) Electromagnetic Field Theory. Bo Thidé, Swedish Institute of Space Physics and Department of Astronomy and Space Physics, Uppsala University, Swe-

den, 184 p.

[2] Ryder, L.H. (1996) Quantum Field Theory, University of Kent at Canterbury, 2nd Section, (Reprinted 1997, 1998, 1999, 2001). Published by the Press Syndicate of The University of Cambridge, The Edinburgh Building, Cambridge.

[3] https://www.wolframalpha.com

[4] Schulten, K. (2000) Notes on Quantum Mechanics. Department of Physics and Beckman Institute, University of Illinois at Urbana-Champaign, Urbana, IL, 328-329. https://www.ks.uiuc.edu/Services/Class/PHYS480/qm_PDF/QM_Book.pdf

[5] Silberstein, L. (1907) Elektromagnetische Grundgleichungen in bivektorieller Behandlung. *Annalen der Physik*, **327**, 579-586. https://onlinelibrary.wiley.com/doi/10.1002/andp.19073270313 https://doi.org/10.1002/andp.19073270313

[6] Hacyan, S. (2018) The Riemann-Silberstein Vector in the Dirac Algebra. *Revista Mexicana de Física*, **65**, 65-68. arXiv:1811.09684v1 [physics.gen-ph] https://doi.org/10.31349/RevMexFis.65.65

[7] Belkovich, I.V. and Kogan, B.L. (2016) Utilization of Riemann Silberstein Vectors in Electromagnetics. *Progress in Electromagnetics Research B*, **69**, 103-116. https://www.jpier.org/issues/reader.html?pid=16051809 https://doi.org/10.2528/PIERB16051809

[8] Zaregonbadi, R., Farhoudi, M. and Riazi, N. (2016) Dark Matter from f(R, T) Gravity. *Physical Review D*, **94**, 084052. arXiv:1608.00469v2 [gr-qc] https://doi.org/10.1103/PhysRevD.94.084052

[9] Harko, T., Lobo, F.S.N., Nojiri, S. and Odintsov, S.D. (2011) f(R,T) Gravity. *Physical Review D*, **84**, 024020. https://doi.org/10.1103/PhysRevD.84.024020