Generativity of Self-Organizing Processes and Their Correlative Description in Terms of a Formal Language of Meta-Ordinal Generative Nature, in the Light of the Maximum Ordinality Principle and the Explicit Solution to the "Three-Body Problem"

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Abstract

The main objective of this paper is to demonstrate that the internal processes of Self-Organizing Systems represent a unique and singular process, characterized by their specific generativity. This process can be modeled using the Maximum Ordinality Principle and its associated formal language, known as the "Incipient" Differential Calculus (IDC).

Keywords

Maximum Ordinality Principle, Solution to the "Three-Body Problem", Generativity of Self-Organizing Processes, Formal Language of Ordinal Generativity, Formal Language of Meta-Ordinal Generativity

1. Introduction

The fundamental motivation of this paper is to show that the Description of a Self-Organizing System, when done on the basis on the Maximum Ordinality Principle, does not require the recourse to *any form* of "Forces", but it only requires the Appropriate Definition of its *Internal Genarativity*, described in terms of an "Incipient" Derivative, characterized by an *appropriate* Ordinality, specific for each considered System. Consequently, the present paper, by starting from the description of the *internal* Processes of some Biological Systems in terms of Incipient Differential Calculus (IDC), will focus, in particular, on the *possibility*

of the Adoption (understood as a "Suggestion", or better, as an "Invitation") of a correlative *Formal Language of Meta-Ordinal Generativity* {F,L, M-O, G}, which is the "Reflex", and at the same time the "Cypher", of a Different Gnoseological Perspective with respect to the Traditional One, characterized by a "Different" Modality of "*Thinking*", "*Decision Making*", and "*Acting*" (in Symbols $\left\{ \widetilde{Th}, \widetilde{DM}, \tilde{A} \right\}$), because directly Respectful and, at the same time, *Adherent* to that "*Irreducible Emerging Quality*" of Self-Organizing Systems, of *any* Nature they might be.

In fact, as already shown in References [1] [2] [3] [4] [5], Self-Organizing Systems can be described on the basis of the introduction of the Concept of the "*Incipient*" Derivative, characterized by a specific Ordinality, which allows us to formulate the Maximum Ordinality Principle [1], whose the First Fundamental Equation can be formulated as follows, which reproduces Equations (4.1) and (4.1.1) of **Appendix A** (par. 4.1), respectively:

$$\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}} \{\tilde{r}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{1.1}$$

$$\left(\tilde{m}/\tilde{n}\right) \rightarrow Max \rightarrow \left\{\tilde{2}/\tilde{2}\right\} \uparrow \left\{\tilde{N}/\tilde{N}\right\} \tag{1.1.1}$$

where $\{\tilde{r}\}$ is the *Relational Space* of the System under consideration (see paragraph 5.1), $\{\tilde{m}/\tilde{n}\} = \left\{k,\left(\tilde{m}/\tilde{n}\right)\right\}$ represents its corresponding Ordinality, while $\left(\tilde{m}/\tilde{n}\right)$ indicates the *Ordinal Genetic Relationships* characterized by $\tilde{m}$ Ordinal Co-productions and $\tilde{n}$ Ordinal Interactions, and *the Maximum Ordinality is reached when* $\left(\tilde{m}/\tilde{n}\right)$ equals $\left\{\tilde{2}/\tilde{2}\right\} \uparrow \left\{\tilde{N}/\tilde{N}\right\}$ (as indicated in Equation (1.1.1)).

Consequently, as we already know, the Maximum Ordinality Principle (M.O.P.) is apt at describing both *non-Living* Systems, *Living* Systems and *Conscious* Systems, by "opening", at the same time, a New Perspective in Modern Science [2].

This is made possible because the Ordinality $\left(\tilde{m}/\tilde{n}\right)$ of the "Incipient" derivative $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}}$ can *each time* be specified in accordance to the Nature of the (*non-Living, Living, Conscious*) System Analyzed.

In this way it is possible to give a description in Adherence to that "Emerging Quality" of Self-Organizing Systems, understood as their specific "*Irreducible*" *Property*.

On this respect, we would here suggest the Reader to *first read* the Appendices of this Paper, because in this case *they play a Fundamental "Role"*. In fact, they will enable us to show the very *Fundamental Property* of the Self-Organizing Systems, that is: the aforementioned *Internal Generativity*, characterized by its specific *Property of Diffusivity* to all the order of Derivatives (up to the order *n* − 1), by generating, in this way, the (so called) "*Harmony Relatioships*" of the Systems, which clearly describe the Self-Organization of the same as a *Unique and Sole* Process, as specifically represented by the *Ordinal Roots of Unity*.

The aforementioned Basic Concepts, in fact, widely and clearly illustrated in **Appendix A**, **Appendix A1**, **Appendix A2**, although here reproposed in the original form of Appendices of Ref. [3], (with its particular References [6] [7] [8] [9]),

they in reality represent the "*Central Part*" *of this Paper*, given the *prevailing Formal Approach* of the same. Consequently, the Reader is "*warmly*" invited to read them before continuing the reading of the Paper. In this way, the Reader can easily be aware of not only the *formal reasons* on the basis of which we succeeded in finding the Explicit Solution of a Self-Organizing System made up of *any* number of bodies, but also, and at the same time, by taking into account that the same Appendices are the Appendices of the Paper dealing with "Three-Drug Therapies" [4], they result as being valid in the case of *any* Self-Organizing System, of *any Nature* it might be (see [3] and [4]).

So that, after such *a due Premise*, we can start with considering the *Central Aspect* of this Paper, which is *essentially of Formal Nature*.

2. The Formal Language of *Ordinal* and *Meta-Ordinal* Generativity: $\left\{\tilde{F},\tilde{L},\tilde{O},\tilde{G}\right\}$) and $\left\{\tilde{F},\tilde{L},\tilde{M}\tilde{O},\tilde{G}\right\}$, Respectifully

The aforementioned Formal Aspects which are clearly illustrated in **Appendix A**, **Appendix A1**, **Appendix A2**, (and at this point the Reader is supposed to have already read the Appendices), and in particular Equation (3.2.9) of **Appendix A**, show that, from a *General Point of View*, it is possible to consider a *Formal Language* based on an *Ordinal Generativity* (in symbols $\left\{\tilde{F},\tilde{L},\tilde{O},\tilde{G}\right\}$) which can be, *each time*, characterized by a *Specific Generativity*, of *Ordinal Nature*, according to the *Field of Analysis* considered, with reference to its specific "Phenomenological" characteristics, when dealing with *non-Living* Systems, *Living Systems*, *Conscious* Systems.

However, it is also possible to consider *a more General Version* of the same Language, in particular, when the Specific Generativity of the System can be considered as being *Over-Structured* with respect to the Basic Version of Equation (3.2.9) of **Appendix A**. This is the case, for example, of Biological Systems such as Proteins, which are made of several Amino Acids, and which, in turn, are made up of Peptides.

In such a case, on the Basis of the same M.O.P., if the considered Protein is supposed formed by n Amino Acids, and it is described from a *Forma*l Ordinal Point of view, the same Protein can be represented (as it is clearly shown in **Appendix A**), at a preliminary *first step*, by a Matrioska of Ordinality n, which is thus representing a System made up of n elements, each one representing an Amino Acid.

In turn, *any* Amino Acid, made up of m Peptides, can analogously be represented, from a *Forma*l Ordinal Point of view, by a Sub-Matrioska of ordinality m, that is made up of m elements, each one representing a specific Amino Acid. In such case the considered Protein, if understood at a *Second Level* of description, can be represented by a Matrioska made up of m Sub-Matrioskas.

Consequently, under these conditions, we can say that the considered Biological System is characterized by an *Over*-Ordinality (or *Meta-Ordinality*) that can

be represented as follows

$$\left\{ \tilde{q} = \left\{ \tilde{n}/\tilde{n} \right\} \uparrow \left\{ \tilde{m}/\tilde{m} \right\} = \left\{ k, \left\{ \tilde{n}/\tilde{n} \right\} \uparrow \left\{ \tilde{m}/\tilde{m} \right\} \right\} \right\}. \tag{2.1}$$

Analogously we can proceed if we want to consider, and thus model, *each* Amino Acid of the considered Protein as being structured in terms of Peptides.

In this case, for more generality, *the Ordinality of the Specific Generativity* of the System can be written as

$$\left\{ \tilde{q} = \left\{ \tilde{n}/\tilde{n} \right\} \uparrow \left\{ \tilde{m}/\tilde{m} \right\} \uparrow \left\{ \tilde{p}/\tilde{p} \right\} = \left\{ k, \left\{ \tilde{n}/\tilde{n} \right\} \uparrow \left\{ \tilde{m}/\tilde{m} \right\} \uparrow \left\{ \tilde{p}/\tilde{p} \right\} \right\} \right\} \tag{2.2}$$

in which *p* represents the number of Peptides that form each Amino Acid.

In such a more General Case, we can speak of a *Formal Language of a Meta-Ordinal Generativity*, in symbols {*F, L, MO, G*}, or better, in order *to clearly point out* the Concept of *Meta*-Ordianality, we can also introduce the "tilde" Notation:

$$\left\{ \tilde{F}, \tilde{L}, \tilde{M}\tilde{O}, \tilde{G} \right\}. \tag{2.3}$$

As an example of such a *Formal Language of Meta-Ordinal Generativity* we can consider the case of its Adoption in Biology, in particular, with reference to those special Self-Organizing Systems represented by Proteins.

3. Adoption of the $\left\{ \tilde{F}, \tilde{L}, \tilde{M}\tilde{O}, \tilde{G} \right\}$ in the Analysis of Proteins as Self-Organizing Systems

A preliminary Analysis of the Protein Folding and Protein-Protein Interaction (PPI) has already been done in Ref. [5], and here it is worth recalling the basic *innovative* Aspects concerning their description, by starting from the case of Protein Folding.

In such a paper, in fact, we showed the possibility of dealing with the Protein Folding by considering, as an Ostensive Example, the case of the diabetic therapy.

It is well-known in fact that human Insulin has a reduced affinity with Blood Albumin, so that the subcutaneously injected Insulin cannot efficiently be conveyed by Blood Albumin in the various parts of the body. The therapy then consists in adopting *a modified form of Insulin*, which presents a higher affinity with Blood Albumin. The modified form of Insulin usually adopted is *Insulin Detemir*, also termed as *Levemir*.

In such an Ostensive Example we considered the Ordinal *three-dimensional structure* of human Insulin (51 Amino Acids), obtained on the Basis of the M.O.P. and its associated *EQS Simulator*, that was run on a simple PC (109 Flops), in less than 1 s.

In turn, we analogously got the three-dimensional structure of Blood Albumin, made up of 585 Amino Acids. Its spatial configuration was also obtained by means of the same EQS Simulator, run on the same PC, in a computation time

of about 1 s.

As in the previous case, such a 3D structure could easily be compared with the corresponding spatial configurations available both in Literature and in Protein Data Banks.

At this stage, by considering the interaction process between the two afore-mentioned Proteins, we obtained that: 1) human Insulin has a very reduced affinity with Albumin (about 1%); 2) at the same time, our analysis explained why human Albumin is usually modified in the form of Levemir, in order to achieve a higher affinity.

Levemir insulin, in fact, differs from human insulin in that the Amino Acid in position B30 is omitted, and a C14 fatty acid chain, termed as *Myristic Acid*, is attached to the Amino Acid B29.

The results consequently showed that the modified Protein has an *inverse chirality* with respect to its primary form of Insulin. This aspect generally favors the Interaction Process. Consequently, the Interaction Process between Levemir and Albumin (obtained by means of the same EQS Simulator in less than 2 s) gave origin to a *Final Compound* characterized by a higher "excess" of *virtual work*.

This result clearly showed that such a modified form of human Insulin presents an affinity of about 20% with respect to blood Albumin. A value that allows Levemir to be conveyed by Albumin, without preventing, however, its subsequent release in the various parts of the body.

4. Informatics Advances of the Description Adopted

The improvements here considered are directly referable to the *formal properties* that are intrinsic to the *mathematical models adopted*. In fact, *any* System modeled on the basis of the M.O.P., as is well-known, always presents *an Explicit Solution* in terms of Incipient Differential Calculus.

This means that the methodology proposed has the capacity of predicting the 3D structure of the Resulting compound essentially because the latter is understood as a Self-Organizing System of *Ordinal Nature*, and thus it is intrinsically "irreducible" to functional relationships between its parts.

This correlatively also means: 1) a reduced number of computations; 2) a reduced need of High Performance Computing (HPC); 3) a reduced incidence of special numerical methods to be adopted to get the corresponding solution.

What's more, the Explicit Solutions so obtained can also be termed as being "*Emerging Solutions*", because they always show an *Ordinal Information Content* which is much higher than the corresponding content of the initial formulation of the problem. This is because the M.O.P. is specifically finalized to describe "Self-Organizing" Systems according to a *Holistic Approach*, in which, as is well-known, "*the whole is much more than the sum of its parts*".

5. Adoption of $\left\{ \tilde{F}, \tilde{L}, \widetilde{MO}, \tilde{G} \right\}$ in the Analysis of Biological Systems, Usually Dealt with through Informatics Methods

In this paper we do not want to show some *specific* Ostensive Examples of Adoption of the aforementioned Formal Language to some Biological Problems, such as, for instance, Protein Folding, but we only want to point out its possibility of Adoption, *in Principle*, of that *Formal Language*, in the *majority* of Biological Problems *usually dealt with through informatics methods*.

This is because the Paper is *mainly and essentially* finalized to show *the Nature* of the Advantages of the afore-mentioned Language *by itself*, and *not only* from an Informatics Point of View.

This is why we will *only* show its specific *Modalities of Adoption* in *two specific cases* concerning Proteins, when understood as Self-Organizing Systems of *Generative Nature*:

1) Firstly, we will show that *any* Protein Folding, from a *Formal Point* of View, is the "*Exit*" of a *Unique and Sole* Self-Organizing Process, characterized by a Unique and Sole *Specific Ordinal Generativity*;

2) Secondly, that the Process of Protein-Protein Inter-Action (PPI) can *always* be solved, *in analogy*, on the basis of the same *Formal Language*, when the aforementioned Inter-Action is understood as being a Self-Organizing System which leads to a Final System characterized by an Over (or *Meta*)-Ordinality.

So that we will limit our exposition to the *sole correlative Formal Aspects* of the two mentioned Biological Processes.

5.1. Description of the Folding of a Protein in Terms of $\left\{ \tilde{F}, \tilde{L}, \tilde{M}\tilde{O}, \tilde{G} \right\}$

In the case of a Protein we can distinguish *Three Levels* of Description:

5.1.1. Protein Folding of a Protein Made up of *n* Amino Acids

In such a case the Ordinality $\tilde{q}$ will be then represented as

$$\tilde{q} = \left\{ k \uparrow \tilde{2}/\tilde{2}; (\tilde{n}/\tilde{n}) \right\} \tag{3.1.1}$$

in order to have, in such a way, an *adherent representation* of the *Internal Generativity* of the System under consideration, while the Initial and Boundary Conditions of the Folding Process will be given according to the assignation modality as specified at par. 5.4 of **Appendix A**.

In this way, the Protein Folding of the considered Protein will result (as there described) as being the "Exit" of *One Sole and Unique Ordinal Process* with reference to the couple "12" of Amino Acids, which is assumed as basic reference of the Process.

5.1.2. Protein Folding of a Protein Made up of *n* Amino Acids, Each One Made up of *m* Peptides

In this case the Ordinality $\tilde{q}$ will be then represented as

$$\tilde{q} = \left\{ k \uparrow \tilde{2}/\tilde{2}; (\tilde{n}/\tilde{n}) \right\} \uparrow \left\{ l \uparrow \tilde{2}/\tilde{2}; (\tilde{m}/\tilde{m}) \right\} \tag{3.1.2}$$

in order to have, in such a way, the most *adherent representation* of the *Internal*

Generativity of the System under consideration, while the Initial and Boundary Conditions of the Folding process will be given, in analogy to the previous case, according to the *assignation modality* as specified at par. 5.4 of **Appendix A**.

Also in this case, the Protein Folding of the considered Protein will result as being the "*Exit*" *of One Sole and Unique Meta-Ordinal Process* with reference to the couple "12" of Peptides assumed as basic reference of the Amino Acid, in turn assumed as Reference the Self-Organizing Process.

5.1.3. Protein Folding of a Protein Made up of *n* Amino Acids, Each One Made up of *m* Peptides, Each One in Turn Made up of *p* Constitutive Elements

In such a case the Ordinality $\tilde{q}$ will be then represented as

$$\tilde{q} = \left\{ k \uparrow \tilde{2}/\tilde{2}; (\tilde{n}/\tilde{n}) \right\} \uparrow \left\{ l \uparrow \tilde{2}/\tilde{2}; (\tilde{m}/\tilde{m}) \right\} \uparrow \left\{ r \uparrow \tilde{2}/\tilde{2}; (\tilde{p}/\tilde{p}) \right\} \qquad (3.1.3)$$

where *r* is an integer number. Even more clearly, by indicating with *N*1 the Number of Amino Acids, with *N*2 the Number of Peptides and *N*3 the Number of the component of each Peptide, the *Meta-Ordinality* of the *Generative Process* can be represented as

$$\tilde{q} = \left\{ k \uparrow \tilde{2}/\tilde{2}; \left(\widetilde{N1}/\widetilde{N1}\right) \right\} \uparrow \left\{ l \uparrow \tilde{2}/\tilde{2}; \left(\widetilde{N2}/\widetilde{N2}\right) \right\} \uparrow \left\{ r \uparrow \tilde{2}/\tilde{2}; \left(\widetilde{N3}/\widetilde{N3}\right) \right\}, \quad (3.1.4)$$

while, still *in analogy* to the previous case, the Initial and Boundary Conditions of the Folding Process will be given according to the *assignation modality* as specified at par. 5.4 of **Appendix A**.

In this way we have that, *even in this case*, the Protein Folding of the considered Protein will result as being the "Exit" of One Sole and *Unique Meta-Ordinal* Process with reference to the couple "12" of the components of a specific Peptide, assumed as basic reference of the Self-Organizing Process.

In all cases whatsoever, the *Meta-Ordinal* Process of Folding will be described, as shown in **Appendix A**, **Appendix A1**, **Appendix A2**, *without any recourse to the "functional" and "cardinal" concepts* characteristic of the Traditional Perspective, such as, for instance, the concepts of Energy, Exergy, Entropy, free Energy, etc., while *the Evolution* of the Self-Organizing *Meta-Ordinal* Process will be happen in the "*Proper Space*" and "*Proper Time*" of the Protein, because they are *specific of the* considered Protein, *and it will not* be an "evolution" in an "*absolute*" *space* and in "*absolute*" *time* as in the case of the Traditional Perspective.

5.2. Description of the Protein-Protein Inter-Action (PPI) in Terms of $\left\{ \tilde{F}, \tilde{L}, \tilde{M}\tilde{O}, \tilde{G} \right\}$

In analogy to the previous cases, the *Inter-Action Process* can be described at three different Levels of Analysis, corresponding to the three different Levels of description of each Protein.

For the sake of simplicity, we will only consider the *Fist Level* of Analysis in which, the Tow Proteins, if considered as being made up of *N*1 and *N*2 Amino Acids respectively, they will be characterized by a *Specific Generativity* of Ordi-

nality which, according to Equation (3.1.1), can be represented as

$$\tilde{q}1 = \left\{ k1 \uparrow \tilde{2}/\tilde{2}; \left(\widetilde{N1}/\widetilde{N1} \right) \right\} \tag{4.1.1}$$

$$\text{and} \quad \tilde{q}2 = \left\{ k2 \uparrow \tilde{2}/\tilde{2}; \left(\widetilde{N2}/\widetilde{N2} \right) \right\} \tag{4.1.2}.$$

The correlative PPI Process, because of the representation of the Proteins in terms of Exponential Matrioskas (see Equation (5.2.2) of **Appendix A**), will give "Origin" to a *Final Compound* characterized by a *Metal-Ordinal Matrioska*, which will be the "Exit" of a *Feed-Back Process* between the Two Specific Matrioskas of the Two given Generative Proteins, whose Relational Spaces are described as

$$\{\tilde{r}\}_{s1} = e^{\begin{vmatrix} \tilde{\alpha}_{11}(t) & \tilde{\alpha}_{12}(t) & \cdots & \tilde{\alpha}_{1n}(t) \\ \tilde{\alpha}_{21}(t) & \tilde{\alpha}_{22}(t) & \cdots & \tilde{\alpha}_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\alpha}_{n1}(t) & \tilde{\alpha}_{n2}(t) & \cdots & \tilde{\alpha}_{nn}(t) \end{vmatrix}} 1 \tag{4.1.3}$$

$$\{\tilde{r}\}_{s2} = e^{\begin{vmatrix} \tilde{\alpha}_{11}(t) & \tilde{\alpha}_{12}(t) & \cdots & \tilde{\alpha}_{1n}(t) \\ \tilde{\alpha}_{21}(t) & \tilde{\alpha}_{22}(t) & \cdots & \tilde{\alpha}_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\alpha}_{n1}(t) & \tilde{\alpha}_{n2}(t) & \cdots & \tilde{\alpha}_{nn}(t) \end{vmatrix}} 2 \tag{4.1.4}.$$

Consequently, the "*Exit*" of the Process will be *formally* represented as

$$\{\tilde{r}\}_{s1} \left\{ \circledR \uparrow \tilde{2}/\tilde{2} \right\} \{\tilde{r}\}_{s2} \tag{4.1.5},$$

where the Symbol $\left\{ \circledR \uparrow \tilde{2}/\tilde{2} \right\}$ represents a *Feed-Back Process* of Ordinality $\tilde{2}/\tilde{2}$, which can be "interpreted" as representing the *Generation* of a "*Unique System*" of "*Two Brothers*" that operate *in Syntony*, such as in a *real and proper* "*Singing Duet*".

Such a Feed-Back Process, however, because of the given exponential representation of the Two Inter-Acting Proteins, it will become more properly represented as a Feed-Back Process *between the Two respective Matrioskas* in Equations (4.1.3) and (4.1.4) respectively.

Obviously, as in the case of the Folding of a single Protein, the *Meta-Ordinal* PPI Process that leads to the *Final System* will be described *without any recourse to the "functional" and "cardinal" concepts* characteristic of the Traditional Perspective, such as, as already mentioned, the concepts of Energy, Exergy, Entropy, free Energy, etc., while the Evolution of the Self-Organizing *Meta-Ordinal* PPI Process will happen in its "*Proper Space*" and its "*Proper Time*", *and not* in an "*absolute*" space and in an "*absolute*" time as supposed in the Traditional Perspective [33] [34]

6. Over-Conclusion. Adoption of the $\left\{ \tilde{F}, \tilde{L}, \tilde{M}\tilde{O}, \tilde{G} \right\}$ in Other Fields of Analysis

The *Formal Language of Meta-Ordinal Generativity* $\left\{ \tilde{F}, \tilde{L}, \tilde{M}\tilde{O}, \tilde{G} \right\}$ can evidently be adopted *in Biology* not only with reference to Self-Organizing Systems such as Proteins and their PPI Processes, but also in several *other* Sectors which

are *specific* of Biology.

However, the previous Ostensive Examples show the possibility that the $\{\tilde{F}, \tilde{L}, \tilde{MO}, \tilde{G}\}$ can *also* be adopted in *Medicine*. For instance, with reference to the DNA, to its correct sequence (or not) of Chromosomes, in order to find New Specific Therapies, because they will be now based on a Meta-Ordinal Modality of "*Thinking*", "*Decision Making*" an "*Acting*" (in Symbols $\{\widetilde{Th}, \widetilde{DM}, \tilde{A}\}$), which is represented, as a "*Reflex*" and at the same time as a "*Cypher*", by the $\{\tilde{F}, \tilde{L}, \tilde{MO}, \tilde{G}\}$, and which is *thus* capable to describe the considered Field of Analysis in *Terms* of that "*Irreducible*" Quality which is characteristic of the same. In this sense the $\{\tilde{F}, \tilde{L}, \tilde{MO}, \tilde{G}\}$ can also adopt to studying, for instance, the *genetic defects*, in order to find *New Therapies* for *Genetic Diseases*, "*Rare*" *Diseases* and so on.

Nonetheless, the $\{\tilde{F}, \tilde{L}, \tilde{MO}, \tilde{G}\}$ can also be adopted in *Literature*, by considering that, from a "*Phenomenological*" Point of View, any given "Article", or a "Chapter of a Book", or one "Entire Book", can be considered made up of *specific* "Basic Elements" such as "*Propositions*", which, in turn are made up of "*Terms*", often *united* in one (o more) "*Definitions*" of *Meta-Ordinal* Generativity.

The "Propositions", in turn, can be considered as being forming "*Syllogisms*" of over and over Increasing *Meta*-Ordinality (such as *First Level* Syllogisms, *Second Level* Syllogisms, and so on).

Evidently this can also be done, for instance, in *Philosophy*, in order to "discover", and thus "*Qualify*", the *Meta-Ordinality* Level of the *Perspective in Terms of* $\{\widetilde{Th}, \widetilde{DM}, \tilde{A}\}$), as proposed by a *given Philosopher*. A Perspective that can even be "compared" with the *Meta-Ordinality* Level of other Perspectives, or can even "*guide*" to recognize their Reciprocal *Similarities* or "*Mutual Dependences*".

Furthermore, the $\{\tilde{F}, \tilde{L}, \tilde{MO}, \tilde{G}\}$ can also be adopted, for instance, in *Music*, where the "*Fenomenological*" "constitutive Elements" are represented by the well-known "notes", the "musical chords", usually made up of "Three Notes", and thus potentially "modeled" and "analyzed" in analogy to the "Three Body Problem". But it is possible to consider some characteristic "refrains", or the "entire song" or even an "entire symphony".

It can evidently also be adopted in *Theology*, in order to recognize the Specific *Level of Meta-Ordinality* of the *Tests of the Holy Scriptures*, such as, the *Books of the Bible*, in order to possibly "reach" a more "*accurate*" "*Reading*", for example, of "*The Gospels*".

A *preliminary Ostensive Example* of this latter case can easily be found in the Author's Web-Site https://www.ordinality.it/, in the Section devoted to "*I Vangeli in Logica Generativa*" (in English, "*The Gospels in Generative Logic*"), where the Two well-known sentences pronounced by Jesus during the Last Supper: "This is my Body", "This is my Blood", are interpreted *in the Form of a Syllogism* of the *First* Metal-Ordinal Generative Level of Analysis. But evidently, on the Basis of the previous Exposition, their correlative Analysis *does not end up* at

this First Level.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A. The Maximum Ordinality Principle: From the "Incipient" Derivative to EQS Simulator

This Appendix, articulated in three parts, presents a *synthesis* of the developments concerning the Maximum Ordinality Principle with reference to a Self-Organizing Systems made up of *an arbitrary number* of Bodies.

All the various developments, according to what is indicated in the title, have been illustrated in the various Papers presented at the Biennial Emergy Conferences (University of Florida) from 1999 to 2020.

In addition, the present Appendix will also present two Aspects of particular Relevance:

i) The Process of Genesis of *The Harmony Relationships* and

ii) The Process of Genesis of *The Ordinal Roots of Unity*.

The Maximum Ordinality Principle [1], in fact, is nothing but the reformulation of the Maximum Em-Power Principle [10] [11] [12], given however in a more general form by means of a new concept of derivative, *the* "*incipient*" *derivative*, whose mathematical definition has already been presented in [13]-[19].

In this way both Emergy and Transformity are replaced by the concept of Ordinality. This is the reason why the Principle was renamed as the Maximum Ordinality Principle.

Consequently, on the basis of the Mathematical Formulation of the Maximum Ordinality Principle [1] and, in particular, its adoption as "One Sole Reference Principle" [20], we can now present, in more details, the radically New Perspective that such a Principle offers to Modern Science. That is: "*Every System is a Self-Organizing System*".

In order to give a clear presentation of the fundamental differences between such a New Perspective with respect to the Traditional Scientific Approach, this Appendix will start from the consideration of a synoptic picture of the basic characteristics of the two mentioned Scientific Approaches (see Table 1), successively analyzed and compared, in more detail, in the context of the Appendix.

1. Fundamental Characteristics of the Two Scientific Approaches

In this respect, it is worth starting from recalling that Self-Organizing Systems and their "emerging properties" began to be studied by L. Boltzmann toward the end of XIX century [21]. Several other Authors (e.g. A. Lotka [22] [23] [24]) dealt with such a theme. However, Self-Organizing Systems received the most significant contribution by H.T. Odum (from 1955 on), with the genial introduction of a more appropriate formal language.

The consequential faithful developments of Odum's approach have led us to the formulation of a Unique General Principle, the Maximum Ordinality Principle (M.O.P.), which is able to describe, by itself, the behavior of any given System as a Self-Organizing System: both "*non-living*" Systems, "*living*" Systems and "*thinking*" Systems too (e.g. Human Systems).

Such a conclusion then results as being deeply different from that of Modern Science, which, from Newton on, is persistently orientated at describing any known system as it were a "mechanism".

Table 1. Synoptic comparison between the basic presuppositions of the two *differential Formal Languages* and their main corresponding fundamental characteristics.

Basic Presuppositions	"*Emerging Quality*" of Self-Organizing Systems
1) causality principle (efficient causality)	1') Generative Causality
2) classical logic (necessary logic)	2') Adherent Logic (Emerging Conclusions)
3) functional relationships	3') Ordinal Relationships

Development of an appropriate Language

d/dt is the corresponding formal translation
$f(t)$ represents a *functional relationship*

- L. Boltzmann, A. Lotka
- H. T. Odum: <u>Emergy Algebra</u> and <u>M. Em-P. P.</u>
- Further developments in transient conditions
- Introduction of the "Incipient" derivative $\tilde{d}/\tilde{d}t$

- Thermodynamic Principles (1st, 2nd, 3rd)
- Physical Laws (specific for each Discipline)
<u>Every System is a "Mechanism</u>

$$\text{Hypotheses} \downarrow \text{Mathematical Formalization} \downarrow \text{Conclusions} \downarrow \text{Confirmation by experimental results}$$

The Maximum Ordinality Principle
- is applicable to any Field of analysis: non-living Systems, living Systems, "thinking" Systems (e.g. Human Systems)
- at any space-time scale and in variable conditions
- it also offers a more appropriate description of any given System and its surrounding habitat
<u>Every System is a "Self-Organizing System"</u>

The present Appendix, after having synthetically recalled the formulation of the M.O.P. and after having pointed out its corresponding descriptive advantages, will focus on the intrinsic new perspective offered by the M.O.P. especially *in Thinking, Decision Making and Acting*, with respect to the Traditional Approach. In particular, with reference to any form of relationship between Man and his surrounding environment.

In particular, and with reference to this fundamental aspect, the basic differences between the two afore-mentioned perspectives will be brought out by comparing, on the one hand, "*side effects*" (related to the Traditional Approach) and, on the other hand, the "*Emerging Exits*" (specifically pertaining to the New Approach).

Let us then consider first the Traditional Approach that characterizes Modern Science.

1.1. The Traditional Scientific Approach

Modern Science is characterized by a persistent and progressively ascendancy toward ever more general Physical Laws and Principles.

However, before any formulation of a single hypothesis or a physical theory, Modern Science (let us say, from Newton on) adopts three fundamental *pre-suppositions* (see Table 1): the *causality principle* (also termed as "efficient causality"), *classical logic* (also termed as "necessary logic"), and *functional relationships* (between the various parts of any System analyzed).

On the basis of such fundamental presuppositions, and only after having developed a strictly conform consequential *formal language* (that is the Traditional Differential Calculus (TDC)), Modern Science progressively ascends toward ever

more general Physical Laws and Principles:

i) from Phenomenological Laws (e.g. Kepler's Laws); ii) to Physical Laws specific of each Discipline (e.g. Newton's Laws, Maxwell's Equations, etc.); iii) up to the three well-known Thermodynamic Principles.

Such a progressive development has given origin to a hierarchy of a multiplicity of *quantitative* Physical Laws and Principles, in particular as a consequence of the first basic presupposition: the *causality principle*. This Principle, in fact, has led Modern Science to introduce "different causes" in different Disciplines. The Principle of causality, in fact, tends to "sub-divide" the entire phenomenology (at present known) in different "branches", precisely because, on the basis of such a presupposition, it leads Scientists to research for the most "appropriate causes" pertaining each specific set of phenomena each time considered.

In this way, Modern Science persistently propends to show that: "*Every System is a mechanism*".

Such a conclusion, however, although confirmed by experimental results, can be considered as being valid *only* from an *operative* point of view, but not from an *absolute point of view*. This is because "necessary logic" (second basic presupposition) does not admit any form of "*perfect induction*" (see Popper's *Falsification Principle*).

In fact, as synthetically illustrated in Table 1, in the strict contest of "necessary logic":

i) after having formulated a single or more hypotheses (such as in the case of a Theory);

ii) after having formalized them in an appropriate formal language (faithfully conform to the three above-mentioned basic presuppositions);

iii) after having drawn the consequential conclusions

iv) and after having also obtained experimental confirmations of the previous formal conclusions;

v) it is impossible, *in any case whatsoever*, to assert the *uniqueness* of the *inverse* process. That is: it is impossible to show that the hypotheses adopted are the *sole* and *unique* hypotheses capable to explain those experimental results.

This is precisely because of the *absence*, in "*necessary*" logic, of any form of *perfect induction*.

In fact, only in the presence of a *perfect induction* it would be possible to assure the *uniqueness* of the *inverse* process and, thus, to transform the adopted hypotheses into an *absolute* perspective.

This means that Modern Science, precisely because based on *necessary logic*, should always be "open" to recognize that *there always exist* many other *possible* Approaches (in principle *infinitive*) capable to interpret the same experimental results.

At this stage, after having synthetically recalled the basic characteristics of Modern Science, we can analyze in more detail the fundamental properties of the New Perspective, synthetically indicated in parallel (for a better comparison) in the right side of Table 1.

1.2. "Emerging Quality" of Self-Organizing Systems and Adoption of New Mental Categories

After having synthetically recalled the basic characteristics of Modern Science and its corresponding formal language, we can now analyze the fundamental properties of a New Scientific Perspective, which leads to the introduction of a new Formal Language, the *Incipient Differential Calculus* (IDC). As anticipated, the fundamental properties we are referring to are synthetically indicated in parallel (for a better comparison) in the right side of Table 1.

Such a New Scientific Perspective is based on the *phenomenological* "Emerging Quality" of Self-Organizing Systems [25]. This represents the fundamental aspect that leads to the adoption of the corresponding *new mental categories* (shown in Table 1).

The expression "*Emerging Quality of Self-Organizing Systems*" refers to the fact that Self-Organizing Systems always show an unexpected "*excess*" with respect to their phenomenological premises. So that they usually say: "*The Whole is much more than its parts*".

Such an "excess" can be termed as *Quality* (with a capital Q) because it cannot be understood as being a simple "property" of a given phenomenon. This is because it is *never reducible* to its phenomenological premises in terms of traditional mental categories: *efficient causality*, *logical necessity*, *functional relationships*.

This evidently suggests a *radically new* gnosiological perspective, which corresponds to recognize that: "*There are processes, in Nature, which cannot be considered as being pure "mechanisms*".

This also leads, *in adherence*, to the adoption of "*new mental categories*"[1] and, correspondently, to the development of a completely *New Formal Language*, so that the description of Self-Organizing Systems might result as being faithfully conform to their "Emerging Quality".

2. The Progressive Development of an Appropriate Formal Language

L. Boltzmann was the first who attempted at describing Self-Organizing Systems in more appropriate formal terms, by proposing the adoption of a new Thermodynamic Principle: The Principle of Maximum Exergy *Inflow* to the System [21].

Some years later, A. Lotka [22] reformulated such a Principle in the form of: The Principle of Maximum Exergy *Flow through* the System (Lotka, [23] [24]).

Both such attempts were not perfectly successful, because still based on the concept of Exergy, which is a quantity that is strictly pertaining to Classical

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[1]These "*new mental categories*" can no longer be termed as "pre-suppositions", because they are not defined "a priori" (as in the case of Traditional Approach). In fact, they are chosen only "a posteriori", on the basis of the "Emerging Quality" previously recognized. "*Generative Causality*", in fact, refers to the *capacity* of a Self-Organizing System to manifest an "irreducible excess"; "*Adherent Logic*", correspondently, refers to the capacity of our mind to draw "*emerging conclusions*". That is, "conclusions" whose information content is much higher than the information content corresponding to their logical premises, although persistently "adherent" to the latter. "Ordinal Relationships", in turn, refer to particular relationships of *genetic nature*, which will be illustrated in more details later on, with reference to any *Generative Process*.

Thermodynamics. Consequently, it re-proposes, by itself, the concepts of *efficient causality*, *logical necessity*, *functional relationships*.

A really *new formal language* only appears with H. T. Odum, with the genial introduction of Emergy (*Em*), defined as Exergy (*Ex*) by Transformity (*Tr*)

$$Em = Ex \cdot Tr \tag{1}$$

Equation (2.1) clearly shows that Emergy is *still* based on "Exergy". However:

i) *Quality Factor Tr* "*Transforms*" *Ex* into a *new physical quantity*: <u>Emergy</u>;

ii) The latter in fact is not defined in "functional terms", but only by "*assignation Rules*" [26]);

iii) This is precisely because *Tr* is expressed by means of a *non-conservative Algebra*;

iv) Consequently, the output "excess" of the three Fundamental Process in Emergy Analysis (Co-Production, Inter-Action, Feed-Back) is always understood as being "irreducible" to its specific inputs in *mere functional terms*.

This means that <u>Emergy</u> is able to represent the "Emerging Quality" of Self-Organizing *Processes.* Consequently, the general enunciation of the *Maximum Em-Power Principle* (Odum [10] [11] [12]) can *equally be referred*, at a phenomenological level, to the *corresponding maximization tendency* of the "Emerging Quality" on behalf of *Self-Organizing Systems*.

The Maximum Em-Power Principle, however, had not a corresponding and specific formulation under *variable conditions*. On the other hand, such a formulation in *variable conditions* could not be given in terms of the Traditional Differential Calculus, because the traditional derivatives, as a consequence of their conceptual basic presuppositions (see **Table 1**), are not properly apt at representing the "generative" behavior of "Self-Organizing Systems", and consequently they tend to partially "filter" such a "generative" behavior.

This is why, in order to achieve an appropriate mathematical formulation of the Maximum Em-Power Principle, we introduced the concept of "*Incipient Derivative*" ([13] [14] [15] [16] [17]), defined as

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{\tilde{q}} f(t) = \underset{\tilde{\Delta}t:0 \to 0^+}{Lim} \circ \left(\frac{\tilde{\tilde{\delta}}-1}{\tilde{\Delta}t}\right)^{\tilde{q}} \circ f(t) \quad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n}, \tag{2}$$

a definition that will be illustrated in detail in the next paragraph.

However, it is already possible to anticipate that such a definition shows that the "*Incipient Derivative*" is not an "operator", like the traditional derivative (*d*/*dt*), but it could be termed as a "*generator*", because it describes a Process *in its same act of being born* ([18]).

The Mathematical Formulation of the M. Em-P. Principle in terms of *Incipient Derivatives* was preliminarily given in [14], and afterwards, in a more articulated form, in a specific book co-financed by the Center for Environmental Policy ([15]).

During the successive eight years (2002-2010), such a mathematical formulation was adopted in several Disciplines, such as *Classical Mechanics*, *Quantum*

Mechanics, *General Relativity*, *Chemistry*, *Biology*, *Economics* and the corresponding results were reunited *in two books* (titled: "*Lightness of Quality*" [27] and "*Ascendency of Quality*" [28]).

At the end of this wide range of applications, I realized that it was possible to give a more general formulation of the Maximum Em-Power Principle, in the form of the "*Maximum Ordinality Principle*" [1].

For the sake of clearness, the Rational of such a generalization process, articulated in a few logical steps, is recalled in the next sections.

3. The Incipient Derivative of Ordinality $\tilde{q}$

The "Incipient" Derivative of a given Ordinality $\tilde{q}$, whose definition previously introduced is here recalled for the sake of clarity

$$\left(\frac{\tilde{d}}{dt} \right)^{\tilde{q}} f(t) = \underset{\tilde{\Delta}t:0\to 0^{+}}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta} - 1}{\tilde{\Delta}t} \right)^{\tilde{q}} \circ f(t) \quad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n} \tag{3.1}$$

will be illustrated by considering first its *general properties* and, immediately after, its more *specific properties*.

To this purpose it is worth preliminary pointing out that the concept of "Ordinality" refers to *two* "*distinct*" *concepts*, which however are considered as being *one sole entity*, that is as a *Whole*. These are: its "*cardinality*" and its "*ordinal genetic relationships*". This means that the Ordinality $\tilde{q}$, synthetically represented as $\tilde{q} = \tilde{m}/\tilde{n}$ (as in Equations (2.2) and (3.1)), in reality it has to be more properly understood as

$$\{\tilde{m}/\tilde{n}\} = \{k, (\tilde{m}/\tilde{n})\} \tag{3.2}$$

in which:

- k represents its *cardinality*
- while $(\tilde{m}/\tilde{n})$ represents its *Ordinal Genetic Relationships*, where the *round brackets* expressly indicate that they represent *only a part* of the concept of Ordinality, understood as a Whole. In fact, the first member of Equation (3.2) is represented in *curly brackets*, precisely because this symbol is usually adopted to indicate *the concept of a Whole*.

The Ordinal Genetic Relationships $(\tilde{m}/\tilde{n})$ can also more synthetically termed as "Ordinal Relationship", not only because they are not "functional" Relationships, but especially because the adjective "Ordinal" also indicates that they are precisely those Relationships that give the most significant contribution to the definition of the general concept of Ordinality understood as a Whole.

3.1. General Properties of the "Incipient" Derivative of Ordinality $\tilde{q}$

Definition (2.2) clearly shows what we have synthetically anticipated, that is: the "*Incipient Derivative*" is not an "operator", like the derivative (d/dt) in the Traditional Differential Calculus (TDC), but it could be termed as a "*generator*", because it describes the *Generativity* of a given Process, *in its same act of being born* ([13]-[18]). In fact:

i) The sequence of the symbols is now interpreted according to the *direct*

priority of the three elements that constitute its definition (*from left to right*). This is the reason why they acquire *a completely new different meaning* with respect to the traditional one;

ii) The three symbols, in fact, do not represent "three" distinct operations, but a *unique and sole* Generative Process;

iii) The symbol $\tilde{Lim}$, whose etymological origin comes from the Latin word "Limen" (which means a "threshold"), represents the "*threshold*" of that "*ideal window*" from which we observe and describe the considered phenomenon;

iv) The symbol $\tilde{\Delta}t : 0 \rightarrow 0^+$ now indicates not only the initial time of our registration, but also the proper "*origin*" (in its etymological sense) of *something new* which we observe (and describe) *in its proper act of being born*, as a Generative Process;

v) It is then evident that the "operator" $\tilde{\delta}$ now registers the variation of the observed property $f(t)$, not only in terms of quantity, but also, and especially, in terms of Quality (as the symbol "tilde" would expressly remind). So that the ratio which appears in Equation (3.1) indicates not only a quantitative variation in time, but both the variation in *Quality* and *quantity*;

vi) Consequently, when we take the incipient (or "prior") derivative of Ordinality $\tilde{q}$ of any $f(t)$, the *Exit* of such a process will keep "*memory*" of its *genetic origin*. This is because, besides its quantity, it will result as being Ordinally structured (as shown at the next paragraph 3.2.2) according to the indication of such an exponent. The latter in fact precisely expresses how each part of the output is *genetically Ordered* to the Whole and, at the same time, *how each part is related to all the others* in terms of *Ordinal Harmony Relationships* (illustrated at paragraph 5.6);

vii) In this way **the "*incipient*" derivative** represents the *Generativity of the considered Process*, that is the *output "excess"* (per unit time) characterized by both its *Ordinal Genetic Relationships* and its related *cardinality*, while the sequence of the symbols in its definition can be interpreted as representing **a unique Inter-Action Process** between the same;

viii) The above-mentioned reasons clearly show why **the "*Incipient*" Derivative**, precisely because of such properties, is able to *Unify* (and, at the same time, to *Specify*) the description of the various Self-Organizing Processes of the surrounding World, when they are explicitly understood *in terms of Quality*.

3.2. Specific Properties of the "Incipient" Derivative of Ordinality $\tilde{q}$

Let us start from considering first its *specific cardinality k*.

3.2.1. The "Incipient" Derivative of cardinality k

On the basis of Definition (3.1), the exit of the incipient derivative of Ordinality *k* is ([16])

$$\frac{\tilde{d}^k}{\tilde{dt}^k} f(t) = \left(f'(t) \tilde{/} f(t) \right)^k \cdot f(t).$$

(3.2.1)

In fact, through successive formal passages, we have that

$$\left(\tilde{\delta}-1\right)f\left(t\right)=f\left(t+\tilde{\Delta}t\right)-f\left(t\right)=f\left(t\right)+f'\left(t\right)\tilde{\Delta}t-f\left(t\right)=f'\left(t\right)\cdot\tilde{\Delta}t \qquad (3.2.2)$$

and, consequently

$$\left(\tilde{\delta}-1\right)\big/\tilde{\Delta}t=\left\{f'\left(t\right)\big/f\left(t\right)\right\}. \qquad (3.2.3)$$

Such an expression, when introduced in the Definition (3.1), gives

$$\tilde{\lim_{\tilde{\Delta}t:0\rightarrow 0^{+}}}\left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{k}\cdot f\left(t\right)=\left(f'\left(t\right)\big/f\left(t\right)\right)^{\tilde{k}}\cdot f\left(t\right). \qquad (3.2.4)$$

Such an explicit formal process shows that the definition of the "*Incipient*" Derivative of cardinality k is based on a concept of limit, which however it is "*prior*" with respect to the considered function. In fact, it is specifically referred to the considered function only *after* the corresponding evaluation of the latter.

It is also worth adding that in Equations (3.2.2) and (3.2.3) we have adopted the simple notation $f'\left(t\right)$, which in reality is more typical of TDC. It is thus now particularly important to point out that, apart from the similarity of the symbol, the traditional derivative $f'\left(t\right)$ presents specific differences with respect to the "Incipient" Derivative $\tilde{f'}\left(t\right)$.

In fact, if we consider the "Incipient" Derivative of cardinality k of the exponential function, that is, if we assume that $f\left(t\right)=e^{\alpha\left(t\right)}$, on the basis of Equation (3.2.4) we get

$$\left(\frac{\tilde{d}}{\tilde{dt}}\right)^{k}e^{\alpha\left(t\right)}\overset{*}{=}e^{\alpha\left(t\right)}\cdot\left[\overset{\circ}{\alpha}\left(t\right)\right]^{k} \qquad (3.2.5)$$

in which the specific symbology adopted $\overset{\circ}{\alpha}\left(t\right)$ is finalized to point out that, even if on the basis of Equation (3.2.4) the first order "Incipient" Derivative (now indicated with $\overset{\circ}{\alpha}\left(t\right)$) coincides with the traditional derivative $\alpha'\left(t\right)$, the *logical processes* that lead to such identical (quantitative) results are radically different. A difference which, in particular, is also pointed out by the adoption of the symbol $\overset{*}{=}$, which reminds us that **any "*Incipient*" Derivative is always the "*Exit*" of a *Generative Logical Process* and not of a *necessary* logical process**.

Equation (3.2.5) can thus preferentially be adopted **as the *General Definition* of the "*Incipient*" Derivative of cardinality k**. This is because any function $f\left(t\right)$ **can always be written in the form** $f\left(t\right)=e^{\ln f\left(t\right)}=e^{\alpha\left(t\right)}$.

Such a formal representation, in fact, leads to the same result as that of Equation (3.2.5). However, such a formal representation will reveal the "Ostensive" Valence of the "Incipient" Derivative of cardinality k when, in the next paragraphs, we will introduce the general definition of Relational Space and, even more, when we will deal with the explicit solution to the Maximum Ordinality Principle.

At the same time, such a definition is also particularly apt at showing the deep differences between the cardinal values of the "Incipient" Derivatives and those pertaining to the traditional derivatives.

In fact, if we compare the traditional derivative of order n of the function

$e^{\alpha(t)}$, evaluated according to Faà di Bruno's formula

$$\left(\frac{d}{dt}\right)^n e^{\alpha(t)} = e^{\alpha(t)} \sum \frac{n!}{k_1!k_2!\cdots k_n!} \cdot \left(\frac{\dot\alpha}{1!}\right)^{k_1} \left(\frac{\ddot\alpha}{2!}\right)^{k_2} \cdots \left(\frac{\alpha^{(n)}}{n!}\right)^{k_n} \qquad (3.2.6)$$

with the "Incipient" Derivative of the corresponding cardinality n

$$\left(\frac{\tilde d}{\tilde{dt}}\right)^n e^{\alpha(t)} \overset{*}{=} e^{\alpha(t)} \cdot \left[\overset{\circ}{\alpha}(t)\right]^n, \qquad (3.2.7)$$

we can easily recognize that they are *deeply different*. And, even if in some cases the two derivatives of the same order k coincide (for instance when $\alpha(t)$ is linear), such a coincidence has always to be seen in the light of the symbol $\overset{*}{=}$ in Equation (3.2.7), which reminds us **that any "*Incipient*" *Derivative* is always the "*Exit*" of a *Generative Logical Process* and not of a *necessary* logical process**. A concept that is contextually and specifically underlined in Equation (3.2.7) by the explicit adoption of the "notation" $\left[\overset{\circ}{\alpha}(t)\right]^n$.

3.2.2. The Ordinal Genetic Relationships $(\tilde m/\tilde n)$ of the "Incipient" Derivative of Ordinality $\tilde q$

As already anticipated, beside its proper cardinality k, the "Incipient" Derivative of Ordinality $\tilde q$, according to Equation (3.1), is characterized by *the genesis* of its corresponding *Ordinal Genetic Relationships*, whose specific indication is represented by $(\tilde m/\tilde n)$.

In this respect, it is worth pointing out that the symbol $f(t)$ does not represent anymore a simple "function", such as in the case of TDC, but it represents a *Physical Entity*, of *Generative Nature*. Consequently, a more appropriate symbol should be $\tilde f(t)$, where the "tilde" notation specifically reminds us its *Generative Nature*.

More specifically, **in the general context of Self-Organizing Systems**, the symbol $\tilde f(t)$ **will be more properly understood as being representing** the *Relational Space* **of a given System**, as it will be shown in the next paragraphs.

After these due premises, we can assert that **the "*Incipient*" *Derivative* of Ordinality $\{\tilde q\} = \{k,(\tilde m/\tilde n)\}$ describes a *Generative Process* which, with reference to a given System, is characterized by both its *cardinal* and "*internal genetic properties*", and it can be represented as follows**

$$\left(\frac{\tilde d}{\tilde{dt}}\right)^{\{k,(\tilde m/\tilde n)\}} e^{\alpha(t)} \overset{*}{=} e^{\alpha(t)} \cdot \left\{ \left(\begin{bmatrix} \left[\overset{\circ}{\alpha}_{11}(t)\right]^k \\ \left[\overset{\circ}{\alpha}_{21}(t)\right]^k \\ \vdots \\ \left[\overset{\circ}{\alpha}_{m1}(t)\right]^k \end{bmatrix} \right), \left(\begin{bmatrix} \left[\overset{\circ}{\alpha}_{12}(t)\right]^k \\ \left[\overset{\circ}{\alpha}_{22}(t)\right]^k \\ \vdots \\ \left[\overset{\circ}{\alpha}_{m2}(t)\right]^k \end{bmatrix} \right) \cdots \left(\begin{bmatrix} \left[\overset{\circ}{\alpha}_{1n}(t)\right]^k \\ \left[\overset{\circ}{\alpha}_{2n}(t)\right]^k \\ \vdots \\ \left[\overset{\circ}{\alpha}_{mn}(t)\right]^k \end{bmatrix} \right) \right\} \qquad (3.2.8)$$

where:

- k represents the *cardinality* of the "Incipient" Derivative;

- $\overset{\circ}{\alpha}_{ij}(t)$ are the *genetic characteristics* of the considered system, which are highlighted by the *Generative Process* described by the "Incipient" Derivative. For this reason, they should more properly be represented as being characterized by a "tilde" notation. However, for the sake of a simpler notation, *the "tilde" notation has been omitted*, and thus *it is simply understood*;

- such genetic characteristics $\overset{\circ}{\alpha}_{ij}(t)$ are generally referred to the ***specific properties*** of the ***Relational Space*** $\alpha(t)$ and are evidently characterized by the initial and boundary conditions of the System;

- at the same time the "***matrix***" which appears in the second member of Equation (3.2.8) *is not a "traditional matrix"*. In fact, it is an "*Ordinal* Matrix", whose various elements are related between them through *Ordinal Relationships*, of *Genetic Nature*, in the form N Co-Generated genetic properties (vertical columns), further related between them in the form of N Interaction Ordinal Relationships (parallel sequence of the N column). The "*Ordinal*" Matrix thus represents an *Ordinal Cooperation* of N Co-Productions and their associated N Inter-actions.

In this way **the various elements form *One Sole Entity***, faithfully represented by the abovementioned *Ordinal Matrix*. A concept that is explicitly pointed out, also in this case, by the adoption of *curly brackets*.

In addition, in order to distinguish such an *Ordinal Matrix* from a traditional matrix, from now on, for the sake of brevity, it will be simply termed by means of the single term "*Matrioska*".

The structure of the "Incipient" Derivative (3.2.8) is then able to Ostend even more clearly the concepts previously anticipated. That is:

- the symbol $(\tilde{m}/\tilde{n})$ represents the *Ordinal Genetic Relationships* that characterize the "Incipient" Derivative, where the round brackets expressly indicate that they represent only *a part* of the concept of Ordinality, which vice versa is understood, by itself, *as a Whole*;

- In fact, for this reason, in Equation (3.2.8) the latter concept is represented by means of the adoption of *curly brackets*;

- **The *Ordinal Genetic Relationships* can also more synthetically be termed as "*Ordinal Relationship*", both because they *are not*, in themselves, "*functional*" *Relationships*, but especially because the adjective "Ordinal" clearly indicates that they are precisely those that give *the most significant contribution* to the definition of the "*General Concept*" of Ordinality;**

- In addition, Equation (3.2.8) allows us to point out that, when we preliminary introduced the concept of *cardinal* "Incipient Derivative", this was represented as a simple and proper *mathematical concept*, which, in this sense, has some similarities with that of a traditional derivative. This is why it was possible to continue to adopt the term "function" and the correlative symbol $f(t)$, even if it was well clear the profound difference between the correlative Logical Process adopted;

- Vice versa, when we consider *the "Incipient" Derivative of Ordinality* $\tilde{q}$, its meaning, when considered in the descriptive context of Self-Organizing Systems, is more properly referable as *the description of a "Generative Process"*;

- Consequently, in such a case it is more appropriate **to consider Equation (3.2.8) as representative of a** *Generative Process*, **which highlights the "Genetic Properties" of a** *Physical Entity* **that, in the case of a Self-Organizing System, it is usually represented by the proper** *Relational Space* **of the System;**

- So that, to take into account the abovementioned different aspects between the two considered Derivatives, in general **it is preferable to adopt the synthetic "tilde" notation** $\tilde{f}(t)$, **in order to more specifically indicate, in addition, that the considered System is already the "Exit" of** *a previous* **"Generative Process".**

3.2.3. Specific Properties of the "Incipient" Derivative when understood of *Higher Ordinality*

The Ordinality of the "Incipient" Derivative, as previously defined (see Equation (3.2)), represents the most frequent form of Ordinality of the Self-Organizing Systems usually considered.

However, in particularly cases (especially in "Living" Systems), it may be characterized by a more "articulated" structure. For example, its cardinality can directly be associated to a correlative Ordinality $\tilde{2}/\tilde{2}$, corresponding to an "*additional*" Coproduction-Interaction Process.

In such a case the Ordinality $\tilde{q}$ will be then represented as

$$\tilde{q} = \left\{ k \uparrow \tilde{2}/\tilde{2} ; (\tilde{m}/\tilde{n}) \right\} \tag{3.2.9}$$

in order to have, in such a way, a more adherent representation of **the "*Internal Generativity*" of the System** under consideration.

In this respect, however, some examples of more articulated forms of "Incipient" Derivative, with reference to *particularly complex* "Living" System, are illustrated in [29], and in particular *dealt with* in the introductive test of this paper.

4. Mathematical Formulation of the Maximum Ordinality Principle

The Maximum Ordinality Principle (M.O.P.), whose verbal enunciation asserts that "*Every System tends to maximize its Ordinality, including that of its surrounding habitat*", is formulated by means of two fundamental equations, which are so *strictly related to each other*, so as to form *a Whole* ([17] [20] [25]):

4.1. The First Fundamental Equation of the Maximum Ordinality Principle

On the basis of the previous concept of "Incipient" Derivative, the First Fundamental Equation is formulated as follows

$$\left(\underline{\tilde{d}/\tilde{d}t} \right)_s^{\{\tilde{m}/\tilde{n}\}} \{\tilde{r}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{4.1}$$

$$\left(\tilde{m}/\tilde{n}\right) \rightarrow Max \rightarrow \left\{\tilde{2}/\tilde{2}\right\} \uparrow \left\{\tilde{N}/\tilde{N}\right\} \qquad (4.1.1)$$

where $\{\tilde{r}\}$ is the *Relational Space* of the System under consideration (see paragraph 5.1), while $\{\tilde{m}/\tilde{n}\} = \{k,(\tilde{m}/\tilde{n})\}$ represents its corresponding Ordinality, while $(\tilde{m}/\tilde{n})$ indicates the *Ordinal Genetic Relationships* characterized by $\tilde{m}$ Ordinal Co-productions and $\tilde{n}$ Ordinal Interactions, and *the Maximum Ordinality is reached when* $(\tilde{m}/\tilde{n})$ equals $\{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\}$ (as indicated in Equation (4.1.1)).

In this respect, it is worth noting that:

i) The **underlined** symbol $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s$ explicitly indicates that **the "*Generative Capacity*" of the System (more appropriately termed as *Generativity*) is "*internal*" to the same System**. This is because it is precisely **that which *gives origin* to its *Self-Organization* as a *Whole*;**

ii) The symbol " $\overset{[\rightarrow}{=}\{\tilde{0}\}$ " represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact, it now represents, at the same time:

- the **specific "*origin and habitat*" conditions** associated to the considered Ordinal Differential Equation (4.1);

- while the symbol " $\overset{[\rightarrow}{=}$ " indicates that the System, during its *Generative Evolution*, **is persistently "*adherent*"** to its "origin and habitat" conditions.

4.2. The Second Fundamental Equation of the Maximum Ordinality Principle

It is formulated as follows

$$\left(\tilde{d}/\tilde{d}t\right)^{(\tilde{2}/\tilde{2})} \left\{ \{\tilde{r}\} \circledR \left(\tilde{d}/\tilde{d}t\right)^{(\tilde{2}/\tilde{2})} \{\tilde{r}\} \right\} \overset{[\rightarrow}{=} \{\tilde{0}\} \qquad (4.2)$$

and it can be considered as representing a *global "Feed-Back Process"* of *Ordinal Nature*, which is *internal* to the same System. Equation (4.2), in fact, asserts that the *Relational Space* of the System $\{\tilde{r}\}$, which "emerges" as a solution from the First Equation, interacts in the form of the Relational Product $\circledR$ (defined at paragraph 5.1) with *its proper Generative Capacity* $\left(\tilde{d}/\tilde{d}t\right)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$. In such a way as **to originate a "*comprehensive Generative Capacity*", which *at any time*, is always *adherent* to the origin and habitat conditions of the Second Fundamental Equation.**

This is an aspect which is particular important for the *Ordinal Stability* of the System, especially when the latter interacts with other surrounding Systems understood as being part of its proper habitat.

The Maximum Ordinality Principle, in its two fundamental equations, *always* presents an *Explicit Solution*.

The latter will be presented:

a) by preliminarily illustrating its basic elements

b) then by formulating the correlative solution in explicit terms

c) finally, at the end of the Appendix, the general explicit solution to the M.O.P. will also be presented and structured in a corresponding *operative form*,

so that it may result as being more directly and easily adopted in analyzing any System under consideration.

5. Explicit Solution to the Mathematical Formulation of the Maximum Ordinality Principle

In order to show the explicit solution to the Maximum Ordinality Principle, it is worth recalling the fundamental concepts pertaining to the *Relational Space* of a System.

5.1. The Relational Space of a System (and the "Proper Space" of the System)

In this respect, it is fundamental to recall that the symbol $\{\tilde{r}\}$ in Equation (4.1) represents the *Relational Space* of the System, which obviously depends on the Nature of the System analyzed.

We can then start from the consideration of a System whose *Relational* Space is characterized, for example, by the following three topological coordinates $\{\tilde{\sigma}, \tilde{\varphi}, \tilde{\vartheta}\}$.

Such a hypothesis is surely valid in the case of a "*non-Living*" System. Nonetheless, it is also valid in the case of a "*Living*" System too. Whereas, in the case of "Conscious" Systems, the three coordinate will surely be different.

For example, in the case of the Economic Analysis of European Community, with its 27 States, the variables could be (K, L, N), that is *Kapital*, *Labour* and *Natural Resources*, as shown in [30].

In all cases whatsoever, the three topological coordinates $\{\tilde{\sigma}, \tilde{\varphi}, \tilde{\vartheta}\}$ are always considered as *the Exit of a Generative Process* (this is the reason for the tilde notation), and we always have that

$$\{\tilde{r}\}_s = e^{\tilde{\alpha}(t)} = e^{\{\tilde{\sigma} \circledR \tilde{i} \oplus \tilde{\varphi} \circledR \tilde{j} \oplus \tilde{\vartheta} \circledR \tilde{k}\}}. \tag{5.1.1}$$

This is because, on the basis of a generalized form of De Moivre representation, it is always possible to write

$$\{\tilde{r}\}_s = \{\tilde{\rho} \circledR \tilde{i} \circledR e^{\tilde{\varphi} \circledR \tilde{j}} \circledR e^{\tilde{\vartheta} \circledR \tilde{k}}\} = \{e^{\tilde{\sigma} \circledR \tilde{i}} \circledR e^{\tilde{\varphi} \circledR \tilde{j}} \circledR e^{\tilde{\vartheta} \circledR \tilde{k}}\},$$
$$= e^{\{\tilde{\sigma} \circledR \tilde{i} \oplus \tilde{\varphi} \circledR \tilde{j} \oplus \tilde{\vartheta} \circledR \tilde{k}\}} = e^{\tilde{\alpha}(t)} \tag{5.1.2}$$

where the traditional versors $\vec{i}, \vec{j}, \vec{k}$ are now replaced by three unit *spinors* $\tilde{i}, \tilde{j}, \tilde{k}$, which are defined in such a way as to satisfy the following *Relational Product* Rules:

$$\tilde{i} \circledR \tilde{i} = \oplus 1 \quad \tilde{i} \circledR \tilde{j} = \tilde{j} \quad \tilde{i} \circledR \tilde{k} = \tilde{k} \tag{5.1.3}$$

$$\tilde{j} \circledR \tilde{i} = \tilde{j} \quad \tilde{j} \circledR \tilde{j} = \ominus 1 \quad \tilde{j} \circledR \tilde{k} = \tilde{k} \tag{5.1.4}$$

$$\tilde{k} \circledR \tilde{i} = \tilde{k} \quad \tilde{k} \circledR \tilde{j} = \tilde{k} \quad \tilde{k} \circledR \tilde{k} = \ominus 1 \tag{5.1.5}$$

where the symbols $\oplus$ and $\circledR$ express more intimate relationships between the same spinors: both in terms of sum $\oplus$ and in terms of (relational) product $\circledR$ with respect to the case of traditional versors $\vec{i}, \vec{j}, \vec{k}$.

So that representation (5.1.1) is similar (albeit not strictly equivalent) to a system of three complex numbers, characterized by one real unit ($\tilde{i}$) and two im-

aginary units ($\tilde{j}$ and $\tilde{k}$).

5.2. The "Generative Capacity" of the System

As already anticipated, the Incipient Derivative $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}}$, when it is *underlined*, explicitly indicates that the "*Generative Capacity*" of the System (more appropriately termed as *Generativity*) is "*internal*" *to the same System*.

This is precisely because under these conditions **it represents** *the* "*Self-Organization*" of the System *as a* "*Whole*".

At the same time this is also the reason why, differently from the traditional "incipient" derivative, in our case the "Incipient" Derivative <u>is directly *referred to the exponent*</u> of the Relational Space, that is

$$e^{\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}}\left\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\right\}} . \tag{5.2.1}$$

In addition, it is also important to underline that **such an exponent**, according to the same symbols adopted, **is understood as a** *Whole* (see the curly brackets, together with the symbols $\oplus$ and $\circledR$).

This means that **the corresponding derivative have to be taken with reference to such a** *Whole*. Otherwise, its corresponding value will be generally "underestimated".

If now, for the sake of clarity we synthetically indicate $\left\{\tilde{\sigma}\circledR\tilde{i}\oplus\tilde{\varphi}\circledR\tilde{j}\oplus\tilde{\vartheta}\circledR\tilde{k}\right\}=\tilde{\alpha}(t)$, the explicit solution to Equation (4.1) will result in the form (5.2.2), when it is given in terms of an External Representation. That is, when the coordinates of the various elements of the System are referred to a Reference System of coordinates whose origin is *external* to the System under consideration.

$$\left\{\tilde{r}\right\}_s = e^{\begin{vmatrix} \tilde{\alpha}_{11}(t) & \tilde{\alpha}_{12}(t) & \cdots & \tilde{\alpha}_{1n}(t) \\ \tilde{\alpha}_{21}(t) & \tilde{\alpha}_{22}(t) & \cdots & \tilde{\alpha}_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\alpha}_{n1}(t) & \tilde{\alpha}_{n2}(t) & \cdots & \tilde{\alpha}_{nn}(t) \end{vmatrix}} . \tag{5.2.2}$$

The "Matrioska" in Equation (5.2.2) also shows that, as consequence of the Internal Generativity of the System $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}}$, *when the System reaches its Maximum Ordinality*, as a consequence of the Self-Organization Process, the initial internal structure $(\tilde{m}/\tilde{n})$ **becomes of the form** $(\tilde{n}/\tilde{n})$. While the various $\tilde{\alpha}_{ij}(t)$ evidently depend on the initial and boundary conditions, and in the next paragraphs we will show how it is possible to find *their explicit expressions*.

5.3. Explicit Expression of the Internal Generativity $\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{m}/\tilde{n}\}}$

Let assume that, under the conditions previously described, the explicit expression of the Ordinality $\{\tilde{m}/\tilde{n}\}$, in Equation (3.2), equals

$$\left\{\tilde{N}/\tilde{N}\right\} = \left\{k,\left(\tilde{N}/\tilde{N}\right)\right\} . \tag{5.3.1}$$

Equation (4.1) then becomes

$$\left(\underline{\tilde{d}/\tilde{d}t}\right)_s^{\{\tilde{N}/\tilde{N}\}} e^{\{\tilde{\alpha}(t)\}} = e^{\begin{Vmatrix} \left(\left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{11}(t)\right) & \left(\left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{12}(t)\right) & \cdots & \left(\left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{1N}(t)\right) \\ \left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{21}(t) & \left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{22}(t) & \cdots & \left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{2N}(t) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{N1}(t)\right) & \left(\left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{N2}(t)\right) & \cdots & \left(\left(\underline{\tilde{d}/\tilde{d}t}\right)^k \tilde{\alpha}_{NN}(t)\right) \end{Vmatrix}} \overset{\longmapsto}{=} \left\{\tilde{0}\right\}, \tag{5.3.2}$$

where the symbol "$\stackrel{[\rightarrow}{=}\{\tilde{0}\}$", as previously anticipated, represents, at the same time:

- the specific "*origin and habitat*" conditions associated to the considered Ordinal Differential Equation (4.1);
- while the symbol "$\stackrel{[\rightarrow}{=}$" indicates that the System, during its *Generative Evolution*, is persistently "adherent" to its "*origin and habitat*" conditions.

5.4. The Initial and Boundary Conditions

Given the particular structure of Equation (5.3.2), it is possible to directly explicit the term $\stackrel{[\rightarrow}{=}\{\tilde{0}\}$ in exponential form, so that it can be written as follows

$$e^{\left\{\left[\begin{pmatrix}(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{11}(t)\\(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{21}(t)\\\vdots\\(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{N1}(t)\end{pmatrix}\begin{pmatrix}(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{12}(t)\\(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{22}(t)\\\vdots\\(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{N2}(t)\end{pmatrix}\begin{pmatrix}\cdots\\\cdots\\\vdots\\\cdots\end{pmatrix}\begin{pmatrix}(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{1N}(t)\\(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{2N}(t)\\\vdots\\(\tilde{d}/\tilde{d}t)^k \tilde{\alpha}_{NN}(t)\end{pmatrix}\right]\right\}} \stackrel{[\rightarrow}{=} e^{\left\{\begin{pmatrix}\tilde{\beta}_{11}(t)\\\tilde{\beta}_{21}(t)\\\vdots\\\tilde{\beta}_{N1}(t)\end{pmatrix}\begin{pmatrix}\tilde{\beta}_{12}(t)\\\tilde{\beta}_{22}(t)\\\vdots\\\tilde{\beta}_{N2}(t)\end{pmatrix}\begin{pmatrix}\cdots\\\cdots\\\vdots\\\cdots\end{pmatrix}\begin{pmatrix}\tilde{\beta}_{1N}(t)\\\tilde{\beta}_{2N}(t)\\\vdots\\\tilde{\beta}_{NN}(t)\end{pmatrix}\right\}}, \quad (5.4.1)$$

which shows that, *in principle*, its explicit solution can be obtained by solving $N \times N$ corresponding differential equations of the form

$$\left(\tilde{d}/\tilde{d}t\right)^k \tilde{\alpha}_{ij}(t) = \tilde{\beta}_{ij}(t). \quad (5.4.2)$$

In reality, as we will see, **from an "*operative point of view*"** <u>it is sufficient the integration of the *sole* "*couple of reference*"</u>, generally indicated as $\tilde{\alpha}_{12}(t)$. This is because, as shown later on, all the other couples are related to the *reference couple* $\tilde{\alpha}_{12}(t)$ in the form of "*Assignation*" conditions, **according to the Harmony Relationships** (see paragraph 5.6), This means that the next paragraph, concerning the explicit solution in terms of Equations (5.4.2), in reality is here presented only for *generality* of exposition, that is before the "Emerging Property" of the *Harmony Relationships*.

5.5. Explicit Solution to Equation (5.4.1), understood in terms of External Description

Equation (5.4.1) **generally presents an *explicit solution***. This is because in the majority of the most frequent Self-Organizing Systems (both "*non-Living*", "*Living*" and "*Conscious*" Systems), the general structure of the initial conditions can be assumed as being equal to

$$\beta_{ij}(t) = \left(a_{ij} + b_{ij} \cdot t\right)^p, \quad (5.5.1)$$

in which *p* can also be a fractional number.

Such initial conditions always lead to the explicit solution of any unknown $\tilde{\alpha}_{ij}(t)$ that appears in Equation (5.4.1). This is because by considering the general definition of the cardinal "Incipient Derivative" (3.2.1), we have that

$$\frac{\tilde{d}^k}{\tilde{d}t^k} f(t) = \left(\frac{\tilde{\widetilde{f'(t)}}}{f(t)}\right)^k \cdot f(t) = \beta(t), \quad (5.5.2)$$

in which $\beta(t)$ now represents the initial condition for the generic function $f(t)$.

Consequently, through successive formal passages we have

$$f(t)^{1-k} \cdot f'(t)^{k} = \beta(t), \tag{5.5.3}$$

from which

$$f(t)^{\frac{1-k}{k}} \cdot f'(t) = \beta(t)^{\frac{1}{k}}, \tag{5.5.4}$$

whose integral

$$\int_{0}^{t} f(t)^{\frac{1-k}{k}} \cdot \tilde{f}'(t) \cdot dt = \int_{0}^{t} (\beta(t))^{\frac{1}{k}} \cdot dt, \tag{5.5.5}$$

leads to

$$f(t)^{1/k} \cdot k = \int_{0}^{t} (\beta(t))^{\frac{1}{k}} \cdot dt, \tag{5.5.6}$$

and, consequently, we have

$$f(t) = 1/k \cdot \left\{ \int_{0}^{t} (\beta(t))^{\frac{1}{k}} \cdot dt \right\}^{k}, \tag{5.5.7}$$

where $f(t)$ now represents any $\alpha_{ij}(t)$, while $\beta(t)$ represents the corresponding associated initial condition $\beta_{ij}(t)$.

The explicit solution of the generic $\alpha_{ij}(t)$ is then given by

$$\alpha_{ij}(t) = \frac{1}{k} \cdot \left\{ \int_{0}^{t} (\beta_{ij}(t))^{\frac{1}{k}} \cdot dt \right\}^{k} = \frac{1}{k} \cdot \left\{ \int_{0}^{t} \left((a_{ij} + b_{ij} \cdot t)^{p} \right)^{\frac{1}{k}} \cdot dt \right\}^{k}$$
$$= \frac{1}{k \cdot b_{ij}} \cdot \left(\frac{k}{p+k} \cdot (a_{ij} + b_{ij} \cdot t)^{\frac{p}{k}+1} \right)^{k} \tag{5.5.8}$$

However, as already anticipated, such a "*formal procedure*" it is not specifically required to evaluate all the couples $\alpha_{ij}(t)$ that characterize the explicit solution to the First Fundamental Equation (4.1). This is because *The Explicit Solution does not end up at this level.*

The General Solution to Equation (4.1), in fact, is characterized by an *additional contribution*. That is, the contribution of the *Harmony Relationships*, which represent an "Emerging Solution" that, correspondently, shows an "*Emerging Property*" of the Self-Organizing Systems: that is, the "*Diffusive Generativity*", among the various elements of the same System, and which represents the *proper origin* of the *Harmony Relationships*.

5.6. The Harmony Relationships, as the "Exit" of a "*Diffusive Generativity*" of the System

The *Process of Genesis* of the *Harmony Relationships* can be shown by adopting two different *descriptive modalities*, that is: by adopting an External Representation or, alternatively, an Internal Representation.

The two Representations are substantially equivalent between them. However, the adoption of an Internal Representation is able to Ostend *much more clearly* the abovementioned "*Excess of Quality*" on behalf of the System

analyzed.

This is because, as already anticipated, an External Representation is the one in which each element of the System is referred to a system of coordinates characterized by an origin which is external to the System analyzed. **Whereas, in the case of an *Internal Representation*, the various elements of the System are referred to a system of coordinates which is internal to the System analyzed.**

In the latter case, each element $\tilde{\alpha}_{ij}(t)$ of the System, at its Maximum Ordinality, **is preferably referred to the corresponding element of the main diagonal belonging the same row *i,*** and, this leads to the following Representation

$$\{\tilde{r}\}_s = e^{\begin{bmatrix} 0 & \tilde{\alpha}_{12}(t) & \cdots & \tilde{\alpha}_{1N}(t) \\ \tilde{\alpha}_{21}(t) & 0 & \cdots & \tilde{\alpha}_{2N}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\alpha}_{N1}(t) & \tilde{\alpha}_{N2}(t) & \cdots & 0 \end{bmatrix}}, \tag{5.6.1}$$

in which all the elements of the main diagonal are evidently equal to zero, whereas all the other elements $\tilde{\alpha}_{ij}(t)$ assume *a binary-duet structure*, and thus satisfy the following *Specularity* Relationships

$$\left\{\tilde{\alpha}_{ij}(t)\right\}^{\{\bar{2}/\bar{2}\}} \overset{*}{=} \left\{\tilde{\alpha}_{ji}(t)\right\}^{\{\bar{2}/\bar{2}\}}, \tag{5.6.2}$$

which represent *a much more profound concept* with respect to the traditional symmetry (the symbol "$\overset{*}{=}$", in fact, does not represent an equality, but a simple *assignation condition*).

Such a Representation then allows us to show **the *Generative Process* that leads the System to its Maximum Ordinality and, at the same time, to its Maximum Stability conditions**, because it *restructures the internal relationships* between the various elements in such way as these show **an additional "*emerging property*"**, which is *initially* based on the following "topological" Relationships:

$$\tilde{\lambda}_{12} \oplus \tilde{\alpha}_{12}(t) \overset{*}{=} \tilde{\lambda}_{1j} \oplus \tilde{\alpha}_{1j}(t) \quad \text{for} \quad j = 3, \cdots, N \tag{5.6.3}$$

together with all their associated *incipient* derivatives, up to the order $N-1$

$$\left\{\overset{\circ}{\tilde{\lambda}}_{12} \oplus \overset{\circ}{\tilde{\alpha}}_{12}(t)\right\}^{\tilde{k}} \overset{*}{=} \left\{\overset{\circ}{\tilde{\lambda}}_{1j} \oplus \overset{\circ}{\tilde{\alpha}}_{1j}(t)\right\}^{\tilde{k}} \quad \text{for} \quad k = 1, \cdots, N-1, \tag{5.6.4}$$

where $\tilde{\lambda}_{ij}$ represent their corresponding *internal reciprocal "Correlating Factors"*, which are clearly distinct from the values of the initial conditions, because the latter are already included in the correlative expressions $\tilde{\alpha}_{ij}(t)$.

Such properties represent *the bases* of the previously mentioned Property of the "*Diffusive Generativity*", which is <u>faithfully represented by the following *Harmony Relationships*</u>

$$\left\{\tilde{\alpha}_{1,j+1}(t) \oplus \tilde{\lambda}_{1,j+1}(t)\right\} \overset{*}{=} \left(\sqrt[N-1]{\{\tilde{1}\}}\right)_j \otimes \left\{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\right\} \quad \text{for} \quad j = 1,2,3,\cdots,N-1, \tag{5.6.5}$$

whose <u>explicit *Process of Genesis* is illustrated in Appendix A1</u>, while <u>the</u>

associated "Ordinal Roots of Unity" $\left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_j$ are illustrated in Appendix A2.

If we now take into account the Harmony Relationships (5.6.5), together with their *specific structure* and the *correlative symbology* adopted, **the Solution to the First Fundamental Equation pertaining to the System analyzed can be represented as follows**

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\right\} \circ \left\{\left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_1, \left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_2, \cdots \left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_{N-1}\right\}}, \qquad (5.6.6)$$

which reflects *the Self-Organization* of the Systems in terms of "*couples*", according to an *Internal Description*.

At the same time, it shows that the basic "*topological*" structure in terms of the reference couple "12" (see Equation (5.6.3)) has been correspondently "transformed" and, at the same time, "updated", as a consequence of the Diffusive Generative Process which leads to **the Harmony Relationships that, as anticipated, are substantially based on** *the sole reference couple* **"12".**

6. Explicit Solution to the Two Fundamental Equations of the M.O.P, understood as *a Whole*

The M.O.P., considered in its two Fundamental Equations understood *as a Whole*, differently from the problems formulated in TDC, *always* **presents an** *Explicit Solution*. This is especially due to IDC and, in particular, both to the solution to the First Fundamental Equation in the form of Matrioska and the associated Harmony Relationships, which allow to represent the System in the form of "couples", by assuming *one arbitrary couple* of elements as a reference.

So that, precisely because of such specific characteristics, the M. O. P. enabled us to reconsider and explicitly solve some "particular" problems, generally dealt with in literature in terms of TDC, which are generally considered as being "unsolvable", "intractable", "with a drift". The solutions of which ended up by showing that the Maximum Ordinality Principle has an extremely general validity ([20] [25]).

The *Explicit Solution* to the Two Fundamental Equations of the M.O.P, understood as a Whole, can be obtained by introducing the solution to the First Fundamental Equation (4.1), previously shown,

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\right\} \circ \left\{\left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_1, \left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_2, \cdots \left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_{N-1}\right\}}, \qquad (5.6.6)$$

into the Global Feed-Back Process represented by the Second Fundamental Equation (4.2). The latter consequently transforms into a typical Riccati's Equation of *Ordinal Nature*, whose explicit solution is given by

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\tilde{B}(t)\right\} \circ \left\{\left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_{13}, \left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_{14}, \cdots \left(^{N-1}\sqrt{\{\tilde{1}\}} \right)_{1N}\right\}}, \qquad (6.1)$$

where

$$\tilde{B}(t) = \left\{ \begin{pmatrix} \oplus \tilde{A}(t) \\ \Theta \tilde{A}(t) \end{pmatrix}, \begin{pmatrix} \Theta \tilde{A}(t) \\ \oplus \tilde{A}(t) \end{pmatrix} \right\} \qquad (6.2)$$

and

$$\tilde{A}(t) = \left\{ \left\{ \tilde{\alpha}_{12}(0) \right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{ \tilde{\lambda}_{12}(0) \right\}^{\{\tilde{2}/\tilde{2}\}} \right\} \circ \left\{ \left({}^{N-1}\!\sqrt{\left\{ \tilde{1} \right\}} \right)^{\uparrow \{\tilde{N}/\tilde{N}\}} \right\}^{(\tilde{2}/\tilde{2})} \oplus \ln \left(\tilde{c}_1 \oplus \left\{ \tilde{c}_2, t \right\} \right), \quad (6.3)$$

where the term $\ln\left(\tilde{c}_1 \oplus \left\{\tilde{c}_2, t\right\}\right)$ accounts for the *origin and habitat conditions* of the Feed-Back Equation and, at the same time, also represents an *Over-Ordinality* contribution specifically due to the same Feed-Back Process.

This latter contribution, as already anticipated, is particularly important for *the System stability*, especially when the System interacts with another System of its surrounding Habitat.

Equation (6.1), together with Equations (6.2) and (6.3), then represents *the Explicit* "*Emerging Solution*" *to the Maximum Ordinality Principle*, formulated in two "Incipient" Differential Equations (4.1) and (4.2)), when the latter are properly considered *as being a Whole*.

7. General Validity of the Explicit Solution to the Maximum Ordinality Principle

Equation (6.1), considered with the associated Equations (6.2) and (6.3), has a *general validity* because, at the same time, **it is *valid* not only for *non-Living* Systems, but also for *Living* Systems and *Human* Systems too.**

What's more, the same fact that solution (6.1) is ***always an Explicit Solution*** represents ***a* "*very general property*"** that evidently has a huge relevance from an *operative* point of view.

In addition, Solution (6.1) introduces some further fundamental novelties of *gnoseological nature*, which enabled us to clearly assert that "The "Emerging Quality" of Self-Organizing Systems, when modeled according to the Maximum Ordinality Principle (M.O.P.), offers a *Radically New Perspective to Modern Science*" ([25]).

This is exactly what also suggested a possible of reformulation of such a Solution into a corresponding version in *operative terms*.

8. Explicit Solution to the M.O.P. reformulated *in operative terms* by means an EQS Simulator

In order to have an explicit solution that may result much easier to be programmed on a computer and, in particular, on a simple PC, the previous Explicit Solution can be restructured in more *operative terms*, in order to realize an "Emerging Quality Simulator" (EQS), which, however, is not "equivalent", by itself, to a traditional computer program.

This is because, even if conceived for *operative finalities*, EQS always *keeps memory* of the *genetic Ordinality* of the Processes analyzed. So that the various forms of Ordinality, although considered in operative terms, will always be accounted for in terms of their "correlative associated cardinalities".

If we then suppose for example that the *Relational Space* of the System is represented by the following three generative coordinates $\left\{\tilde{\sigma}, \tilde{\varphi}, \tilde{\vartheta}\right\}$, characteristic of a "non-Living" System, the fundamental Relationships of EQS are shown

here below:

a) $\tilde{\rho}_{1j}(t) = A \cdot e^{\tilde{S}_l(t)}$ (7.1)

with

$$\tilde{S}_l(t) = \psi_{1,1} \cdot E_{l,1} \cdot \left[B_l \cdot \tilde{\Sigma}_0(t) - C_l \cdot \left(\tilde{\Phi}_0(t) + \tilde{\Theta}_0(t) \right) \right]$$ (7.1.1)

b) $\tilde{\varphi}_{1j}(t) = \psi_{1,2} \cdot E_{l,2} \cdot \left[B_l \cdot \tilde{\Phi}_0(t) + C_l \cdot \tilde{\Sigma}_0(t) \right]$ (7.2)

c) $\theta_{1j}(t) = \psi_{1,3} \cdot E_{l,3} \cdot \left[B_l \cdot \tilde{\Theta}_0(t) + C_l \cdot \tilde{\Sigma}_0(t) + C_l \left(\tilde{\Phi}_0(t) + \tilde{\Theta}_0(t) \right) \right]$ (7.3)

where

$$E_{l,i} = \frac{\varepsilon_{l,i} + 4\pi \cdot l}{N-1} \quad B_l = \cos\left(\sqrt{2} \cdot \psi_l\right) \quad C_l = D_l = \frac{1}{\sqrt{2}} \sin\left(\sqrt{2} \cdot \psi_l\right)$$ (7.4)

and

$$\psi_l = \psi_2 \cdot \frac{\varepsilon_2 + 2\pi \cdot l}{N-1}$$ (7.5)

in which:

i) $\tilde{\Sigma}_0, \tilde{\Phi}_0, \tilde{\Theta}_0$ synthetically represent the Ordinal coordinates of the *reference couple*, generally termed as "*couple 12*", which, on the other hand, can be arbitrarily chosen. So that the symbols $\Sigma_0, \Phi_0, \Theta_0$ stand for $\{\tilde{\sigma}_{12}, \tilde{\varphi}_{12}, \tilde{\vartheta}_{12}\}$.

Such coordinates, however, considered *in transient conditions*, will correspond to the solution to the Equation (5.6.5), with reference to *the sole couple* "12".

Consequently, in adherence to the symbology previously adopted, those coordinates can be represented as $\Sigma_0(t), \Phi_0(t), \Theta_0(t)$;

ii) the Ordinal factors $\psi_{1,i} \cdot E_{l,i}$ originate from the assumption that the Harmony Relationships, here reproduced for the sake of clearness

$$\left\{ \tilde{\alpha}_{1,j+1}(t) \right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{ \tilde{\lambda}_{1,j+1}(t) \right\}^{\{\tilde{2}/\tilde{2}\}} \stackrel{*}{=} \left({}^{N-1}\!\!\sqrt{\{\tilde{1}\}} \right)_j \circledR \left\{ \tilde{\alpha}_{12}(t) \right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{ \tilde{\lambda}_{12}(t) \right\}^{\{\tilde{2}/\tilde{2}\}}$$

for $j = 1, 2, 3, \cdots, N-1$ (5.6.5)

are modulated by the correlative Ordinal terms $\left\{ \tilde{\lambda}_{1,j+1}(t) \right\}^{\{\tilde{2}/\tilde{2}\}}$, which, apart from specific cases of given Habitat conditions, can be considered "null", because the *initial* topological "assignation" of the Correlative Factors is "Transfigured" by the *Diffusive Generative Process*.

In this respect, the terms $\left\{ \tilde{\alpha}_{1,j+1}(t) \right\}^{\{\tilde{2}/\tilde{2}\}}$, after a previous *reduction of the Ordinality* $\{\tilde{2}/\tilde{2}\} \to 1$, are characterized by three different periodicities $E_{l,i} = \frac{\varepsilon_{l,i} + 4\pi l}{N-1}$, each one *specific for each coordinate*, which originate from the explicit expression of the Ordinal Roots of *Unity* and, at the same, are characterized by the specific factors $\psi_{1,i}$;

iii) In fact, after having rewritten the Ordinal Relationships in the following form

$$Exp\left\{ \tilde{\sigma}_{1j}(t_0), \tilde{\varphi}_{1j}(t_0), \tilde{\vartheta}_{1j}(t_0) \right\} \stackrel{*}{=} Exp\left[\left({}^{N-1}\!\!\sqrt{\tilde{1}} \right)_l \circledR \left\{ \tilde{\sigma}_{12}(t_0), \tilde{\varphi}_{12}(t_0), \tilde{\vartheta}_{12}(t_0) \right\} \right]$$ (7.6)

iv) and after having assumed the explicit expression of the *Ordinal Roots of Unity*, illustrated in **Appendix A2** (Equations (A2.5) and (A2.6)), here explicitly recalled for the sake of clarity

$$\left({}^{N-1}\!\sqrt{\tilde{1}} \right)_l^* = Exp\left\{ \tilde{\alpha} \circledast \tilde{i} \oplus \tilde{\beta} \circledast \tilde{j} \oplus \tilde{\gamma} \circledast \tilde{k} \right\}, \tag{A2.5}$$

where

$$\alpha = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1} \quad \beta = \frac{\varepsilon_2 + 2\pi \cdot l}{N-1} \quad \gamma = \frac{\varepsilon_3 + 2\pi \cdot l}{N-1}, \tag{A2.6}$$

the expansion series of Equation (A2.5), together with the contextual adoption of the Rules of the Ordinal Product (5.1.3), (5.1.4), (5.1.5), leads to the Ordinal Relationships (7.1), (7.1.1), (7.2), (7.3), initially introduced, with the associated coefficients given by Equations (7.4), (7.5).

For the sake of completeness, it is worth adding that:

- The symbol $\left\{\tilde{1}\right\}$ represents *the Unity of the System* (understood as *a Whole*) by means the representation of the *Unity* of its *Proper Space of Relations*;
- $\varepsilon_1, \varepsilon_2, \varepsilon_3$ characterize the spatial orientation of the System as a Whole, with reference to its Ordinal Proper Space and, more specifically, with respect to the Reference "Couple 12";
- the "periodicity" of the "spinor" $\tilde{i}$ is assumed equal to 4π, because it is expressed in steradians;
- while the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to 2π radians, because these spinors are always "orthogonal", both between them and with respect to the spinor $\tilde{i}$. An "orthogonality" that can be seen as a form of *reciprocal* "*irreducibility*" (as also indicated by the same Relational Products);
- while the Factor "$\tilde{A}$" represents an *Internal Ordinal Factor* according to which all the *radial* "*Uniances*" of the various Couples are appropriately referred to the *radial* "Uniance" of the Reference Couple "12". This latter concept of "Uniance" will clearly be illustrated in a next section specifically titled "*Distance and Uniance*".

At the same time, by means of the *Internal Ordinal Factor* "$\tilde{A}$", the cardinalities "associated" to the various "Uniances" are all expressed in terms of a desired scale of measure.

9. General Considerations on the Explicit Solution reformulated in *operative terms* by EQS

From the previous exposition, it should result as being evident that the ***Harmony Relationships*** (further illustrated in **Appendix A1**) **represent an "*Irreducible Excess*"**. That is an "Exceeding" manifestation of the "*Generativity*" **of the System**, where the latter is at the same time *Self-Organizing*, of *Ordinal Nature*, and understood as *a Whole*.

This means that *the same Explicit Solution* reformulated in *operative terms*, precisely because obtained through an Ordinal Deductive Process from the Harmony Relationships and the Ordinal Roots of Unity (further illustrated in **Appendix A2**), represents an "*Emerging Solution*" from the Maximum Ordinal-

ity Principle.

Consequently, even if the single Relationships refer to each single couple "*1j*", and thus to the three "distinct" variables $\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}$, the latter do not represent a simple traditional "vector", **but an "*Ordinal vector*". That is a *unique and sole* "*Relational Entity*", which is usually represented in *curly brackets*, such as** $\left\{ \tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j} \right\}$**, precisely because it is understood as *a Whole*.**

This means that the three variables $\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}$, although recognizable as being "distinct", **they are *not conceptually "separable" between them*.**

Such an assertion is also even truer (and especially) with reference to the *various triples of variables* pertaining to *all the couples which compose* the System, which *a fortiori* are not conceptually "separable" between them precisely because the System is understood *as a Whole*.

In other words, the Fundamental Relations pertaining to EQS previously shown do not only furnish the $N − 1$ single *Ordinal vectors* $\left\{ \tilde{\rho}_{1j}(t), \tilde{\varphi}_{1j}(t), \tilde{\vartheta}_{1j}(t) \right\}$ that characterize each single couple of the System, but **they also represent, even more, a "*Unified Ordinal Description*" of the System understood as a "*Whole*".**

In other terms, the coordinates furnished by the *Operative Solution* are not conceptually "separable" between them, *neither with reference to each single couple, nor* with reference to *all the various couples* of the System as a Whole.

This leads us to point out another important aspect always in the Light of the Maximum Ordinality Principle.

9.1. Distance and "Uniance"

A direct and correlative consequence is that, even if at a "preliminary and intuitive" interpretation, such Ordinal Relationships could be thought as giving the "distances" between the various couples of the System analyzed, in reality, in adherence to the M.O.P, such an interpretation (and the corresponding "terminology"), should be substantially modified. In particular, by adopting a more appropriate term, such as "*Uniance*", instead of that of "distance".

This is because, as already anticipated, the concept of "*distance*" tends more to *divide*, than to *unify*. In fact, the same *etymology* of the word (from Latin "*dis-stant*") indicates that "one element *stays here* and the other one *stays there*" or, equivalently, "*one is here and the other one is there*".

Consequently, in an Ordinal Perspective the term "distance" should preferably be replaced by a different term, possibly able to indicate the concept of "*union*" of two elements, more than their "distance".

In this respect, by introducing a *neologism* (that "rhymes" with the term "distance", but it exactly indicates the opposite meaning), we could say that the same value that in a "functional" perspective represents a "*distance*", in an Ordinal Perspective indicates a "*Uniance*". That is, it indicates that the two elements form "*one sole thing*" of *Ordinal Nature*, precisely because they are the Exit of the same Generative Process. So that the term "Uniance" expresses an *Ordinal concept*, and not a mere cardinal concept, such as that of "distance". **Any "Uniance", in fact, is characterized by its own Ordinality.**

As a simple example, let us think of a couple of elements $\tilde{\alpha}_{ij}(t)$ whose Relationship is characterized by a *Binary-Duet* Ordinality $\left\{\tilde{\alpha}_{ij}(t)\right\}^{\{2/2\}}$. Such a specific and proper *Ordinality* is precisely that which represents *the Ordinal "Unity"* between two elements of the System. While, at the same time, its "associated cardinality" only indicates their topological distribution in the *Relational Space* of the System.

Consequently, **when all the various "Uniances" are considered in the context of the Harmony Relationships, they reveal that the System is a *Whole of Ordinal Nature*, in perfect adherence to the Maximum Ordinality Principle**.

In addition, such an assertion has also an *even more general sense*, that is: **it is precisely the *Generativity* of the Self-Organizing System the one which, with its proper "*Diffusivity*", characterizes all the elements of the System in terms of "Ordinal Relationships". In that sense, such Ordinal Relationships are all of *Genetic Nature*, like in the case of "brothers"**.

In fact, as previously anticipated, "brothers" are termed as such not because of their "direct reciprocal relationships", but because of their *direct reference* to *the same genetic principle*: their father (or their mother or both).

Consequently, in perfect "*Adherence*", **the term "*Uniance*" *synthesizes* the concept of an *Ordinal Unity of Genetic Nature*.**

9.2. Proper Space and Proper Time

Another important aspect that has to be underlined, always in the Light of the Maximum Ordinality Principle, is precisely that synthetically indicated in the title.

The Maximum Ordinality Principle, in fact, shows that Each Self-Organizing System, precisely because characterized by its own "Emerging Quality", evolves in a "*time*" and a "*space*" which are *exclusive and specific* of each System analyzed. Consequently, the latter can be more faithfully termed as "*Proper Time*" and "*Proper Space*" of the System ([29]).

This is an aspect that is radically different from the case of the Traditional Scientific Approach, in which *time* and *space* are assumed as being *absolute*.

Such a difference, however, *does not represent a real "obstacle"* with specific reference to the interpretation of the output of EQS Simulator. What is important, in fact, is to know that such a "difference" exists and, at the same time, to be aware of their correlative *different Nature*. In this case, in fact, such a "difference" can always be dealt with in perfect analogy with the "reduction" of the *Uniances*, when the latter have to be compared with the correlative *distances*.

In addition, such a "difference" is so specific and characteristic of the Self-Orgnizing Systems, that it cannot even be "reduced" to the *space-time* conception of General Relativity.

General Relativity, in fact, introduces the concept of "*space contraction*" and "*time dilatation*" between two reference systems in a reciprocal movement, according to the following relationships ([31])

$$\Delta x' = \Delta x \cdot \sqrt{1 - V^2/c^2} \tag{9.2.1}$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - V^2/c^2}} \ . \tag{9.2.2}$$

It is then possible to show that Einstein's "*space-time conception*" represents a *particular modality* at introducing the concept of the *second order* "*incipient*" *derivative* ([29]). Such a particular modality, however, by itself manifests at *a simple cardinal level*, corresponding to a "reduction" process of the *Proper Space* and *Proper Time* of a given System.

This means that Einstein's *space-time conception* in reality corresponds to the introduction of the *second order* "*incipient*" *derivative*, considered, however, at its mere "*cardinal level*" [ib.].

9.3. The Relationship between "Forces" and "Diffusive Generativity"

In the context of Classical Mechanics, the relationships between the various Bodies are described in terms of "forces", understood as "efficient causes", of mechanical nature, such in the case of the "Three-body System".

In the Ordinal Context, on the contrary, the General Tendency of the Systems to the **Maximum Ordinality**, and the correlative Relationality between the Bodies, is **"Guided" by the "*Diffusive Generativity*" of the System understood as a Whole.**

As a consequence, **it is precisely such a *Diffusive Generativity* that "first" generates the "Binary-Duet" Couples between the various Bodies, and *then* establishes between them the "Harmony Relationships", in order to *successively* re-organize their Ordo-cardinalities in terms of "*Ordinal Roots of Unity*".**

This in fact is precisely the main characteristic that qualifies a System as a "Self-Organizing" System.

10. Two "*com-possible*" Scientific Approaches, albeit "not equivalent" between them

The two above mentioned Scientific Approaches, with their corresponding formal languages, TDC and IDC, respectively, when considered with reference to their corresponding "presuppositions" (that is the subjacent "way of thinking") result as being two different descriptive modalities which are always "*com-possible*". In the sense that they *do not exclude each other*. **They simply co-exist**.

This is because, as already anticipated, the Traditional Scientific Approach, which leads to TDC, *cannot exclude* (in principle) the adoption of a different mental categories and their corresponding formal language (e.g. IDC), because of the *absence* in its presuppositions (especially "necessary" logic) of any form of *perfect induction*.

On the other hand, the same happens in the case of the adoption of IDC, precisely because of the *same reason*, although the latter is based on mental categories characterized by a different form of Logic (e.g. the "Generative" Logic).

Consequently, the two formal languages, TDC and IDC, can *always* be adopted independently from one another. **Although this "compossibility" does not mean that they are "equivalent" between them** (as it happens in the case

of the "Three-body Problem").

Their "in-equivalence", in fact, can easily be shown by comparing the different *consequences* of their respective adoption, when such consequences are obviously considered in the light of their corresponding "mental categories".

In fact, *beside* the Traditional Scientific Approach, which affirms that "Every System is a *mechanism*" (at a phenomenological level), there is also the possibility of a different Approach, according to which "*Every System is a Self-Organizing System*" (always at a phenomenological level). This is the fundamental reason why they lead to the adoption of *two corresponding different formal languages*, with some associated important consequences.

In the first case, in fact, the adoption of TDC leads to:

i) *Unsolvable Problems* in explicit formal terms (as in the case of the "Three-body Problem");

ii) *Intractable Problems* even by adopting the most advanced computers (as in the case of Protein Folding);

iii) *Problems characterized by experimental "drifts"*, which always represent an indication of possible "side effects";

iv) In addition, it is worth pointing out that TDC can present some "side effects" even in the case of accurate experimental confirmations. Such "side effects", in fact, can result as being "*masked*" by the same fact that all the experimental confirmations are always based on the adoption of *methods, instrumentation and measurements* that are conceived (and designed) in a perfect conformity with the fundamental presuppositions of TDC ([25]).

Vice versa, the adoption of IDC does not present such problems, whereas, in turn, it presents several advantages.

In fact, as already anticipated, the adoption of IDC is finalized to describe the "Emerging Quality" of "Self-Organizing Systems". This leads to the formulation of the M.O.P., which is able to offer *a radically New Perspective* to Modern Science. That is: "*Every System is a Self-Organizing System*" (see Table 1).

This is because IDC results as being the most appropriate language able to describe the fundamental characteristics of "Self-Organizing Systems". In fact, the "Incipient Differential Calculus" (IDC):

i) is able to represent, in appropriate formal terms, the "Emerging Quality" of Self-Organizing Systems as an "*Irreducible Excess*";

ii) In this way IDC enabled us to formulate a very General Principle, the Maximum Ordinality Principle (M.O.P.), which can be understood as "*One Sole Reference*" Principle ([20]);

iii) The latter in fact results as being valid *in any Field* of Analysis (from *non-Living* Systems, to *Living* Systems and "*Conscious*" (*Human*) Systems too);

iv) In addition, the adoption of IDC *always* leads to *explicit formal solutions* (such as in the case of the "*Three-body Problem*");

v) At *any topological scale* (e.g. from atoms (*Quantum Mechanics*) to Galaxies (*Celestial Mechanics*));

vi) Both *under steady state and variable conditions*;

vii) What's more, the corresponding Solution to *any* mathematical model based on the M.O.P. (and thus formulated in terms of IDC) **always results as being an "*Emerging Solution*"**. That is, a **Solution whose "*Ordinal Information content* is always *much higher* than the Ordinal content corresponding to the initial formulation of the Problem"**;

viii) As a direct consequence, this leads to the fact that any "Emerging Solution" *can never be reduced* to mere "functional relationships" (as shown in the case of the "Three-body Problem");

ix) This is also means that the adoption of IDC *does not require any* specific reference to the traditional Physical Laws or to the well-known Thermodynamic Principles (precisely because the latter are always understood as "functional relationships"). In this respect, see also previous paragraph 9.3 concerning *the Relationship* between "*Forces*" and "*Diffusive Generativity*";

x) Finally, the adoption of IDC never leads to "side effects". This is because, even when an "Emerging Solution" might manifest some related "Emerging Exits" ([25]), the latter can always be interpreted as being corresponding "Extra Benefits", initially not recognized as such. This leads to point out another fundamental aspect, always in the Light of the Maximum Ordinality Principle and the correlative maximization of the Ordinality of the surrounding Habitat.

11. More general *in-equivalence* between the Two Scientific Approaches, especially with reference to the relationships between Man and the Environment

Although from a general point of view the *in-equivalence* between the two formal languages can preliminarily be recognized at the level of "Thinking", such an in-equivalence is even much more marked at the level of "Decision Making and Acting". Especially when considering, as a basic reference criterion, the corresponding different concepts of "inter-relationships" *between Man and the Environment* ([25]).

This is because the adoption of TDC always "reflects" the general idea that "every system is a *mechanism*", while the "com-possible" formal language IDC is always orientated at describing any system as a "*Self-Organizing System*". This is the fundamental reason for the adoption of the three *new mental categories* (shown in Table 1), which are radically different from the three basic presuppositions of the former.

This easily leads to recognize that the most profound "in-equivalence" between TDC and IDC situates at the level of Decision Making and Acting, in particular with respect to *the Environment*. In fact:

i) At the level of "Decision Making" the two formal languages will evidently lead to make decisions (that will become consequential future *actions*) in a perfect *conformity* with their respectively different way of thinking: TDC, in conformity with its "*aprioristic*" presuppositions; IDC, vice versa, in conformity with the *new mental categories* that, on the contrary, are adopted "*a posteriori*".

Consequently, in both cases the two formal languages will suggest "decisions" in perfect *conformity* with their corresponding concepts of "*surrounding habitat*": understood as a "*set of mechanisms*", in the case of TDC or, respectively, as "*a unique Self-Organizing System*" in the case of IDC [25]);

ii) At the level of Action, however, it is exactly *where* it is possible to recognize the most marked differences between the two Scientific Approaches. This is because, in such a case, the specific different *origin* of each formal language, together with the associated *powerful expressive capacity that any formal language is able to manifest*, represent the fundamental aspect that systematically "guides" (sometimes even "forces") the research for specific *practical solutions* to the various problems and their subsequent actual implementation, in particular with respect to the Environment.

In other terms, the profound differences between the two Scientific Approaches, characterized by their corresponding formal languages, TDC and IDC, respectively, become particularly evident at the "level of Action", because the corresponding formal solutions *become consequential facts* ([32]).

In this respect, the Ostensive Examples previously considered in the various Biennial Emergy Conferences (from 2010 to 2020), are sufficiently clear to show the profound differences that may result, *in practice*, when adopting the one or the other descriptive formal language.

In addition, *an ulterior* and more *radical form* of *in-equivalence* will be analyzed in the next paragraph.

12. Radical *In-equivalence* between Falsification and Relaunce

Another aspect that points out even more clearly the *in-equivalence* between the Traditional Approach and the Ordinal Approach is the fact that the first one is characterized by "*confirmation/falsification*" processes whereas the second one is characterized by "*Emerging Exits*".

The "confirmation" processes, in fact, are strictly necessary in the case of Traditional Theories, which are adopted "*a priori*", and are specifically based on those mental categories previously recalled. In particular, *necessary logic*.

At the same time, *the absence of experimental confirmations* of the corresponding conclusion of Traditional Theories represents a valid argumentation for their "*falsification*" (according to Popper's Falsification Principle).

On the contrary, the Ordinal Approach based on the "Emerging Quality" of Self-Organizing Systems, strictly speaking *does not require* correlative "confirmation processes" in order to be accepted as being a "valid" Approach.

This is because the Ordinal Approach is adopted "*a posteriori*", that is downstream the recognition of the *Manifestation* of Quality as an "Irreducible Excess", and consequential adoption of the new correlative Mental Categories.

So that, the research for the "*Maximum Adherence*" of the correlative Over-Deductions (in Generative Logic) to experimental results, does not represent, properly speaking, the research for a "confirmation". But, paradoxically, it represents the "confirmation" of a "*denial*". Or better, "a confirmation" that can be termed as

being "*not less than*".

In fact, it is exactly such circumstance the one that properly generates the concept of *Relaunce*.

The latter in fact consists in recognizing that the description of the "Emerging Quality", as performed at a preliminary given stage, if characterized by "*Emerging Exits*", can be recognized as being "not less than". Thus, the description can be re-proposed at a *Higher Level of Ordinality* with respect to the one initially supposed and assumed to describe the Process (or Phenomenon) analyzed.

At this stage, the profound "*in-equivalence*" previously shown between the two formal languages, which mainly and clearly manifests at the level of "facts", may suggest, as a possible conclusion, the consideration of an extremely important question: "where are we going", as a consequence of the adoption of *one* or *the other* descriptive formal language": TDC or IDC?

13. Conclusion. Where are we going?

The afore-mentioned differences between the two Scientific Approaches and their correlative formal languages, TDC and IDC, which can preliminarily be recognized at a gnoseological level and, even more, at the level of their respective *practical* consequences, enable us to draw some general conclusions that can be synthetically summarized as follows.

From a general point of view, in fact, it is possible to delineate three possible answers to the previous question:

i) Modern Science is so radically rooted in TDC (and in its corresponding presuppositions) that it is extremely improbable to hypothesize, in spite of the afore-mentioned intrinsic *limitations* of such a formal language, a rapid change of the corresponding paradigm (as the case of the "Three-body Problem", for example, would suggest). In this sense, we have to expect a generalized persistence in the adoption of the traditional formal approach (TDC);

ii) This fact, however, does not prevent from thinking that some Scientists, with reference to some specific problems (related, for instance, to the "Three-body Problem"), will decide to *preferentially* adopt the innovative IDC approach;

iii) Even if, more probably, because of the afore-mentioned "*com-possibility*" between TDC and IDC, it may be expected the adoption of both formal approaches *at the same time*, so as to choose the optimal *operative* solutions on the basis of the corresponding experimental results.

By always taking into account, however, that TDC translates, in formal terms, a "*self-referential*" gnoseological approach, while IDC represents, always in formal terms, a "*hetero-referential*" gnoseological approach (as previously illustrated and synthetically summarized in Table 1).

Appendix A1. Process of Genesis of the Harmony Relationships

This **Appendix A1** points out, in more explicit terms, what synthetically previously asserted, that is: the Harmony Relationships **represent, by themselves, an "*Emerging Solution*"** which, in addition, it is also "*Exceeding*" with re-

spect to the Solution to the First Fundamental Equation.

In fact, what we presented at paragraph 5.6 of **Appendix A** are nothing but the *basic presuppositions* for the formulation of the Harmony Relationships, which, however, do not represent a "*necessary consequence*" of those presuppositions, because they manifest an "*Extra*", or better, an "*Irreducible Excess*" with respect to them.

Let us thus recall the basic elements that will enable us to show that the Harmony Relationships precisely represent an "Emerging Extra" of *Generative Nature*.

We have seen in fact that the Emerging Solution to the First Fundamental Equation allow us to write the following *topological* "*Assignation* Relationships"

$$\left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left\{\tilde{\alpha}_{1j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \quad \text{for} \quad j = 3, 4, \cdots, N, \quad \text{(A1.1)}$$

and, at the same time, their corresponding *topological* "Assignation Relationships", written in terms of "Incipient" Derivatives in the form

$$\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\right\}^{\tilde{k}} \overset{*}{=} \left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}} \quad \text{for} \quad k = 1, 2, \cdots, N-1, \quad \text{(A1.2)}$$

in which, for simplicity of notation, the Ordinalities $\left\{\tilde{2}/\tilde{2}\right\}$, which appear in Equation (A1.1), are thought as being included in the symbols of the quantities to which they refer to.

More specifically, Equations (A1.2) cannot be interpreted as a "necessary consequence" of Equations (A1.1), because the latter are obtained on the basis of "Incipient" Derivatives. Consequently, they are all of *Generative Nature*.

In fact, if rewritten in the following form

$$\frac{\left\{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\right\}^{\tilde{k}}}{\left\{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}}} \overset{*}{=} \tilde{1} \quad \text{for} \quad k = 1, 2, \cdots, N-1, \quad \text{(A1.3)}$$

they allow us to assert that the considered System is already characterized by a proper and specific "*Interior Unit*", of *Generative Nature*, formally represented by the symbol "$\tilde{1}$".

Such a "Unity", however, is still in the form of "*Not Less Than*". This is because:

- in a Generative Contest, they are certainly not *the parts* that, through the Relationships "*between*" them, give "Origin" to the "Excess of Unity"
- because it is exactly true *the opposite*: in fact, it is the *Generative Unit* of the System that, with its *proper* "*Excess*", *Qualifies* the Relationships "*between*" *the parts*.

Consequently, the most Adherent Formulation of the Self-Organizing Generative Process is that which can be obtained by re-proposing Equations (A1.3) in the form

$$\frac{\left\{\mathring{\tilde{\alpha}}_{12}(t) \oplus \mathring{\tilde{\lambda}}_{12}\right\}^{\tilde{k}}}{\left\{\mathring{\tilde{\alpha}}_{1j}(t) \oplus \mathring{\tilde{\lambda}}_{1j}\right\}^{\tilde{k}}} \overset{*}{=} \left\{\tilde{1}\right\} \quad \text{for} \quad \forall k \,, \tag{A1.4}$$

or better, even more properly, as follows

$$\frac{\left\{\mathring{\tilde{\alpha}}_{12}(t) \oplus \mathring{\tilde{\lambda}}_{12}\right\}}{\left\{\mathring{\tilde{\alpha}}_{1j}(t) \oplus \mathring{\tilde{\lambda}}_{1j}\right\}} \overset{*}{=} \left\{\tilde{1}\right\}^{\frac{1}{(N-1)}} \,, \quad j = 2, \cdots, N \,, \tag{A1.5}$$

in which the symbol $\left\{\tilde{1}\right\}$ now formally represents the *Generative Whole*, which, at the same time, is *Self-Organizing* and of *Ordinal Nature*. While its *unique* and *sole* exponent $1/(N-1)$ explicitly represents the fundamental concept previously anticipated, that is: it is the "Whole", with its *proper* Generative "*Excess*", the one that properly "*Qualifies*" the Relationships "*Between*" the parts.

This is obviously true not certainly in the sense of Relationships understood "two by two", but as the specific Reflex of an *Ordinal Unit*, which, in any case, represents an "Irreducible Excess" with respect to the simple "composition" of the single "parts".

Consequently, Relation (A1.5), can also be written in the form

$$\left\{\mathring{\tilde{\alpha}}_{1j}(t) \oplus \mathring{\tilde{\lambda}}_{1j}\right\} \overset{*}{=} \left\{\tilde{1}\right\}^{\frac{1}{\{N-1\}}} \circ \left\{\mathring{\tilde{\alpha}}_{12}(t) \oplus \mathring{\tilde{\lambda}}_{12}\right\} \quad \text{for} \quad j = 2, \cdots, N \,, \tag{A1.6}$$

which, reinterpreted in terms of "*Progenitor Relationships*", finally leads to the formal expression of the Harmony Relationships. The latter, written in the form

$$\left\{\tilde{\alpha}_{1,j+1}(t) \oplus \tilde{\lambda}_{1,j+1}(t)\right\} \overset{*}{=} \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_{j} \circledR \left\{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\right\} \quad \text{for} \quad j = 1, 2, \cdots, N-1, \tag{A1.7}$$

clearly show that the *Diffusive Generativity* "updates", *by Assignation*, *all the couples* at the first member, and, contextually, the *same reference couple* "12".

Equations (A1.7) then clearly show that **all the elements of the Ordinal Matrix (5.6.1) can be obtained on the basis of *one sole couple* $\tilde{\alpha}_{ij}(t)$ assumed as reference and their associated Correlating Factors.**

In this respect, it is also worth noting that condition (A1.2) is properly the one that represents the *fundamental presupposition* of what could be termed as an *Intensive Whole*, precisely **because of the "*consonance*" between all the *Generative Derivatives* up to the order $N-1$, due to the "*Generative Diffusivity*" of the Self-Organizing System.**

This is the specific reason why, by means of the M. O. P., and its correlative Harmony Relationships, it was possible to reconsider some "particular" problems that, in the Traditional Scientific Literature, are generally known as being "*unsolvable*" (such as, for example, the "Three-body Problem"), or "*intractable*", or "*with a drift*". Whose solutions ended up by showing that the Maximum Ordinality Principle has an extremely general validity ([20]).

Appendix A2. The Ordinal Roots of Unity $\left\{\tilde{1}\right\}$

In this respect it is worth observing that previous Relationships (A1.7) are written in such a form only for reasons of clarity and exposition simplicity. In such a form, in fact, it could "seem" that the various elements that characterize the System are "still" related, "between" them, according to Relationships of the type "two by two".

In reality, if one makes explicit the term $\left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_j$ according to its more specific meaning, that is as $\left\{\tilde{1}\right\}^{\frac{1}{\{N-1\}}} \equiv \left\{\tilde{1}\right\}^{\frac{1}{\{N-1,(\tilde{N-1})\}}}$, in which $N-1$ refers to the cardinality, while $(\tilde{N-1})$ refers to the Internal Ordinal $(N-1)$-ary Relationship, it is possible to more appropriately write (by pointing out the Ordinalities $\left\{\tilde{2},\tilde{2}\right\}$, previously underwritten)

$$\left\{\tilde{\alpha}_{1j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1j}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \left\{\tilde{1}\right\}^{\frac{1}{\{N-1,(\tilde{N-1})\}}} \circ \left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}, \quad (A2.1)$$

that is, even more explicitly, in the form

$$\left\{\tilde{\alpha}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{1,j+1}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \begin{pmatrix} \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_1 \\ \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_2 \\ \vdots \\ \left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_{N-1} \end{pmatrix} \circ \left\{\tilde{\alpha}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}} \oplus \left\{\tilde{\lambda}_{12}(t)\right\}^{\{\tilde{2}/\tilde{2}\}}, (A2.2)$$

from which it is possible to recognize that the single "cardinal" values that in Equation (A1.7) appear as they were "distinct", and, in addition, as being "separated", **in reality they are the *Formal "Reflex" of an "Ordinal Unit"* that transcends them, and relates them in the form of an $(N-1)$-ary Relationship**.

This is the aspect that (more than others) *clearly manifests that the Harmony Relationships represent an "Excess"* with respect the initial Assignation Relationships (5.6.3) and (5.6.4).

In addition, as far as the "explicit" meaning of the Ordinal Routs of Unity is concerned, previously synthetically indicated in the form

$$\left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_j \quad \text{for} \quad j=1,2,3,\cdots,N-1, \quad (A2.3)$$

it is worth expressly pointing out that **the symbol $\left\{\tilde{1}\right\}$ represents the *"Unity"* of the System** (understood as a *Whole*), with *specific reference* to the *Unity of its Proper Space* (as well as its *Relational Space*).

Such a Fundamental Unit can be then expressed by the following Relationship

$$\left\{\tilde{1}\right\} = e^{\left\{\alpha \otimes \tilde{i} \oplus \beta \otimes \tilde{j} \oplus \gamma \otimes \tilde{k}\right\}}. \quad (A2.4)$$

Consequently, the Ordinal Roots $\left(\sqrt[\tilde{N-1}]{\left\{\tilde{1}\right\}}\right)_l$ will be represented in the following form

$$\left\{ \tilde{1} \right\}_l = e^{\frac{\left\{ \alpha \circledast \tilde{i} \oplus \beta \circledast \tilde{j} \oplus \gamma \circledast \tilde{k} \right\}}{N-1}}, \tag{A2.5}$$

where:

- $\tilde{i}, \tilde{j}, \tilde{k}$ are the <u>fundamental spinors of the Relational Space</u>, **understood** *in their more general sense*, that is, as the <u>*specific foundation*</u> of *any* given System

- *α, β, γ* are respectively equal to

$$\alpha = \varepsilon_1 + \frac{4\pi \cdot l}{N-1} \quad \beta = \varepsilon_2 + \frac{2\pi \cdot l}{N-1} \quad \text{e} \quad \gamma = \varepsilon_3 + \frac{2\pi \cdot l}{N-1}, \tag{A2.6}$$

- where the "periodicity" of the "spinor" $\tilde{i}$, as we already know, is equal to 4π, because expressed in *steradians*;

- while the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to 2π radians (each), because these spinors are always "orthogonal", both between them, and with respect to the spinor $\tilde{i}$ (an orthogonality that can be understood, inter alia, *as a form of reciprocal "irreducibility"*);

- the quantities ε_1, ε_2, ε_3 represent specific "parameters" of the *Relational Space* each time considered, *with specific reference to the "couple 12"*.

Sometimes (for example in the case of Protein Folding), for an easier "topological" representation Equations (A2.6) can also represented as

$$\frac{\alpha}{N-1} = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1} \quad \frac{\beta}{N-1} = \frac{\varepsilon_2 + 2\pi \cdot l}{N-1} \quad \frac{\gamma}{N-1} = \frac{\varepsilon_3 + 2\pi \cdot l}{N-1}, \tag{A2.7}$$

which however can always re-proposed in the previous form (A2.6) through an appropriate choice of the parameters ε_1, ε_2, ε_3.

On the basis of the previous exposition, it should be even clearer that the Harmony Relationships represent an "*Irreducible Excess*", that is an "*Exceeding*" Manifestation of a *Generative System*, which, at the same time, is *Self-Organizing*, of *Ordinal Nature*, **and, above all, it is** *understood as a Whole "from the very beginning"*, and *not* vice versa.